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WINTER COLLEGE ON
LASER PHYSICS: SEMICONDUCTOR LASERS
AND INTEGRATED OPTICS

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ELECTRO-OPTIC, ACOUSTO-OPTIC AND MAGNETO-OPTIC EFFECTS IN MATERIALS

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Electro-optic, Acousto-optic and Magneto-optic effects in materials

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References:

Ammon Yariv, Pochi Yeh: Optical Waves in crystals, J. Wiley & Sons, 1984

Flügge: Handbuch der Physik

Dielectric properties of materials

Polarisation :

$$\vec{P} = \varepsilon_0 \chi \vec{E}$$

Isotropic medium :

χ is a scalar

Anisotropic medium :

$$\begin{cases} P_x = \varepsilon_0 [\chi_{11} E_x + \chi_{12} E_y + \chi_{13} E_z] \\ P_y = \varepsilon_0 [\chi_{21} E_x + \chi_{22} E_y + \chi_{23} E_z] \\ P_z = \varepsilon_0 [\chi_{31} E_x + \chi_{32} E_y + \chi_{33} E_z] \end{cases}$$

χ is a tensor [electric susceptibility]

$$P_i = \varepsilon_0 \chi_{ij} E_j$$ [summation over repeated indices]

Displacement

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}$$

where

$$\varepsilon = \varepsilon_0 [1 + \chi]$$

$$\begin{cases} D_x = \varepsilon_{11} E_x + \varepsilon_{12} E_y + \varepsilon_{13} E_z \\ D_y = \varepsilon_{21} E_x + \varepsilon_{22} E_y + \varepsilon_{23} E_z \\ D_z = \varepsilon_{31} E_x + \varepsilon_{32} E_y + \varepsilon_{33} E_z \end{cases}$$

or

$$D_i = \varepsilon_{ij} E_j$$

if $x, y, z \rightarrow$ principal axes of the crystal

$$\begin{cases} P_x = \varepsilon_0 \varepsilon_{11} E_x \\ P_y = \varepsilon_0 \varepsilon_{22} E_y \\ P_z = \varepsilon_0 \varepsilon_{33} E_z \end{cases} \qquad \begin{cases} D_x = \chi_{11} E_x \\ D_y = \chi_{22} E_y \\ D_z = \chi_{33} E_z \end{cases}$$

In a <u>lossless</u> medium

$$\underline{\varepsilon_{ij} = \varepsilon_{ji}^*} \qquad \text{Hermitian}$$

<u>homogeneous</u> and <u>magnetically isotropic</u>

$$\underline{\varepsilon_{ij} = \varepsilon_{ji}} \qquad \text{real symmetric}$$

<u>Wave propagation</u>

$$\left| \begin{array}{l} E \exp[j(\omega t - \vec{k}\cdot\vec{r})] \\ H \exp[j(\omega t - \vec{k}\cdot\vec{r})] \end{array} \right. \qquad \text{plane monochromatic wave}$$

$$\underline{\vec{k} = \frac{\omega}{c} n \vec{s}}$$

$\underline{\omega}$ angular frequency
$\underline{c}$ speed of light in vacuo
$\underline{n}$ refractive index
$\underline{\vec{s}}$ unit propagation vector

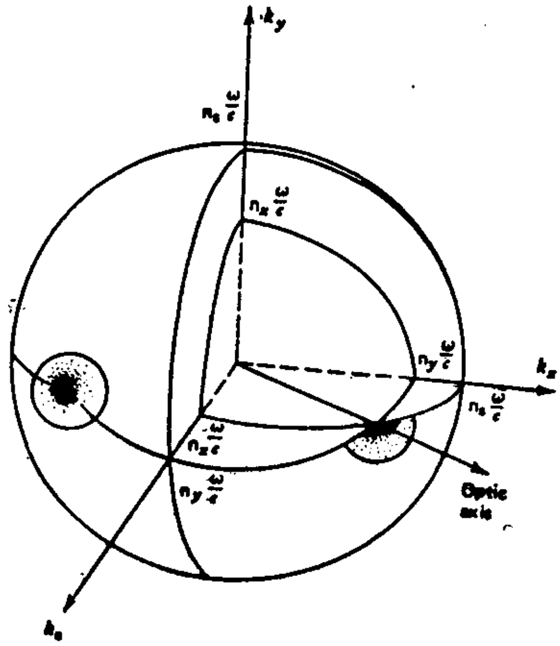
$$\left| \begin{array}{l} \nabla \times \vec{E} + \dfrac{\partial \vec{B}}{\partial t} = 0 \\ \nabla \times \vec{H} - \dfrac{\partial \vec{D}}{\partial t} = \vec{J} \end{array} \right. \qquad \text{Maxwell eq.s}$$

$$\left| \begin{array}{l} \vec{k} \times \vec{E} = \omega \mu \vec{H} \\ \vec{k} \times \vec{H} = -\omega \varepsilon \vec{E} \end{array} \right.$$

$$\underline{\vec{k} \times (\vec{k} \times \vec{E}) - \omega^2 \mu \varepsilon \vec{E} = 0}$$

$$\begin{pmatrix} \omega^2\mu\varepsilon_x - k_z^2 - k_y & k_x k_y & k_x k_z \\ k_y k_x & \omega^2\mu\varepsilon_y - k_x^2 - k_z^2 & k_y k_z \\ k_z k_x & k_z k_y & \omega^2\mu\varepsilon_z - k_x^2 - k_y^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

(in the principal coordinate system)



The normal surface

$$\det \begin{vmatrix} \omega^2\mu\varepsilon_x - k_y^2 - k_z^2 & k_x k_y & k_x k_z \\ k_x k_y & \omega^2\mu\varepsilon_y - k_x^2 - k_z^2 & k_y k_z \\ k_z k_x & k_z k_y & \omega^2\mu\varepsilon_z - k_y^2 - k_x^2 \end{vmatrix} = 0$$

$$\vec{E}_1, \vec{E}_2 \quad \longrightarrow \quad \begin{pmatrix} \dfrac{k_x}{k^2 - \omega^2\mu\varepsilon_x} \\ \dfrac{k_y}{k^2 - \omega^2\mu\varepsilon_y} \\ \dfrac{k_z}{k^2 - \omega^2\mu\varepsilon_z} \end{pmatrix}$$

Alternatively:

$$\frac{s_x^2}{n^2 - \varepsilon_x/\varepsilon_0} + \frac{s_y^2}{n^2 - \varepsilon_y/\varepsilon_0} + \frac{s_z^2}{n^2 - \varepsilon_z/\varepsilon_0} = \frac{1}{n^2}$$

Fresnel's eq.n

$$D_1 \cdot D_2 = 0 \qquad D_1 \cdot E_2 = 0 \qquad D_2 \cdot E_1 = 0$$

$$S \cdot D_1 = S \cdot D_2 = 0 \qquad S \cdot (E_1 \times H_2) = S \cdot (E_2 \times H_1) = 0$$

$$\begin{pmatrix} \dfrac{s_x}{n^2 - \varepsilon_x/\varepsilon_0} \\ \dfrac{s_y}{n^2 - \varepsilon_y/\varepsilon_0} \\ \dfrac{s_z}{n^2 - \varepsilon_z/\varepsilon_0} \end{pmatrix}$$

The index ellipsoid

the <u>impermeability tensor</u> η

$$\eta_{ij} = \varepsilon_0 \left(\varepsilon^{-1}\right)_{ij}$$

$$\vec{E} = \frac{1}{\varepsilon_0}\,\eta\,\vec{D}$$

$$\vec{s} \times \left[\vec{s} \times \eta\vec{D}\right] + \frac{1}{n^2}\vec{D} = 0 \qquad \text{wave eq.}$$

coordinate system s_1, s_2, s_3 : <u>s_3 parallel to $\vec{s}$</u>

$$\vec{s} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ 0 & 0 & 0 \end{pmatrix} \cdot \vec{D} = \frac{1}{n^2}\vec{D}$$

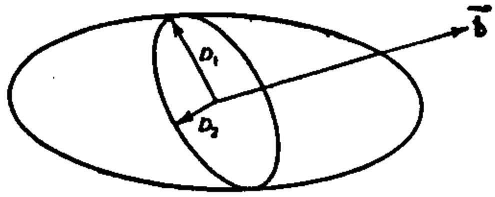
<u>Transverse impermeability tensor</u>

$$\eta_t = \begin{pmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{pmatrix}$$

$$\boxed{\left(\eta_t - \frac{1}{n^2}\right)\vec{D} = 0} \qquad \text{wave equation}$$

two solutions n_1, n_2
two eigenvectors $\vec{D}_1, \vec{D}_2$

index ellipsoid
(optical indicatrix)



Ellipsoid

$$\eta_{11}\xi_1^2+\eta_{22}\xi_2^2+\eta_{33}\xi_3^2+2\eta_{12}\xi_1\xi_2+2\eta_{13}\xi_1\xi_3+2\eta_{23}\xi_2\xi_3=1$$

the intersection with the plane normal to the propagation vector $\vec{s}$ $(\xi_3=0)$:

ellipse

$$\eta_{11}\xi_1^2+\eta_{22}\xi_2^2+2\eta_{12}\xi_1\xi_2=1$$

the principal axes of the ellipse:
- directions of D_1, D_2
- lengths n_1, n_2

In the principal coordinate system x, y, z, the index ellipsoid

$$\boxed{\frac{x^2}{n_x^2}+\frac{y^2}{n_y^2}+\frac{z^2}{n_z^2}=1}$$

where

$$n_x^2=\frac{\varepsilon_x}{\varepsilon_0}\qquad n_y^2=\frac{\varepsilon_y}{\varepsilon_0}\qquad n_z^2=\frac{\varepsilon_z}{\varepsilon_0}$$

principal indices of refraction

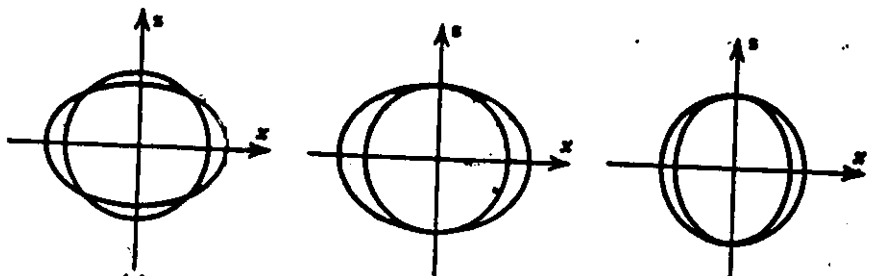
Biaxial crystals: $n_x<n_y<n_z$

Uniaxial crystals: $n_x=n_y=n_o$ $n_z=n_e$

positive: $n_o<n_e$

negative: $n_o>n_e$

...



(a) (b) (c)

Optical Symmetry	Crystal System	Point Groups	Dielectric Tensor
Isotropic	Cubic	$\bar{4}3m$ 432 m3 23 m3m	$\varepsilon = \varepsilon_0 \begin{pmatrix} n^2 & 0 & 0 \\ 0 & n^2 & 0 \\ 0 & 0 & n^2 \end{pmatrix}$
Uniaxial	Tetragonal	4 $\bar{4}$ 4/m 422 4mm $\bar{4}2m$ 4/mmm	$\varepsilon = \varepsilon_0 \begin{pmatrix} n_o^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}$
	Hexagonal	6 $\bar{6}$ 6/m 622 6mm $\bar{6}m2$ 6/mmm	
	Trigonal	3 $\bar{3}$ 32 3m $\bar{3}m$	
Biaxial	Triclinic	1 $\bar{1}$	$\varepsilon = \varepsilon_0 \begin{pmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{pmatrix}$
	Monoclinic	2 m 2/m	
	Orthorhombic	222 2mm mmm	

Isotropic	CdTe		2.69	
	NaCl		1.544	
	Diamond		2.417	
	Fluorite		1.392	
	GaAs		3.40	
Uniaxial:		n_o		n_e
positive	Ice	1.309		1.310
	Quartz	1.544		1.553
	BeO	1.717		1.732
	Zircon	1.923		1.968
	Rutile	2.616		2.903
	ZnS	2.354		2.358
negative	$(NH_4)H_2PO_4$(ADP)	1.522		1.478
	Beryl	1.598		1.590
	KH_2PO_4(KDP)	1.507		1.467
	$NaNO_3$	1.587		1.336
	Calcite	1.658		1.486
	Tourmaline	1.638		1.618
	$LiNbO_3$	2.300		2.208
	$BaTiO_3$	2.416		2.364
	Proustite	3.019		2.739
Biaxial		n_x	n_y	n_z
	Gypsum	1.520	1.523	1.530
	Feldspar	1.522	1.526	1.530
	Mica	1.552	1.582	1.588
	Topaz	1.619	1.620	1.627
	$NaNO_2$	1.344	1.411	1.651
	SbSI	2.7	3.2	3.8
	$YAlO_3$	1.923	1.938	1.947

The electro-optic effect

the impermeability tensor η_{ij} (principal values $1/n_x^2$, $1/n_y^2$, $1/n_z^2$)
is modified by an electrical field $\vec{E}$:

$$\eta_{ij}(\vec{E}) = \eta_{ij}(0) + \Delta\eta_{ij}$$

$$\Delta\eta_{ij} = r_{ijk} E_k + s_{ijkl} E_k E_l$$

r_{ijk} : linear electrooptic coefficient. (Pockels)

s_{ijkl} : quadratic electrooptic coefficients (Kerr)

crystal symmetry conditions:

$$r_{ijk} = r_{jik} \qquad \text{18 independent}$$

$$\left.\begin{array}{l} s_{ijkl} = s_{jikl} \\ s_{ijkl} = s_{ijlk} \end{array}\right\} \text{36 independent}$$

Shortened notation:

$$\begin{array}{l}
r_{1k} = r_{11k} \\
r_{2k} = r_{22k} \\
r_{3k} = r_{33k} \\
r_{4k} = r_{23k} = r_{32k} \\
r_{5k} = r_{13k} = r_{31k} \\
r_{6k} = r_{12k} = r_{21k}
\end{array}$$

Centrosymmetric ($\bar{1}$, 2/m, mmm, 4/m, 4/mmm, $\bar{3}$, $\bar{3}$m 6/m, 6/mmm, m3, m3m):

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Triclinic:

$$\begin{array}{c} 1 \\ \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \end{array}$$

Monoclinic:

$$\begin{array}{cc} 2 \quad (2 \parallel x_2) & 2 \quad (2 \parallel x_3) \\ \begin{pmatrix} 0 & r_{12} & 0 \\ 0 & r_{22} & 0 \\ 0 & r_{32} & 0 \\ r_{41} & 0 & r_{43} \\ 0 & r_{52} & 0 \\ r_{61} & 0 & r_{63} \end{pmatrix} & \begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ r_{41} & r_{42} & 0 \\ r_{51} & r_{52} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} \end{array}$$

$$\begin{array}{cc} \mathrm{m} \quad (\mathrm{m} \perp x_2) & \mathrm{m} \quad (\mathrm{m} \perp x_3) \\ \begin{pmatrix} r_{11} & 0 & r_{13} \\ r_{21} & 0 & r_{23} \\ r_{31} & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & r_{53} \\ 0 & r_{62} & 0 \end{pmatrix} & \begin{pmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ r_{31} & r_{32} & 0 \\ 0 & 0 & r_{43} \\ 0 & 0 & r_{53} \\ r_{61} & r_{62} & 0 \end{pmatrix} \end{array}$$

Orthorhombic:

$$\begin{array}{cc} 222 & 2\mathrm{mm} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} & \begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

Tetragonal:

$$\overset{4}{\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}} \quad \overset{\bar{4}}{\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & -r_{13} \\ 0 & 0 & 0 \\ r_{41} & -r_{51} & 0 \\ r_{51} & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}} \quad \overset{422}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}}$$

$$\overset{4mm}{\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}} \quad \overset{\bar{4}2m\ (2 \parallel x_1)}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}}$$

Trigonal:

$$\overset{3}{\begin{pmatrix} r_{11} & -r_{22} & r_{13} \\ -r_{11} & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ -r_{22} & -r_{11} & 0 \end{pmatrix}} \quad \overset{32}{\begin{pmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}}$$

$$\overset{3m\ (m \perp x_1)}{\begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}} \quad \overset{3m\ (m \perp x_2)}{\begin{pmatrix} r_{11} & 0 & r_{13} \\ -r_{11} & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}}$$

Hexagonal:

$$\overset{6}{\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}} \quad \overset{6mm}{\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}} \quad \overset{622}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}}$$

$$\overset{\bar{6}}{\begin{pmatrix} r_{11} & -r_{22} & 0 \\ -r_{11} & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & -r_{11} & 0 \end{pmatrix}} \quad \overset{\bar{6}m2\ (m \perp x_1)}{\begin{pmatrix} 0 & -r_{22} & 0 \\ 0 & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}} \quad \overset{\bar{6}m2\ (m \perp x_2)}{\begin{pmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}}$$

Cubic:

$$\overset{\bar{4}3m,\ 23}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{pmatrix}} \quad \overset{432}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}$$

*The symbol over each matrix is the conventional symmetry-group designation.

Substance	Symmetry	Wavelength λ (μm)	Electro-optic Coefficients[a] r_{lk} (10^{-12} m/V)		Index of Refraction n_i	n^3r (10^{-12} m/V)	Dielectric Constant[a] $\varepsilon_i(\varepsilon_0)$
$LiIO_3$	6	0.633	(S) $r_{13} = 4.1$ (S) $r_{41} = 1.4$	(S) $r_{33} = 6.4$ (S) $r_{51} = 3.3$	$n_o = 1.8830$ $n_e = 1.7367$		
Ag_3AsS_3	3m	0.633	(S) $n_e^3 r_c = 70$ (S) $n_o^3 r_{22} = 29$		$n_o = 3.019$ $n_e = 2.739$		
$LiNbO_3$ ($T_c = 1230°C$)	3m	0.633	(T) $r_{13} = 9.6$ (T) $r_{22} = 6.8$ (T) $r_{33} = 30.9$ (T) $r_{51} = 32.6$ (T) $r_c = 21.1$	(S) $r_{13} = 8.6$ (S) $r_{22} = 3.4$ (S) $r_{33} = 30.8$ (S) $r_{51} = 28$	$n_o = 2.286$ $n_e = 2.200$		(T) $\varepsilon_1 = \varepsilon_2 = 78$ (T) $\varepsilon_3 = 32$ (S) $\varepsilon_1 = \varepsilon_2 = 43$ (S) $\varepsilon_3 = 28$
		1.15	(T) $r_{22} = 5.4$ (T) $r_c = 19$		$n_o = 2.229$ $n_e = 2.150$		
		3.39	(T) $r_{22} = 3.1$ (T) $r_c = 18$	(S) $r_{33} = 28$ (S) $r_{22} = 3.1$ (S) $r_{13} = 6.5$ (S) $r_{51} = 23$	$n_o = 2.136$ $n_e = 2.073$		
$LiTaO_3$	3m	0.633	(T) $r_{13} = 8.4$ (T) $r_{33} = 30.5$ (T) $r_{22} = -0.2$ (T) $r_c = 22$	(S) $r_{13} = 7.5$ (S) $r_{33} = 33$ (S) $r_{51} = 20$ (S) $r_{22} = 1$	$n_o = 2.176$ $n_e = 2.180$		(T) $\varepsilon_1 = \varepsilon_2 = 51$ (T) $\varepsilon_3 = 45$ (S) $\varepsilon_1 = \varepsilon_2 = 41$ (S) $\varepsilon_3 = 43$
		3.39	(S) $r_{33} = 27$ (S) $r_{13} = 4.5$ (S) $r_{51} = 15$ (S) $r_{22} = 0.3$		$n_o = 2.060$ $n_e = 2.065$		
$AgGaS_2$	$\bar{4}2m$	0.633	(T) $r_{41} = 4.0$ (T) $r_{63} = 3.0$		$n_o = 2.553$ $n_e = 2.507$		

CsH_2AsO_4 (CDA)	$\bar{4}2m$	0.55	$(T)\ r_{41} = 14.8$ $(T)\ r_{63} = 18.2$		$n_o = 1.572$ $n_e = 1.550$	
KH_2PO_4 (KDP)	$\bar{4}2m$	0.546	$(T)\ r_{41} = 8.77$ $(T)\ r_{63} = 10.3$		$n_o = 1.5115$ $n_e = 1.4698$	$(T)\ \epsilon_1 = \epsilon_2 = 42$ $(T)\ \epsilon_3 = 21$
		0.633	$(T)\ r_{41} = 8$ $(T)\ r_{63} = 11$		$n_o = 1.5074$ $n_e = 1.4669$	$(S)\ \epsilon_1 = \epsilon_2 = 44$ $(S)\ \epsilon_3 = 21$
		3.39	$(T)\ r_{63} = 9.7$ $(T)\ n_o^3 r_{63} = 33$			
KD_2PO_4 (KD*P)	$\bar{4}2m$	0.546	$(T)\ r_{63} = 26.8$ $(T)\ r_{41} = 8.8$		$n_o = 1.5079$ $n_e = 1.4683$	$(T)\ \epsilon_3 = 50$ $(S)\ \epsilon_1 = \epsilon_2 = 58$
		0.633	$(T)\ r_{63} = 24.1$		$n_o = 1.502$ $n_e = 1.462$	$(S)\ \epsilon_3 = 48$
$(NH_4)H_2PO_4$ (ADP)	$\bar{4}2m$	0.546	$(T)\ r_{41} = 23.76$ $(T)\ r_{63} = 8.56$		$n_o = 1.5266$ $n_e = 1.4808$	$(T)\ \epsilon_1 = \epsilon_2 = 56$ $(T)\ \epsilon_3 = 15$
		0.633	$(T)\ r_{41} = 23.41$ $(T)\ n_o^3 r_{63} = 27.6$		$n_o = 1.5220$ $n_e = 1.4773$	$(S)\ \epsilon_1 = \epsilon_2 = 58$ $(S)\ \epsilon_3 = 14$
$(NH_4)D_2PO_4$ (AD*P)	$\bar{4}2m$	0.633	$(T)\ r_{41} = 40$ $(T)\ r_{63} = 10$		$n_o = 1.516$ $n_e = 1.475$	
$BaTiO_3$ ($T_c = 395$ K)	4mm	0.546	$(T)\ r_{51} = 1640$ $(T)\ r_c = 108$	$(S)\ r_{51} = 820$ $(S)\ r_c = 23$	$n_o = 2.437$ $n_e = 2.365$	$(T)\ \epsilon_1 = \epsilon_2 = 3600$ $(T)\ \epsilon_3 = 135$
$KTa_xNb_{1-x}O_3$ (KTN), $x = 0.35$ ($T_c = 40$–$60°C$)		0.633	$(T)\ r_{51} = 8000\ (T_c - 28)$ $(T)\ r_c = 500\ (T_c - 28)$ $(T)\ r_{51} = 3000\ (T_c - 16)$ $(T)\ r_c = 700\ (T_c - 16)$		$n_o = 2.318$ $n_e = 2.277$ $n_o = 2.318$ $n_e = 2.281$	
$Ba_{0.25}Sr_{0.75}Nb_2O_6$ ($T_c = 395$ K)	4mm	0.633	$(T)\ r_{13} = 67$ $(T)\ r_{33} = 1340$	$(T)\ r_{51} = 42$ $(S)\ r_c = 1090$	$n_o = 2.3117$ $n_e = 2.2987$	$\epsilon_3 = 3400$ (15 MHz)

The index ellipsoid

$\vec{E} = 0 \longrightarrow \dfrac{x^2}{n_x^2} + \dfrac{y^2}{n_y^2} + \dfrac{z^2}{n_z^2} = 1$ principal coordinate system

$\vec{E} \neq 0 \longrightarrow$

$$\left(\frac{1}{n_x^2} + r_{1k} E_k\right) x^2 + \left(\frac{1}{n_y^2} + r_{2k} E_k\right) y^2 + \left(\frac{1}{n_z^2} + r_{3k} E_k\right) z^2 +$$
$$+ 2yz\, r_{4k} E_k + 2zx\, r_{5k} E_k + 2xy\, r_{6k} E_k = 1$$

An example

KDP (Potassium dehydrogen Phosphate KH_2PO_4)

- 1 4-fold symm. axis (z or optic axis)
- 2 2-fold " axes (x, y)
- uniaxial, class $\bar{4}2m$
- coeff.s $\neq 0$: $r_{41}, r_{52} = r_{41}, r_{63}$

the index ellipsoid, in presence of the field $\vec{E}$

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2 r_{41} E_x yz + 2 r_{41} E_y xz + 2 r_{63} E_z xy = 1$$

a) - $\vec{E}$ parallel to z :

$E_x = E_y = 0 \quad E_z \neq 0$

b) - E parallel to x :

$E_x \neq 0 \quad E_y = E_z = 0$

— a) : $E_x = E_y = 0 \qquad E_z \neq 0$

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2\, r_{63}\, E_z\, xy = 1$$

problem : find a <u>new coordinate system</u> such that

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1$$

$$\left\| \begin{array}{l} x = x' \cos 45^\circ - y' \sin 45^\circ \\ y = x' \sin 45^\circ + y' \cos 45^\circ \end{array} \right.$$

$$\left(\frac{1}{n_o^2} + r_{63} E_z\right) x'^2 + \left(\frac{1}{n_o^2} - r_{63} E_z\right) y'^2 + \frac{z'^2}{n_e^2} = 1$$

principal axes : $x', y', z' = z$

principal indices of refraction :

$$\frac{1}{n_{x'}^2} = \frac{1}{n_o^2} + r_{63} E_z \;, \quad \frac{1}{n_{y'}^2} = \frac{1}{n_o^2} - r_{63} E_z \;, \quad \frac{1}{n_e^2}$$

$r_{63} E_z \ll 1/n_o^2$: $\qquad dn = -\frac{1}{2} n^3\, d\left(\frac{1}{n^2}\right) = -\frac{1}{2} n^3 r_{63} E_z$

$$\left\| \begin{array}{l} n_{x'} = n_o - \frac{1}{2} n_o^3 r_{63} E_z \\ n_{y'} = n_o + \frac{1}{2} n_o^3 r_{63} E_z \\ n_z = n_e \end{array} \right.$$

- b) $\underline{E_x \neq 0 \qquad E_y = E_z = 0}$

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2yz\, r_{51} E_x = 1$$

$$\begin{cases} y = y'\cos\vartheta - z'\sin\vartheta \\ z = y'\sin\vartheta + z'\cos\vartheta \end{cases}$$

$$\frac{x^2}{n_o^2} + \left(\frac{1}{n_o^2} + r_{51} E_x \tan\vartheta\right) y'^2 + \left(\frac{1}{n_e^2} - r_{51} E_x \tan\vartheta\right) z'^2 = 1$$

where

$$\tan 2\vartheta = \frac{2 r_{51} E_x}{1/n_o^2 - 1/n_e^2}$$

$$\begin{cases} n_{x'} = n_o \\ n_{y'} = n_o - \frac{1}{2} n_o^3 r_{51} E_x \tan\vartheta \\ n_{z'} = n_e + \frac{1}{2} n_e^3 r_{51} E_x \tan\vartheta \end{cases}$$

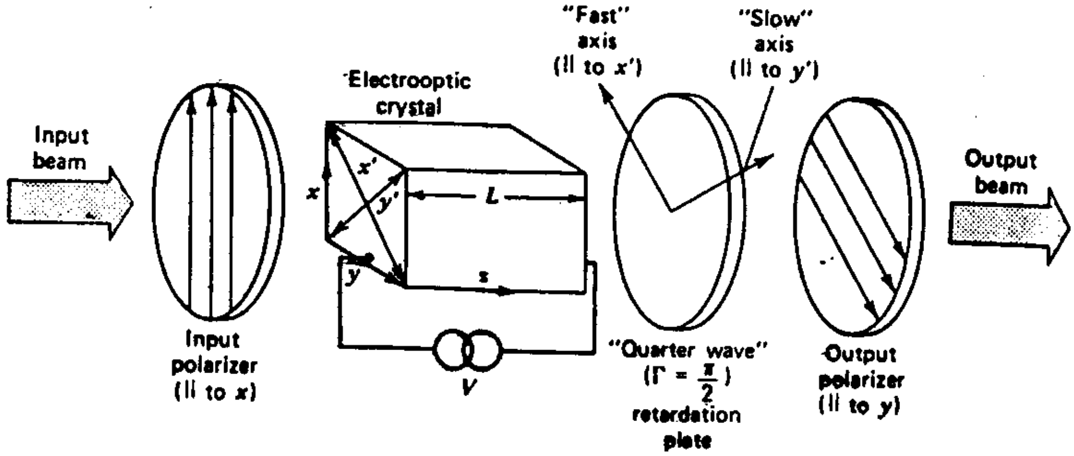
- c) $\underline{LiNbO_3}$: Lithium Niobate - class 3m

$\underline{E_x = E_y = 0 \qquad E_z \neq 0}$ $\qquad$ (c axis)

$$x^2\left(\frac{1}{n_o^2} + r_{13} E\right) + y^2\left(\frac{1}{n_o^2} + r_{13} E\right) + z^2\left(\frac{1}{n_e^2} + r_{33} E\right) = 1$$

$$\begin{cases} n_x = n_o - \frac{1}{2} n_o^3 r_{13} E \\ n_y = n_o - \frac{1}{2} n_o^3 r_{13} E \\ n_z = n_e - \frac{1}{2} n_e^3 r_{33} E \end{cases}$$

Amplitude modulation



$$n_{y'} - n_{x'} = n_o^3 r_{63} E_z$$

$$\Gamma = \frac{\omega}{c}(n_{y'} - n_{x'})L = \frac{2\pi}{\lambda} n_o^3 r_{63} E_z L$$ optical retardation

$$= \frac{2\pi}{\lambda} n_o^3 r_{63} V$$

$$V_{\pi} = \frac{\lambda}{2 n_o^3 r_{63}} \qquad (\Gamma = \pi)$$ half-wave voltage

Intensity transmission

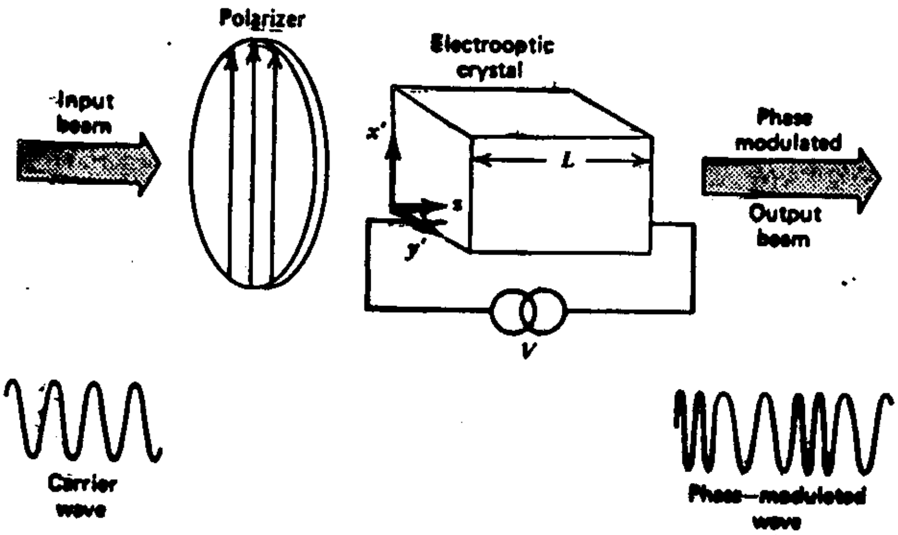
$$T = \sin^2 \frac{\Gamma}{2}$$

where $\Gamma = \underbrace{\tfrac{1}{2}\pi}_{\text{bias}} + \underbrace{\Gamma_m \sin \omega_m t}_{\text{modulation}}$

$$\frac{I_o}{I_i} = \sin^2\left(\frac{1}{4}\pi + \frac{1}{2}\Gamma_m \sin \omega_m t\right) = \frac{1}{2}\left[1 + \sin(\Gamma_m \sin \omega_m t)\right]$$

$$\frac{I_o}{I_i} \approx \frac{1}{2}\left[1 + \Gamma_m \sin \omega_m t\right] \qquad (\Gamma_m \ll 1)$$

Phase modulation



$$n_{x'} = n_o - \frac{1}{2} n_o^3 r_{63} E_z$$

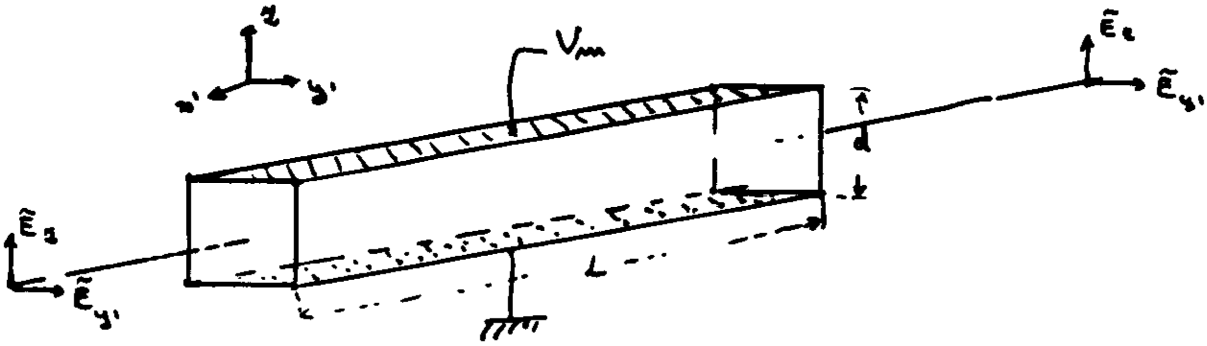
$$\phi_{out} = \frac{\omega}{c}\left(n_o - \frac{n_o^3}{2} r_{63} E_m \sin \omega_m t\right) L$$ output phase

$$E_{out} = A \cos\left[\omega t + \delta \sin \omega_m t\right]$$

where

$$\delta = \frac{\omega n_o^3 r_{63} E_m L}{2c} = \frac{\pi n_o^3 r_{63} V}{\lambda}$$ phase-modulation index

Transverse modulation



a) - **Phase modulation** : $\tilde{E}_{in} = \tilde{E}_{y'}$

$$\underline{n_{y'} = n_o + \frac{1}{2} n_o^3 r_{63} E_z}$$

$$\phi_{out} = \frac{\omega}{c}\left(n_o - \frac{n_o^3}{2} r_{63} E_m \sin \omega_m t\right) L$$ output phase

$$\tilde{E}_{out} = A \cos\left[\omega t + \delta \sin \omega_m t\right]$$

$$\underline{\delta = \frac{\pi n_o^3 r_{63}}{\lambda} V_m \frac{L}{d}}$$ phase-modulation index

b) - **Amplitude modulation** : $\tilde{E}_{in} = \begin{pmatrix} \tilde{E}_{x'} \\ E_{y'} \end{pmatrix}$

$$\underline{n_{y'} - n_z = (n_o - n_e) + \frac{1}{2} n_o^3 r_{63} E_z}$$

$$\Gamma = \frac{\omega}{c}\left[(n_o - n_e) + \frac{1}{2} n_o^3 r_{63} E_z\right] L$$ optical retardation

$$= \underbrace{\frac{2\pi}{\lambda}(n_o - n_e) L}_{\text{bias}} + \underbrace{\frac{\pi}{\lambda} n_o^3 r_{63} V_m \frac{L}{d}}_{\text{modulation}}$$

Transit time impairment :

$$\frac{\sin \Delta L}{\Delta L} \qquad \Delta = \frac{\omega_m}{2}\left(\frac{1}{v_m} - \frac{1}{v_o}\right)$$

Coupled-mode analysis

$$\nabla \times \left(\frac{1}{\mu}\nabla \times E\right) + \varepsilon \frac{\partial^2 E}{\partial t^2} = 0$$

- time dependence : $e^{j\omega t}$
- propagation along $\vec{\zeta}$: $\nabla \to \vec{\zeta}\frac{\partial}{\partial \zeta}$
- $\vec{\zeta}$ parallel to $\vec{z}$

$$\eta_t \frac{\partial^2}{\partial \zeta^2} D = -\left(\frac{\omega}{c}\right)^2 D$$

$$\frac{\partial^2}{\partial \zeta^2} D = -\left(\frac{\omega}{c}\right)^2 N^2 D$$

where $N^2 \eta_t = 1$

$$\frac{\partial}{\partial \zeta} D = -j\frac{\omega}{c} N D$$

$$\frac{\partial}{\partial \zeta} D = j\frac{\omega}{c} N D$$

N: refractive index matrix

In the absence of perturbation :

$$\eta_t = \begin{pmatrix} 1/n_1^2 & 0 \\ 0 & 1/n_2^2 \end{pmatrix} \qquad N = \begin{pmatrix} n_1 & 0 \\ 0 & n_2 \end{pmatrix}$$

1,2 : axes of normal modes

In the presence of perturbation

$$\eta_t = \begin{pmatrix} 1/n_1^2 & 0 \\ 0 & 1/n_2^2 \end{pmatrix} + \Delta\eta \qquad N = \begin{pmatrix} n_1 - \frac{1}{2}n_1^3\Delta\eta_{11} & -\frac{n_1^2 n_2^2}{n_1+n_2}\Delta\eta_{12} \\ -\frac{n_1^2 n_2^2}{n_1+n_2}\Delta\eta_{21} & n_2 - \frac{1}{2}n_2^3\Delta\eta_{22} \end{pmatrix}$$

for $\Delta\eta \ll 1/n_i^2$

$$\Delta N = \begin{pmatrix} -\frac{1}{2}n_1^3\Delta\eta_{11} & -\frac{n_1^2 n_2^2}{n_1+n_2}\Delta\eta_{12} \\ -\frac{n_1^2 n_2^2}{n_1+n_2}\Delta\eta_{21} & -\frac{1}{2}n_2^3\Delta\eta_{22} \end{pmatrix}$$

refractive-index perturbation matrix

Non-stationary waves

$$\left[\frac{\partial}{\partial \xi} + \frac{1}{c} N \frac{\partial}{\partial t}\right] D = 0$$

Displacement

$$D = A_1(\xi,t)\,\vec{d}_1\, e^{j(\omega t - \kappa_1 \xi)} + A_2(\xi,t)\,\vec{d}_2\, e^{j(\omega t - \kappa_2 \xi)}$$

$\vec{d}_1, \vec{d}_2$ polarization vectors of normal modes
A_1, A amplitude of normal modes
κ_1, κ_2 propagation wave numbers : $\kappa_{1,2} = \frac{\omega}{c} n_{1,2}$

we get :

Coupled-mode equations

$$\left(\frac{\partial}{\partial \xi} + \frac{n_1}{c}\frac{\partial}{\partial t}\right) A_1 = -j\frac{\omega}{c}\,\Delta N_{11} A_1 - j\frac{\omega}{c}\,\Delta N_{12} A_2\, e^{j(\kappa_1-\kappa_2)\xi}$$

$$\left(\frac{\partial}{\partial \xi} + \frac{n_2}{c}\frac{\partial}{\partial t}\right) A_2 = -j\frac{\omega}{c}\,\Delta N_{21} A_1\, e^{-j(\kappa_1-\kappa_2)\xi} - j\frac{\omega}{c}\,\Delta N_{22} A_2$$

where

$$\underline{\underline{\Delta N}} = \begin{pmatrix} -\frac{1}{2} n_1^3 \Delta \eta_{11} & -\frac{n_1 n_2}{n_1+n_2}\Delta\eta_{12} \\ -\frac{n_1 n_2}{n_1+n_2}\Delta\eta_{21} & -\frac{1}{2} n_2^3 \Delta\eta_{22} \end{pmatrix}$$

electro-optic $\quad \underline{\underline{\Delta \eta_{\alpha\lambda}}} = r'_{\alpha\lambda 1} E_1 + r'_{\alpha\lambda 2} E_2 + r'_{\alpha\lambda 3} E_3$

elastooptic $\quad \underline{\Delta \eta_{ij}} = p_{ijkl} S_{kl}$

where S_{kl} – strain tensor
p_{ijkl} → strain-optic tensor

$$
\begin{cases}
\left(\dfrac{\partial}{\partial \zeta} + \dfrac{n_1}{c}\dfrac{\partial}{\partial t}\right) A_1 = -j\,\dfrac{\omega n_1^3}{2c}\,\Delta n_{11}\,A_1 \;-\; j\,\dfrac{\omega n_1^2 n_2^2}{c(n_1+n_2)}\,\Delta n_{12}\,A_2\,e^{j(\kappa_1-\kappa_2)\zeta} \\[2ex]
\left(\dfrac{\partial}{\partial \zeta} + \dfrac{n_2}{c}\dfrac{\partial}{\partial t}\right) A_2 = -j\,\dfrac{\omega n_1^2 n_2^2}{c(n_1+n_2)}\,\Delta n_{21}\,A_1\,e^{-j(\kappa_1-\kappa_2)\zeta} \;-\; j\,\dfrac{\omega n_2^3}{2c}\,\Delta n_{22}\,A_2
\end{cases}
$$

<u>Phase modulation</u> :

$\underline{\Delta n_{\alpha\beta} = 0}$

$\underline{\Delta n_{\alpha\alpha} = r'_{\alpha\alpha\gamma}\,E_{m\gamma}\,\sin(\omega_m t - \kappa_m \zeta)}$

<u>Amplitude modulation</u>

$\underline{\Delta n_{\alpha\alpha} = 0}$

$\underline{\Delta n_{\alpha\beta} = r'_{\alpha\beta\gamma}\,E_{m\gamma}\,\sin(\omega_m t - \kappa_m \zeta)}$