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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONE: 2240-1
CABLE: CENTRATOM - TELEX 460392 - I

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Physics of Ionizing Radiation

G. BONSIGNORI
Universita' degli Studi, Bologna, Italy

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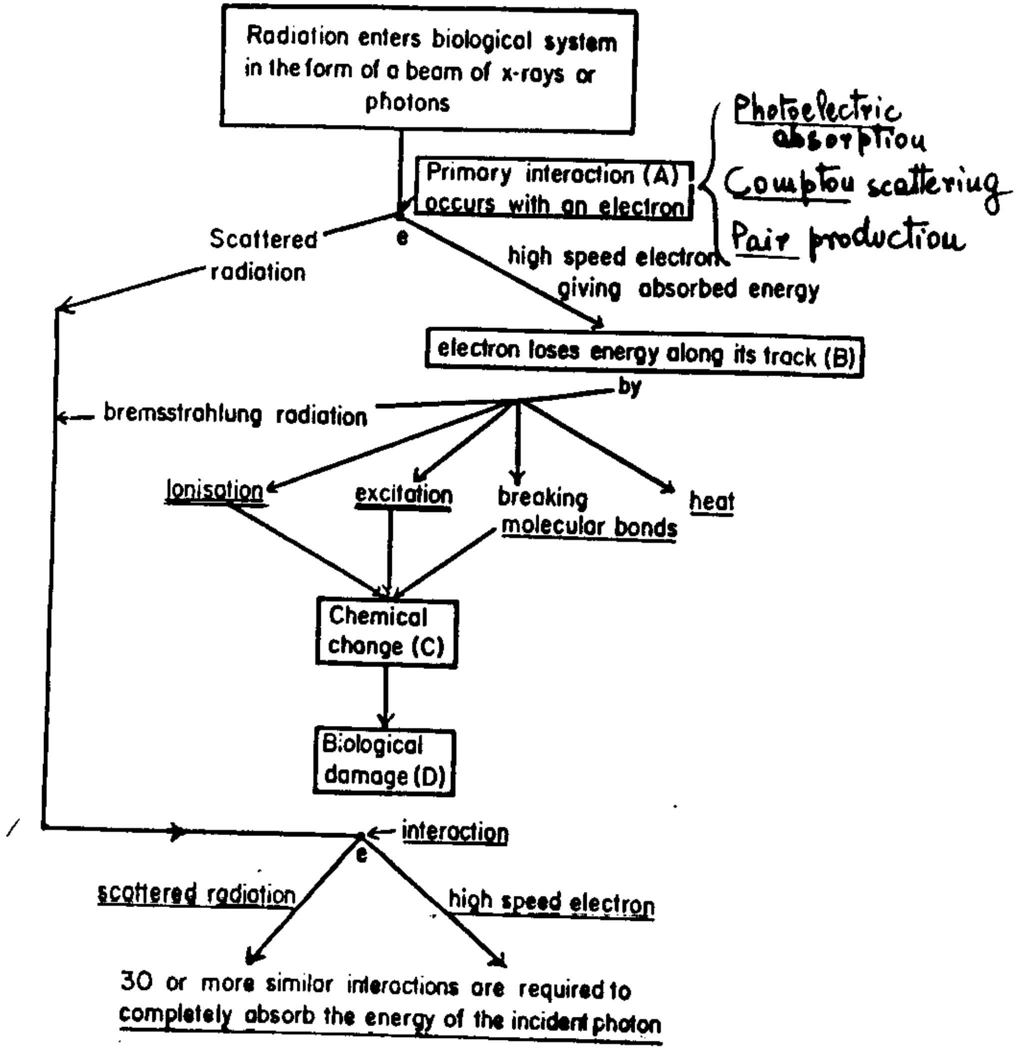
PHYSICS OF IONIZING RADIATION

TABLE V-1

Degradation of Energy



ELECTROMAGNETIC INTERACTION

MAIN CONSERVED QUANTITIES:
- ENERGY
- MOMENTUM
- ANGULAR MOMENTUM
- CHARGE
- PARITY
- ⋮

ENERGY

Classical Kinetic $E = \frac{1}{2} m_0 v^2$

Relativistic $E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$

mass/energy equivalence $v = 0$ $E = m_0 c^2$

Ex. e^- $m = 9.1084 \cdot 10^{-28}\, g$, $c = 3 \cdot 10^{10}\, cm/s$ $E = 0.511\, MeV$

$1\, eV = 1.6 \cdot 10^{-12}\, ergs = 1.6 \cdot 10^{-19}\, joules$

Kinetic $T = E - m_0 c^2$

Photon $E = h\nu$ $\nu = \frac{c}{\lambda}$

MOMENTUM

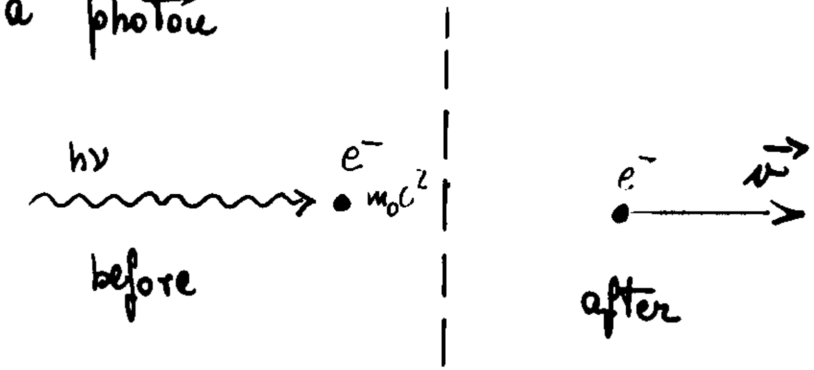
Classical $\vec{p} = m\vec{v}$ relativistic $\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$

Photon $\vec{p} = \frac{h\vec{\nu}}{c}$

Energy/momentum relation $E = \sqrt{p^2c^2 + m^2c^4}$

An application:

An unbound electron cannot absorb a photon



Conservation of energy

$$h\nu + m_0c^2 = \sqrt{p^2c^2 + m_0^2c^4}$$

Conservation of momentum

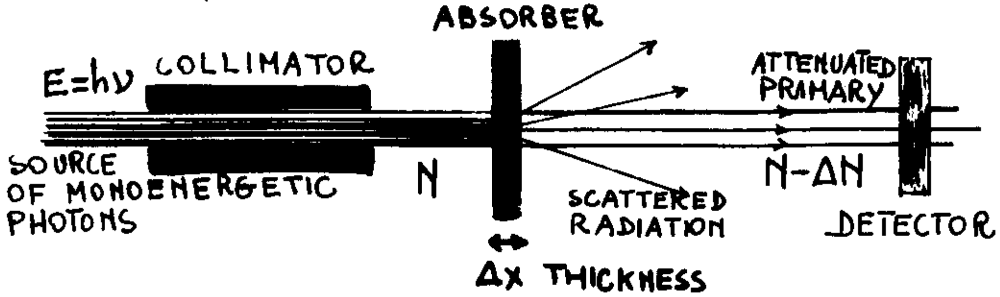
$$\frac{h\nu}{c} = p$$

from these we deduce $h\nu = pc$ and

$$pc + m_0c^2 = \sqrt{p^2c^2 + m_0^2c^4}$$

manifestly impossible! $\Rightarrow$ the process is forbidden!

NARROW-BEAM ABSORPTION



ΔN number of photons "removed" from the beam (absorbed + scattered)

$$\boxed{\Delta N = -\mu \cdot N \cdot \Delta x}$$

μ – **LINEAR ABSORPTION COEFFICIENT**

$$\mu = \mu(Z, E)$$

Meaning of μ: $\mu = -\frac{\Delta N}{N}\frac{1}{\Delta x}$ cm^{-1}

Set $\Delta x = 1$ cm then: μ is numerically equal to the fractional number of photons removed from the beam by 1 cm of absorber

EXPONENTIAL ABSORPTION

$$\boxed{N = N_0 e^{-\mu x}}$$

$e = 2{,}718$

x – thickness of the absorber

N_0 – number of incident photons

MASS ABSORPTION COEFFICIENT

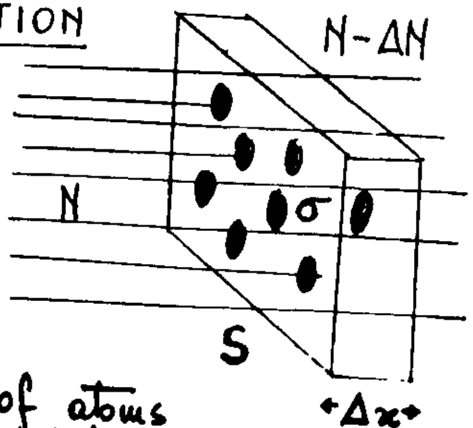
$$N = N_0 e^{-\frac{\mu}{\rho}(\rho x)}$$

ρ – density of the absorber

$\frac{\mu}{\rho}$: $\frac{cm^{-1}}{\frac{g}{cm^3}} = \frac{cm^2}{g}$ — **MASS ABSORPTION COEFFICIENT**

$\rho \cdot x$: $\frac{g}{cm^3} \cdot cm = \frac{g}{cm^2}$ — **THICKNESS IN g/cm^2**

ATOMIC CROSS SECTION



$\rho \cdot S \cdot \Delta x$ – mass of the target

A – atomic weight

$\mathcal{N} = 6 \cdot 10^{23}$ Avogadro number

$\frac{\rho \cdot S \cdot \Delta x}{A} \cdot \mathcal{N}$ number of atoms in the target

$\sigma \cdot \frac{\rho S \Delta x}{A} \mathcal{N}$ total "target area" of the atoms

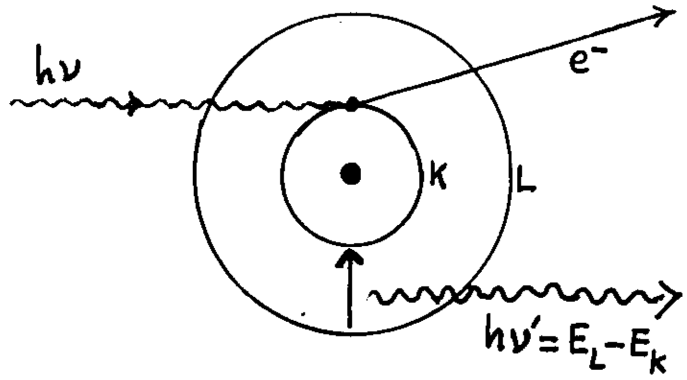
photons removed $\Delta N = -\frac{N}{S} \cdot \frac{\sigma \rho S \Delta x \mathcal{N}}{A}$

by comparing with $\Delta N = -\mu N \Delta x$

$$\mu = \frac{\sigma \rho \mathcal{N}}{A}$$

σ – ATOMIC CROSS SECTION

PHOTOELECTRIC ABSORPTION (Einstein 1905



Energy imparted to the electron $E = h\nu - E_K$ ← binding energy in the K shell

The quantum disappears

All the energy of the quantum is given to the photoelectron

Within $\sim 10^{-8}$ s. another electron will fill the K shell and produce characteristic radiation which will be radiated from the absorber as fluorescent radiation.

Example: $h\nu = 50$ keV in tissue

$E_K \simeq 500$ eV

Energy of the photoelectron $E = 49.5$ keV

Energy of the emitted photon $h\nu' = 500$ eV

immedi…

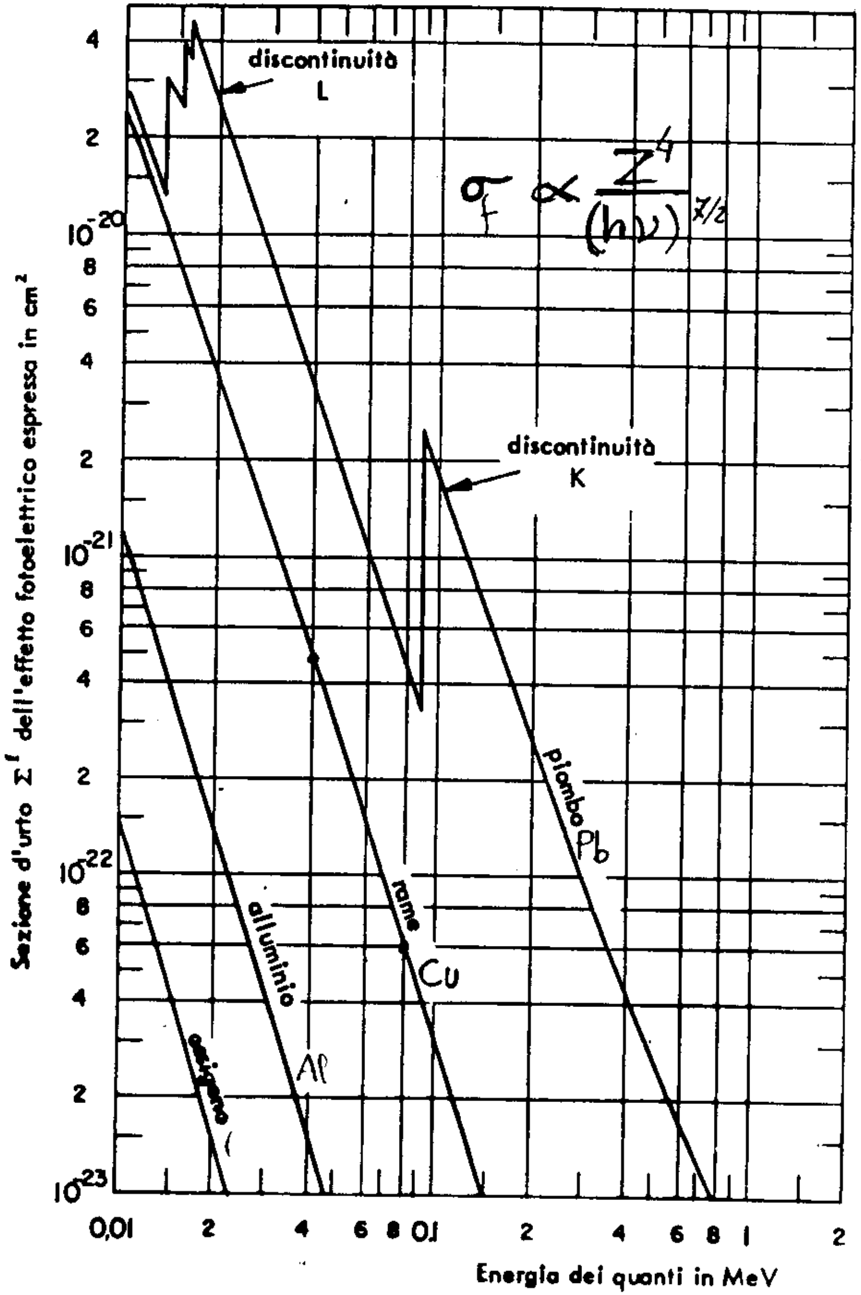


Figura 9.1. Sezioni d'urto dell'effetto fotoelettrico in diversi elementi in funzione dell'energia dei quanti. Si noti il rapido diminuire della sezione d'urto con l'energia e l'aumentare, a parità di energia, con il numero atomico del bersaglio.

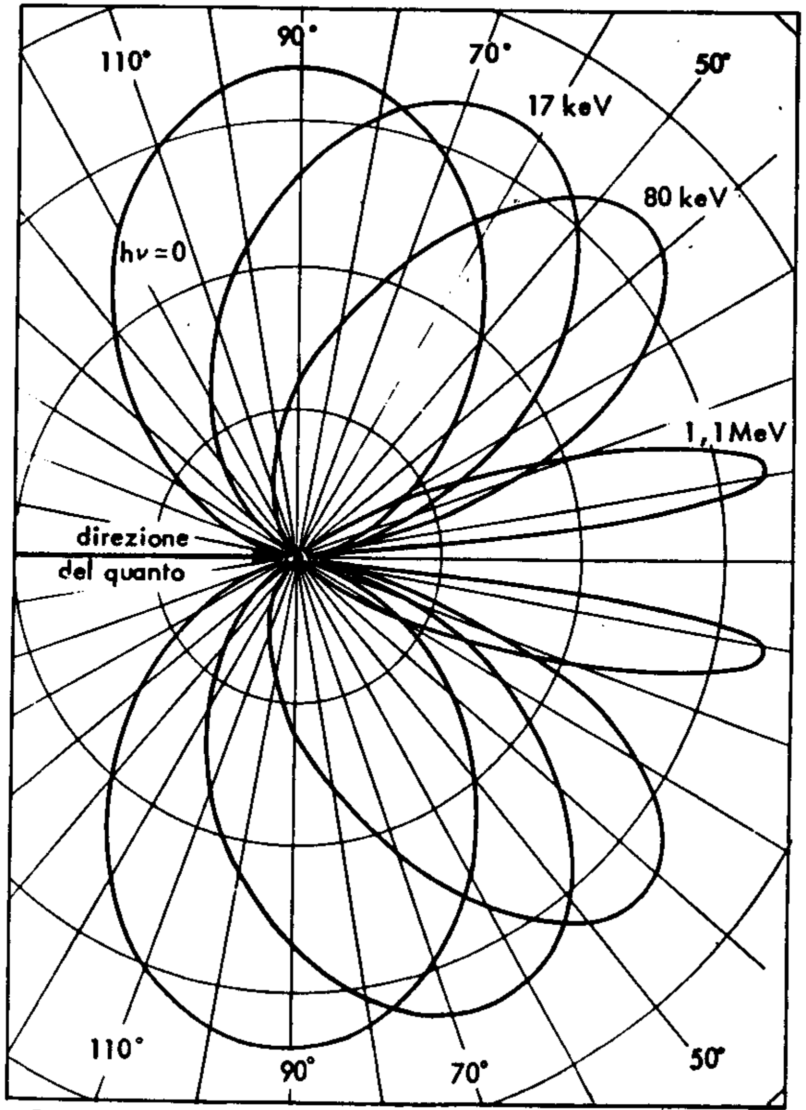


Figura 9.2. Distribuzione angolare dei fotoelettroni; le curve si riferiscono a diverse energie dei quanti incidenti. In questo "grafico polare" il numero di fotoelettroni, che è prodotto in una certa direzione, è proporzionale alla lunghezza del segmento che congiunge il centro del grafico con la curva relativa all'energia scelta; dalla figura si ricava, per esempio, che in avanti e all'indietro non sono emessi fotoelettroni.

SUMMARY

The photoelectric process involves bound electrons

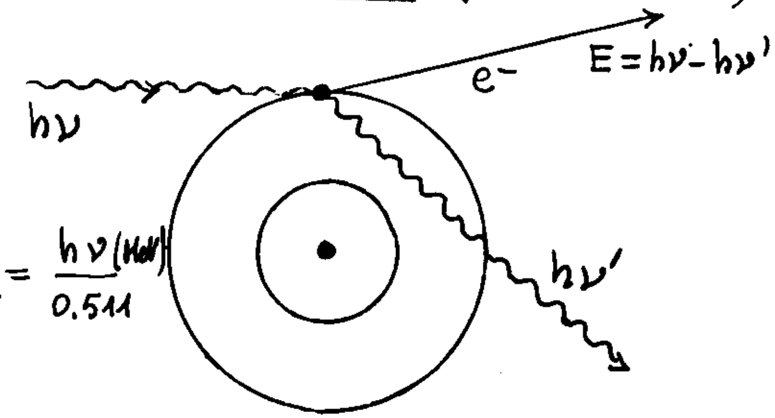
The probability of ejection is maximum if the photon has just enough energy to knock the electron from its shell.

The photoelectric cross section varies with energy and atomic number as

$$\sigma_f \propto \frac{Z^4}{(h\nu)^3}$$

In soft tissue is the dominant process up to about 60 keV photon energy (Radiodiagnosis)

COMPTON SCATTERING (Compton 1923)



$$\alpha = \frac{h\nu}{m_0c^2} = \frac{h\nu(\text{MeV})}{0.511}$$

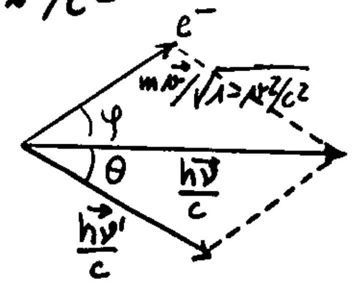
An incident photon of energy $h\nu$ collides with an electron to produce a recoil electron and a scattered photon with reduced energy $h\nu'$ (and increased wavelength λ').

Conservation of Energy

$$h\nu + m_0c^2 = h\nu' + \frac{m_0c^2}{\sqrt{1-v^2/c^2}}$$

Conservation of Momentum

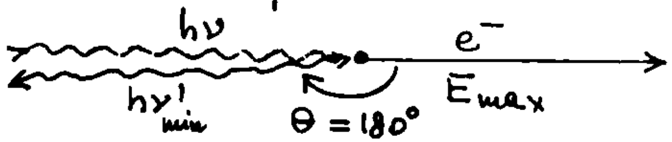
$$\frac{\vec{h\nu}}{c} = \frac{\vec{h\nu'}}{c} + \frac{m\vec{v}}{\sqrt{1-v^2/c^2}}$$



Energy of the electron $$E = h\nu\,\frac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}$$

Energy of the scattered photon $$h\nu' = h\nu\,\frac{1}{\ldots\theta)}$$

If the photon is scattered backword ($\theta = 180°$) than the electron acquires the maximum energy



$$E_{max} = h\nu \frac{2\alpha}{1+2\alpha}$$

$$h\nu'_{min} = h\nu \frac{1}{1+2\alpha}$$

Suppose $h\nu = 5.11$ keV $= 0.00511$ MeV

$\alpha = 0.01$ and

$E_{max} = 0.10$ keV $\quad h\nu'_{min} = 5.01$ keV

If $h\nu = 5.11$ MeV then $\alpha = 10$

$E_{max} = 4.87$ MeV $\quad h\nu'_{min} = 0.24$ MeV

Conclusion:

In a collision of a low energy photon no appreciable amount of energy is transferred into electronic motion.

In a collision of a high energy photon the recoil electron acquires most of the energy of

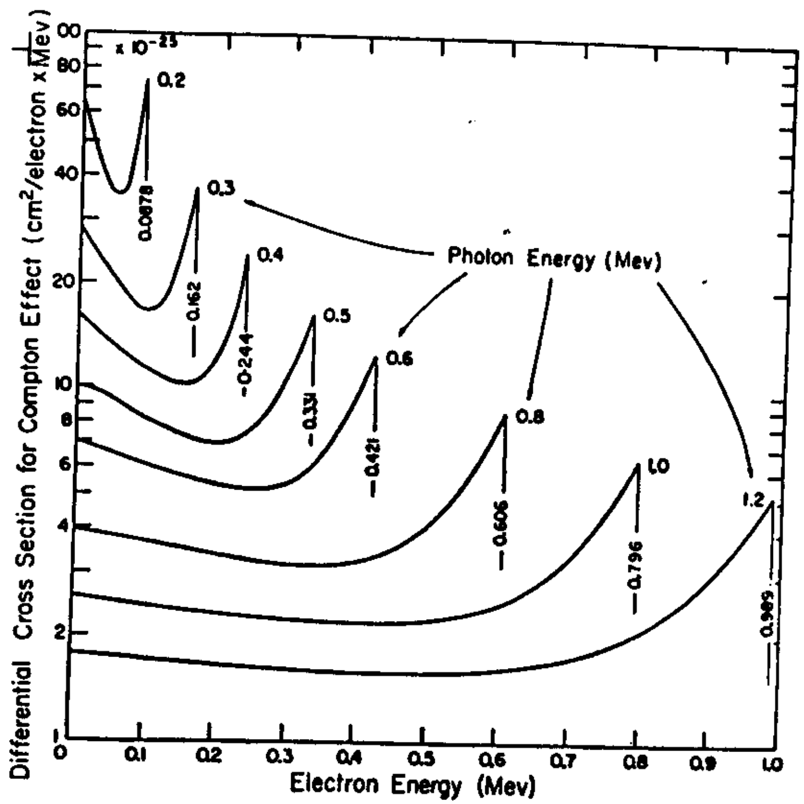


Fig. VI-5. Graphs showing the distribution of electron energies which are produced by monoenergetic photons of energies 0.2 to 1.2 Mev. The ordinates are adjusted so that the area under any one of the curves is equal to the total Compton cross section for that photon energy. The numbers appearing along the vertical lines at the ends of each curve are the maximum energy which the recoil electron may acquire. For example, a 0.8 Mev photon may produce electrons with all energies from 0 to 0.606 Mev. (Courtesy of Radiation Dosimetry (6).)

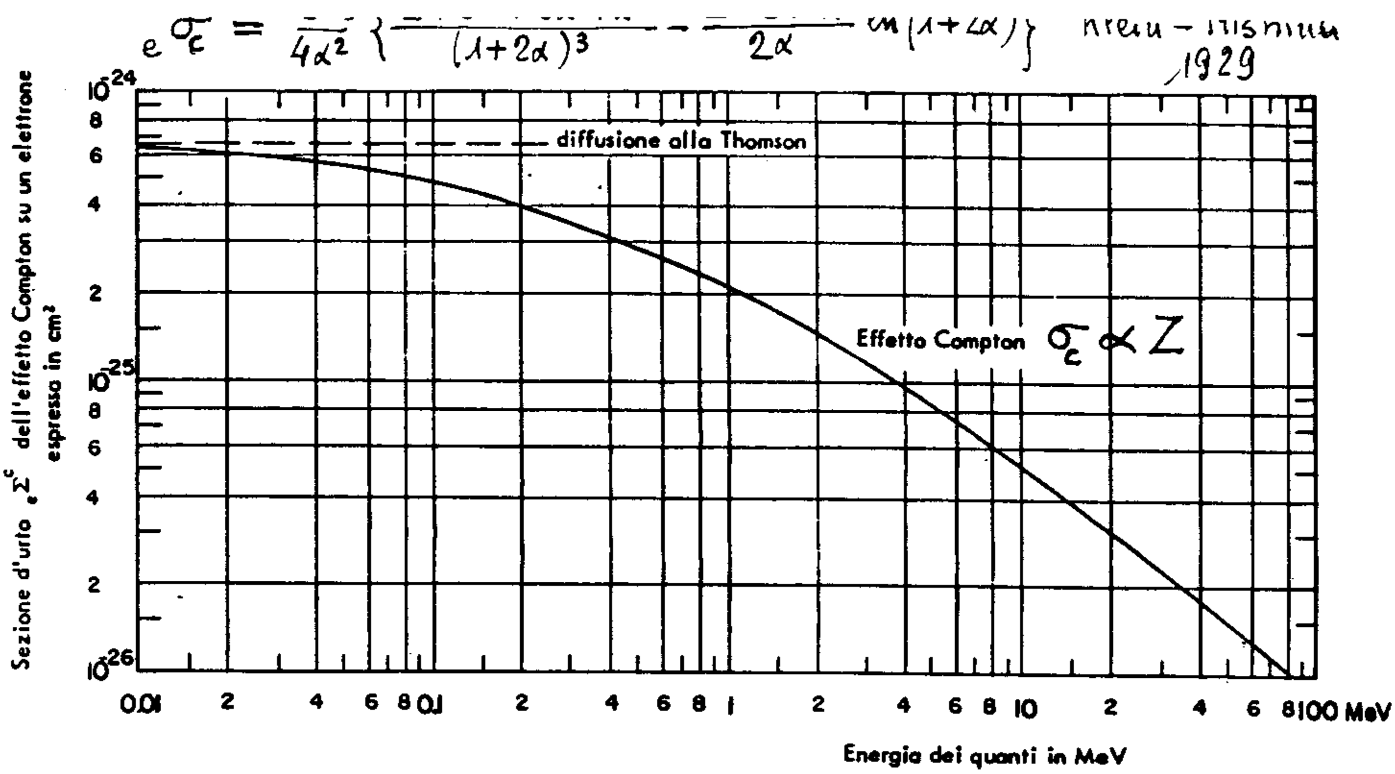


Figura 9.6. Sezione d'urto dell'effetto Compton su un elettrone libero in funzione dell'energia dei quanti. A bassa energia, la sezione d'urto Compton tende al valore della sezione d'urto della diffusione alla Thomson; è questa una prova del fatto che, a piccole energie dei quanti, il modello ondulatorio classico è soddisfacente.

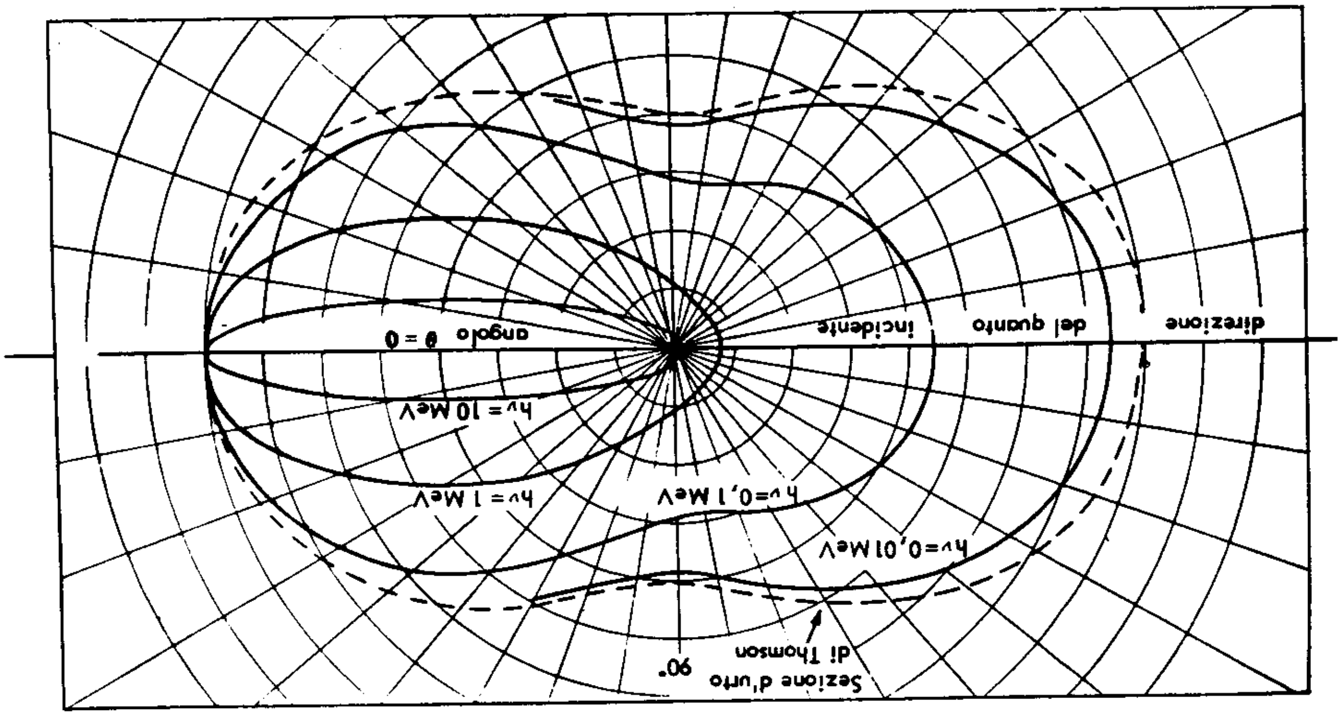


Figura 9.7. Distribuzione angolare dei quanti diffusi Compton per diverse energie dei quanti incidenti. La curva tratteggiata, che è simmetrica rispetto ai 90°, è la sezione d'urto classica di Thomson; dall'andamento delle curve continue si ricava che, al crescere dell'energia, i quanti vanno sempre più in avanti.

SUMMARY

The Compton process involves an interaction between a photon and a 'free, or 'unbound, electron.

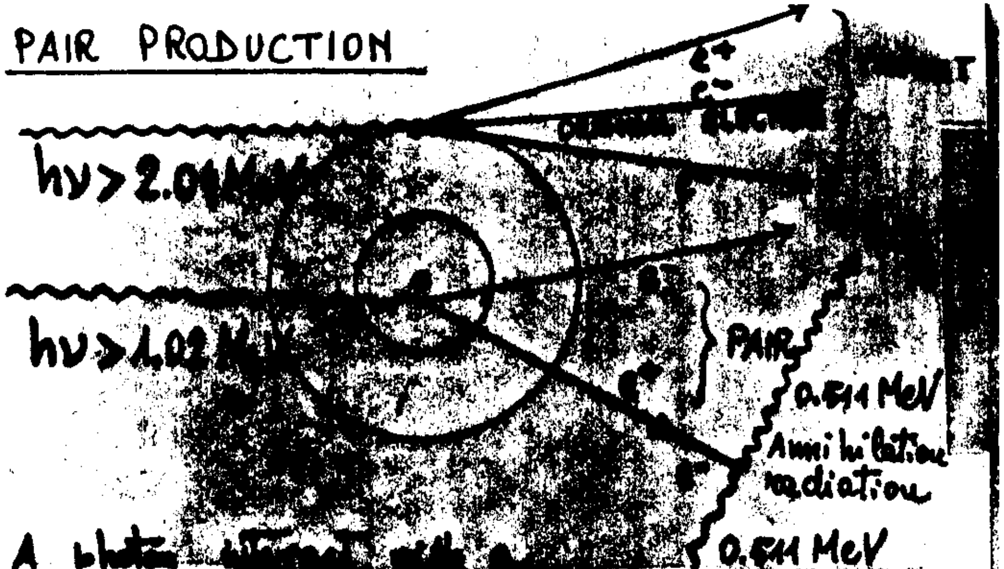
In each collision some energy is scattered and some absorbed, the amount depending on the angle of the collision.

The atomic cross section is ∝ to Z and it decreases with increase in energy.

On the average, the fraction of energy absorbed in a collision increases with increase in energy. It is very small for low energy photons.

In soft tissue is the dominant process in the range from about 100 keV to 10 MeV of the photon energy.

PAIR PRODUCTION



A photon interacts with a nucleus and disappears.

An electron-positron pair is created. Charge conserved

Energy threshold $2 \times 0.511 = 1.02$ MeV

Energy conservation: $h\nu = 1.02 + E_+ + E_-$

The positron, after coming at rest, will combine with an electron and both annihilate producing two oppositely directed γ rays

Energy, momentum and charge conserved.

Much the same is Triplet production except that the interaction occur in the field of an electron.

Threshold: $4 m_0 c^2 = 2.04$ MeV

Less frequent in comparison with pair production

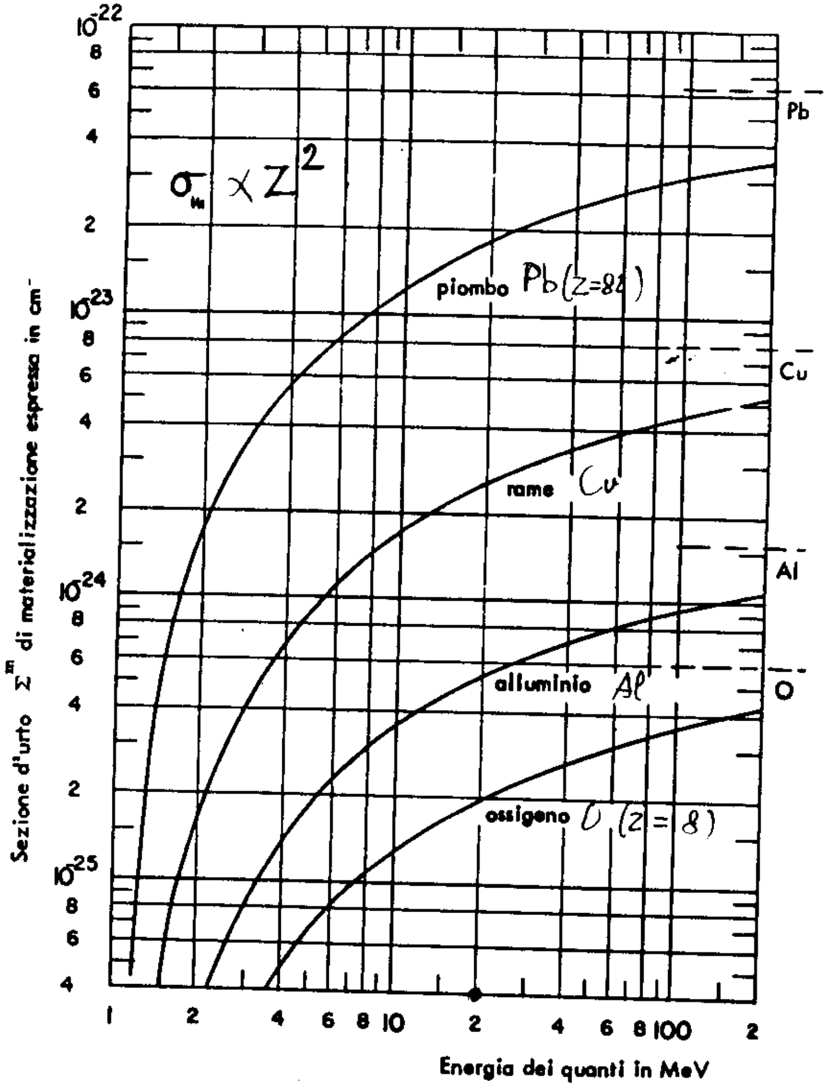


Figura 9.12. Sezioni d'urto dell'effetto materializzazione in funzione dell'energia dei quanti. In tutti i materiali le sezioni d'urto sono nulle allorché l'energia del quanto è uguale a 1,02 MeV e crescono con l'energia tendendo a valori non lontani dai limiti calcolati qualitativamente applicando il modello semiclassico e rappresentati dai tratti orizzontali tratteggiati.

SUMMARY

It involves the interaction of a photon and a nuclear charge.

The threshold for the process is 1.02 MeV

It increases rapidly with energy above this threshold

It increases with atomic number as Z^2

The energy absorbed is less than the energy of the incident photon by 1.02 MeV

Two annihilation photons, each of 0.511 MeV, are produced and radiated in opposite direction from the absorber.

In soft tissue is the dominant process above 10 MeV energy.

$$\sigma_{tot} = \sigma_f + \sigma_c + \sigma_m$$

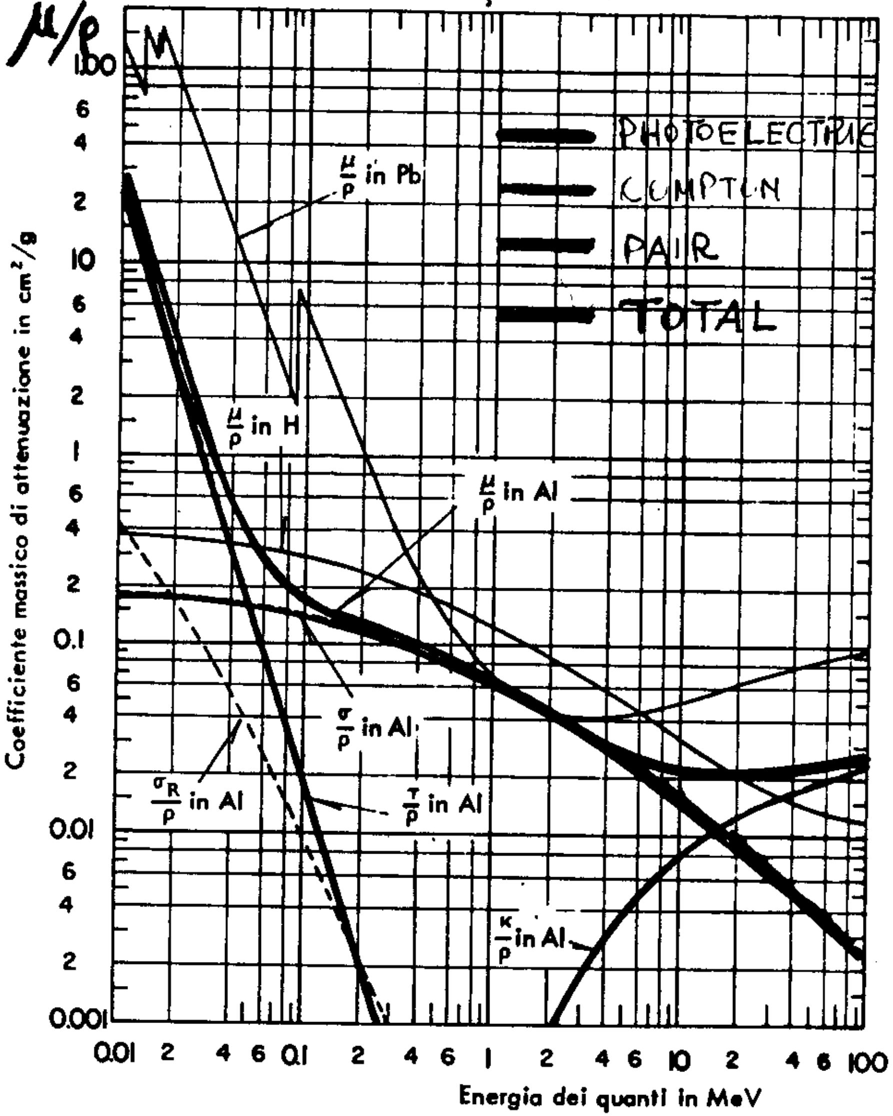
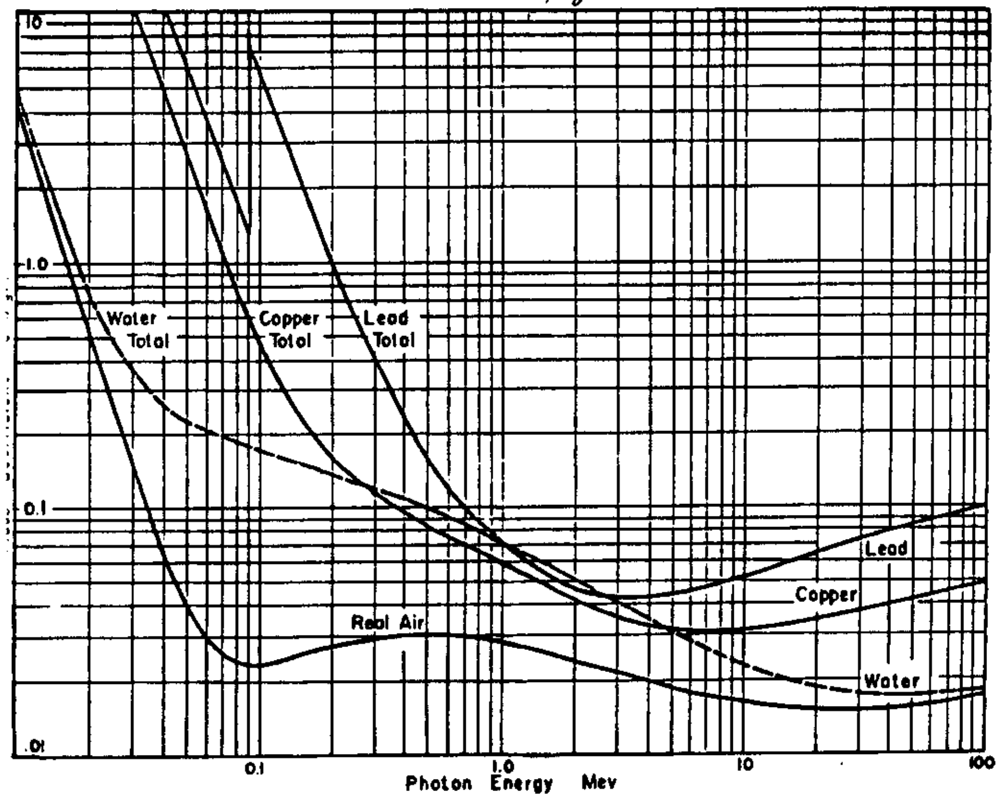


Figura 9.16. Coefficienti massici di attenuazione in idrogeno, alluminio e piombo. Per l'idrogeno (H) e il piombo (Pb) sono disegnati i soli coefficienti di attenuazione massici totali μ/ρ. Per l'alluminio (Al) le curve tratteggiate rappresentano i quattro contributi al coefficiente massico totale.

$\sigma_f \propto Z^4$ $\quad \mu_f/\rho \propto Z^4/A \simeq Z^3$

$\sigma_c \propto Z$ $\quad \mu_c/\rho \propto Z/A \simeq$ constant

$\sigma_m \propto Z^2$ $\quad \mu_m/\rho \propto Z^2/A \simeq Z$



9. Graph showing *total* mass absorption coefficient for water, copper and lead an the real absorption coefficient for air as a function of energy.

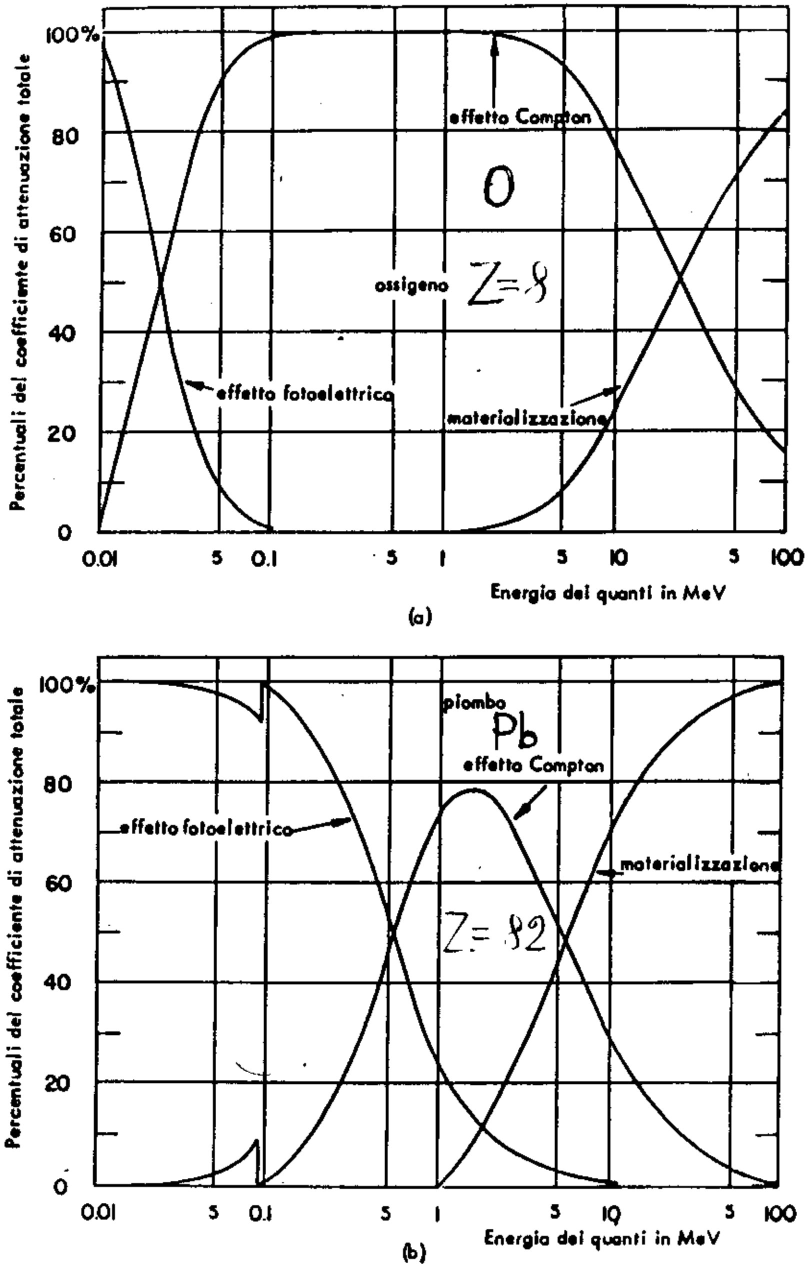


Figura 9.18. Percentuali del coefficiente di attenuazione totale dovuto ai tre effetti in ossigeno e piombo.

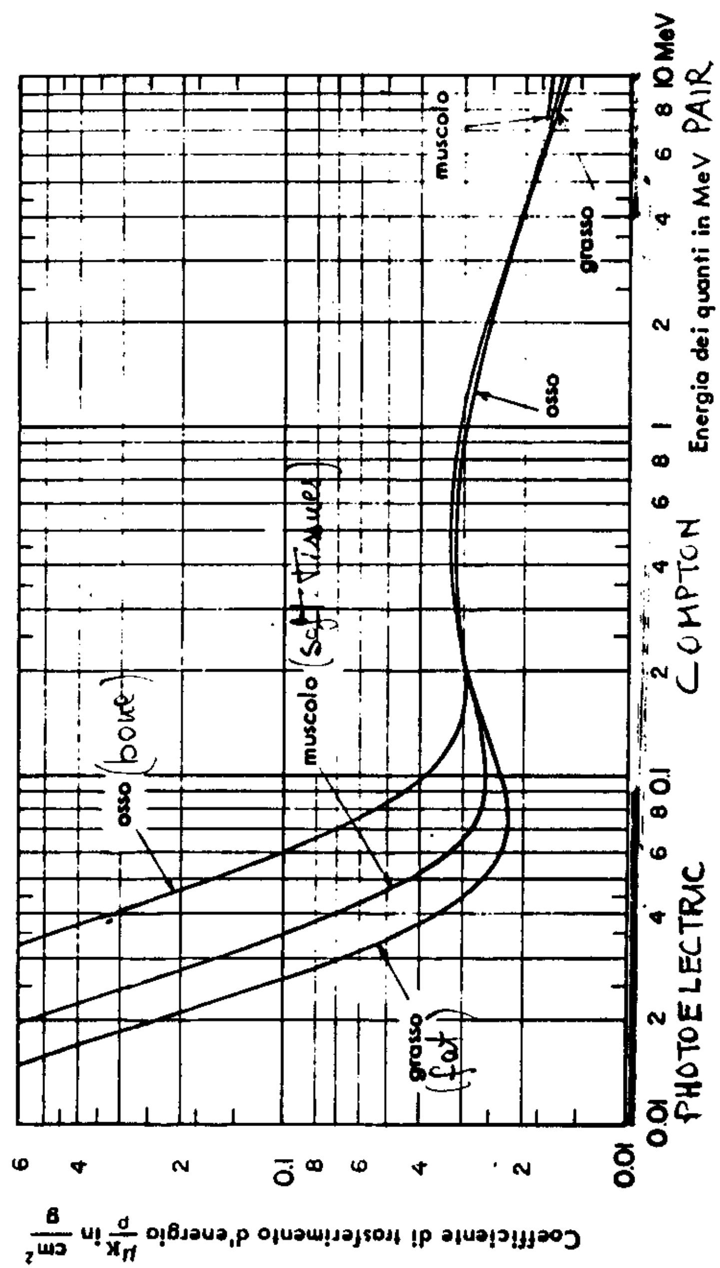


Figura 9.23. Coefficienti di trasferimento d'energia nel grasso, nel muscolo e nell'osso. Le differenze tra i vari coefficienti sono notevoli al di sotto dei 100 keV, perché alle basse energie predomina il trasferimento d'energia per effetto fotoelettrico che ha una forte dipendenza dal numero atomico del materiale.

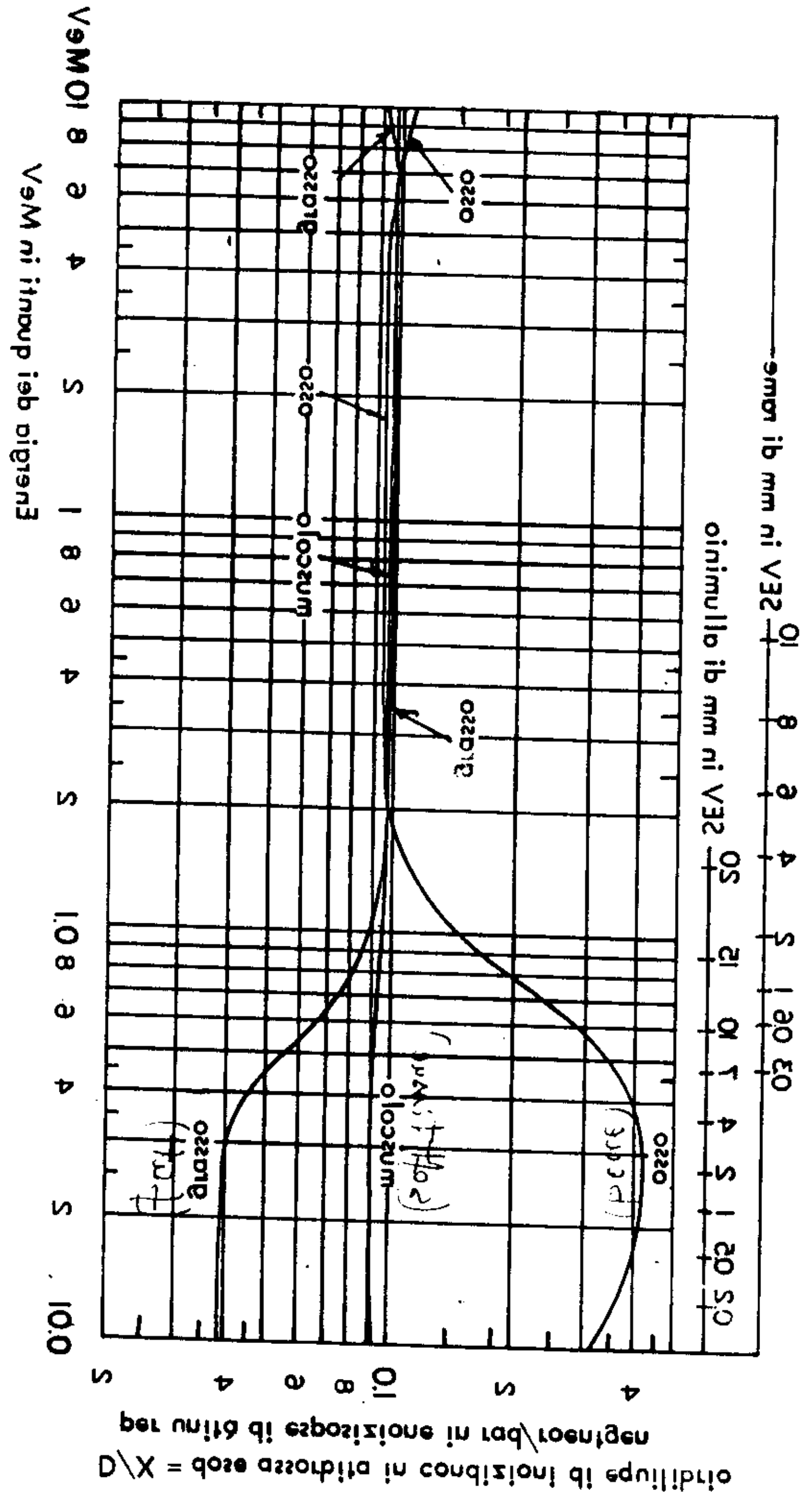
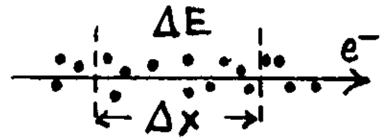


Figura 6.30. Dose assorbita nel caso di equilibrio alle particelle cariche per l'esposizione di 1 roentgen.

COLLISIONAL ENERGY LOSS

Collision energy loss is usually measured as

$$\frac{\Delta E}{\Delta x} \qquad \frac{keV}{\mu}$$



- EXCITED ATOMS
- IONIZED ATOMS

Ex. Electron with energy in the range 400 keV – 100 MeV in water and soft tissue $\frac{\Delta E}{\Delta x} \simeq 0.2 \frac{keV}{\mu}$

We may also introduce a mass collision energy loss defined as

$$\frac{1}{\rho} \frac{\Delta E}{\Delta x} \qquad \frac{keV\ cm^3}{\mu\ g}$$

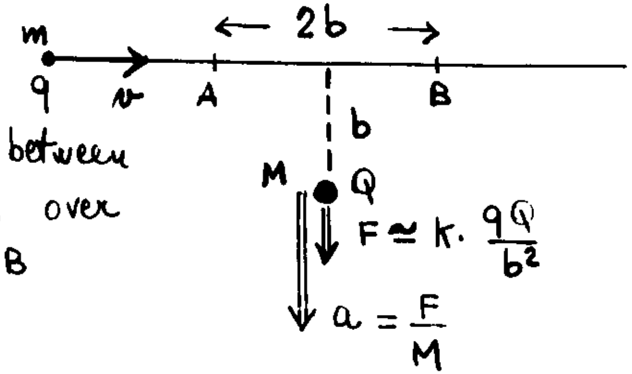
ρ – density of the absorber

Ex. 100 keV electrons in Pb have a mass energy loss of $0.2 \frac{keV\ cm^3}{\mu \cdot g}$; $\rho = 11.34 \frac{g}{cm^3}$

thus $\frac{1}{\rho} \frac{\Delta E}{\Delta x} = 0.2 \qquad \frac{\Delta E}{\Delta x} = \rho \cdot 0.2 = 2{,}27 \frac{keV}{\mu}$

their maximum range would be

A SIMPLE MODEL FOR COLLISIONAL ENERGY LOSSES



Average force between the two charges over the distance AB

$$F \simeq k \frac{qQ}{b^2}$$

Interaction time $t \simeq \frac{2b}{v}$

Acquired acceleration $a = \frac{F}{M}$

Final velocity $V = a \cdot t$

Energy loss by the moving charge = Energy acquired by the rest particle =

$$\Delta E = \frac{1}{2} M V^2 = 2k^2 \frac{q^2 Q^2}{M v^2 b^2}$$

COMMENTS

$$\Delta E = 2k^2 \frac{q^2 Q^2}{M v^2 b^2}$$

The energy loss is inversely proportional to the mass of the collided particle M ⇒ the energy loss is only due to the ELECTRONS in the absorber. Energy loss with nuclei are negligeably small.

The energy losses are inversely proportional to the square of the velocity of the incident particle ⇒ slow electrons lose much more energy than fast electrons. Energy loss of relativistic electrons cannot anymore increase with increasing energy since the velocity is already c.

The energy loss is inversely proportional to the square of the impact parameter b ⇒ Small energy losses are more likely than large ones

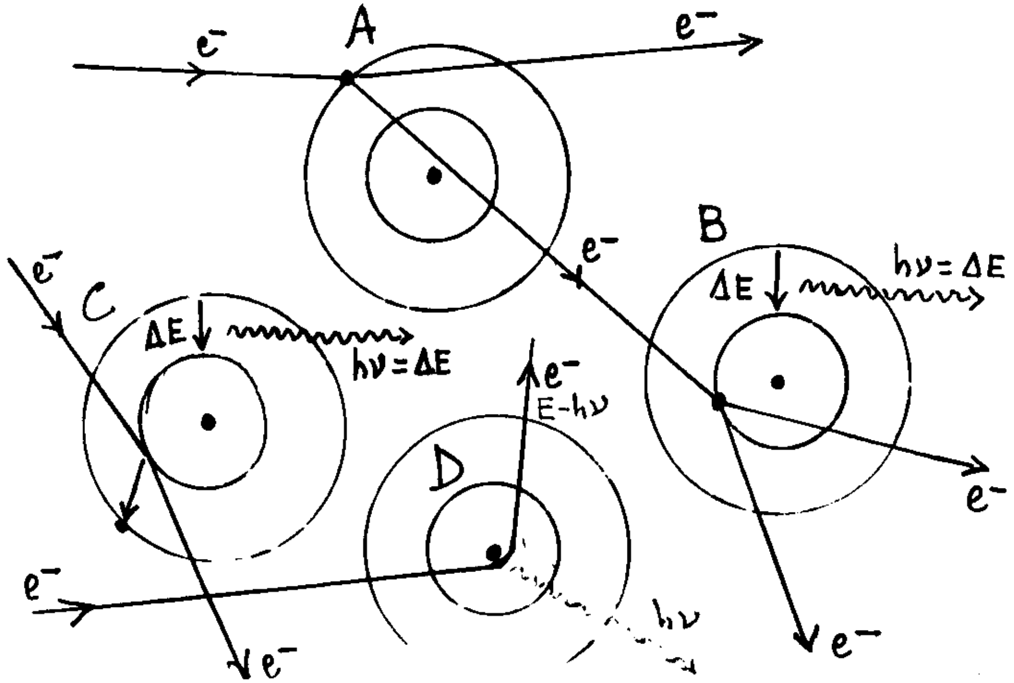
The energy loss is directly proportional to the square of the charge of the incident particle. We expect heavy ionization losses by

ELECTRON INTERACTIONS

Two fundamentally different energy loss processes may occur:

① COLLISIONAL LOSSES { IONIZATIONS, EXCITATIONS } ABSORBED ENERGY ⇓ DOSE ⇓ BIOLOGICAL DAMAGE

② RADIATION LOSSES



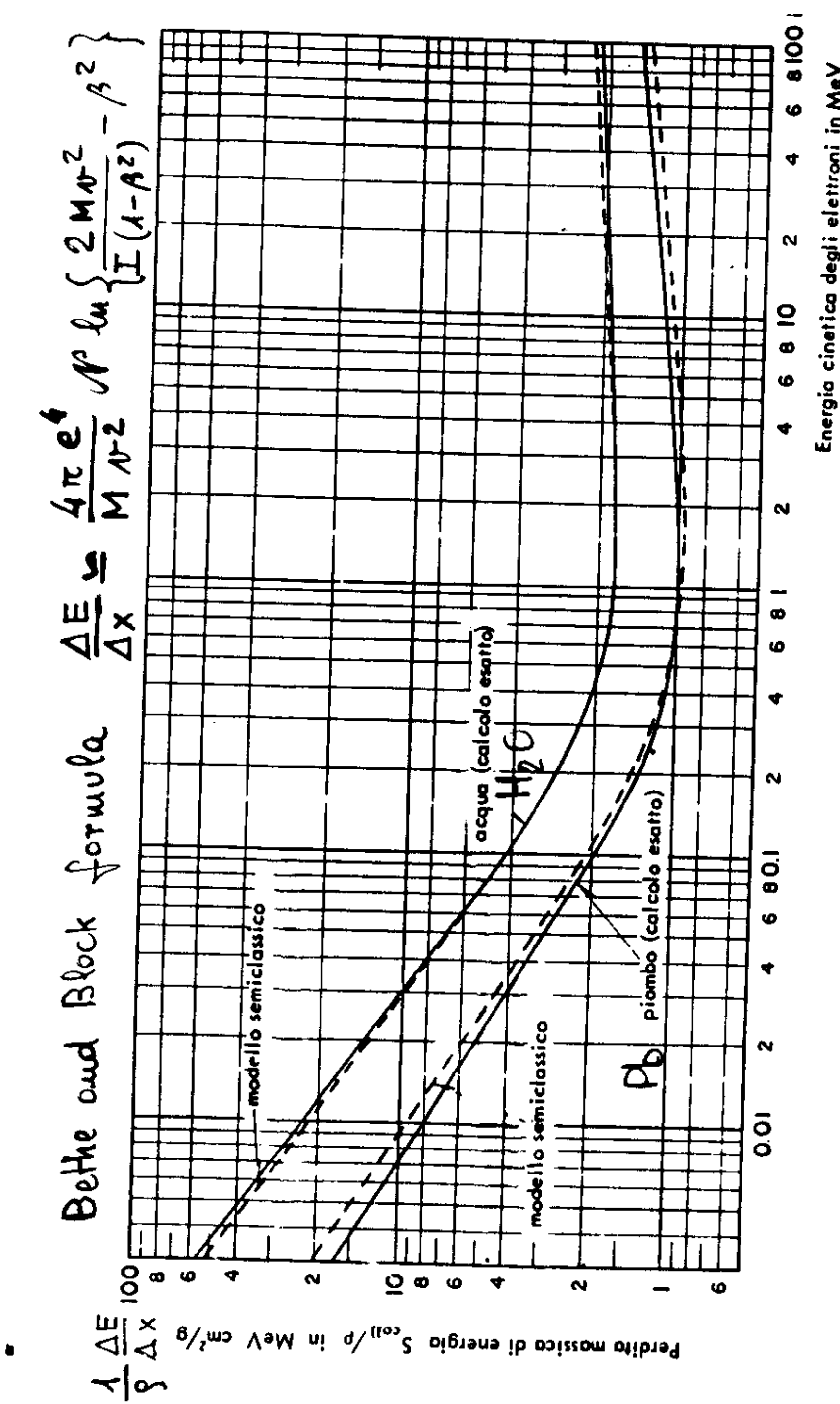


Figura 8.4. Potere frenante massico in collisioni di elettroni in acqua e piombo. Il modello semiclassico è in buon accordo con il calcolo esatto.

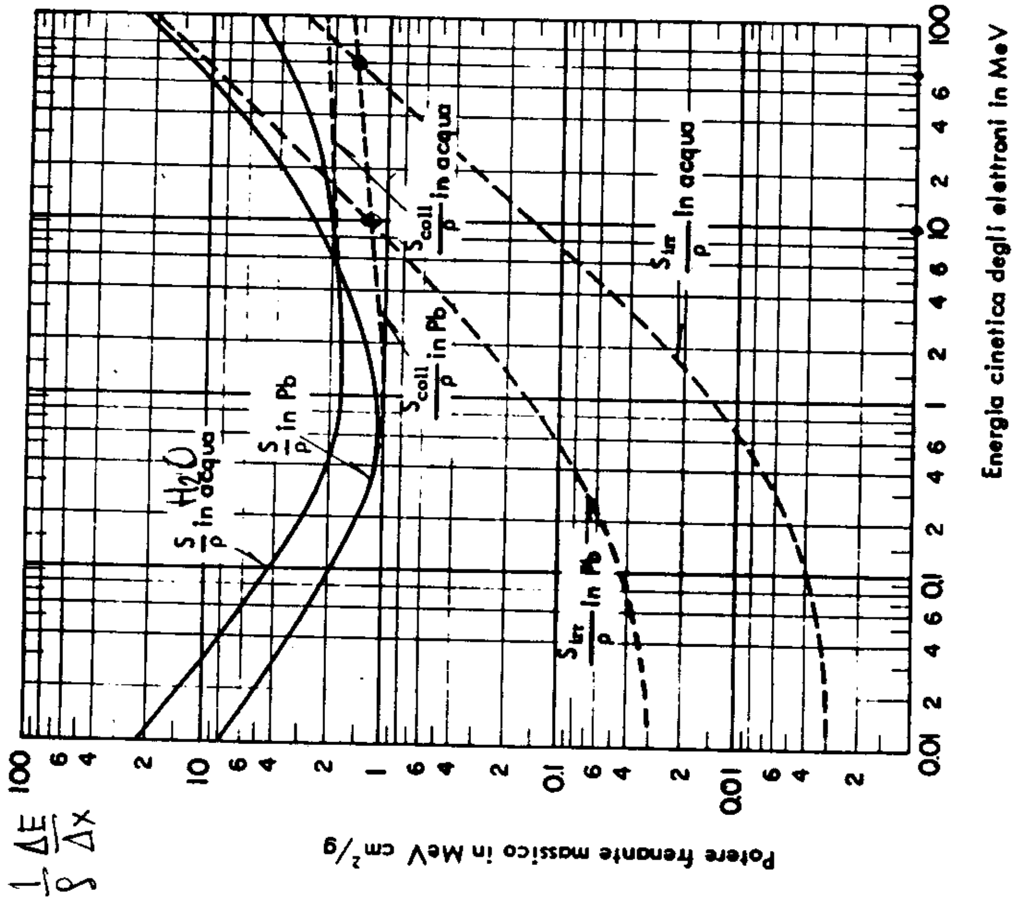


Figura 8.14. Potere frenante massico totale di elettroni in acqua e piombo. Le curve tratteggiate rappresentano i contributi dovuti alle collisioni e all'irraggiamento e le curve continue i poteri frenanti totali.

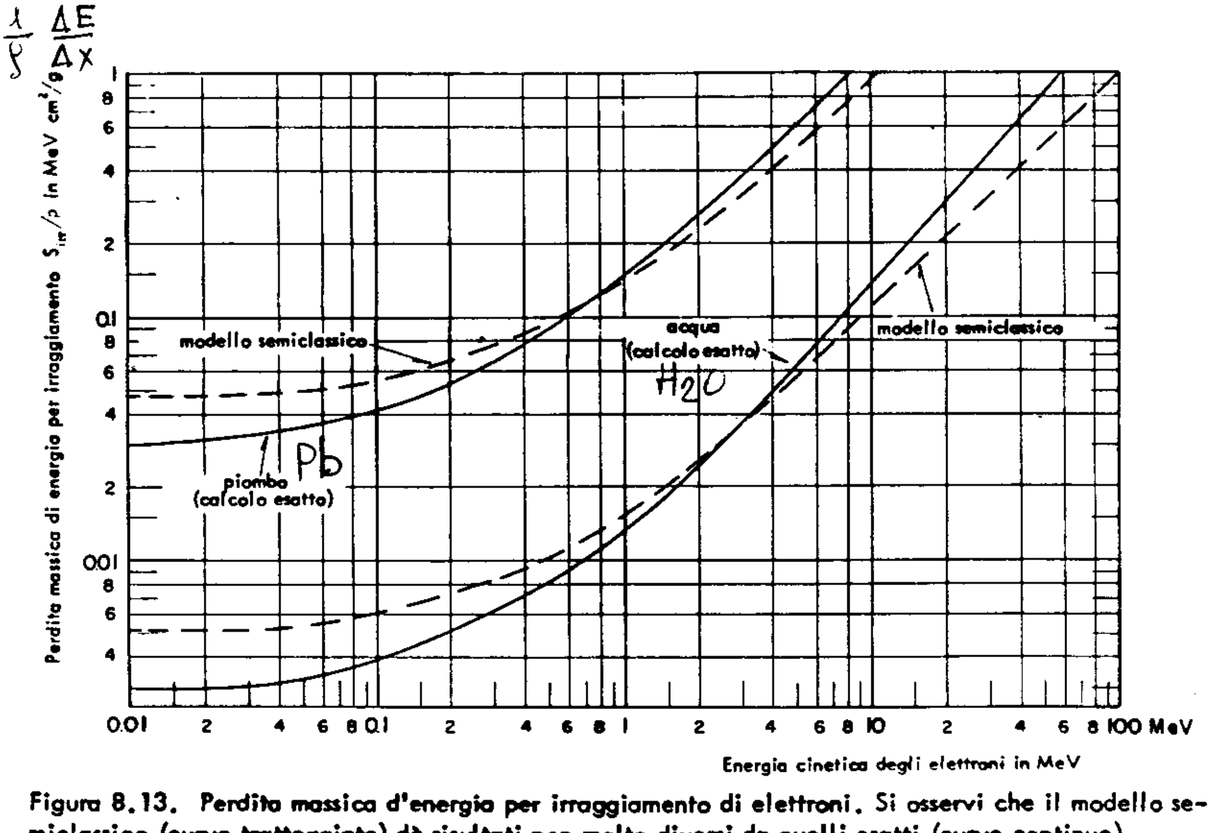


Figura 8.13. Perdita massica d'energia per irraggiamento di elettroni. Si osservi che il modello semiclassico (curve tratteggiate) dà risultati non molto diversi da quelli esatti (curve continue).