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# Fundamentals of Acoustic Wave Theory

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# FUNDAMENTALS OF ACOUSTIC WAVE THEORY

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## SOUND WAVES

Ultrasound radiation, which is included among Non-Ionizing Radiation (NIR) with electromagnetic radiations such as radiofrequency, microwaves, visible light etc., differs from these because of its mechanical nature. Sound waves are produced as a result of disturbances taking place in a material medium: these disturbances cause the particles from which the medium is formed to be set into vibration. The vibration of particles is an essential characteristic of acoustic propagation and, for this reason, it is impossible for sound to travel through a vacuum.

Sound propagates in waves, or disturbances whose amplitude varies with space and time according to the wave equation. For a displacement u of a particle this equation takes, in the simplest case (one dimension), the following form:

$$\frac{d^2u}{dx^2} = \frac{1}{c}\frac{d^2u}{dt^2} \qquad (1)$$

It could be shown that eq.(1) is satisfied by a whole class of solutions, namely by any function having an argument x+ct or x-ct, i.e.:

$$f(x+ct) \qquad \text{or} \qquad f(x-ct) \qquad (2)$$

or any linear combination of such functions. The parameter c represents the propagation speed of the wave. $f(x_0, t_0)$, the function at a given point $x_0$ and time $t_0$, takes the same value at a later time $t_0 + \Delta t$ in a point $x_0 + \Delta x$, where $\Delta x = \pm c \Delta t$ (the sign depending on the propagation direction).

Of all possible solutions of a form given in (2), periodic functions have special importance from a practical point of view. Consideration is limited to sinusoidal functions without any loss of generality because any periodic function can be expressed as a suitable combination of sinusoidal functions, through an analytical technique known as Fourier analysis.

Consider the following special solution of eq.(1):

$$u(x,t) = u_0 \sin k(x-ct) \qquad (3)$$

representing a harmonic oscillation of amplitude $u_0$ ; k is called wave number, and $\lambda = 2\pi/k$ is the wavelength. It is possible to obtain directly from eq.(3) the well known expressions relating to the oscillation period T and frequency f:

$$T = \lambda/c \qquad f = c/\lambda \qquad (4)$$

Thus a sinusoidal wave is fully characterized by three independent parameters: propagation speed, frequency and amplitude. In the following paragraphs is more detail on the meaning of each of these quantities for sound waves.

## CHARACTERISTICS OF SOUND

The transmission of sound consists of an ordered and periodical movement of the molecules of a medium, which may be solid, liquid, or gas. As a consequence of an external perturbation, a number of molecules oscillate in phase, thus transmitting their kinetic energy to neighboring molecules. In this way, energy is transferred from one molecule to another, with no associated transfer of matter. The direction of energy propagation may be parallel or perpendicular to the direction of oscillation of the particles; the corresponding waves are termed longitudinal and transverse, respectively.

Transverse waves are especially important in solids, but other waves such as shear waves, torsion waves, flexural (or Lamb) waves, surface Rayleigh waves, and Love waves also exist. Among these, only shear waves, also called rotational waves, are of interest in ultrasonics. Transverse waves travel only through solids, because liquids and gases do not support shear stresses under normal conditions.

Longitudinal waves on the contrary can pass through all types of media and are more important with regard to interactions with biological systems. In longitudinal waves, the collective motion of particles creates alternate regions of compression and rarefaction, i.e. a periodical pressure variation. This variation has the same propagation speed and frequency of oscillations of particles. The sound wave can therefore be described in terms of the pressure p:

$$p(x,t) = p_0 \sin k(x-t) \qquad (5)$$

Eq.(5) does not give the absolute value of pressure at a given point; it gives the time varying pressure term responsible for the sound, to which the imperturbed pressure should always be added in order to obtain the total pressure value. The following paragraphs address this varying term, which is called acoustic pressure.

It is important to point out that eq.(5) represents a special case of vibration, namely a wave propagating in one direction from a source (assumed to be a point). In practice, a sound wave is emitted from an extended source radiating energy in all directions, and the distribution pattern may be quite complex.

In principle, the problem of an extended source of finite dimensions could be solved by considering it as the sum of a large number of point sources, each emitting waves in all directions with the same intensity.



Such waves, radiating isotropically in space, are called spherical waves and are mathematically described by an equation similar to eq.(5), but in three dimensions:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad (6)$$

For the symmetry, any solution of eq.(6) will be given by a function taking the same value in all points at the same distance from the source. If r is the radius of the surface of a sphere which is the locus of these points, the sinusoidal solution is:

$$p(r,t) = p_0 \sin k(r-ct) \qquad (7)$$

The direction of the radius r changes from point to point on the spherical surface. For large distances from the source (compared to the sound beam wavelength), the propagation axes may be considered to be parallel for all waves within a given region. This corresponds to replacing a portion of spherical surface with a plane and to approximating molecular vibrations with parallel oscillations, called plane waves. In practice, the plane wave approximation is applicable when the distance between wavefront and source is much greater than the wavelength.

Summing the contributions from point sources is simple in principle, but in practice the overlapping of a virtually infinite number of spherical waves may give rise to enormous mathematical difficulties. How to manage the problem of sources of finite dimensions will discussed later in more detail. However, if the distance of the wavefront from the source is large enough, the latter may be considered a point.

Let us now consider in more detail the parameters which characterize a sound wave, namely its propagation speed, frequency and amplitude.

### a. Propagation speed

The propagation speed of sound in a medium depends on the physical properties of the medium itself. For longitudinal waves, the following relations:

$$c = \sqrt{\gamma p/\rho} \qquad c = \sqrt{1/\beta\rho} \qquad (8)$$

express the propagation speed in gases and in liquids, respectively, where $\gamma = C_p/C_v$ is the ratio of specific heats at constant pressure and volume, $\beta$ is the liquid compressibility, and $\rho$ is the density of the medium.

In solids, the sound speed is:

$$c = \sqrt{\frac{E}{\rho}\,\frac{1-\nu}{(1+\nu)(1-2\nu)}} \qquad c = \sqrt{G/\rho} \qquad (9)$$

for longitudinal and transverse waves, respectively. Here, E is the Young's modulus for the medium, G is the rigidity modulus, and $\nu$ is the Poisson's ratio, i.e. the ratio of the transverse contracting strain to the elongation strain (typical values range from 0.2 to 0.5). In a solid rod with cross-sectional dimensions considerably smaller than the wavelength the transverse strain may be neglected, and the first of eqs.(9) reduces to:

$$c = \sqrt{E/\rho} \qquad (10)$$

Because both the density and the elastic moduli vary with the temperature, the sound speed will also vary, generally increasing with increasing temperature.

The speed of sound in solids is higher than in liquids, and very much higher than in gases. Table 1 gives the approximate values of the acoustic speed of some commonly used materials.

Table 1. Approximate values of the acoustic speed (m/s) of some materials at room temperature and standard pressure

| Material | Longitudinal waves | Transverse waves |
|---|---|---|
| Aluminium | 6400 | 3100 |
| Steel | 6000 | 2900 |
| Copper | 4700 | 2300 |
| Water | 1400 | -- |
| Hydrogen | 1300 | -- |
| Air | 330 | -- |

It is also seen in the table that, in solids, transverse waves travel at about half the speed of longitudinal waves. These differences in sound speed give rise to noticeable effects at interfaces between different media, such as air and biological tissue.

b. Frequency

Sound waves are divided into infrasound (frequency ranging between 0 and 16-20 Hz), audible sound (from 16-20 Hz to 16-20 kHz) and ultrasound (above 16-20 kHz). The limits are not clearly defined because of the variability in response of the human ear. This distinction is more important from the physiological point of view than the physical description, and this is the reason why one speaks of sound in general rather than of ultrasound in particular.

In optics, light radiation of a single frequency corresponds to a pure colour (called monochromatic); in the same way, a perfectly sinusoidal acoustic wave corresponds to a pure tone. Whereas many sources of monochromatic light do exist, the same is not true for acoustic sources. Sound emitted from a real source results from the overlapping of many vibrations of different frequency, amplitude and duration. To quantitatively describe sound is therefore an extremely complicated task, which can be solved in an approximate way by averaging the different physical quantities over finite intervals of time, space, or frequency.

For practical reasons the whole spectrum of acoustic frequencies is divided into frequency bands. If $f_1$ and $f_2$ are the lower and upper limit of each band, the partition into proportional bands occurs where the ratio $f_2/f_1$ is the same for each band. Limiting frequencies are in a geometric, rather than arithmetic, progression. This criterion allows one to cover



an extremely wide frequency range with a limited number of bands and, what is most important, corresponds to the physiological response of the human ear.

According to the geometric progression of frequencies, a centre frequency is defined for each band, as the geometric mean of the limiting frequencies, i.e.:

$$f_0 = (f_1 f_2)^{1/2} \qquad (11)$$

An octave is defined as an interval corresponding to the doubling of frequency. An octave band incorporates those frequencies between $f_1$ and $f_2$ where $f_2 = 2f_1$. In a similar manner, a third octave band is characterized by $f_2 = 2^{1/3} f_1$; in general, a n-th octave band is such that $f_2 = 2^{1/n} f_1$. It follows from the above definition that the centre frequency is 1/2n-th octave above $f_1$ and below $f_2$:

$$f_2 = 2^{1/2n} f_0 \qquad f_0 = 2^{1/2n} f_1 \qquad (12)$$

Eqs.(12) show that any proportional frequency band is completely defined by its centre frequency and by n.

c. Amplitude

As mentioned above, a sound wave is described by an equation giving the acoustic pressure as a function of space coordinates and time, so that the distribution of any complex sound is described in terms of a time-varying pressure field. Sound amplitude, or its root mean square, could therefore be measured in pascal. For reasons related to the physiological response of the human ear, sound amplitude is usually expressed through a quantity varying as the logarithm of acoustic pressure. This quantity, defined as:

$$L_p = 10 \log \frac{p^2}{p_{ref}^2} = 20 \log \frac{p}{p_{ref}} \qquad (13)$$

is termed sound pressure level (SPL) and is measured in decibels (dB).

In eq.(13) p is the acoustic pressure, whereas $p_{ref}$ is a reference pressure which must be specified when dealing with absolute levels; the value generally chosen is 20 μPa. However, differences between SPLs are independent of the reference pressure, since:

$$L_{p_2} - L_{p_1} = 10 \log \frac{p_2^2}{p_1^2} \qquad (14)$$

Eq.(14) shows that equal differences between SPLs correspond to equal ratios between sound pressures. For example, a 20 dB difference between two levels corresponds to a ratio of 10 between rms pressures.

ACOUSTIC ENERGY

The propagation of a sound wave corresponds to an energy transfer without a transfer of matter. This can be deduced from the wave equation (5). It is an expression of the energy conservation principle, and may be regarded as the equivalent in acoustics to Poynting's theorem of electromagnetic radiation. If w is the acoustic energy density, i.e. the energy per unit volume, eq.(5) may be expressed as:

$$\frac{\partial w}{\partial t} + \vec{\nabla} \vec{I} = 0 \qquad (15)$$

where $\vec{I} = p\vec{v}$, and $\vec{v}$ is the velocity of particles.

The vector $\vec{I}$ represents the energy flow per unit time through the unit surface perpendicular to the propagation direction, and is termed acoustic energy flux or, more frequently, acoustic intensity. This quantity is of fundamental importance in acoustics. Using the analogy of other branches of physics, we can define a field as an intermediary in interactions between physical systems (whose properties are functions of the space coordinates and time). The "acoustic field" (or "ultrasonic field", in the case of ultrasound) can be expressed in terms of acoustic intensity.

From the fundamental equations of fluid dynamics, it can be shown that the energy density w is made of two terms:

$$w = \frac{1}{2}\left(\rho\, v^2 + \frac{p^2}{\rho c}\right) \qquad (16)$$

the first of which is called the acoustic kinetic energy, the second the acoustic potential energy.

The interpretation of eq.(15) as an energy conservation law is more immediate if it is integrated over a finite volume. If W is the total acoustic energy enclosed in the volume, we obtain:

$$\frac{dW}{dt} = \frac{d}{dt}\int_V w\, dV = -\int_S \vec{I}\cdot\vec{n}\, dS = -\,\Phi_s(\vec{I}) \qquad (17)$$

This shows that the variation of W per unit time equals the flux of vector $\vec{I}$ through the external surface ($\vec{n}$ is the unit vector outgoing perpendicularly from the surface element dS).

Eq.(15) may be interpreted in terms of another important physical quantity, namely the acoustic power, which is defined as the energy variation per unit time:

$$\mathcal{P} = dW/dt \qquad (18)$$

If a volume V encloses a sound source, then the acoustic energy flux through the limiting surface gives the instantaneous power of the source.

Eqs.(15) and (16) become simpler and more meaningful in the case of plane waves. Here the velocity v of particles is related to the sound propagation speed c by the expression:

$$v = p/\rho c \qquad (19)$$

and one obtains:

$$\frac{1}{2}\rho\, v^2 = \frac{1}{2}\frac{p^2}{\rho c^2} = \frac{1}{2}w \qquad I = cw \qquad (20)$$

The first of eqs.(20) states that in a plane wave the kinetic and potential energy are the same, the second that the acoustic energy travels with speed c along the propagation direction.

Units for the acoustic power and intensity are respectively the watt (W) and the watt per square meter ($W/m^2$). A sound intensity level $L_I$ and a sound power level $L_W$ are also defined, respectively, as:

$$L_I = 10\log\frac{I}{I_{ref}} \qquad L_W = 10\log\frac{W}{W_{ref}} \qquad (21)$$

Commonly used reference values are 1 picowatt (1 pW = $10^{-12}$ W) and 1 picowatt per square meter, respectively.



## CONTINUOUS AND PULSED WAVES

An ideal wave, defined by eq.(7), is characterized by a single frequency, fixed amplitude and unlimited duration. Such a wave is called a continuous wave. In reality a wave can be considered continuous when its amplitude variation is lower than a given value (e.g. 5%) and its duration is very long with respect both to its period and to the response time of any interacting system.

Non-continuous waves may be amplitude modulated waves or pulsed waves. In the first case the amplitude varies periodically in time, whereas in the second it periodically goes to zero.

Pulsed waves are characterized by several parameters. The time duration of each pulse is known as the pulse length or pulse width. The pulse repetition period (or pulse period) is the time interval between two consecutive pulses, and the pulse repetition frequency (or pulse frequency) is the number of pulses propagated per second. In general, the pulse period is much longer than the wave period. Finally, the ratio of the pulse width to the pulse period, i.e. the fraction of pulse period where the amplitude differs from zero, is termed the duty cycle, or duty factor.

In order to obtain pulses of finite width, the source must be damped, so that the oscillations disappear after the required number of waves has been propagated. As shown in Fig.1, the heavier the damping, the further are the waves from the condition of a pure tone, and the larger is the number of components involved in the Fourier analysis. Therefore, pulsed waves are constituted by oscillations which do not take place at a single frequency, but at frequencies extending over a continuous range, known as the frequency bandwidth. Any increase of the damping results in an increase of the frequency bandwidth.

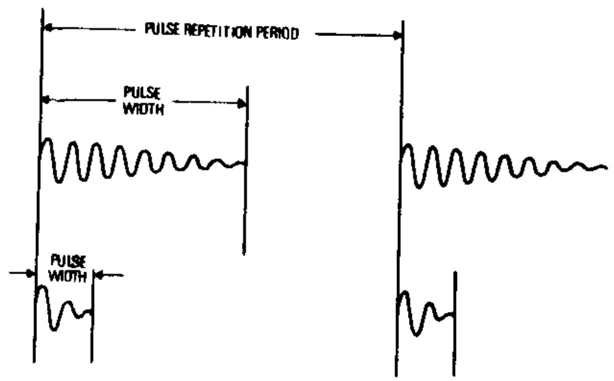


Fig.1 - Example of pulsed waves having the same pulse period, but different dampings.

## SOUND PROPAGATION

Since both optics and acoustics deal with wave propagation, the corresponding physical laws are in many respects analogous. Some differences exist which are related to the different nature of physical systems and to the wavelength (for sound it is several orders of magnitude longer than for light). The latter is responsible for the fact that in acoustics the concept of "ray", and the "geometric" representation of sound propagation are seldom exploited. It is well known that such representation is satisfactory only when all physical systems interacting with the radiation (obstacles, limiting surfaces, slits) have linear dimensions much greater than the wavelength.

As shown in Table 2, human dimensions are much larger than optical wavelengths, but are comparable with acoustic wavelengths. For ultrasound, the case may occur where the ratio of the dimensions of objects of interest (in particular the human body or its parts) to the wavelength is such that geometric laws are satisfied to first approximation. In this case one can speak in terms of sound rays, or beams, propagating in straight lines and obeying laws similar to those governing geometric optics.

Table 2. Typical wavelengths of some acoustic and optical radiations propagating in air (meters)

| | |
|---|---|
| Grave tone (audibility limit) | 16 |
| Shrill tone (audibility limit) | $1.6 \times 10^{-2}$ |
| Ultrasound in medicine | $2 \times 10^{-5} - 3 \times 10^{-4}$ |
| Red light | $8 \times 10^{-7}$ |
| Violet light | $3.5 \times 10^{-7}$ |

The propagation of a sound beam may be described rather simply at large distances from the source, but is complex in its proximity because of the finite dimensions of any transducer. To better understand this point, let us consider in some detail the ultrasonic field of a transducer, represented for the sake of simplicity as a cylindrical oscillating piston (Fig.2a). As already mentioned, the ultrasonic field may be described as the overlapping of elemental spherical waves emitted in phase from each point of the piston base. This overlapping gives rise to interference of sound waves in the surrounding space: moving along the piston axis, i.e. along the direction of the sound beam, points are encountered where interference gives rise to a maximum in intensity, and points where the intensity is reduced to zero.

The intensity distribution pattern is as shown in Fig.2b. The position of the last maximum depends on the piston diameter D and on the wavelength through the relation:

$$x = \frac{D^2 - \lambda^2}{4\lambda} \qquad (22)$$

and divides the space into two regions. The one nearer to the source, where interference effects occur, is called the near field, or Fresnel, region; the farther one is called the far field, or Fraunhofer, region.

In the far field region the sound intensity, measured along the propagation axis, attenuates (apart from possible absorption and obstacles) because of the diffraction. This effect causes the sound beam to spread

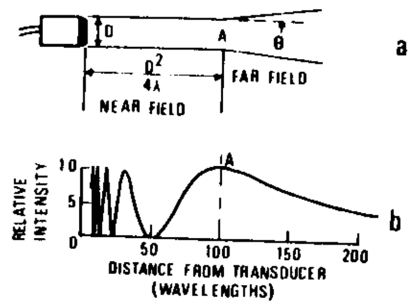


Fig.2 - The ultrasonic field produced by a cylindrical source (a) and its intensity vs.distance (b).

out or diverge; apart from a number of lobes of very much reduced intensity, the main part of the beam travels within a cone whose aperture is called the divergence of the beam. The divergence depends on the diameter of the transducer and on the wavelength through the relationship:

$$\sin\theta = 1.22\ \lambda / D \qquad (23)$$

Thus if the diameter is small compared to λ, then the waves diverge at very short distance from the source, giving rise to spherical waves. On the other hand, when the diameter of the transducer is a large number of wavelengths, the beam is highly directional and keeps nearly parallel over long distances.

Since the cross-section of a cone is proportional to the square of its height, the beam intensity in the far field attenuates with distance according to the classical inverse-square dependence.

## PEAK VALUES AND AVERAGE VALUES

It has been shown that sound intensity, as well as other acoustic quantities, depends on space and time variations, sometimes in a very complicated way. Thus one frequently resorts for practical measurements to space and/or time averaging. One can define and measure a spatial peak intensity I(SP) as the intensity measured along the beam axis or a spatial average intensity I(SA) as the ratio of the total beam power to its area of cross section. With regard to time, one distinguishes between a time peak intensity I(TP) and a time average intensity I(TA); the average is performed over a time interval much greater than the wave or pulse period. In the case of pulsed waves, time average may be referred to a single pulse and the sound intensity is therefore referred to as pulse average intensity I(PA).

In practice one specifies both characteristics for any intensity measurement, indicating for example I(SATA) an intensity averaged over both space and time, or I(SPTA) the time averaged axial peak intensity, etc. When this specification is missing, it is generally assumed a time- and space-averaged intensity, i.e. I(SATA).

## REFLECTION AND TRANSMISSION AT A SURFACE

When a sound wave strikes a surface, it gives rise to reflection and refraction phenomena very similar to the well known effects of optical radiation; in particular, the reflection angle r equals the incidence angle i, whereas the refraction angle t depends on the values of sound speed in both media through the well known Snell's law:

$$c_1/\sin i = c_2/\sin t \qquad (24)$$

If $c_2 > c_1$ , a critical angle $i_c$ exists, for which t is equal to 90 degrees. For incidence angles greater than $i_c$, no refracted wave can pass through the second medium, and the beam is fully reflected. The critical angle is immediately deduced from eq.(24):

$$\sin i_c = c_1/c_2 \qquad (25)$$

Since $c_1$ and $c_2$ may be significantly different, as in the case of an air-solid interface, total reflection may occur even at small incidence angles.

The extent to which the sound intensity is reflected or transmitted depends on the characteristics of both media: the intensity ratios can be expressed in terms of a quantity characteristic of each medium, which is called acoustic impedance and is defined as:

$$Z = p/v = \rho c \qquad (26)$$

where v and c are the velocity of molecules and the sound propagation speed in the medium, respectively. The acoustic impedance is much greater in liquids than in gases, and even greater in solids, since both the density and the sound speed are progressively greater in the three cases. Some typical values, both of biological and inorganic materials, are listed in Table 3.

Table 3. Acoustic impedance of some materials (kg/ms)

| | |
|---|---|
| Steel | $4.7 \times 10^7$ |
| Aluminium | $1.7 \times 10^7$ |
| Perspex | $3.2 \times 10^6$ |
| Bone | $7.8 \times 10^6$ |
| Muscle | $1.7 \times 10^6$ |
| Water | $1.4 \times 10^6$ |
| Air | 430 |
| Hydrogen | 110 |

At normal incidence, the intensities $I_i$ , $I_r$ and $I_t$ of incident, reflected and transmitted beams are related by the expressions:



$$\frac{I_r}{I_i} = \frac{(Z_2-Z_1)^2}{(Z_2+Z_1)^2} \qquad \frac{I_t}{I_i} = \frac{4 Z_1 Z_2}{(Z_2+Z_1)^2} \qquad (27)$$

The reflection is high if the impedances of the two media are very different, and is zero if they are equal. The opposite is true for the transmission. This is especially important in the case of airborne ultrasound beams, which are almost completely reflected by the surface of any solid or liquid. Only about 0.1% of the airborne ultrasound intensity is transmitted into the human body.

The practical importance of eqs.(27) is evident. Impedance matching is needed to transfer energy from a transducer into a medium, in particular into a biological tissue. In practice, good contact between the surfaces of the transducer and of the absorbing body is difficult to achieve, since a layer of air separates them. This results in very little sound energy being transmitted. To overcome this difficulty, a commonly used technique is to insert between the transducer and the body a thin layer of a material with suitable impedance (a gel, oil or water), in order to maximize the energy transfer.

The introduction of a third medium, however, presents some difficulties, and the resultant transmission coefficient may not necessarily be equal to the product of the coefficients for each boundary. Much depends on the thickness of the intervening layer and on the wavelength. If the thickness of the layer is equal to an integer number of half-wavelengths, then sound will be transmitted as though the intervening medium did not exist. This optimum condition is difficult to realize in ultrasound applications because the very short wavelengths require an extremely high degree of precision in determining the layer thickness. Moreover, the transmission does not take place at a single frequency, but over a band of frequencies, which extends to both sides of a centre frequency and may be very wide, especially for pulsed waves. Under these conditions, a perfect coupling is impossible, since a given thickness will be equal to an exact number of half-wavelengths for only one of the frequencies in the band. However, where the frequency band is quite narrow and the impedance of the intervening layer is close to the value of those of the outer media, a departure of not more than one or two decibels from the ideal case my be expected.

A very interesting phenomenon is observed when a beam of longitudinal waves strikes the interface between a fluid and a solid at an oblique angle i. In this case part of the beam is converted into transverse waves, which propagate in the solid. This effect is usually called mode conversion, and may give rise to a double refraction or a double reflection when the incident beam travels in the fluid and in the solid, respectively. In the first case, since in a solid the speed of transverse waves is always less than the speed of longitudinal waves, Snell's law leads to different refraction angles. Referring to Fig.3a, we have:

$$c_1/\sin i = c_2/\sin r = c'_2/\sin r' \qquad (28)$$

Here, the subscripts 1 and 2 refer to the solid and the fluid, respectively, and the prime indicates the transverse waves. Two distinct beams are therefore transmitted into the solid, the longitudinal waves being refracted away from the normal more than the transverse waves. The critical angle is obviously different for either beam. If the incidence angle is greater than the first critical angle $i_{c_1}$, only transverse waves can pass through the solid. Increasing the incidence angle still further, the second critical angle $i_{c_2}$ is reached, for which the transverse waves are refracted at an angle of 90 degrees: in this case surface waves are

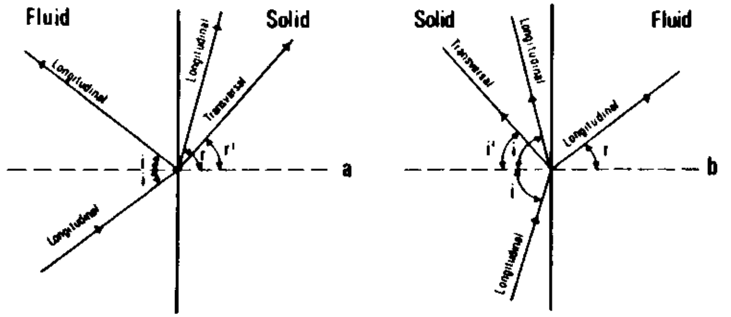


Fig.3 - Mode conversion at a fluid-solid interface. Longitudinal waves are partially converted into transverse waves giving rise to a double refraction (a) or to a double reflection (b).

propagated, whose properties differ from those of bulk waves. For incidence angles more than $i_{c2}$, no wave propagation in the solid occurs.

In the case of a longitudinal wave travelling in a solid and striking a boundary with a fluid at an incidence angle i, a beam of longitudinal waves passes into the fluid at some angle r to the normal, and two beams are reflected back into the solid (Fig.3b). The incidence, reflection and refraction angles are related in this case by the expressions:

$$c_1/\sin i = c'_1/\sin i' = c_2/\sin r \qquad (29)$$

Because of the different sound speeds, the beam of transverse waves is always closer to the normal than the incident beam.

The phenomena of reflection and refraction are exploited to create acoustic images of objects which are opaque to light by means of acoustic mirrors and lenses, i.e. selected materials shaped to reflect or refract sound waves in suitable directions.

STANDING WAVES

When a beam of sound waves strikes at normal incidence a boundary between two media, part of the waves are reflected backwards along the same axis of the incident beam. If the incident and reflected beams are continuous, they will interfere giving rise to standing waves (also called stationary waves). These waves correspond to a solution of the wave equation built up with two terms, describing waves travelling in the positive and negative direction, respectively.

In terms of displacement of the particles, we have:

$$u = u_i \sin k(x-ct) + u_r \sin k(x+ct) \qquad (30)$$

In the special case of perfect (100 per cent) reflection, this corresponds to $u_r = u_i$ and may be written as:

$$u = 2u_i \cos kx \sin kct = u(x) \sin kct \qquad (31)$$

This expression describes a situation where each particle of the medium oscillates sinusoidally with time, the amplitude of the oscillation being time-independent and sinusoidally distributed along the x-axis: waves do not propagate in either direction. The same is true also in the case of partial reflection ($u_r < u_i$), although the mathematical expression of the resulting wave is more complicated than eq.(31).

Standing waves are characterized by the appearance of fixed and equally spaced positions of minimum and maximum amplitude, called nodes and antinodes, respectively (Fig.4). The waveform varies with time, but always lies within the envelope shown in Figure 4. Perfect nodes of zero amplitude would appear only in the ideal case of 100 per cent reflection and no absorption; correspondingly, the amplitude at the antinodes would be twice the amplitude of the incident travelling wave. In practice, there will always be some finite value of amplitude at the nodes, and some reduction at the antinodes; the displacement from the ideal condition is expressed by the standing wave ratio (SWR), defined as the

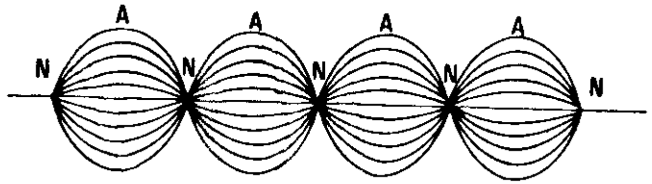


Fig.4 - Structure of standing waves in the case of perfect reflection (N:nodes, A:antinodes).

amplitude at an antinode relative to that at a node:

$$SWR = (u_i + u_r)/(u_i - u_r) \qquad (32)$$

It is seen that the standing wave ratio can never be less than unity. A value of 1 for the SWR would correspond to no reflection, a value of infinity to 100 per cent reflection.

For longitudinal waves, the SWR can also be expressed in terms of pressure:

$$SWR = (p_i + p_r)/(p_i - p_r) \qquad (33)$$

It is interesting in this case to note that nodal positions of pressure correspond to antinodes of displacement, and viceversa.

Standing waves are actually found in any medium of finite size. Due to reflections at the end boundaries, sound waves travel forward and back, while the intensity attenuates because of energy losses on reflection and absorption, unless sustained by an external device.

## SCATTERING

An obstacle in the path of a sound beam causes reflection and refraction as described above, only if its dimensions are large relative to the wavelength. When, however, the obstacle dimensions are comparable with or less than one wavelength, scattering takes place, and secondary sound spreads out in all directions.

Scattering may be caused by particles diffused in a liquid or a gas: an important example is given by red cells in blood. Scattering is also responsible for the spreading of a sound beam which reflects from a rough surface. In particular, at high ultrasonic frequencies, scattering may take place in a solid having a polycrystalline structure, i.e. a solid consisting of a large number of tiny single crystals tightly packed together and oriented at random.

Scattering causes energy to be diverted from a sound beam, so that its intensity attenuates. In low-frequency scattering (or Rayleigh scattering), when the dimensions of the obstacle are much smaller than the wavelength, the amplitude of the scattered wave is proportional to the square of the frequency. The scattered intensity is therefore proportional to the fourth power of the frequency. Thus scattering increases significantly with increasing frequency.

## ABSORPTION

In addition to the energy losses due to reflection, refraction, divergence, and scattering effects, the intensity of a sound beam attenuates because part of the vibrational energy of the particles is converted into heat. This absorption of energy takes place through a variety of mechanisms, mainly by viscous loss and relaxation processes.

The occurrence of absorption can be accounted for by introducing an absorption coefficient in the solution of the wave equation:

$$p(x,t) = p_0\, e^{-\alpha x} \sin k(x-ct) \qquad (34)$$

$\alpha$ is called the amplitudes absorption coefficient. The attenuation of intensity may also be expressed in terms of $\alpha$; an intensity absorption coefficient is however frequently used, whose value is twice the value of $\alpha$ :

$$I = I_0\, e^{-2\alpha x} = I_0 e^{-\mu x} \qquad (35)$$

The absorption coefficients of a material increase with increasing frequency. If viscosity is the only mechanism responsible for absorption of a plane longitudinal wave, then $\alpha$ and $\mu$ are proportional to the square of the frequency. In the most general case, the absorption coefficients vary according to the general expressions:

$$\alpha = \alpha_0\, (f/f_0)^n \qquad \mu = \mu_0\, (f/f_0)^n \qquad (36)$$

where $f_0$ is an arbitrary reference frequency, and n is a parameter which in turn depends on frequency. Within limited frequency ranges it may be assumed as a constant, ranging for most materials between 1 and 2.

The unit currently used for absorption coefficients is the decibel per centimetre (dB/cm).



## THE DOPPLER EFFECT

The Doppler effect consists of a frequency (and consequently of a wavelength) shift of the waves emitted by a source moving relative to the receiver, with a non-zero component of velocity along the axis of the sound beam. The observed frequency increases when the source and the receiver approach each other, and decreases when they move away. The effect is easily explained if we consider that in the first case compression and rarefaction waves arrive at the receiver at a higher frequency, and in the second at a lower frequency, with respect to the condition where the source and the receiver are stationary. Motion of either causes a decreasing or increasing path that each wave must travel.

The phenomenon is clearly depicted in Fig.5 for a moving point source radiating spherical waves. The successively generated spheres are closer together ahead of the source, and farther apart behind it. Since the frequency is determined by the number of waves passing a stationary receiver per unit time, it is clear that the frequency is higher ahead of the source and lower behind it. By simple geometric considerations, it may be deduced that, if a source moves relative to a receiver with velocity V, the perceived frequency f is related to the emitted frequency $f_0$ by the relation:

$$f = \frac{f_0 c}{c - V\cos\theta} \qquad (37)$$

where $\theta$ is the angle between the direction of the velocity vector $\vec{V}$ and the vector joining the source and the receiver. The frequency shift therefore depends on the velocity component directed towards the receiver. The Doppler shift at a given time and position only depends on the source velocity and frequency at the instant of generation of the wave.

In clinical diagnostic applications the Doppler effect is used to determine the speed of a moving target, such as flowing blood.

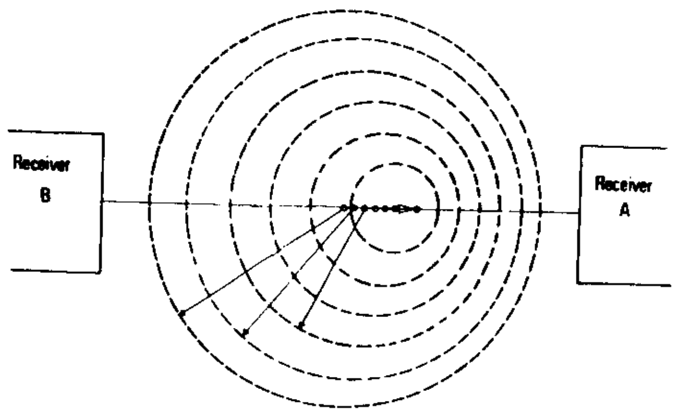


Fig.5 - The Doppler effect. Frequencies experienced at receivers A and B are respectively higher and lower than the frequency emitted by the source.

Sound waves received from a source are subsequently reflected back towards the receiver at rest relative to the source. Under these conditions, a double frequency shift occurs: the first during transmission, with the target acting as a receiver, and the second during reflection from the target, when it acts as a source.

## CAVITATION

In fluid mechanics, the term cavitation indicates the formation of gas- or vapour-filled cavities in a liquid by mechanical forces. A typical example is observed in boiling water. Cavitation is in response to an alternating pressure field. From an acoustic field, it is called acoustic cavitation.

During the rarefaction phase of the acoustic cycle, the local pressure becomes lower than the ambient pressure, and any bubbles preexisting (called "nuclei") in the liquid may begin to grow; during the next half of the cycle the local pressure rises above the ambient pressure, and the bubbles growth reverses. The degree of growth, and the lifetime of bubbles depend on several factors: the acoustic pressure amplitude, the value of the ambient pressure, the frequency of the waves and the duty cycle (if pulsed), and obviously the characteristics of the liquid and the dissolved gases.

Where the acoustic pressure is sufficiently high, the bubbles will collapse suddenly on compression, and will release a large amount of energy almost istantaneously. The minimum acoustic intensity required for the onset of this collective phenomenon is called the cavitation threshold (or threshold intensity). The cavitation threshold increases with increasing frequency and ambient pressure. It also varies with temperature, because of the temperature dependence of the surface and of the saturation vapour pressure of the bubbles.

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