INTERNATIONAL ATOMIC ENERGY AGENCY
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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONE: 2240-1
CABLE: CENTRATOM - TELEX 460392 - I

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WORKSHOP
GLOBAL GEOPHYSICAL INFORMATICS WITH APPLICATIONS TO RESEARCH IN EARTHQUAKE PREDICTIONS AND REDUCTION OF SEISMIC RISK

(15 November - 16 December 1988)

MODELLING SEISMICITY

E. NYLAND

University of Alberta
Dept. of Physics
Edmonton T6G 2J1
Canada

Why Model?

Earthquake sequences appear random, but Some of the randomness may result from "simple" nonlinear mechanisms rather than true complexity.

Randomness

occurs to the extent that something cannot be predicted

is a matter of degree. Some systems are more predictable than others

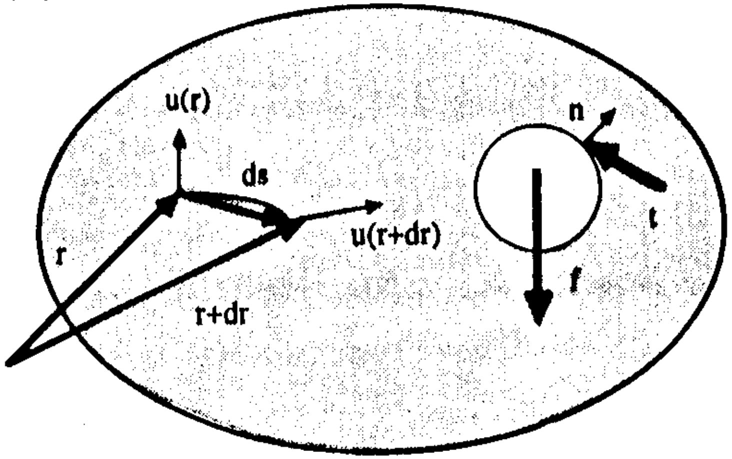
often arises out of a sensitive dependence on initial conditions because the non linear system amplifies noise.

may be associated with attractors of varying dimensions

The theory of random processes is an empirical technique for coping with inadequate information and makes no statements about the causes of randomness.

The only truly fundamental source of randomness is the uncertainty principle of quantum physics.

Basic ideas of Continuum Mechanics



$$\int_S \mathbf{t}\,ds+\int_V \mathbf{f}\,dv=\int_V \rho \frac{d^2\mathbf{u}}{dt^2}\,dv$$

$$\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

Block Mechanics?

$$\begin{bmatrix} \frac{\partial u_x}{\partial x}dx+\frac{\partial u_x}{\partial y}dy+\frac{\partial u_z}{\partial z}dz \\ \frac{\partial u_y}{\partial x}dx+\frac{\partial u_y}{\partial y}dy+\frac{\partial u_y}{\partial z}dz \\ \frac{\partial u_z}{\partial x}dx+\frac{\partial u_z}{\partial y}dy+\frac{\partial u_z}{\partial z}dz \end{bmatrix}$$

$$\varepsilon=\begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{1}{2}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u_y}{\partial x}+\frac{\partial u_x}{\partial y}\right) & \frac{\partial u_y}{\partial y} & \frac{1}{2}\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right) & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

Finite Strain?

Constitutive Equations

1. Linear Elastic theories

$$\sigma_{xx} = \lambda\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right) + 2\mu\frac{\partial u_x}{\partial x}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

$$\nabla\cdot\ \sigma = 0 \qquad \longleftrightarrow \qquad \vec{F} = m\vec{a}$$

2. Biot theory

$$\sigma_{ij} - \alpha\delta_{ij}p = \lambda\delta_{ij}\nabla\cdot\ \mathbf{u} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\nabla\cdot\ \sigma = 0$$

$$p = a_1\theta + a_2\nabla\cdot\ \mathbf{u} \qquad \mathbf{v} = K\nabla P$$

$$\frac{\partial\theta}{\partial t} = -\nabla\cdot\ \mathbf{v}$$

Modify the principal stresses by a fraction of the fluid pressure. This is also a linear theory, originally

developed for soils. Notice we now have diffusion.

3. Temperature dependence

See Aki and Richards page 20 ff

Rate of doing Mechanical Work + Rate of Heating =
= Rate of increase of (kinetic + internal) energy

Rate of doing Mechanical Work =

$$= \int_{\mathbb{V}} \mathbf{f}\cdot\ \mathbf{v}\, d\mathbb{V} + \int_{\mathbb{S}} \mathbf{t}\cdot\ \mathbf{v}\, d\mathbb{S}$$

$$= \int_{\mathbb{V}} \mathbf{f}\cdot\ \mathbf{v}\, d\mathbb{V} + \int_{\mathbb{S}} \mathbf{v}\cdot\ \sigma\cdot\ \mathbf{n}\, d\mathbb{S}$$

$$= \int_{\mathbb{V}} \left(\rho\mathbf{v}\cdot\ \frac{\partial\mathbf{v}}{\partial t} + \nabla\mathbf{v}:\sigma\right) d\mathbb{V}$$

$$= \frac{d}{dt}\int_{\mathbb{V}} \left(\rho\frac{\mathbf{v}\cdot\ \mathbf{v}}{2} + \sigma:\dot{\varepsilon}\right) d\mathbb{V}$$

= Rate of increase of kinetic energy +

?

Rate of Heating $= -\int_{\mathbb{S}} \mathbf{h} \cdot \mathbf{n} d\mathbb{S} = \frac{d}{dt} \int_{\mathbb{V}} Q d\mathbb{V}$

Rate of increase of internal energy per unit volume =

Rate of doing Mechanical Work + Rate of Heating -

Rate of increase of kinetic energy

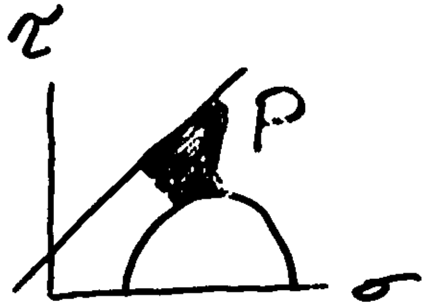
$$\frac{dU}{dt} = -\nabla \cdot \mathbf{h} + \sigma : \dot{\varepsilon} = \frac{dQ}{dt} + \sigma : \dot{\varepsilon} \text{ or}$$

$$dU = dQ + \sigma : d\varepsilon = TdS + \sigma : d\varepsilon$$

Idea $\delta \oint dU = 0$ for Equilibrium Thermodynamics ✓

Is the occurrence of an earthquake to be described by equilibrium thermodynamics?

Probability of "Failure"



What is the form of P?

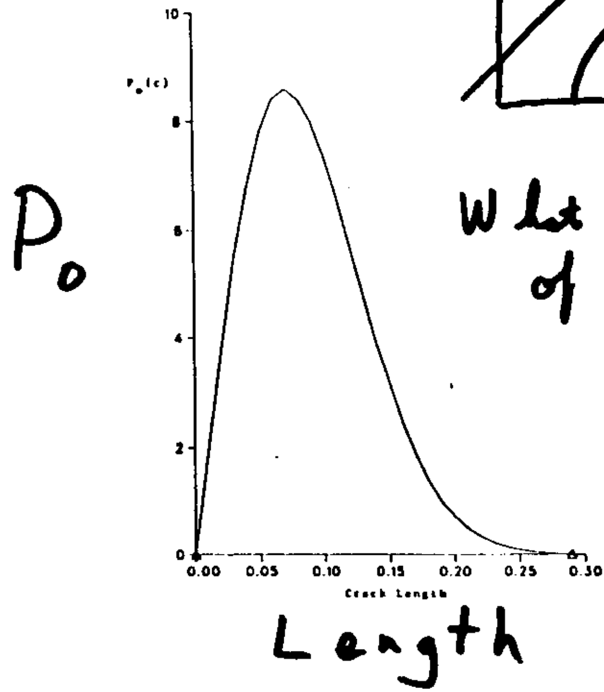
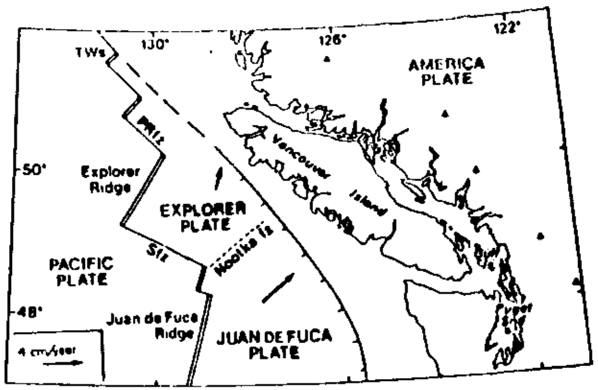


Figure 11... shows the assumed distribution of cracks in a body. $P_o(c)$ is calculated for σ=0.1.

M.Sc. Thesis Li Qing PRC

Ph.D. Antonio Uribe Mexico


TWs
130°
126°
122°
AMERICA PLATE
50°
Explorer Ridge
EXPLORER PLATE
Vancouver Island
Nootka fz
PACIFIC PLATE
48°
Juan de Fuca Ridge
JUAN DE FUCA PLATE
4 cm/year

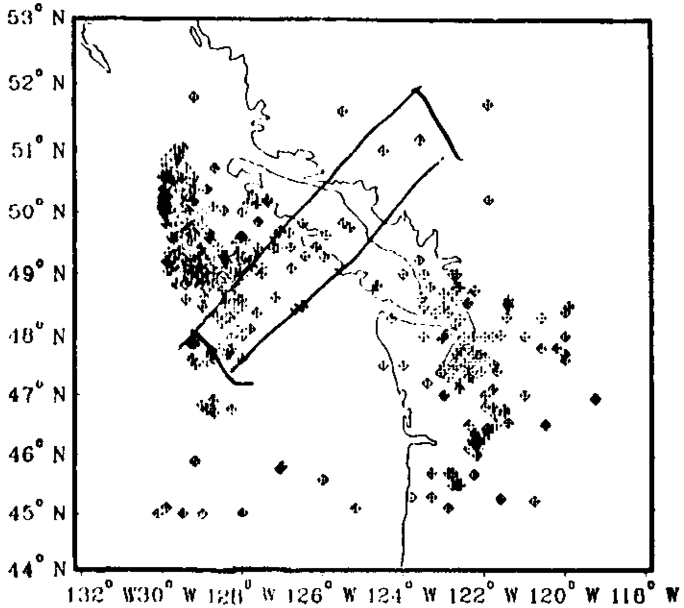

53° N
52° N
51° N
50° N
49° N
48° N
47° N
46° N
45° N
44° N
132° W
130° W
128° W
126° W
124° W
122° W
120° W
118° W

Figure 3

Earthquake Locations

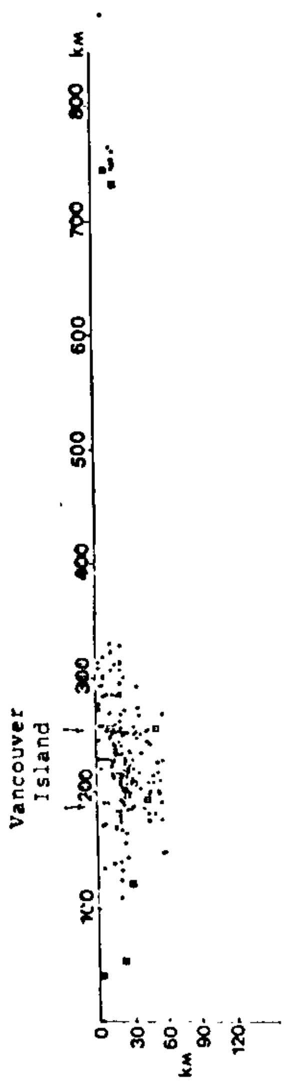
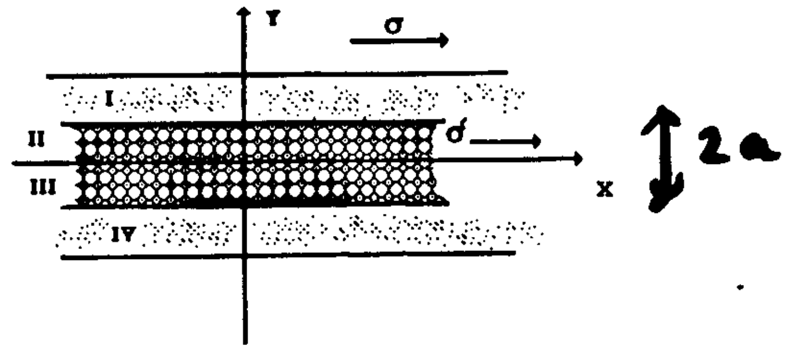
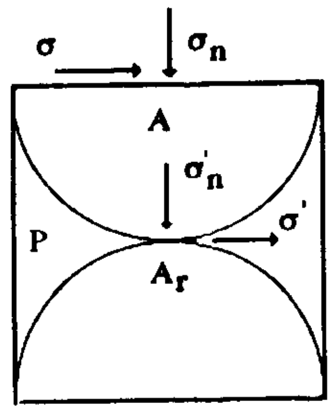


Figure 8... shows the distribution of the foci of the earthquakes within the rectangle in Figure 11.



If a fault zone is a mixture of "blocks" and fluids, what is the stress on the blocks?

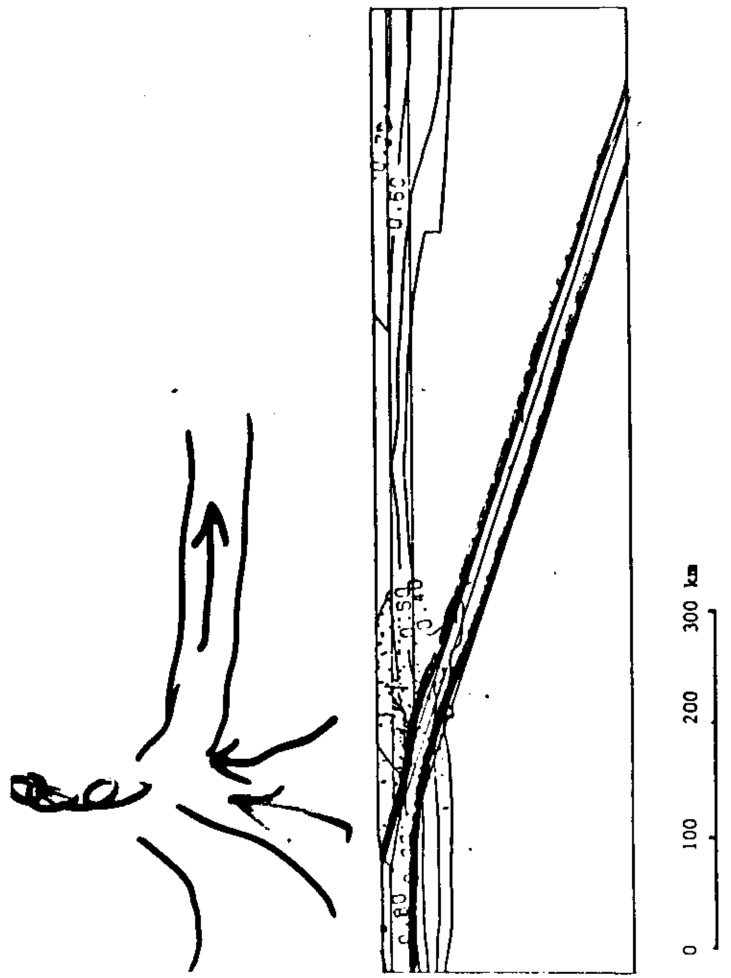


σ is shear stress applied to total area A, σ' is the shear stress applied to the real area of contact A_r, P is pore pressure, σ_n is normal stress applied to A, σ'_n is normal stress applied to A_r

$$\dot{\varepsilon}=D_0 f(\tau) e^{\frac{AT_m}{T}}$$

$$\dot{\varepsilon} = c_0^{-1} e^{\frac{-E}{kT}} \sinh \frac{v\tau}{kT}$$

$$\rho c_p \frac{\partial T}{\partial t} - \frac{\partial^2 T}{\partial x^2} = \tau\dot{\varepsilon}$$



Push

Figure 15 ... shows the distribution of instability for the model with ridge push force exerted on oceanic plate.

$$\rho c_p \frac{dT}{dt} = \tau\dot{\varepsilon} - 2\frac{K}{a^2}(T-T_0)$$

$$\dot{\varepsilon} = \frac{1}{a}\frac{d\Delta}{dt}$$

$$\frac{\partial \Delta}{\partial t} = \frac{a}{c_0} e^{\frac{-E}{kT}} \sinh \frac{v\,\tau}{kT}$$

$$(\eta_{eff})_{\tau\to 0} = C^{-1}\frac{kT}{v} e^{\frac{E}{kT}}$$

$$v_0 \rho T e^{\frac{AT_m}{T}}$$

$$v_0 = B\left\{\frac{1}{2}L_1\left(1+\tanh c'(T-T_m)\right)+L_2\right\}^2$$

$$\frac{dL_1}{dt} = \alpha\left(L_1 - L_{min}\right)\frac{\partial \Delta}{\partial t} - \frac{\left(L_1 - L_{max}\right)}{\tau_0}$$

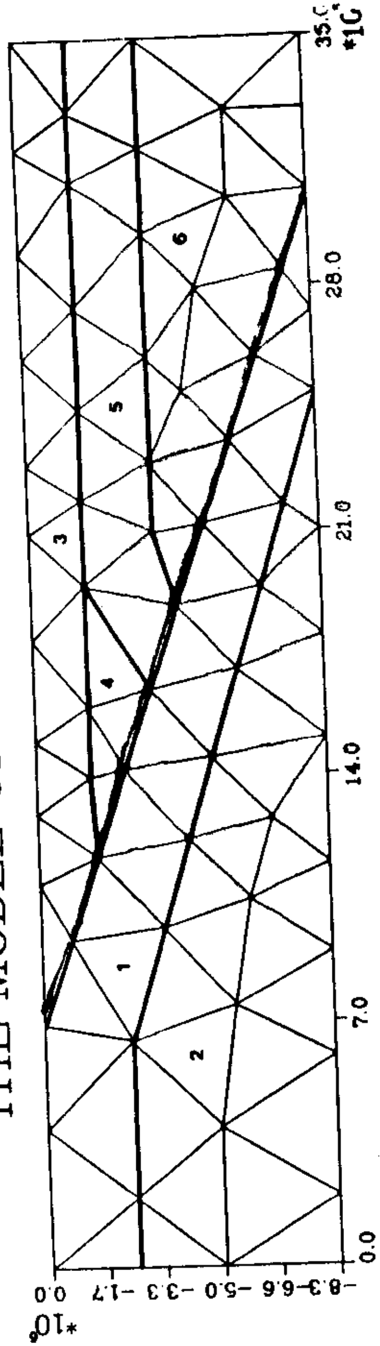
$$\tau_0 = 10^6 e^{\frac{AT_m}{T}}$$

Displacement across a disc~
" " " fault

Materials	E ($\times 10^{12}$ dyn/cm^2)	ν	ρ
1	1.2	0.25	3.3
2	1.04	0.3	2.295
3	0.89	0.25	2.92
4	1.163	0.3	3.3
5	0.71	0.3	2.92
6	1.25	0.3	3.285

Faults	E ($\times 10^{5}$ dyn/cm^2)	ν
1	2.0	0.4

THE MODEL OF VANCOUVER ISLAND



$$\mathbf{Kd=f=A_1b+A_2\delta+A_3g}$$

$$\boldsymbol{\tau}=\mathbf{M_1d+M_2\Delta}$$

$$\frac{d\tau}{dt}=\mathbf{M_1K^{-1}}\left[\mathbf{A}_1\frac{d\mathbf{b}}{dt}+\mathbf{A}_2\frac{d\Delta}{dt}\right]+\mathbf{M}_2\frac{d\Delta}{dt}$$

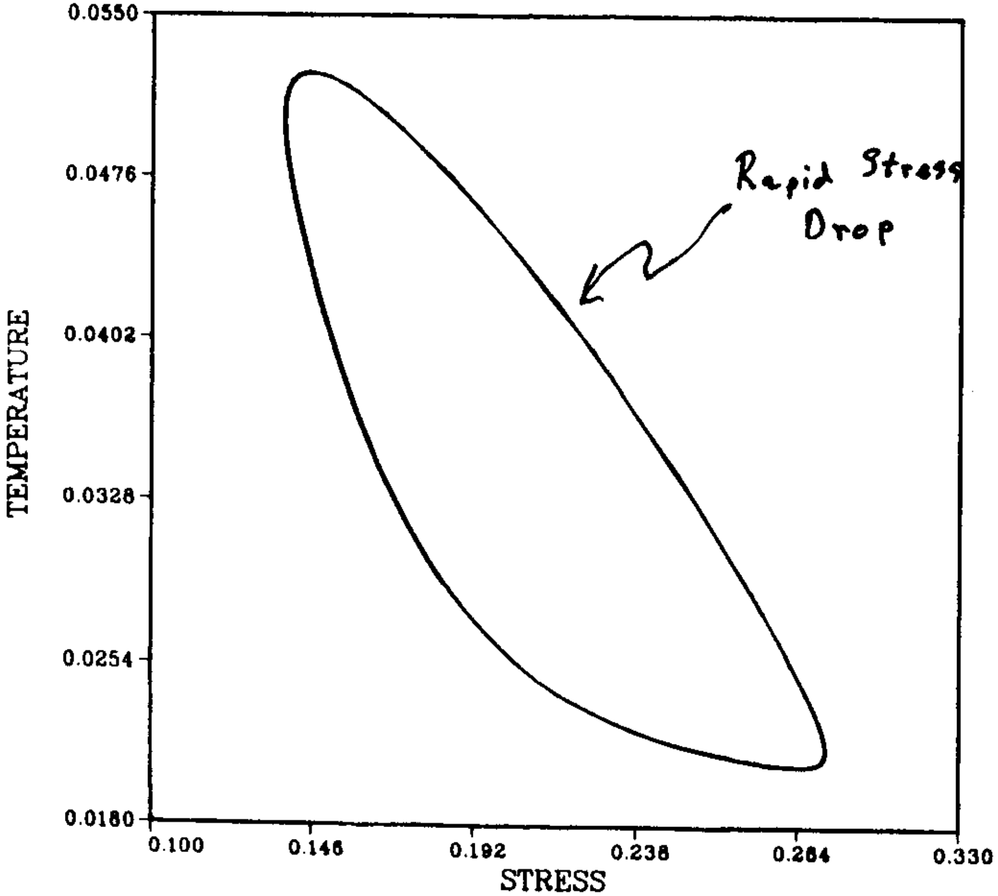
$$\mathbf{b(t)} = \mathbf{b_0} + \mathbf{t}\sum_{k=1}^{K} \mathbf{v_k c_k} = \mathbf{b_0+tCv}$$

$$\frac{\partial\tau}{\partial t}=\left[\mathbf{M_1K^{-1}A_1C}\right]\mathbf{v}+\left[\mathbf{M_1K^{-1}A_2+M_2}\right]\frac{d\Delta}{dt}$$

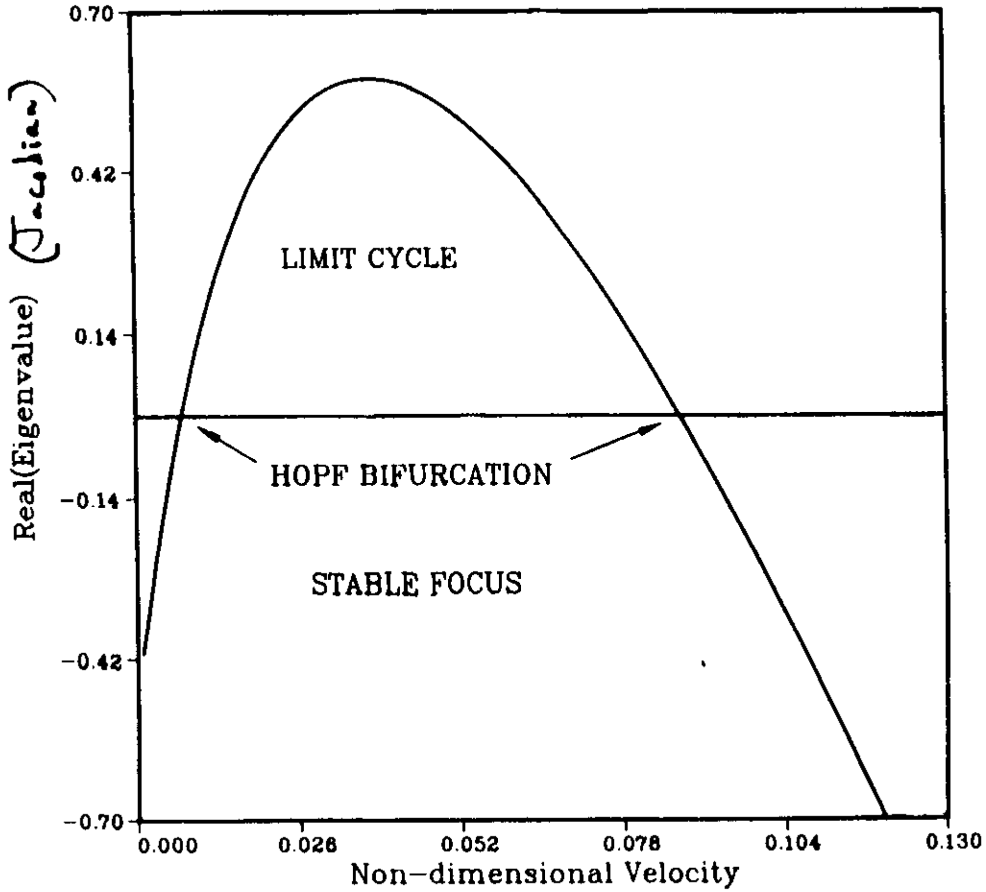
$K(K^{-1}A_1C) = A_1C \qquad K(K^{-1}A_2) = A_2$

Solve a small number of FE problems

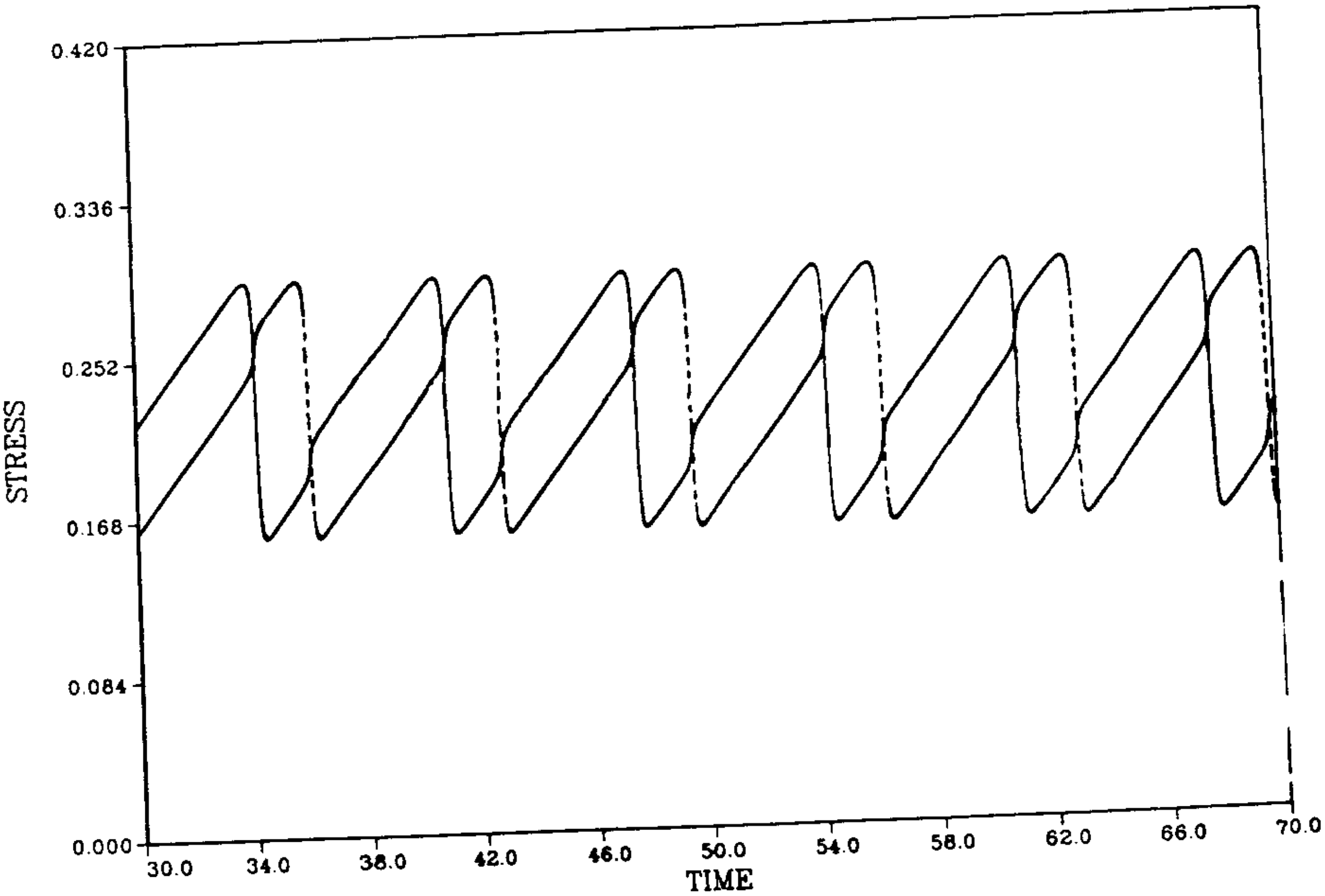
One Element System Phase Plot



One Element System Stability Analysis

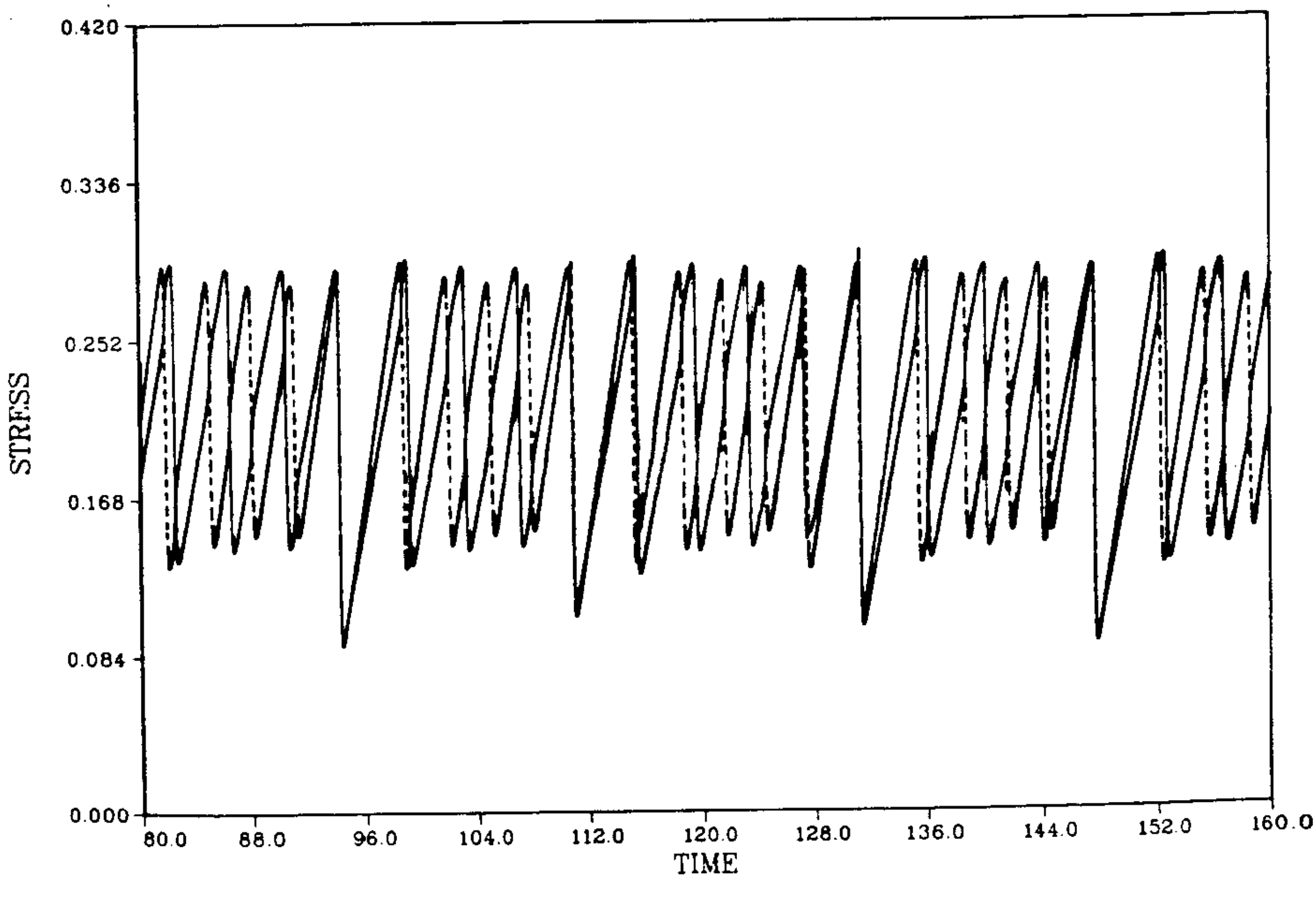


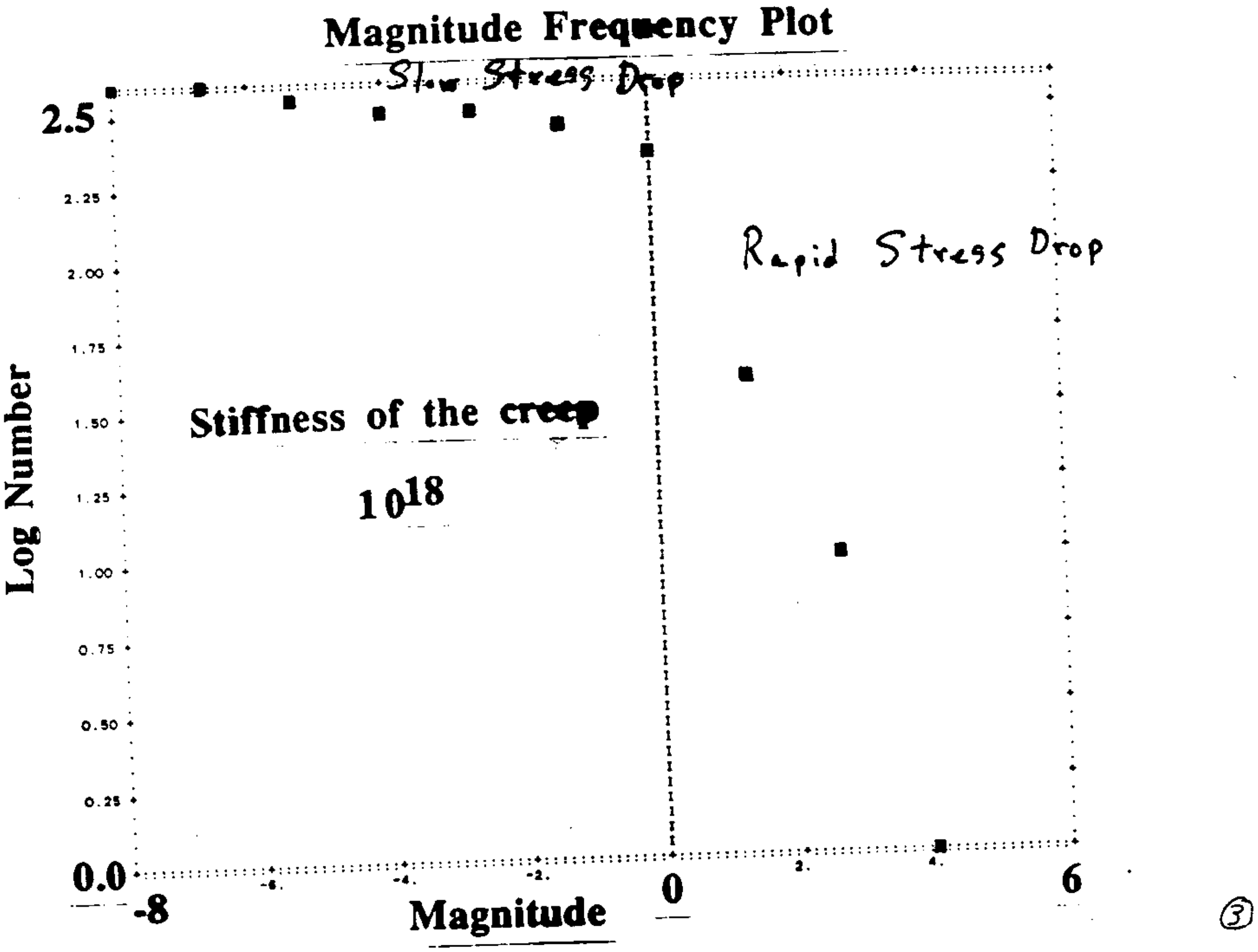
TWO ELEMENTS, EQUAL DRIVES

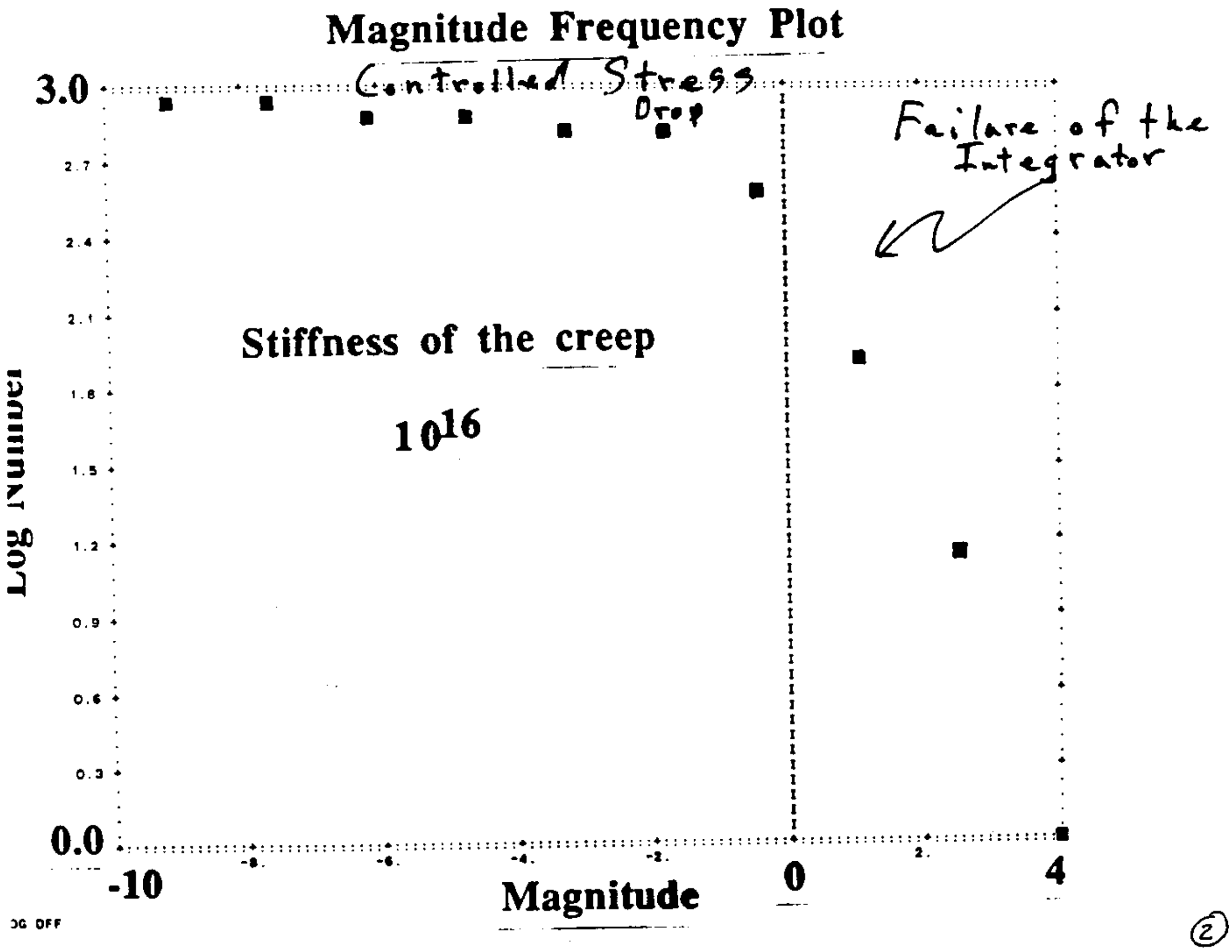


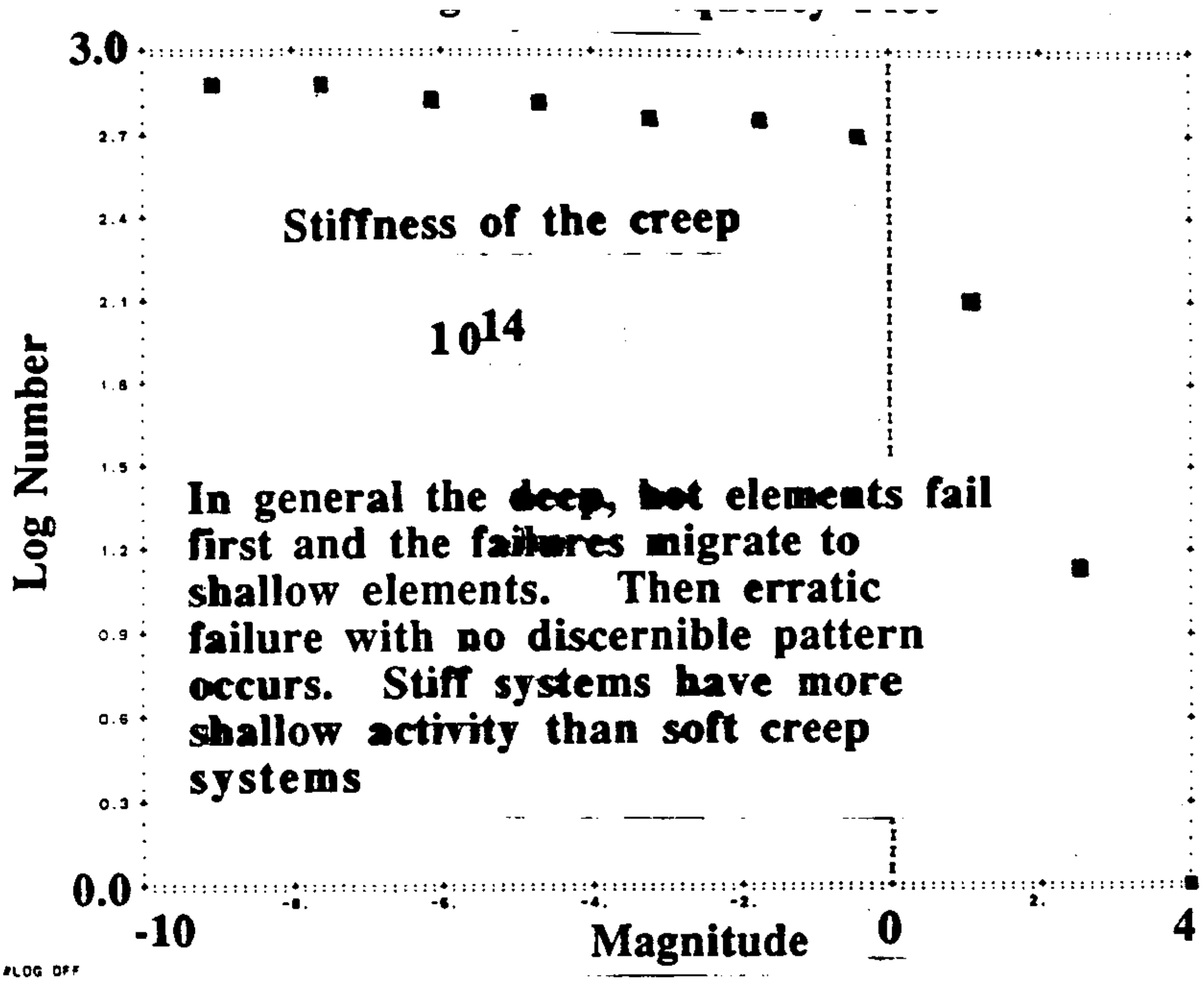
TWO ELEMENTS, UNEQUAL DRIVES

Effect of Breaking Symmetry




Magnitude Frequency Plot
Slow Stress Drop
Rapid Stress Drop
Stiffness of the creep
10^{18}
Log Number
Magnitude
2.5
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.0
-8
-6.
-4.
-2.
0
2.
4.
6
③


Magnitude Frequency Plot
Controlled Stress Drop
Failure of the Integrator
Stiffness of the creep
10^{16}
Log Number
Magnitude
3.0
2.7
2.4
2.1
1.8
1.5
1.2
0.9
0.6
0.3
0.0
-10
-8.
-6.
-4.
-2.
0
2.
4
②


3.0
2.7
2.4
2.1
1.8
1.5
1.2
0.9
0.6
0.3
0.0
Log Number
Stiffness of the creep
10^14
In general the deep, hot elements fail first and the failures migrate to shallow elements. Then erratic failure with no discernible pattern occurs. Stiff systems have more shallow activity than soft creep systems
-10
-8.
-6.
-4.
-2.
0
2.
4
Magnitude

#LOG OFF

12

①