INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONE: 2240-1
CABLE: CENTRATOM - TELEX 460392-1

H4.SMR/303 - 25

WORKSHOP
GLOBAL GEOPHYSICAL INFORMATICS WITH APPLICATIONS TO RESEARCH IN EARTHQUAKE PREDICTIONS AND REDUCTION OF SEISMIC RISK

(15 November - 16 December 1988)

STATISTICAL ANALYSIS OF THE RESULTS OF EARTHQUAKE PREDICTION, BASE ON BURST OF AFTERSHOCKS

G.M. MOLCHAN, O.E. DMITRIEVA
& I.M. ROTWAIN

Institute of Physics of the Earth
Academy of Sciences of the U.S.S.R.
Bolshaya Gruzinskaya 10
123 242 Moscow
U.S.S.R.

Statistical analysis of the results of earthquake prediction, based on bursts of aftershocks.

G. M. Molchan, O. E. Dmitrieva, I. M. Rotwain

Introduction.

Premonitory seismicity pattern named "burst of aftershocks" or, briefly, "pattern B" is defined as a main shock with abnormally large number of aftershocks (Keilis-Borok, et al, 1978). It was hypothethized that this pattern often preceeds the strong earthquake in the same region, with the lead time up to few years.

Retrospective prediction, based on this hypothesis, scored an acceptable success - to - failure ratio in many regions of the world. (Keilis-Borok, et al., 1980). However, it was difficult to evaluate these results statistically, since they were obtained retrospectively, with certain degree of freedom in the choice of some parameters; these difficulties are analysed by Molchan and Rotwain (1983).

At the same time additional evidences in favour of predictive value of pattern B were collected in the applications of a set of premonitory patterns, which includes pattern B. Two versions of such set, defined by algorithms CN and M8, were tested, mainly retrospectively, in wide variety of regions (e.g., Gabrielov et al., 1985). Elimination of pattern B from this set changes for the worst the success - to - failure score; the predictive value of pattern B is thus supported, though remained not proven.

A new attempt of statistical justification of pattern B is entertained in this study. It is made possible by two

developments:

- The experiment on forward predictions, based on pattern B was conducted by Soviet- American team (Dewey et al.,1988). This provided the results, not influenced by retrospective parameters fitting, since all parameters were chosen in advance, in previous publications.

- More data, allowing retrospective prediction, have been accumulated.

There are several reasons to use this opportunity to establish statistical significance of pattern B.

- This will be a first step toward evaluation of wider set of premonitory patterns.

- If confirmed, it will be essential for understanding the process of the preparation of strong earthquake, above and beyond practical purpose of prediction.

- At last, it is obviously tempting to establish statistical significance of some earthquake precursor among the multitude of suggested ones.

The statistical method of testing the significance of the pattern B is suggested by Molchan and Rotwain (1983). Its generalization for forward predictions is given in the present paper. It can be applied to statistical analysis of a wide variety of precursors. Joint and marginal distributions which served as a basis for the present analysis are presented in Appendix A.

1. Patern B: definition.

We remind here the main features and parameters of pattern B.

- Strong earthquakes are the main shocks with $M \geq M_0$;

- Main shocks, i.e. events which are not aftershocks are considered within magnitude interval $M \in [M_0 - a_2, M_0 - a_1]$, where a_1 and a_2 are numerical parameters ;
- Aftershocks of the main shock. They are identified within the following windows: the magnitude range of $M \geq M_0 - 3.5$, distance from a main shock of less than 50 km if registered in a regional catalog and of less than 100 km if the catalog of NOAA is used; time after a main shock up to 0.5 yrs for $M \leq 5.4$, up to 2 yrs for $M > 6.5$ and up to 1 year in all other cases;
- A burst of aftershocks. It is the sequence of aftershocks, which includes an anomalously large number of events $b \geq B_0$ during the first 2 days;
- An alarm which is announced for the period of τ yrs after the burst of aftershocks and is called off at the moment of a strong earthquake if it occurs during this period.

The earthquakes with focal depth up to 100 km within a certain region are considered. M_0 and B_0 are the main parameters which depend on the region. The alarm τ depends on M_0: for M_0 from 6 to 7 τ = 3yrs, for $M_0 > 7.5$ τ = 3 or 4yrs, depending on the region. Forward prediction was made during a time period T^+. The values of parameters M_0, B_0 for forward prediction were chosen on the basis of "pre-history", for preceding period T^-; these values optimize retrospective prediction in each region.

The parameters assumed for each region, are shown in Table 1. They were published earlier (Keilis-Borok, Knopoff, Rotwain, 1980; Negmatullaev et al., 1981; Keilis-Borok et al.,1981; Karpukhina and Rotwain, 1983; Lee and Lukina, 1983; Dewey et al., 1984)

Periods T^- and T^+ do not intersect except in Santa-Crus region, where the period T^- ended in alarm, so that the whole alarm is added to the T^+. As a rule the intervals of monitoring end by the middle of 1985 (for the Caucasus and New Zealand - by 1983). Either regional catalogs or the world NOAA catalog were used, as indicated in Table 1.

A standard magnitude interval of main shocks $[M_0 - 1, M_0 - 0.1]$ chosen for pattern B from the beginning is used for forward predictions. A wider interval of main shocks $[M_0 - 2, M_0 - 0.3]$ is used in the analysis below for retrospecive predictions (see also Molchan and Rotwain, 1983). The difference between the intervals is that in retrospective predictions an interval estimate, not a point estimate of parameters (M_0 ,B_0) is obtained for significance level (for details see Molchan and Rotwain, 1983).

The results of forward predictions can be shortly summed up as follows: 10 out of 14 strong earthquakes in 7 regions are preceded by bursts of aftershocks and the alarm duration is about 56% to total time of monitoring. So the question arises: which significance level do the given results of predictions correspond to ?

2. Estimation of significance; method.

We consider the hypothesis, that patern B is indeed a precursor of strong earthquakes. The natural alternative is the hypothesis that pattern B and strong earthquakes are completely independent. More specific formulation of this alternative is hypothesis H_0:

If K strong earthquakes and N_b bursts of aftershocks occur in time interval T, then the times of the events of these two types represent the independent realizations of K and N_b tosses of a random point with a uniform distribution into the interval T.

The main idea of weak test of precursor B is to construct a functional ζ which depends on the results of predictions and satisfies two following conditions:

1) The properties of precursor B are counted as full as possible. We will define the functionals ζ in such a way, that its large values testify against the hypothesis H_0.

2) The distributions of ζ under the hypothesis H_0 can be determined analytically.

Then the significance of predictions is characterized by an error of the first kind:

$$\varepsilon_{obs} = Pr (\zeta \geq \zeta_{obs} \mid H_0) \qquad (1)$$

i.e. by the probability of rejection of hypothesis H_0 when the observed value of our functional is ζ_{obs}. The smaller is ε_{obs} the more significant are the results of prediction.

The results of predictions for a set of regions $\{ G_i \}$ were summed up (Molchan and Rotwain, 1983) with the help of statistics

$$\chi^2 = -2 \sum_{i=1}^{s} \ln \varepsilon_{obs}(G_i) \qquad (2)$$

Under the hypothesis H_0 and with continious ζ this statistics has the χ^2 - distribution with 2s degrees of freedom. Large values of χ^2 testify in favour of precursor B in most of the regions G_i, as (2) is a monotonous function of the geometric mean of separate significance levels $\varepsilon_{obs}(G_i)$.

For discrete variable ζ the distribution (2) may be not quite adequate. This is just the case with forward predictions

for usually encountered values of ζ. The time intervals considered are so short that just a few strong earthquakes and main shocks may occur during these intervals. Therefore the simplest statistics, say, $\varkappa$ - the number of successful predictions, - will be essentially discrete, having only 2 to 5 possible values.

The statistics (2) is imprecise for the sum of regional predictions. Let us introduce the functional

$$\zeta_\Sigma = \sum_{i=1}^{s} \zeta(G_i) \qquad (3)$$

for a set of regions $\{G_i\}$, here $\zeta(G_i)$ is the functional for a region G_i. The distribution of ζ_Σ is a convolution of distributions of $\zeta(G_i)$. The significance level ε^Σ_{obs} corresponding to ζ_Σ gives the significance of predictions for a set of regions $\{G_i\}$.

Regional functionals ζ are constructed on the basis of two statistics: $\varkappa$ - the number of predicted strong earthquakes (successes); and the number ν, of successful alarms. Obviously $\nu \geq \varkappa$, since an earthquake may be preceded by several patterns B. Joint distribution of variables ($\varkappa,\nu$) is found under the hypothesis H_0 when the number of strong events K and the number of alarms N are fixed (Molchan and Rotwain, 1983). In other words: K, N_b and $\dot{\tau}$ are the parameters of the distribution of ($\varkappa,\nu$), here $\dot{\tau}$ is time of a single alarm divided by the total time of monitoring T.

Knowing the distribution of ($\varkappa,\nu$) we can find distributions of any functionals of $\varkappa$ and ν. In the test of precursor B three following statistics are considered:

$$\varkappa \;:\; \xi_1 = \begin{cases} \varkappa/K + \nu/N_b \\ 0, \text{ if } N_b = 0 \end{cases} ; \quad \xi_2 = \begin{cases} \varkappa/K - (\nu-\varkappa)/(N_b-\varkappa) \\ \varkappa/K \text{ , if } N_b = \varkappa \end{cases}$$

The statistics ξ_1 and ξ_2 take into account not only the relative number of successful predictions but of successful alarms as well. In statistics ξ_1 the number of predicted earthquakes is taken into account twice since in an implicite form it is involved also in second term because of an inequality $\nu \geq \varkappa$. In statistics ξ_2 the successful predictions and the clustering of alarms are represented more equally.

The variable ξ_1 is regarded as the main one. It characterizes pattern B better than $\varkappa$ does and assigns more adequate weights to predicted earthquakes and successful alarms than ξ_2. For purpose of prediction it is more valuable when more strong earthquakes are preceded by alarms, than when successful alarms are clustered before smaller number of strong earthquakes leaving more failures to predict. The functional ξ should emphasize this consideration.

The distribution of $\varkappa$ and ν is considered above with fixed values of parameters K and N_b not only for convenience of calculations but also to use the minimum of assumptions about the earthquake flow. The independence of strong earthquakes and bursts of aftershocks is the most important assumption in alternative hypothesis H_0. Additional assumptions - mutual independence and uniform distribution of events of each kind - have been forced although they steam from the usual basic hypothesis on Poissonian structure of the sequence of main shocks. In each particular study it is necessary to check the influence of additional

assumptions. This influence may be substantial because of non-stationaries of earthquake flow, e.g. seismic quiescence, so-called seismic cicle etc., when the period of observations is comparable with the period of these fluctuations. So the test of hypothesis H_0 on the several decades of retrospective prediction is more sensitive to additional assumptions than the test of forward predictions during few years.

Further we will assume the conditional situation, when the regional values of K are fixed and equal to observed ones.

In forward prediction during short time intervals the values N_b are small and their fixation leads to noticable loss of information on pattern B. Indeed if it is really a precursor it must influence the number of bursts of aftershocks:, in other words the occurence of a strong earthquake during a short time interval must be preceeded by a rare event - a burst of aftershocks. In particular, an absence of alarms when K = 0 in many regions speaks in favour of precursor B. The loss of information would be less, if the number of main shocks is fixed and the following extended hypothesis H'_0 is taken as an alternative.

Hypothesis H'_0 : K strong earthquakes and N_{ms} main shocks correspond to independent realizations of K and N_{ms} independent tosses of a random point with a uniform distribution into the interval T . A main shock can happen to be a burst with probability "p".

Evidently the regional parameter "p" can be estimated from retrospective "pre-history", as

$$\hat{p} = N_b^- / N_{ms}^- \qquad (4).$$

Here N_b^- is the number of bursts and N_{ms}^- is the number of main shocks during the "pre-history". Let $P_\zeta(\cdot \mid n, H_0)$ be the distribution of statistics ζ when K and $N_b = n$ are fixed. Then under the extended hypothesis H_0', the distribution of ζ is as follows:

$$P_\zeta(\cdot \mid N_{ms}, H_0') = \sum_{n=0}^{N_1} C_{N_1}^n \, p^n (1 - p)^{N_1 - n} \, P_\zeta(\cdot \mid n, H_0) \quad , \quad N_1 = N_{ms} \quad (5)$$

Replacement of "p" by the estimate (4) makes the distribution (5) biased as $E \hat{p}^n \neq p^n$ if $n \neq 1$. When $N_{ms}^- \geq N_1$ the bias may be eliminated by substitution:

$$p^n (1 - p)^{N_1 - n} \rightarrow (-N_b^-)_n (-N_{ms}^-)_n / (-N_{ms}^-)_{N_1} \quad , (6)$$

where $(a)_n = a(a+1)\cdot \ldots \cdot (a+n-1)$ (see Appendix B).

Under the hypothesis H_0 the averages for total alarm time τ_Σ and useful one τ_Σ^+ may be easily calculated (see Molchan and Rotwain, 1983). τ_Σ^+ is defined as a total length of adjacent alarms which end by a strong earthquake. It is clear that if $\bar{\tau}_\Sigma(n, H_0)$ and $\bar{\tau}_\Sigma^+(n, H_0)$ correspond to the average values of τ_Σ and τ_Σ^+ under the hypothesis H_0, and if $N_b = n$, then under the hypothesis H_0' these values are determined by (5,6) where P_ζ is substituted for $\bar{\tau}_\Sigma$ or $\bar{\tau}_\Sigma^+$ respectively.

3. Evaluation of forward predictions.

The results of forward monitoring of predictions by pattern B are given in the right column of Table 1.

Let us examine the 7 regions where strong earthquakes occured during the period of monitoring, so that the method described above is applicable. The results of the test of hypotheses H_0 and H_0' are given in Table 2.

We will summarize some conclusions:

1) The significance of prediction for each region is in agreement with hypothesis H_0 even for those regions where all strong earthquakes are predicted and all alarms are successful. The values of $\varepsilon(G)$ vary from 30 to 60%. This result is due to discreteness of distribution of the statistics considered; distributions of $\varkappa$ and ν have from 1 to 5 jumps due to the short time of monitoring.

2) After summation of regional statistics for hypothesis H_0 we obtain the significance levels of 11% and 1.4% using statistics $\varkappa_\Sigma$ and ζ_Σ respectively. As it was expected, the statistics ζ_1 takes better account of the properties of precursor B; the statistics ζ_1 and ζ_2 are about equally valuable.

3) The significance level becames better by an order of magnitude, if the extended hypothesis H_0' is tested, i.e. the stohastic nature of number of bursts of aftershocks is allowed for and data on the total number of main shocks are used. The significance levels of hypothesis H_0' for the statistics $\varkappa_\Sigma$ and $\zeta_{1,\Sigma}$ are 2% and 0.4% respectively.

4) The nonstationarity of the earthquake flow may influence the significance level of H_0'. The estimates of parameter "p" (the percent of bursts of aftershocks among the main shocks) for the periods of "pre-history" and of prediction are compared in Table 1. The difference between these two estimates is noticeable for one

of the regions, namely, Southern California, where $N_b^- / N_{ms}^- = 14 / 86$ but $N_b^+ / N_{ms}^+ = 5 / 6$. Statistical significance of this difference is not high so that the estimates of "p" for the two periods are quite in agreement for 7 regions.

5) The previous conclusion concerns the stationarity of the relative number of bursts of aftershocks, but not the stability of the flow of main shocks. Let us consider this stability now, comparing the intensity of the main shocks flow in the periods T^+ and T^-. It is characterized by average number of main shocks per year. These numbers for three regions where the difference is obviously significant are presented below:

REGION	N_{ms}^+ / T^+	N_{ms}^- / T^-
Northern California	22 / 6.6	39 / 31
Caucasus	20 / 7	19 / 19
New Zealand	0 / 6	66 / 33

Out of seven regions with K > 0, only for N. California and Caucasus the difference, threefold, is significant. However, it is not of consequence in our conditional aproach.

6) When the significance of hypothesis H_0' was tested the difference between the estimates obtained with biased (5) or unbiased (5,6) distributions turned out insignificant.

7) The summary alarm times , both total τ_Σ and useful τ_Σ^+, for all 7 regions are the following:

Alarm time	Observed value	Theoretical value for H_0	Theoretical value for H_0'
Total in % of ΣT_i *)	56%	48%	37%
Useful in % of total	64%	57%	57%

*) T_i is the time of forward monitoring for region G_i.

Although the theoretical range of variation of these values has not been determined the data on alarm times may be considered as not consequential. If pattern B is really a precursor the observed values of τ_Σ should be less than the theoretical ones. The actual relation is opposite, but it is most probably due to the short time of predictions. Thus the current alarms in three out of seven regions produce a noticable effect on the summary statistics τ_Σ.

Let us consider also the regions where strong earthquakes didn't occur during the period of monitoring (K = 0).

In these 6 regions the small number of false alarms is the only possible argument in favour of precursor B. Statistics $\zeta = - N_b$ characterizes this situation. In fact there are no alarms in 5 regions; in the region 11 there are two.

However in three regions, Nos 4, 12, 13, there were no main shocks So the absence of pattern B may be explained just by a seismic quiescence.

In other 3 regions the results of monitoring may be summarized as follows:

Region	N_b^+ / N_{ms}^+	N_b^- / N_{ms}^-	T^+/ τ
9. Colombia	0 / 4	3 / 11	1.2
10. New Guinea	0 / 1	2 / 16	0.56
11. Solomon Isl.	2 / 6	3 / 24	0.56

The time of monitoring T^+ is comparable with the duration of a single alarm, so that the estimation of significance ε_{obs} for the values of N_b^+/N_{ms}^+ is made here only to illustrate the method. The significance level ε_{obs} for a region where K=0 is determined in the following way:

$$\varepsilon = \Pr (N_b \le N_b^{obs} \mid N_{ms}, H_0') =$$

$$= \sum_s C_{N_i}^s \, p^s (1 - p)^{N_i - s} , \qquad 0 \le s \le N_b^{obs} , \qquad N_i = N_{ms} .$$

Using (6), with the regional values of p estimated by "pre-history", we calculate for the regions 9, 10, 11 the following unbiased estimates ε_{obs} =33%, 88% and 95% respectively. As to the summary statistics of false alarms, $\varepsilon_{\Sigma}^{obs}$ = 87%. Thus formally the values of N_b/N_{ms} are in agreement with hypothesis H_0.

Returning to the list of conclusions we have:

8) Due to small number of main shocks in the regions where strong events did not occur the data for these regions, specifically - the statistics of false alarm, neither reject nor confirm precursor B.

4. Significance level for retrospective predictions.

It was analysed first by Molchan and Rotwain (1983). Here we use extended data for four regions: Northern California,

Southern California, New Zealand and Northern Japan.

The significance level ε of predictions in a region depend upon the parameters of the precursor , first of all on M_0 and B_0. In forward monitoring parameters are fixed, but in retrospective analysis they are not; so ε is a function of $\alpha = (M_0, B_0)$. The conventional way to choose parameters values is to minimize $\varepsilon(\alpha)$ by trying different values of α. The minimal values $\min \varepsilon(\alpha)$, obtained for "pre-history" may give qualitative characteristics of the precursor but don't measure its significance level. For this reason the upper bound for significance level was used in (Molchan and Rotwain, 1983):

$$\varepsilon^* = \max_{\alpha \in \Omega} \varepsilon (\alpha) \quad .$$

Here Ω is the acceptable domain of α in a region.

The acceptable values of B_0 were estimated by hystogram of values of b for main shocks. Obviously, if pattern B is really a precursor, then the threshold B_0 must divide the mixture of two distributions of b . The first one corresponds to the general ensemble of main shocks, and the second one corresponds to the main shocks accompanied by an anormalously large number of aftershocks. (In terms of hypothesis H_0' the threshold B_0 is the (1-p) -quantile of the distribution of b). In Fig.1 the candidates for the thresholds B_0 in four above mentioned regions are indicated by arrows. They are located at the end of "main part" of a hystogram. The last part of a hystogram shouldn't be of much importance since the "tails" correspond to the strongest earthquakes.

The acceptable values of M_0 are determined by the requirements of "effectiveness" of the precursor . Inexplicitely these requirements are met in the selection of a precursor. They may be summarized as follows:

The total alarm time should be a sufficiently small part of the period considered. For simplicity, we require that 3τ is smaller than the mean interval between the strong events. This limits the M_0 from below.

We require also, that the average time interval Δ between strong earthquakes is not too large, say $\Delta = 10$ yrs or $\Delta = 20$ yrs. This imposed two upper limits for M_0.

The acceptable ranges of M_0 and B_0 are shown in Table 3. The procedure for their choice is described in detail in (Molchan and Rotwain, 1983). The worst results of predictions for the two ranges of M_0 are also given in Table 3. As the change of B_0 within the acceptable ranges doesn't affect significantly the results of predictions, we fixed $B_0 = 15$. For the four regions mentioned above the single alarm time $\tau = 3$ yrs was left.

The hypothesis H_0 has been tested on retrospective predictions. For the summary statistics $\varkappa_\Sigma$ and $\zeta_{i,\Sigma}$ the upper limits for significance level of H_0 are the following: 3% and 0.7% respectively for the first interval of M_0; 82% and 69% for the second interval.

In comparison with the previous results (Molchan and Rotwain, 1983) the significance of pattern B increases by one order of magnitude This is due to two reasons: range of M_0 was narrowed after addition of new data; and significance level is estimated by a more accurate method.

Molchan and Rotwain (1983) estimated ε_Σ according to the χ^2 statistics, (2), without taking into account the discrete nature of

variables $\varkappa$ and ζ_i. This made the estimates very rough even in retrospective analysis, where the statistics ζ_i have rather large spectrums of values. For the first range of M_0 from Table 3 these rough estimates would be $\varepsilon = 20\%$ and 6% according to statistics $\varkappa$ and ζ_i respectively, instead of 3% and 0.7% obtained above. It is a remarkable contrast! We emphasize this, the summing up of regional significance levels is rather popular in statistics of earthquake prediction.

Thus we come to the next conclusion:

9) According to the retrospective predictions pattern B is significant only for the first range of parameter M_0, with $\varepsilon = 0.7\%$. In other words bursts of aftershocks in four regions considered precede the strong earthquakes with reoccurence time about 10 years. But stronger earthquakes, with double reoccurence time, if considered alone, are practically uncorrelated with pattern B.

Alarm time. The mean alarm times, total and useful, for retrospective predictions in four regions are presented in Fig.2. They are calculated for the values of M_0 from the two acceptable intervals. The theoretical curves τ_Σ and $\tau_\Sigma^+ / \tau_\Sigma$ calculated for the hypothesis H_0 are compared with empirical estimates. Fig.2 leads us to the last conclusion:

10) The total alarm time is of little interest as a characteristics of the quality of predictions. The ratio $\tau_\Sigma^+ / \tau_\Sigma$ that is the ratio of useful alarm time to total one is more informative. For the first interval of M_0 the empirical values of $\tau_\Sigma^+ / \tau_\Sigma$ are higher than the theoretical ones.

Conclusion.

At the beginning of the article the problem was formulated - how significant are the results of forward prediction: 10 successes out of 14 strong earthquakes in 7 regions with the 56% of total alarm time. The alarm time proved to be of little value, so the literal answer to the question gives a modest estimate of 11%. However, after we use also the statistics of successful alarms, this estimate becomes by an order of magnitude better; using in addition the statistics of main shocks we obtain significance level - 0.4% ; such it is the probability to get predictions in situation of total randomness (hypothesis H_0') .

The results of predictions in 6 regions where strong earthquakes didn't occur neither reject nor confirm precursor B.

In general, the results of forward predictions by pattern B provide serious argument in favour of its reality as a precursor. Retrospective analysis indicates also that bursts of aftershocks precede not so the very strongest earthquakes but those, which occur once a decade.

For the practical purposes prediction, based on pattern B may be probably reinforced in some regions by allowing for the clustering of these patterns.

Appendix A.

The verification of pattern B under the hypothesis H_0 is based on a general statistical problem, which seems to have a wide set of applications. Let us normalize the observation time to unit. Then the problem could be formalized as follows:

K and N random points $x_1, \dots, x_K$ and $y_1, \dots, y_N$ with a uniform distribution are tossed independently into the interval [0,1] . We say that y_i predicts x_j if $y_i \le x_j \le y_i + \tau$ and there are no X-points inside the interval (y_i , x_j). The points x_i which are predicted in this way determine number of successes, $\varkappa$. The points y_i predicted them determine number of successful alarms, ν. The quality of predictions is characterized by statistics $\xi = f(\varkappa, \nu)$, so its distribution has to be known.

The joint distribution of $(\varkappa, \nu)$ is found in the paper (Molchan and Rotwain, 1983). On this basis we can calculate distributions of any functionals $\xi = f(\varkappa, \nu)$ for two cases: N is fixed; N is random with the given distribution, but K and τ are fixed. Let us recall that N has a binomial distribution when hypothesis H_0' is tested.

Denote $(a)_p = a (a + 1) \cdot \dots \cdot (a + p - 1)$, and $x_+ = x$ if $x \ge 0$, $x_+ = 0$ if $x < 0$.

1°. The distribution $(\varkappa, \nu)$ is located within the range $0 \le \varkappa \le \nu \le N$, $\varkappa \le K$ and is determined as follows:

$$\mathrm{Prob}(\varkappa = m , \nu = n \mid K, N, \tau) = \frac{-(-K)_m}{m!} \frac{(-N)_n}{n!} \sum_{\varkappa=0}^{n} \frac{(-n)_\varkappa}{\varkappa!} S_\varkappa \quad (A1)$$

$$\text{where} \quad S_\varkappa = \sum_{\gamma=0}^{n_\varkappa} \frac{(-\bar{n})_\gamma}{\gamma!} \frac{(-K)_\gamma}{\gamma!} \left[\frac{(K+1)_{\bar{n}}}{\bar{n}!} \right]^{-1} (1 + \gamma\tau)^\varkappa S_\gamma ,$$

$$n_\varkappa = \min(N - n , K), \quad \bar{n} = N - n ;$$

$$S_{\gamma} = \sum_{\bar\rho=0}^{n_\gamma} \frac{(-m)_{\bar\rho}}{\bar\rho\,!} \left[1 - (\gamma + \bar\rho) \tau \right]_{+}^{N+K-\varkappa} \frac{(\bar n + 1)_{\bar\rho}}{(1 + \gamma)_{\bar\rho}} \frac{(-K + \gamma)_{\bar\rho}}{(-K)_{\bar\rho}} S_{\bar\rho}$$

$n_{\gamma} = \min(m , K - \gamma) ;$

$$S_{\bar\rho} = \sum_{\rho=\rho_1}^{m-\bar\rho} \frac{(\bar\rho - m)_{\rho}}{\rho\,!} \frac{(-K + \gamma + \bar\rho)_{\rho}}{(-K + \bar\rho)_{\rho}} \frac{(-K - \bar n)_{\rho}}{(-N - K + \varkappa)_{\rho}} S_{\rho}$$

where $\rho_1 = 1$ if $(\bar\rho = 0,\ n > 0)$ and $\rho_1 = 0$ in the opposit case,

$$S_{\rho} = \sum_{\beta=0}^{\bar\rho} \frac{(-\bar\rho)_{\beta}}{\beta\,!} \frac{(-K + n + \rho)_{\beta}}{(-N - K + n + \rho)_{\beta}} \frac{(-\gamma - \bar\rho)_{\beta}}{(-n - \bar\rho)_{\beta}} \quad .$$

For particular case $\tau = 1$

$$\mathrm{Prob}(\varkappa = m , \nu = n \mid K, N, 1) = \frac{m}{n} C_K^m C_n^m / C_{N+K}^m , \quad m \le n \quad (A2)$$

This case is of some practical interest: it means that an alarm continues until a strong earthquake occurs. In fact for the premonitory pattern "long-range aftershocks" an alarm is called off only in this way (Prozorov and Shreider, 1987).

2°. The marginal distributions are:

$$\left.\begin{aligned} &\mathrm{Prob}(\varkappa = m \mid K, N, \tau) = \frac{(- K)_m}{m\,!} \sum_{\alpha=0}^{m} \frac{(-m)_\alpha}{\alpha\,!} \Im(K - \alpha) , \\ &\qquad\qquad 0 \le m \le \min(K, N) \end{aligned}\right\} (A3),$$

$$\text{where} \quad \Im(p) = \sum_{s=0}^{p} C_N^{s} C_K^{p-s} \left[C_{N+K}^{p} \right]^{-1} (1 - s \tau)_{+}^{N+K}$$

Note, that the distribution of $\varkappa$ is symmetrical for (K, N) :

$$\mathrm{Prob}(\varkappa = m \mid K, N, \tau) = \mathrm{Prob} (\varkappa = m \mid N, K, \tau) \quad .$$

$$\mathrm{Prob}(\nu = n \mid K, N, \tau) = \frac{(-N)_n}{n\,!} \sum_{\alpha=0}^{n} \frac{(-n)_\alpha}{\alpha\,!} \Im(N - \alpha) , \quad (A4)$$

$$\text{where} \quad \Im(p) = \sum_{0 \le s \le \min(p,K)} C_p^{s} C_K^{s} \left[C_{p+K}^{K} \right]^{-1} (1 - s \tau)_{+}^{K+p} \quad .$$

The distributions (A2-4) serve as a control for (A1).

Appendix B.

Let ν be the number of successes in a series of N independent trials, and the probability of a single success is p. We show that

$$(-\nu)_k (-N + \nu)_m / (-N)_{k+m} \qquad (B1)$$

is an unbiased estimator for $p^k (1 - p)^m$. Here $(\alpha)_k = \alpha (\alpha + 1) \cdot \ldots \cdot (\alpha + k - 1)$.

Actually, the mean value:

$$E\, x^{\nu} y^{N-\nu} = y^N E(x / y)^{\nu} = y^N (x p / y + q)^N = (x p + y q)^N$$

where $q = 1 - p$. Let us apply an operator $\dfrac{\partial^{m+k}}{\partial x^k\, \partial y^m}$ to both sides of this expression at the point $x = y = 1$. We obtain the desired result:

$$E(-\nu)_k(-N+\nu)_m = p^k q^m (-N)_{k+m}$$

Note that (B1) depends on sufficient statistics of the results of the trials. Therefore the given estimator is the minimum variance unbiased estimator for $p^k q^m$.

Table 1. Results of predictions.

Region	**)	Parameters of pattern B: Time	M_0	B_0	τ	K^-	N_b^-	N_{ms}^-	Results of *forward* predictions: T^+ (in yrs)	K/K^+	n/N_b^+	N_{ms}^+	τ_Z	τ_Z'
Caucasus	1	1962-1976	6.0	22	3	4	1	19	7	1/1	1/1	20	2	2
N. California	2	1948-1978	6.2	12	3	3(4)	6	39	6.6	2/4	3/3	22	4.3	4.3
S. California	2	1932-1978	6.5	13	3	6(7)	14	36	6.3	2/2	2/5	6	5	1
New Zealand	2	1945-1977	6.6	14	3	5	8	67	6	0/0	0/0	0	0	0
Middle Asia	1	1962-1977	7.0	5	3	1	3	14	8	1/3	2/3	8	5.5	2.5
N. Japan	2	1940-1977	7.7	6	3	3	13	25	7.6	2/2	8/10	15	5.6	3.5
Santa Crus	3	1960-1983	7.7	13	4	4(5)	7	43	5	1/1	2/2	8	2.8	2.8
Chile	3	1960-1980	7.7	17	4	2	5	26	4.7	1/1	1/1	4	0.1	0.1
Colombia	3	1960-1980	7.7	17	4	1	3	11	4.7	0/0	0/0	4	0	0
New Guinea	3	1960-1983	7.7	13	4	2	2	16	2.3	0/0	0/0	1	0	0
Solomon Isl.	3	1960-1983	7.7	13	4	3	3	24	2.3	0/0	0/2	6	0.2	0
Peru	3	1960-1980	7.7	17	4	1	2	5	4.7	0/0	0/0	0	0	0
China	3	1960-1976	7.7	6	3	1	3	5	8.6	0/0	0/0	0	0	0

Notes: *) double events occured at the same place during half a year, are counted as a single event; τ - time duration of a single alarm; **) catalogs: 1 - "Earthquakes *in the USSR*", 2 - a regional catalog, 3 - the world catalog of NOAA.

Table 2. Significance estimations of forward predictions.

Region	τ/T	Parameter "p" for periods: history		in advance	Significance level of hypothesis: H_0 for statistics: $æ$	H_0: ξ_1	H_c' *): $æ$	H_c' *): ξ_1	H_c' *): ξ_2
Caucasus	43%	0.05	0.05	0.05	[34%]	[34%]	29%	15%	28%
N. California	45%	0.15	0.06	0.14	61% **)	41%	58%, 62%	37%, 41%	33%, 37%
S. California	48%	0.39	0.08	0.83	[45%]	48%	18%, 18%	40%, 40%	48%, 48%
Middle Asia	38%	0.21	0.11	0.37	92%	68%	68%, 74%	49%, 51%	45%, 48%
N. Japan	39%	0.56	0.10	0.67	[68%]	15%	59%, 58%	12%, 12%	11%, 11%
Santa Crus	80%	0.16	0.06	0.25	[66%]	[29%]	44%, 46%	28%, 30%	22%, 30%
Chili	85%	0.19	0.08	0.25	[49%]	[49%]	31%, 32%	25%, 26%	31%, 30%
Series of the regions	10 successes out of 14events $\xi_{1,2}/max$ =11.7/14				11%	1.4%	2%, 2%	0.2%, 0.4%	-

Notes: *) estimates obtained using biased (first) or unbiased (second) distributions.
**) cases when the statistics is at its maximum are marked.

Table 3. Analysis of retrospective predictions.

Region	Time (in yrs)	Intervals of acceptable values of parameters: M_0	B_0	First range: M_0	k/K	n/N_b	Sign. level (in %) for $æ$	ξ_1	ξ_2	Second range: M_c	k/K	n/N_b	Sign. level (in %) for $æ$	ξ_1
N. California	37	6.2-6.3(6.8)	8-16	6.3	3/4	3/7	13	14	17	6.8	0/1	0/10	100	100
S. California	53.3	6.5-6.6(6.8)	15-25	6.5	4/5	7/20	34	18	18	6.8	3/3	6/23	31	31
New Zealand	44	6.6-6.7(7.0)	12-16	6.6	3/11	4/6	58	37	25	7	1/5	4/8	93	40
N. Japan	25	7.6-7.7(7.9)	12-17	7.7	4/5	6/8	14	6	6	7.9	1/2	1/11	91	91
Series of the regions				-	14/25	20 41	3	0.7	-	-	5/11	11/52	82	69

The worst results of predictions for two ranges of M_0.

Note : The upper margin of the second interval of M_0 is given in parentheses.

REFERENCES.

Cox, D., Hinkley, D., *Theoretical Statistics* [Russian translation] (Mir, Moscow 1978).

Dewey, J., Keilis-Borok, V.I., Rotwain, I.M. (1984), *Bursts of aftershocks and strong earthquakes of S-W Pacific*, In *Logical and Computat. Methods in Seismology* (Nauka, Moscow) Vychisl. Seismol. [Computational Seismology], no.17, 3-7.

Dewey, J., Hutton, K., Rotwain, I.M., Vorobieva, I.A. (1988), *The test of premonitory seismicity pattern "bursts of aftershocks"*, In *Problems of Seismological Information* (Nauka, Moscow) Vychisl. Seismol. [Computational Seismology], no.21.

Gabrielov, A.M., Dmitrieva, O.E., Keilis-Borok, V.I., Kosobokov, V.G., Kuznetsov, I.V., Levshina, T.A., Mirzoev, K.M., Molchan, G.M., Negmatulaev, S.Kh., Pisarenko, V.F., Prozoroff, A.G., Reinhart, W., Rotwain, I.M., Shebalin, P.N., Shnirman, M.G., Schreider, S.Yu., *Algorithms of long-term earthquake prediction*, Int. School for Research Oriented to Earthquake Prediction Algorithms, Software and Data Handling, Sept.1986, Lima, Peru (available at CERESIS, Apartado 11363 Lima14, Peru).

Karpukhina, O.A., Rotwain, I.M. (1983), *Long-range seismological precursors of strong earthquakes. V. Bursts of aftershocks and the strongest earthquakes of Kamchatka, the Komandor Islands and the Caucasus*, In *Earthquake Prediction and Study of the Structure of the Earth* (Nauka, Moscow) Vychisl. Seismol. [Computational Seismology], no.15, 44-47.

Keilis-Borok, V.I., Rotwain, I.M., Sidorenko, T.V. (1978), *Intensified sequence of aftershocks as a precursor of strong earthquake*, Dokl. AN SSSR 242 (3), 567-569.

Keilis-Borok, V.I., Knopoff, L., Rotwain, I.M. (1980), *Long-range seismological precursors of strong earthquakes in California, Sierra Nevada, New Zealand, Japan, and Alaska*, In *Methods and Algorithms for Interpreting Seismological Data* (Nauka, Moscow) Vychisl. Seismol. [Computational Seismology], no.13, 3-12.

Keilis-Borok, V.I., Lukina, E.N., Rotwain, I.M., Harn, T. (1981), *Long-range seismological precursors of strong earthquakes. III. Bursts of aftershocks and the strongest earthquakes of South America*, In *Mathematical Models of the Earth's Structure and Earthquake Prediction* (Nauka, Moscow) Vychisl. Seismol. [Computational Seismology], no.14, 15-20.

Lee, W.H.K., Lukina, E.N. (1983), *Long-range seismological precursors of strong earthquakes. YI. North China platform zone*, In *Earthquake Prediction and Study of the Structure of the Earth* (Nauka, Moscow) Vychisl. Seismol. [Computational Seismology], no.15, 47-52.

Molchan, G.M., Rotwain, I.M. (1983), *Statistical analysis of long-range precursors of strong earthquakes*, In *Earthquake Prediction and Study of the Structure of the Earth* (Nauka, Moscow) Vychisl. Seismol. [Computational Seismology], no.15, 52-66.

Negmatullaev, S.H., Rotwain, I.M., Sidorenko, T.V. (1981), *Long-term premonitory seismicity patterns. II. Pamir and Tyen-Shan*, In *Mathematical Models of the Earth's Structure and Earthquake Prediction* (Nauka, Moscow) Vychisl. Seismol. [Computational Seismology], no.14, 11-15.

Prozorov, A.G., Schreider, S.Yu. (1987), *Long-range aftershocks as indicators of the place where next strong earthquakes of Greater Caucasus will occur*, In *Numerical modelling and analysis of geophysical processes* (Nauka, Moscow) Vychisl. Seismol. [Computational Seismology], no.20, 111-116.

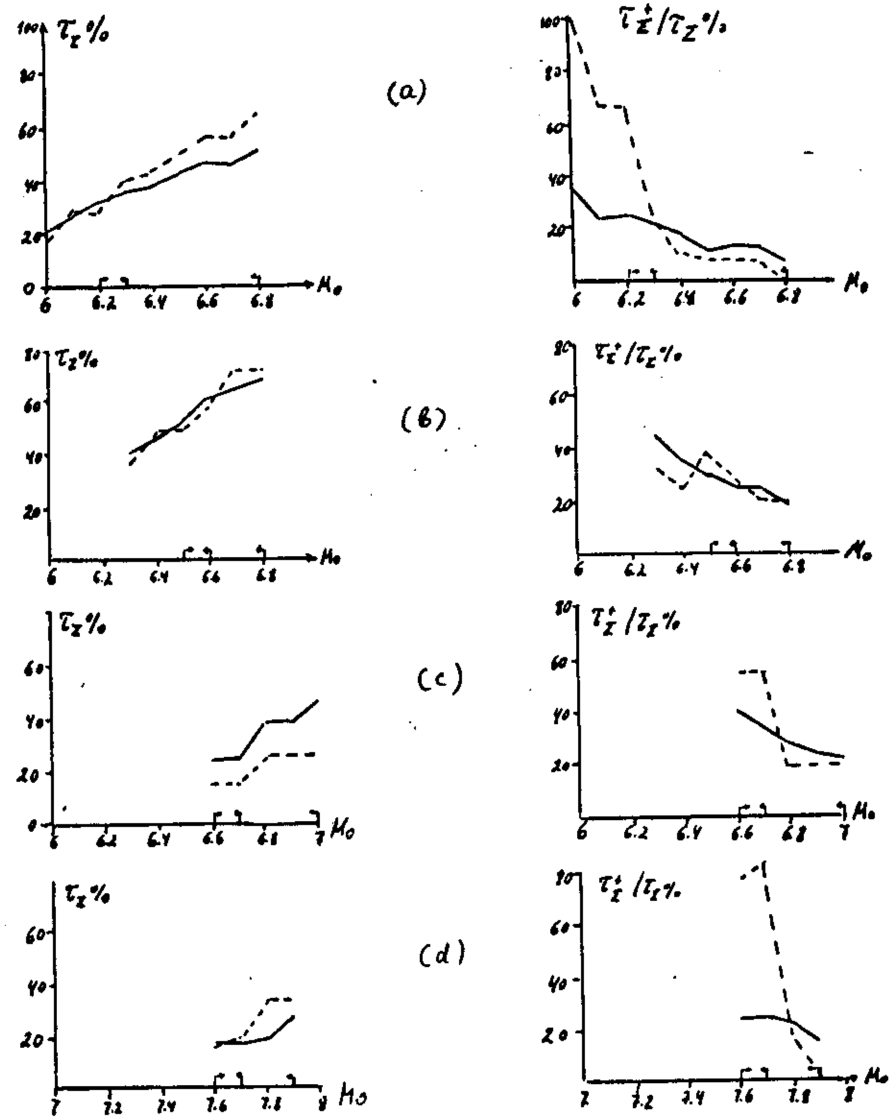


Fig.2

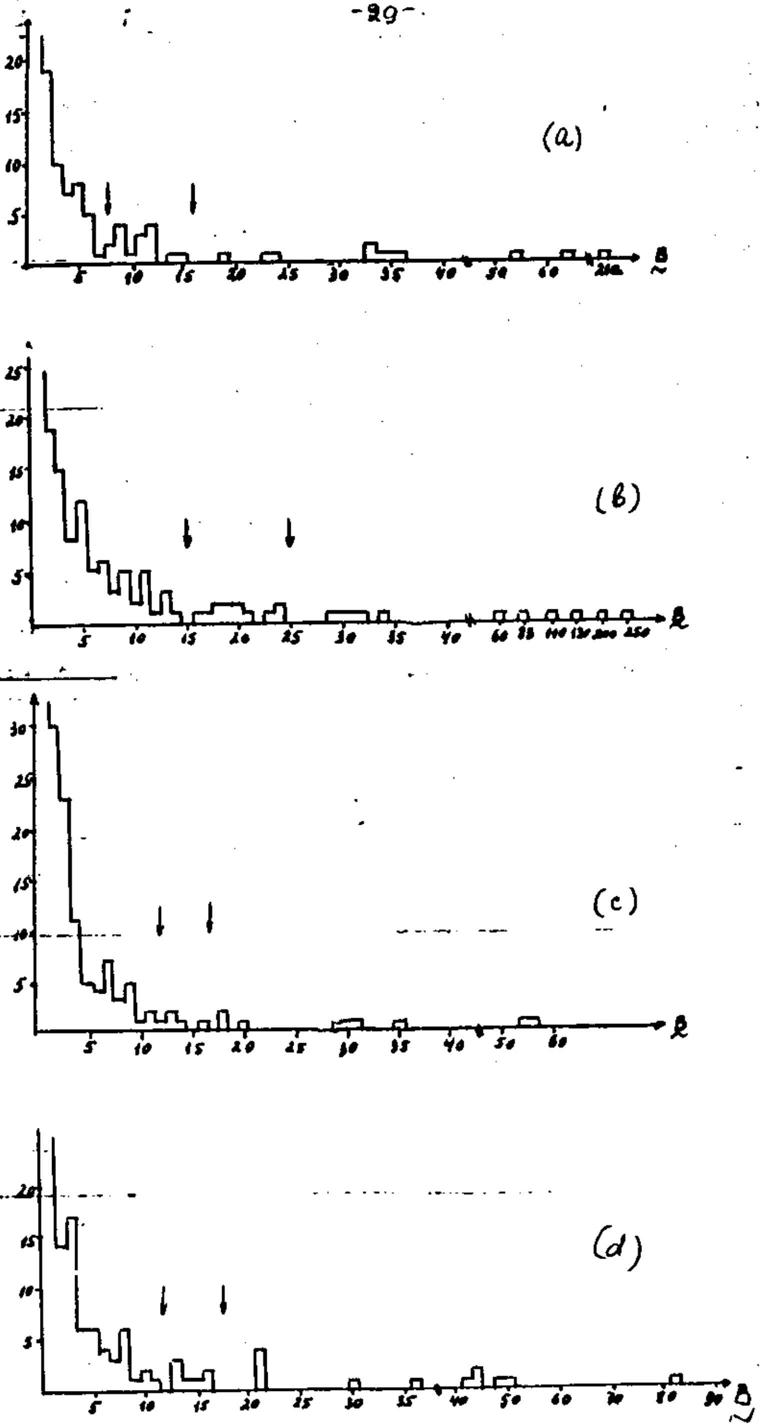


Fig.1

Figures captions.

Fig.1. Histograms of the number of aftershocks b for four regions using regional catalogs: a) Northern California, 1948-84 yrs, b) Southern California, 1932-85 yrs, c) New Zealand, 1940-78 yrs, d) Northern Japan, 1940-84 yrs.

Fig.2. Total alarm time, total τ_Σ and useful τ_Σ^+ in retrospective predictions for the regions: a) Northern California, b) Southern California, c) New Zealand, d) North Japan. B_o = 15. Solid lines: theoretical curves; dashed lines: empirical data. Two intervals of acceptable values of M_o are marked by arrows.

Fig.3 Appendix C.

Calculations of joint distribution ( ) are not very simle, that is why we tabulated distributions of a simplest statistics ( ). The precomputer graphs of the distributions are given on figure 3 (a-f). K, N and are parameters of the distributions (here K and N correspond to numbers of strong earthquakes and numbers of alarms; is normalized to 1). Luckly the distribution of ( ) is symmetrical for (K,N), so we can assume in any case that K < N. Quantiles of the distributions are given in isolines on Fig 3a-3f. -quantile is the smallest root of unequation:

The following ranges of parameters are available: 1 < K < N < 20; = 0.2 - 0.12 (0.2) (figs: 3a-3f); =0.001, 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5 (different values of correspond to different triangle quadrants and solid or dash lines).

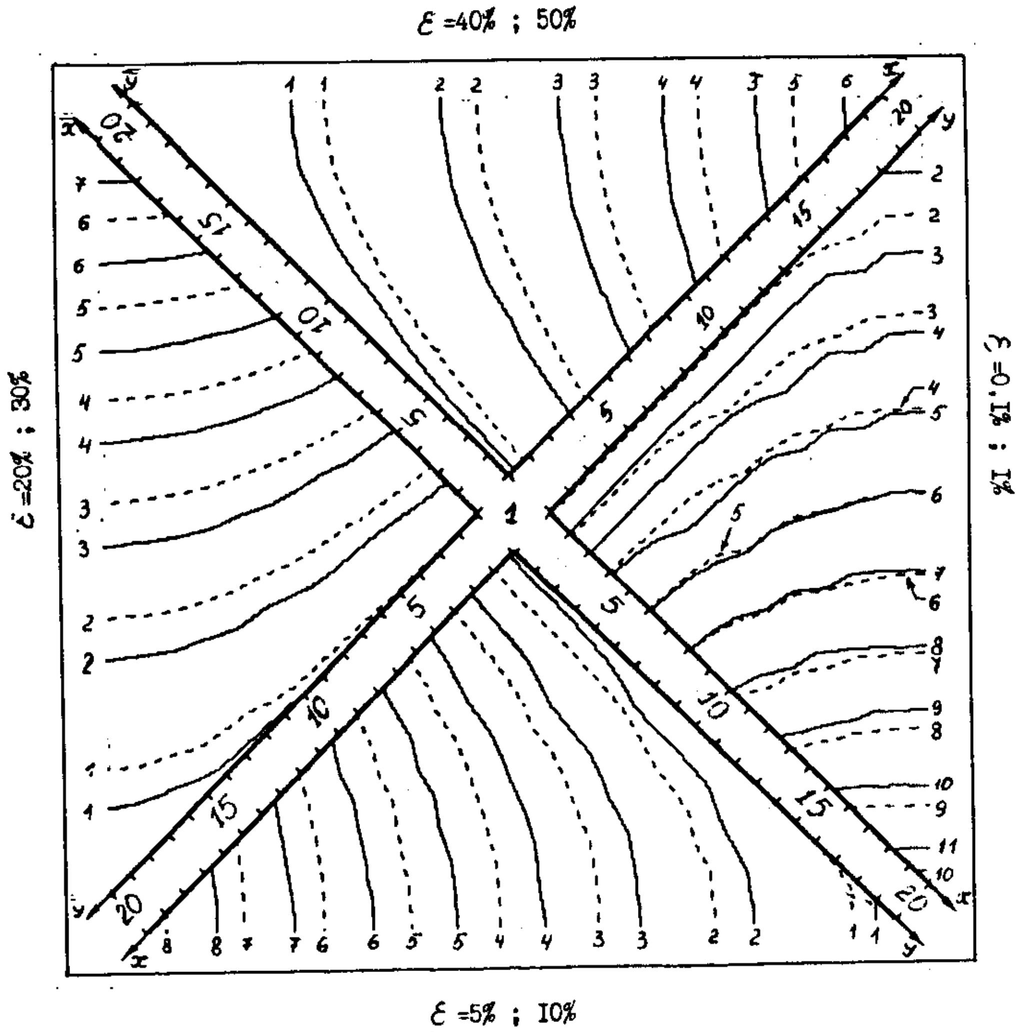
HOW TO USE FIG.3 TO ESTIMATE SIGNIFICANCE OF PREDICTION, FOR STATISTICS (- number of successful predictions).

1) Define time of a single alarm in the share of total time of observation T, = /T. Among fig.3a-3f find that one, which corresponds to this value .
2) Let (k/K, N) be the result of your prediction, i.e. k out of K strong earthquakes are preceded by an alarm, and the total number of precursors observed is equal to N. As 1 < K < N < 20 on fig.3, let's determine

x =min(K,N), y =21-max(K,N)

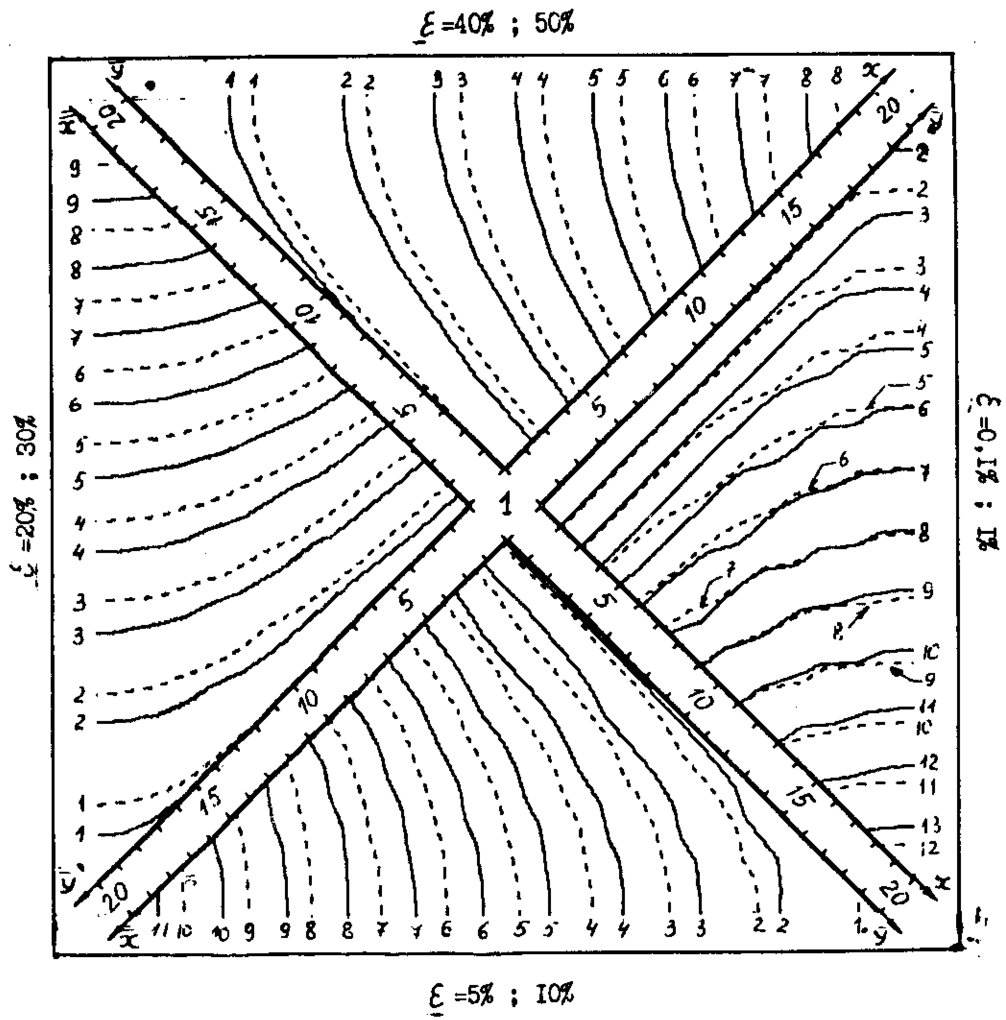
3) Axis for variables (x,y) are placed on the diagonals of the square. Each triangle quadrant corresponds to two significance levels and from the set mentioned above. Level values are marked on the side sides of the square (dash lines correspond to the smallest ones min(,).

Point (x , y) has to be marked in each quadrant of the corresponding square. Let be such that (x ,y) is located between isolines, marked as k -1 and k . Then serves as an upper estimate for results of predictions (k /K, N).



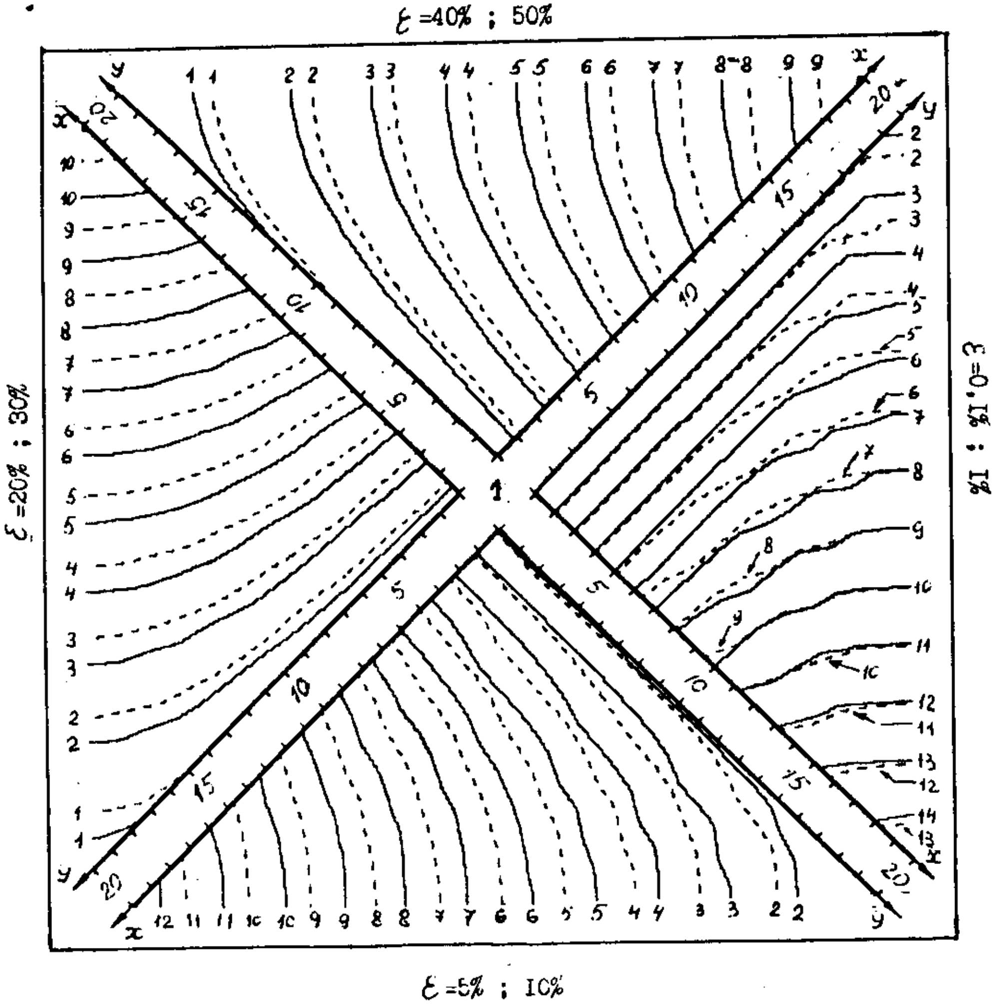
a) τ =0,02

Fig .3



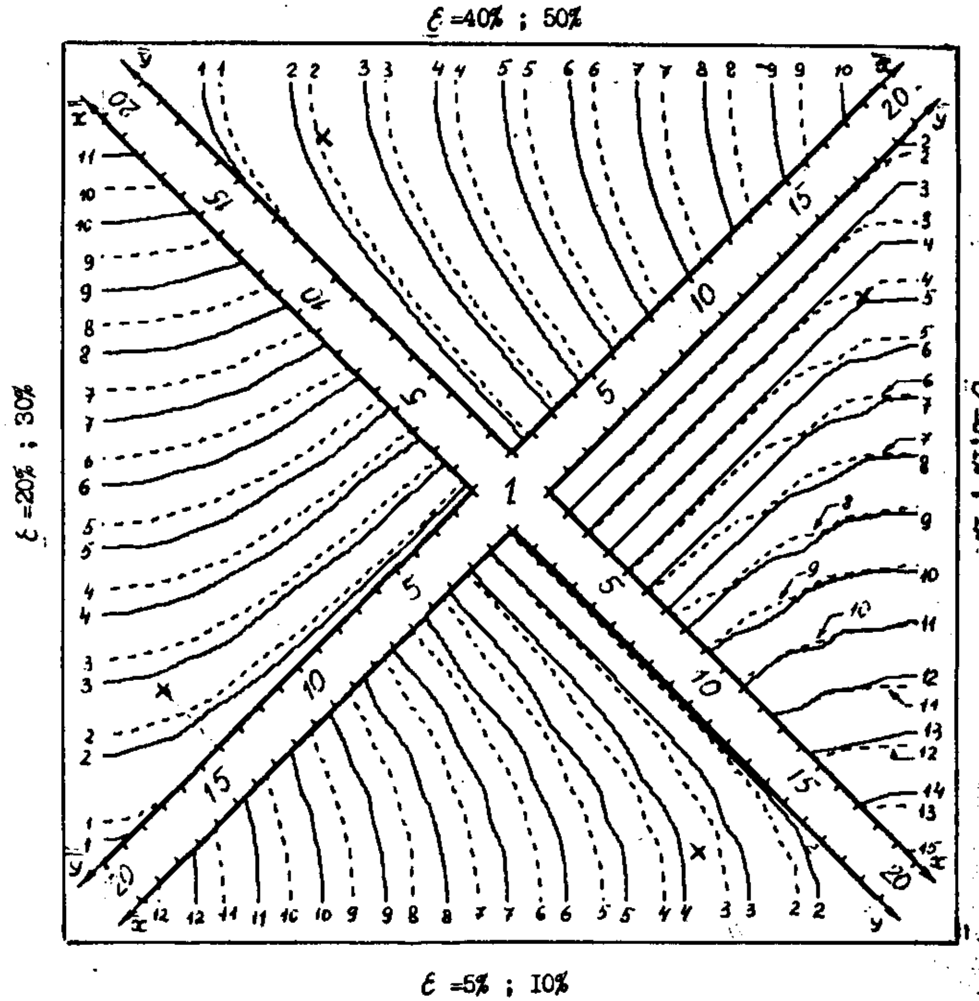
b) τ =0,04

Fig.3



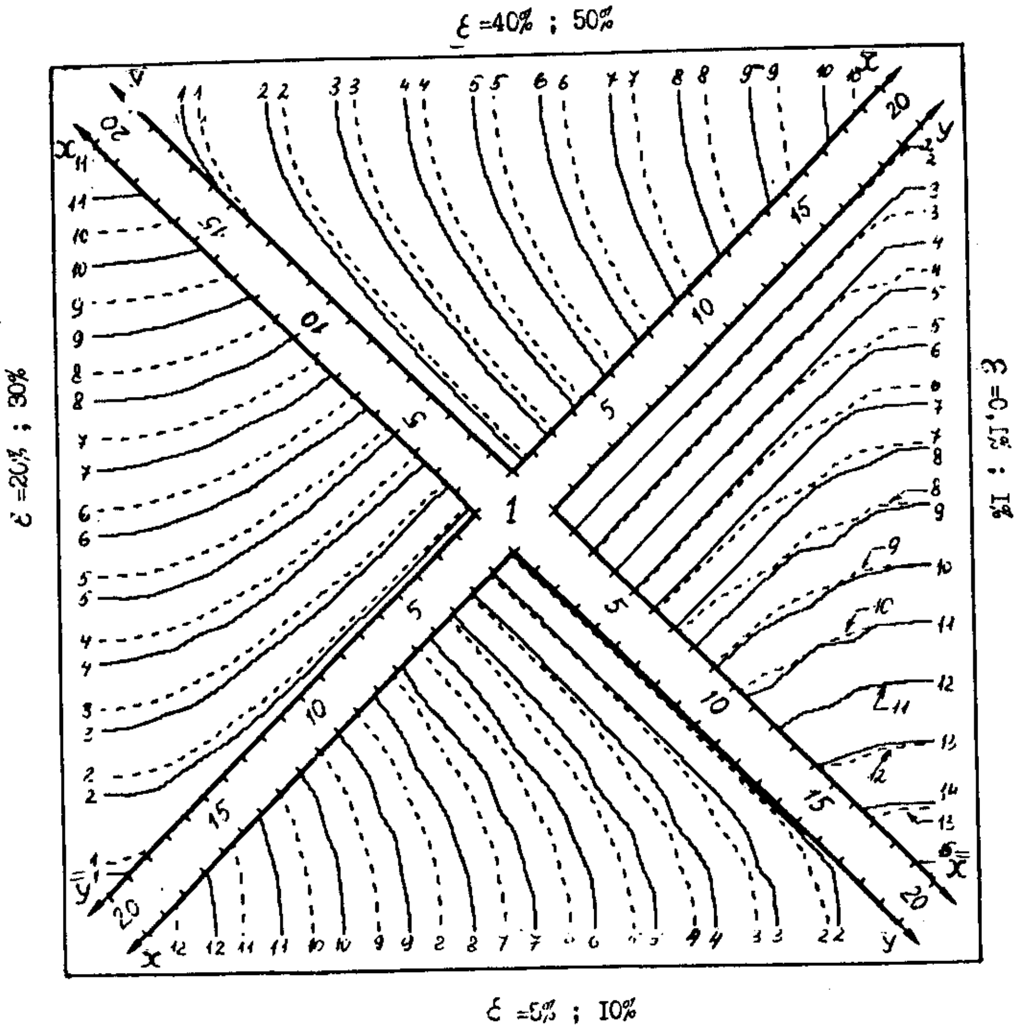
c) τ =0,06

Fig.3

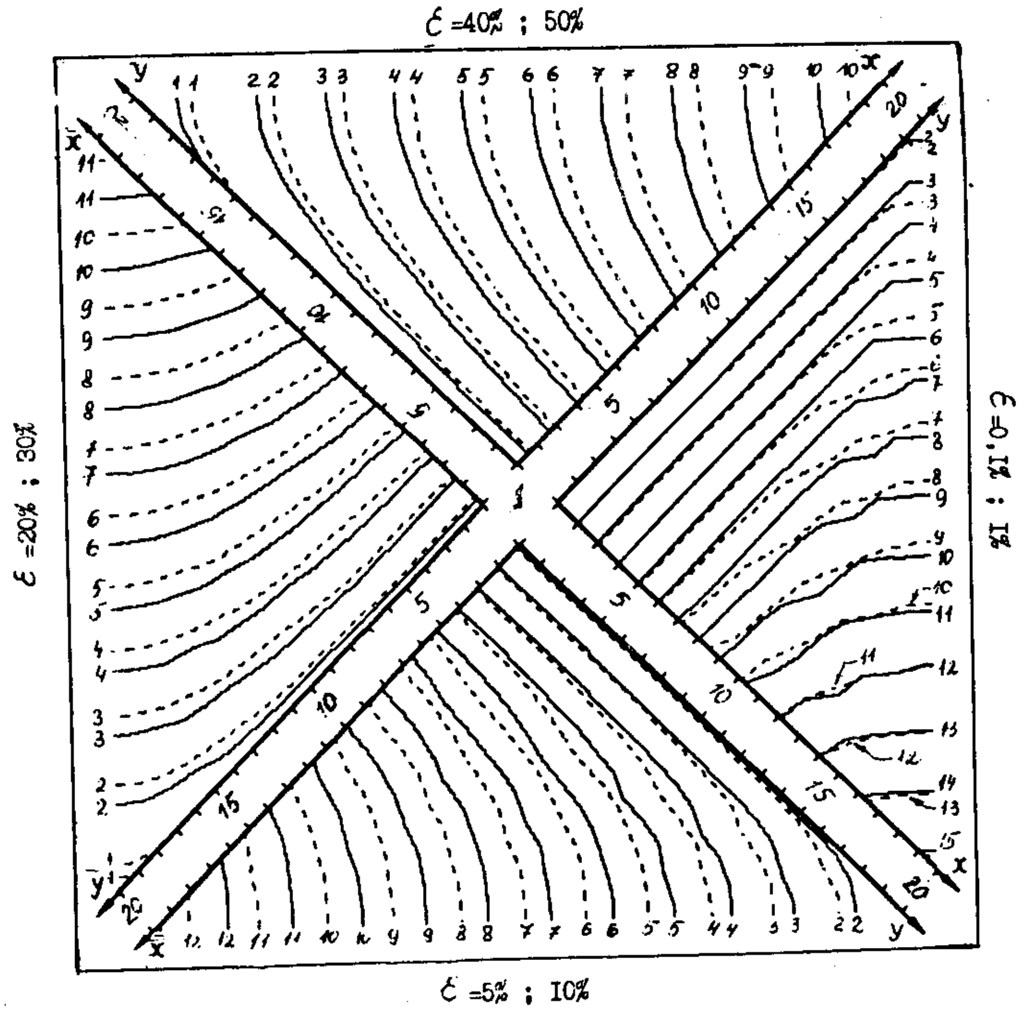


d) τ=0,10

Fig.3



e) $\tilde{\tau}$ =0,10
Fig.3



f) τ =0,12
Fig.3