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WORKSHOP
GLOBAL GEOPHYSICAL INFORMATICS WITH APPLICATIONS TO RESEARCH IN EARTHQUAKE PREDICTIONS AND REDUCTION OF SEISMIC RISK

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THE ANALYSIS OF DEFORMED ARRAY OF MOVEABLE ELEMENTS USING PARTICLES

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Abstract. The system of moveable interacting discs in cramped conditions is regarded as a model geophysical medium. The evolution of the simulated system is described by the solution of a system of dynamic equations. Exposed to the stationary action the system displays a complicated behavior characterized by both chaotic nature and formation of structures. The character of the time course of energy dissipation is subject to qualitative changes with changeable deformation velocity.

Introduction. Modern knowledge of geophysical medium as a hierarchical discret structure (SADOVSKY *et al.*, 1987) lay a natural foundation for the design of a variety o formally analyzable models. The attempts previously made to this end (PISARENKO *et al.* 1985; SHVARTZ and SHNIRMAN.,1986; SHNIRMAN,1987) had a weak point: the absence o moveability of the structural elements. In this paper the general knowledge of th geophysical medium as a hierarchical structure are specified by a system of a larg number of interacting elements. It would be expected that some important features o this system are determined by the interaction of a large number of elements and ir independent of their particular geometry. Therefore, the disc as the simplest shape ha been chosen.

The physical properties of the elements have also been chosen to be rather simpl The discs are assumed to be rigid, i.e. it is presumed that all the deformation i concentrated in the vicinity of the points of the contact between the elements and al the forces arising in different contact points act independently. The idea of relativ compliance of the zones of contact of rocks as compared to the rigidity of the basi massif can also be found in literature (CUNDALL, 1971; WILKE *et al.*, 1985) Thes simplifications allow one without additional computer time to use complicated nonlinea laws of interaction of the elements in contact points which make up for the simplicit of the shapes of basic elements. The schedule of the deformation of the array wit constant volume is determined by a "tangent" movement of the walls at a cer velocity. Spatial heterogeneity of the stress field in the array and the time cours energy dissipation have been studied.

The evolution of the simulated system is described by the solution of a system o dynamic equations. The technique of computer simulation of such systems have long bee developed in the framework of the approach called the method of interacting particle Comprehensive references are given by HOCKNEY and. EASTWOOD (1985). An importan variant of this general approach is the method of molecular dynamics which allowed t obtain significant results in the studies on liquids and phase transitions. It is only th

central forces that are generally considered in the framework of the method of molecular dynamics. The computation experiments with absolutely rigid spheres behaving as a system of billiard balls were first carried out by ALDER and WAINWRIGHT (1959). The behavior of the present system has been based on computations carried out using the explicit finite-difference method developed by CUNDALL (1971,1975,1976), which allows one to take into account the friction forces. The fact that the friction forces are noncentral and have "memory" is what makes the most important difference from the standard procedure of the method of molecular dynamics.

Computation method

1. n discs made of the same material as the walls are enclosed in a rectangle with moveable sides. The velocities of the movement of the walls $u_k(j)=\dot{w}_k(j)$ where k=1,2; j=1÷4 are fixed in the simplest case.

2. The state of the i-th disc is characterized by three constants: r(i) - radius, m(i) - mass, I(i) - inertia momentum; and nine variables $x_1(i)$, $x_2(i)$ - Cartesian coordinates of the disc center, $x_3(i)$- the rotation angle, $v_1(i)$,$v_2(i)$ - velocity components, $v_3(i)$- angular velocity, $a_1(i)$,$a_2(i)$ - acceleration components, $a_3(i)$- - angular acceleration.

3. The computation consists of a number of repeated two-step cycles.

At step 1 of each cycle the new state of all the n discs is computed. Time step is dt.:

$$v_k(i)_{t+dt} = v_k(i)_t + a_k(i)_t\,dt \qquad (1)$$

$$k=1,2,3;\ i=1\div n;$$

$$x_k(i)_{t+dt} = x_k(i)_t + v_k(i)_t\,dt \qquad (2)$$

$$k=1,2,3;\ i=1\div n;$$

The new location of the walls is also computed:

$$w_k(j)_{t+dt} = w_k(j)_t + u_k(j)_t\,dt \qquad (3)$$

$$k=1,2;\ j=1\div 4;$$

At step 2 of each cycle the forces and moments of forces acting upon each disc as a function of the new state of the system are determined.

All the normal forces $\bar{F}_n(i,j)$ are only dependent on the value of the overlapping vector $\bar{h}$ which arose on step 1(see Figure1).

In the simplest case this relationship is linear and is described by an elastic constant C_n:

$$\bar{F}_n(i,j) = -C_n \cdot \bar{h} \qquad (4)$$

$$h = r(i)+r(j)-\sqrt{\sum_{k=1,2}(x_k(j)-x_k(i))^2} \qquad (5)$$

The shear forces $\bar{F}_s(i,j)$ are described in a somewhat more complicated fashion for they are dependent on the shear deformation accumulated since the moment of the contact.

The increment $\Delta\bar{F}_s(i,j)$ is computed as a function of the increment $\Delta\bar{d}$ of shear deformation. In the simplest case this relation is described by one constant of elastic friction C_s :

$$\Delta\bar{F}_s(i,j) = -C_s \cdot \Delta\bar{d} \qquad (6)$$

$$\Delta d = dt\,[(v_1(i)-v_1(j))(x_2(j)-x_2(i))-$$

$$-(v_2(i)-v_2(j))(x_1(j)-x_1(i))]/\sqrt{(x1(j)-x1(i))^2+(x2(j)-x2(i))^2}+$$

$$+v_3(i)\,r(i)\,dt + v_3(j)\,r(j)\,dt \qquad (7)$$

The first term of this sum accounts for the shear deformation related to the noncentral character of the interaction and the two others - for the rotations of the discs. Fig.2 shows the choice of the direction of the vectors in (6) and (7).

Coulomb law (without adhesion):

$$F_s(i,j) \leq C_k F_n(i,j) \qquad (8)$$

introduces another constant C_k and imposes a limitation upon $\bar{F}_s$:

$$\bar{F}_s(i,j)_{t+dt} = \begin{cases} \bar{F}_s(i,j)_t + \Delta\bar{F}_s(i,j) & \text{if (8) is true} \\ -C_k \cdot C_n \cdot h \cdot \dfrac{\Delta\bar{d}}{\Delta d} & \text{if (8) is not true} \end{cases} \qquad (9)$$

The last force constant C_v describes viscous friction for oscillations dumping:

$$\bar{f}_v(i) = -C_v \bar{v}(i) \qquad (10)$$

Then,

$$f_k(i) = \Sigma_j F_{nk}(i,j) + \Sigma_j F_{sk}(i,j) + f_{vk}(i) \qquad k=1,2 \qquad (11)$$

$$f_3(i) = \Sigma_j (F_{s1}(i,j)\, x_2(i,j) - F_{s2}(i,j)\, x_1(i,j)) + f_{v3}(i) \qquad (12)$$

where the summation is taken over all the contact points j of disc i. The coordinates of these points are designated as $x_k(i,j)$.

$f_1(i)$, $f_2(i)$- are the components of the total force,

and $f_3(i)$ is the total moment of the force acting upon the i-th disc.

Finally,

$$a_k(i) = f_k(i)/m(i) \qquad k=1,2\ ;\ i=1 \div n \qquad (13)$$

$$a_3(i) = f_3(i)/I(i) \qquad i=1 \div n \qquad (14)$$

And this is the end of step 2 of the basic computation cycle.

Experimental design. At first a square with the side equal to 20 cm was chosen. Then a random rarefied configuration of discs with unit radius and unit mass was generated. To this end uniformly distributed points - the centers of discs were consecutively thrown into the square. The attempt was considered successful if the next

disc did not overlap with any previous ones and the sides of the square. This procedu was repeated until one thousand of unsuccessful attempts in succession occurred. Tl generated configuration consisted of 60 discs in the considered case. Then the array w consolidated by bringing the walls closer together. A compact compressed packing discs in a new square with the side equal to 14 cm was formed.

The resulting configuration of discs was initial for a number of computati experiments. In these experiments the vertical walls were fixed, the upper wall w moved to the left and the lower one - to the right at the same velocity U. The ti constant and the four force constants were chosen as follows:

$dt=10^{-3}$sec; $C_n=10^4$n/cm; $C_s=10^4$n/cm; $C_k=0.5$; $C_v=10$nxsec/cm; (15

Parameter U responsible for the additional feeding of energy into the system w varied.

In each of the experiments the behavior of the system was observed over dozens seconds, i.e.dozens of thousands of states of the disc system were registered.

In addition to the values that need to be computed for the solution of the init system of dynamic equations and that allow one to obtain a qualitative description of behavior it is also useful to measure the additional physical characteristics, such stress tensor and energy dissipation rate.

Stress field. Every ten time steps mean values of stress tensor were compu for each disc. Let us derive the expression of this value without the assumption that disc is at equilibrium. The Newton's law II in differential form is:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i = \rho a_i \qquad i,j=1,2 \qquad (16)$$

Here x_j and a_i are linear coordinates and accelerations of any point of the disc. usmultiply equation (16) by x_k and integrate it over the disc volume.

$$\int_V \frac{\partial \sigma_{ij}}{\partial x_j} x_k \, dV = \int_V \rho a_i x_k \, dV - \int_V \rho g_i x_k \, dV \qquad (17)$$

$$i,j,k=1,2$$

Let us integrate the left side by parts:

$$\int_V \frac{\partial(\sigma_{ij} x_k)}{\partial x_j} dV = \int_V \sigma_{ij} \frac{\partial x_k}{\partial x_j} dV + \int_V \rho a_i x_k dV - \int_V \rho g_i x_k dV \qquad (18)$$

$$i,j,k=1,2$$

Transforming the volume integral into surface integral in the left side we obtain:

$$\int_S \overline{\sigma_{ij}} x_k \overline{dS_j} = V \cdot \langle \sigma_{ik} \rangle + \int_V \rho a_i x_k dV - \rho g_i \langle x_k \rangle V \qquad (19)$$

$$i,j,k=1,2$$

It should be noted that in formulae (16) to (19) the summation is taken over repeated indices. Replacing the surface integral by the sum over all the contact points we obtain for the mean stress tensor

$$\langle \sigma_{ik} \rangle = \frac{1}{V} \left[\sum_S F_i x_k + m g_i x_k^{center} - m \int_V a_i x_k dV \right] \qquad (20)$$

It should be pointed out that the right side (20) is independent of the choice of the beginning of the coordinate system. The last integral is primitively expressed by the values of coordinates and acceleration of the disc center, angular velocity and angular acceleration. Resuming the previous notations let us write the expression for spatial mean stress tensor of j-th disc, assuming that the disc is homogeneous:

$$\langle \sigma_{ik}(j) \rangle = \frac{1}{V} \left[\sum_l (F_{bi}(j,l) + F_{si}(j,l)) x_k(j,l) + \right.$$

$$+ m(j) g_i x_k(j) - m(j) a_i(j) x_k(j) +$$

$$\left. + b_{ik} I(j)/2 \right], \qquad (21)$$

$$\text{where } b_{ik} = \begin{cases} v_3(j) & \text{when } k=i \\ -a_3(j) & \text{when } k \neq i \end{cases} \qquad (22)$$

The results of the computation of mean stress tensor are graphically shown in Fig.3. The evolution of tangent stresses is apparently rather chaotic. On the other hand, there are grounds to state that specific structures in the form of load-keeping chains appear.The structures may consist of a relatively small number of elements. The vertical chains are oriented towards to load, whereas the horizontal ones tend to distort.

The components of the mean stress tensor provide for each disc a non-invariant description of the stress field. Although the choice of the coordinate system is not random but is related to the orientation of the system boundaries, yet it seems interesting to describe the stress field in terms of directions and values of principal compressions for each disc. In fig.4 this is done for the initial configuration of the phase of stationary loading.

Energy dissipation. As we know the values of forces and relative slip in each contact point at every moment it is easy to compute the energy that dissipated over a time step:

$$\hat{\sum_{i<j}} \bar{F}_s(i,j) \cdot \Delta \bar{d} + \check{\sum} \bar{F}_s(i,k) \cdot \Delta \bar{d} \qquad (23)$$

In the formula (23) the symbol $\hat{\Sigma}$ stands for a summation over all such pairs of elements i and j in whose contact point the condition (8) is not true. The symbol $\check{\Sigma}$ stands for a summation over all such pairs of element i and border k in whose contact point the condition (8) is not true.

Characteristic plots of energy dissipation rate for three values of U equal to 5/0\ 0/5 and 0/05, respectively, are presented in Figs.5-7. It is noteworthy that at high loading velocity the energy dissipation rate becomes quasi-stationary almost immediately after the onset of the process. Long-term observations of the system at medium loading velocity has demonstrated that in spite of significant variations of the energy dissipation rate, the latter still has long quasi-stationary areas (of about 1 sec.).

The behavior of the system at low loading velocity is of greatest interest. In this case long-lasting periods (about 1 sec long) during which there is hardly any energy dissipation at all alternate with relatively short periods of fast dissipation. A detailed analysis of spatial localization of the dissipation process during the fast periods has shown that although the main portion of the energy is lost in one or several closely located contact points, secondary zones of energy loss are simultaneously formed in remote parts of the system. This seems to suggest that the correlation radius in the system increases during these fast periods.

Macromovements. Let us now return to the numerical experiments with U equal to 5.0 which is most suitable for the qualitative description of the structure of macroscopic movements in the system. Fig.8 shows the initial stage of the evolution of the system at U equal to 5 cm/sec. It should be noted that at first rather a simple movement structure appeared. It was a nonuniform circulation of subsurface layers. This circulation was characterized by the alternation of short fast phases with longer and slower ones. After 1.5 sec, however, the structure of the movement began to become dramatically more complicated. Because of the inhibitory action of the friction against the fixed vertical walls and relatively stable internal layers of the discs, the circulation velocity was lower by an average of an order of magnitude than the velocity of the forcing movement of horizontal walls. It should be reminded that the inhibition is significant because the system is in the state of strong overall compression whereas the constant of dry friction was chosen to be rather big. This led to gradual accumulation of discs in the vicinity of the bottom righthand corner and in the top lefthand corner and to the appearance of relative rarefaction in the areas of the two other corners. The structure of the velocity field became complicated firstly, because in the rarefaction zones fast coordinated linear and rotating movements occasionally appeared that were relatively independent of the remaining set of the discs. Secondly, although the shifts of horizontal layers of discs were still localized in the vicinity of the upper and lower boundaries, the

analogous vertical shifts, unlike the initial period, were shorter in average and appeared much further from the vertical boundaries. Thirdly, the elements of the central area were gradually involved in the movement.

Conclusions. The system of a relatively few identical elements with a simple interaction law under the conditions of the stationary action under analysis displays a complicated behavior in which the chaotic character is combined with formation of structures. If this action is weak enough one may reproduce the properties of the seismic process, such as time and space heterogeneity. In the future it seems promising to investigate more complicated multiscale variants of this computation design. For self-similarity reasons it may be expected that the results of such a simulation (probably with choice of various sets of parameters) can be interpreted both in terms of the seismic process and as applies to destruction experiments.

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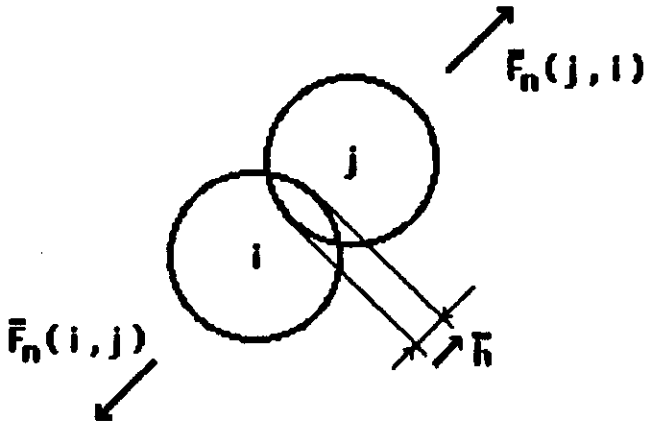
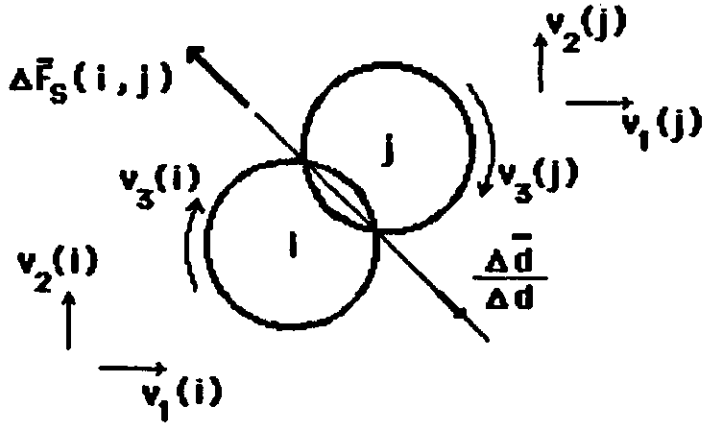
Figures for the paper by Pisarenko V.F., Primakov I.M. and Shnirman M.G.
The analysis of deformed array of moveable elements using particles.


$\bar{F}_n(j,i)$
j
i
$\bar{F}_n(i,j)$
h


$\Delta\bar{F}_S(i,j)$
$v_2(j)$
$v_1(j)$
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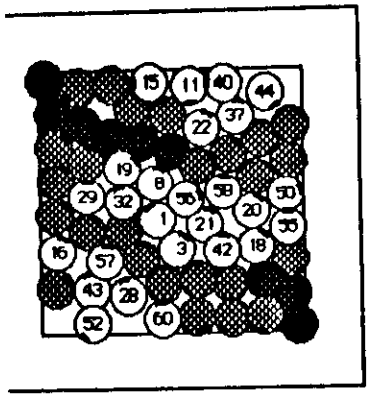
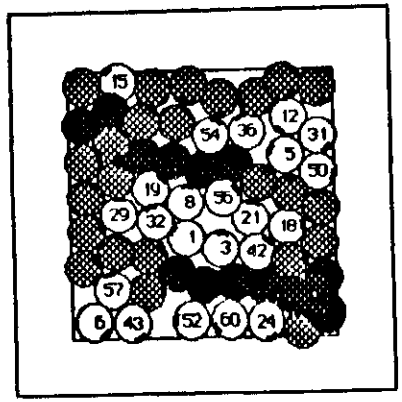
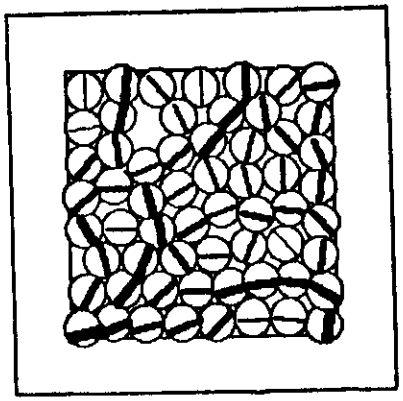
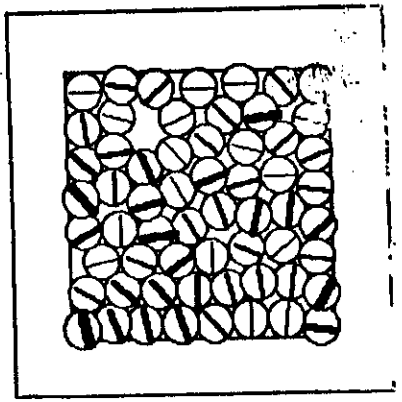
$v_2(i)$
$\frac{\Delta\bar{d}}{\Delta d}$
$v_1(i)$

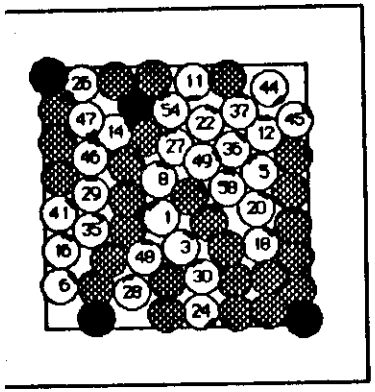


σ_{11}

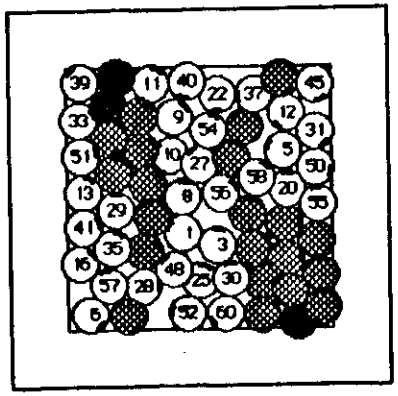


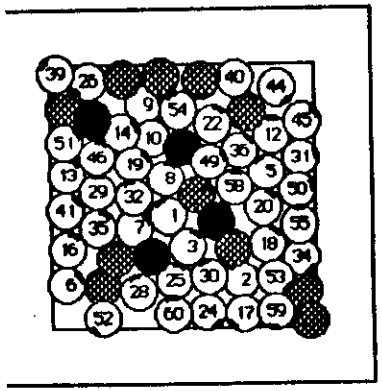




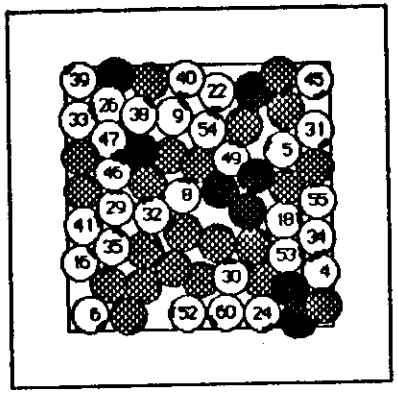


σ_{22}





$|\sigma_{12}|$



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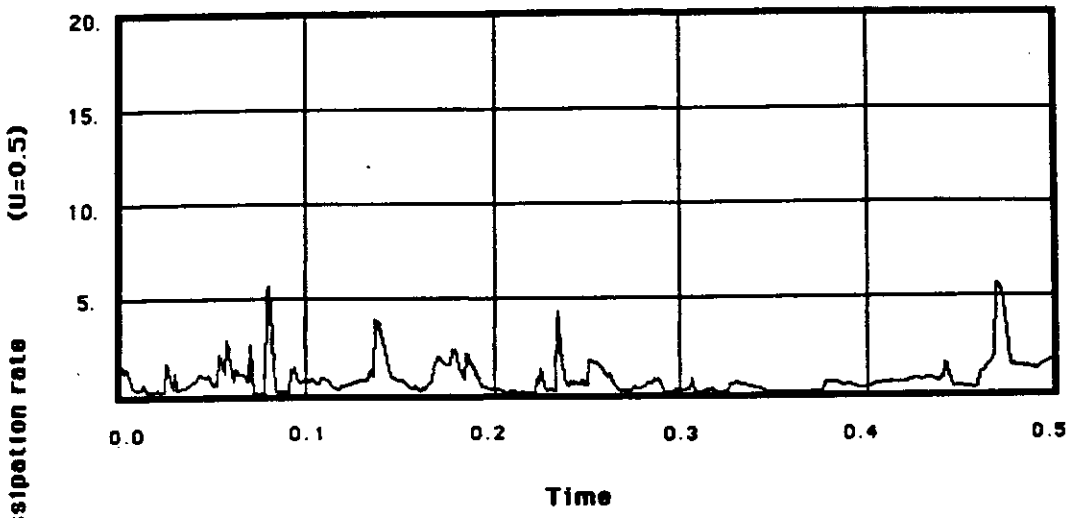
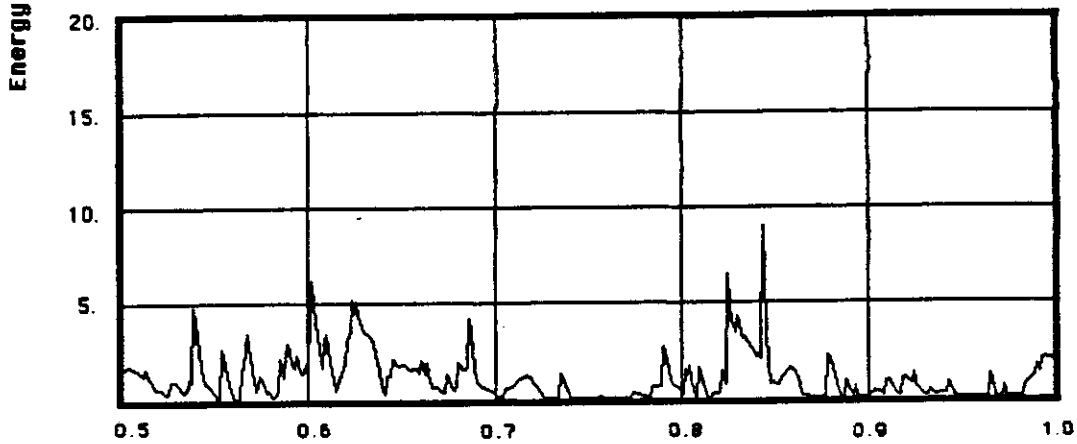
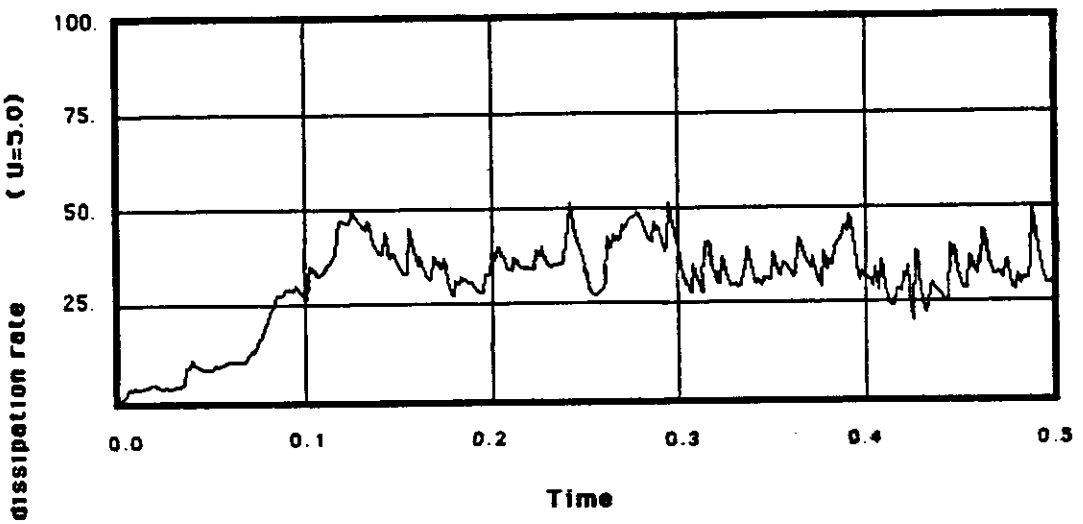
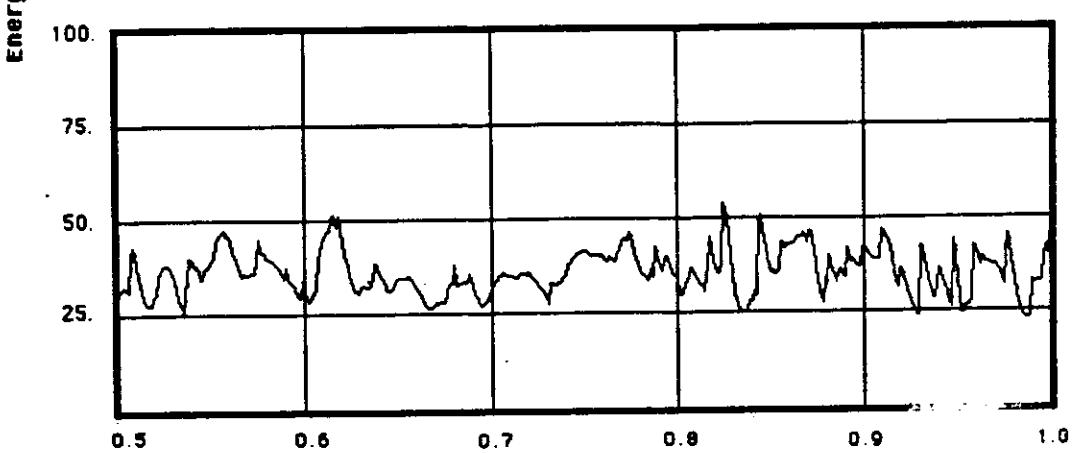

(U=0.5)
Energy dissipation rate
20.
15.
10.
5.
0.0
0.1
0.2
0.3
0.4
0.5
Time


20.
15.
10.
5.
0.5
0.6
0.7
0.8
0.9
1.0


(U=5.0)
Energy dissipation rate
100.
75.
50.
25.
0.0
0.1
0.2
0.3
0.4
0.5
Time


100.
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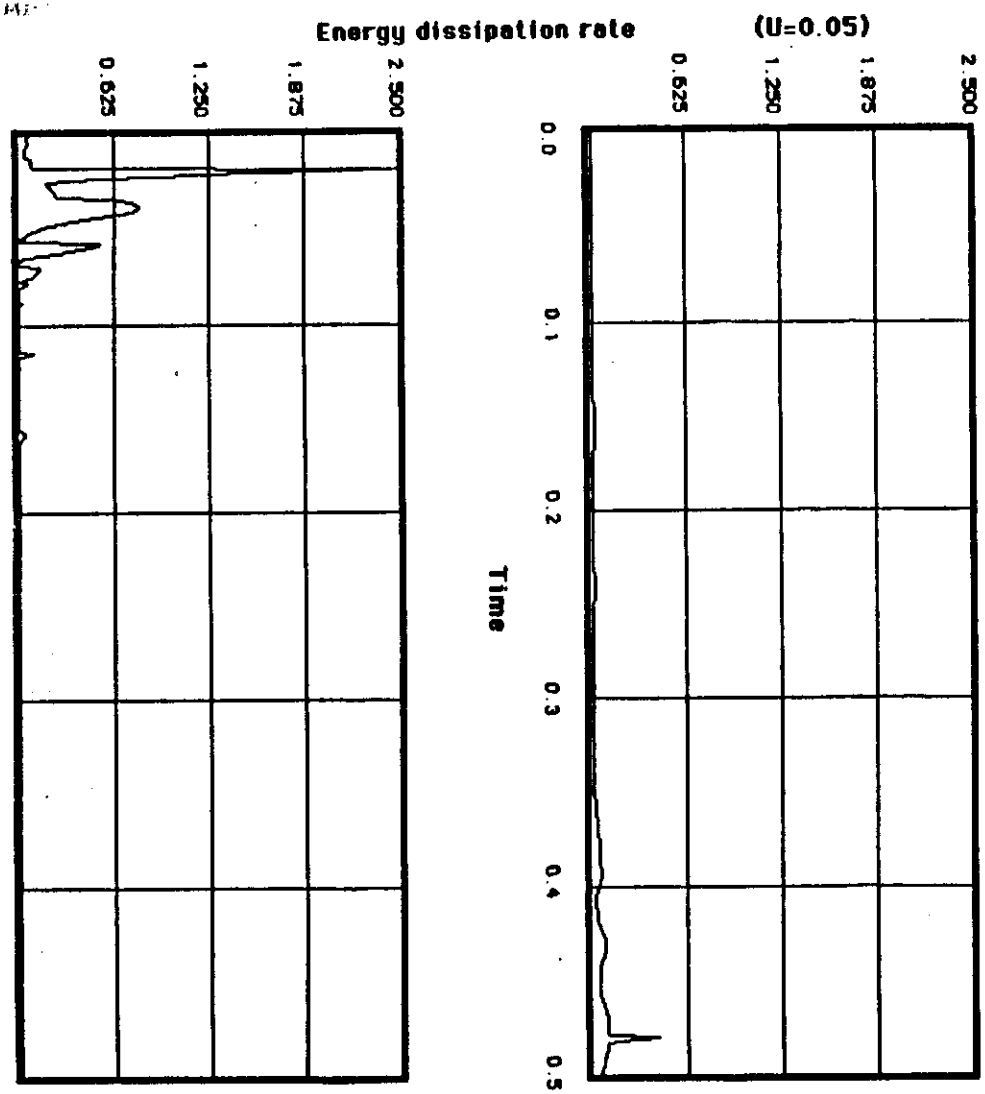

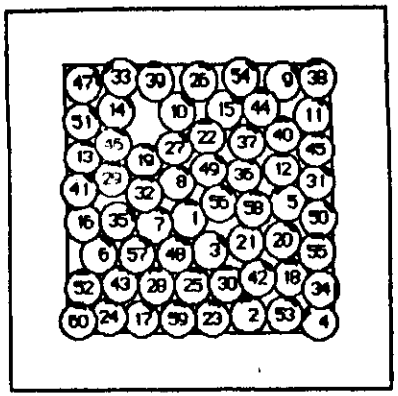

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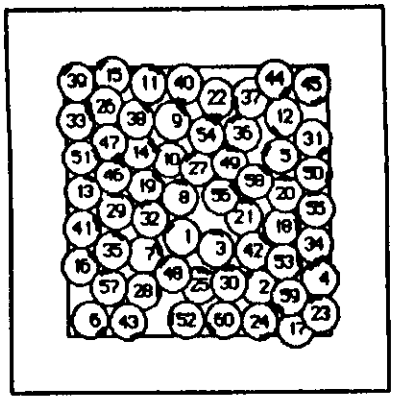

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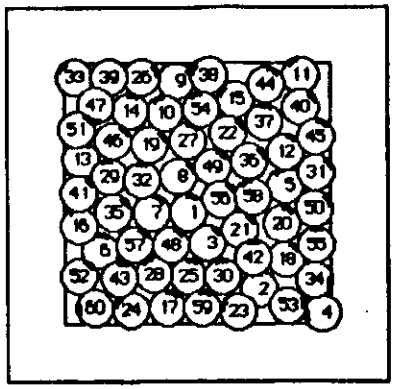

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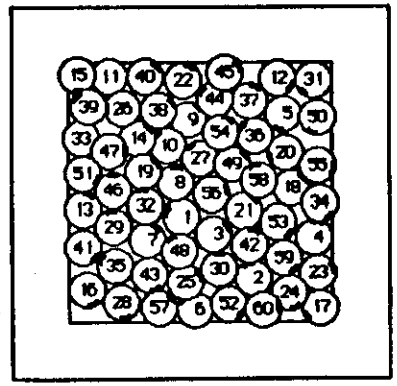

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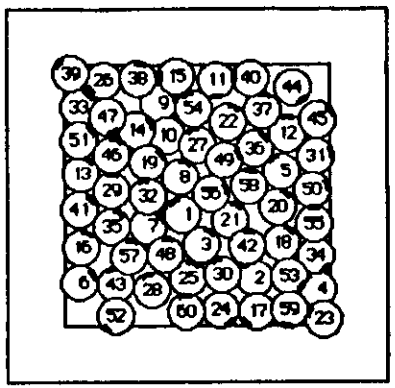

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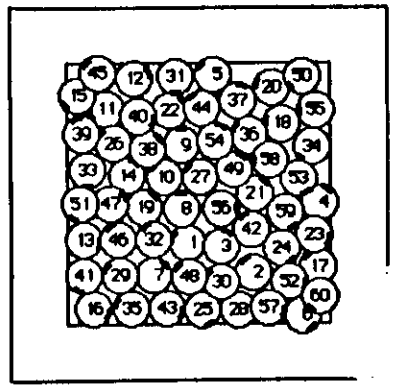

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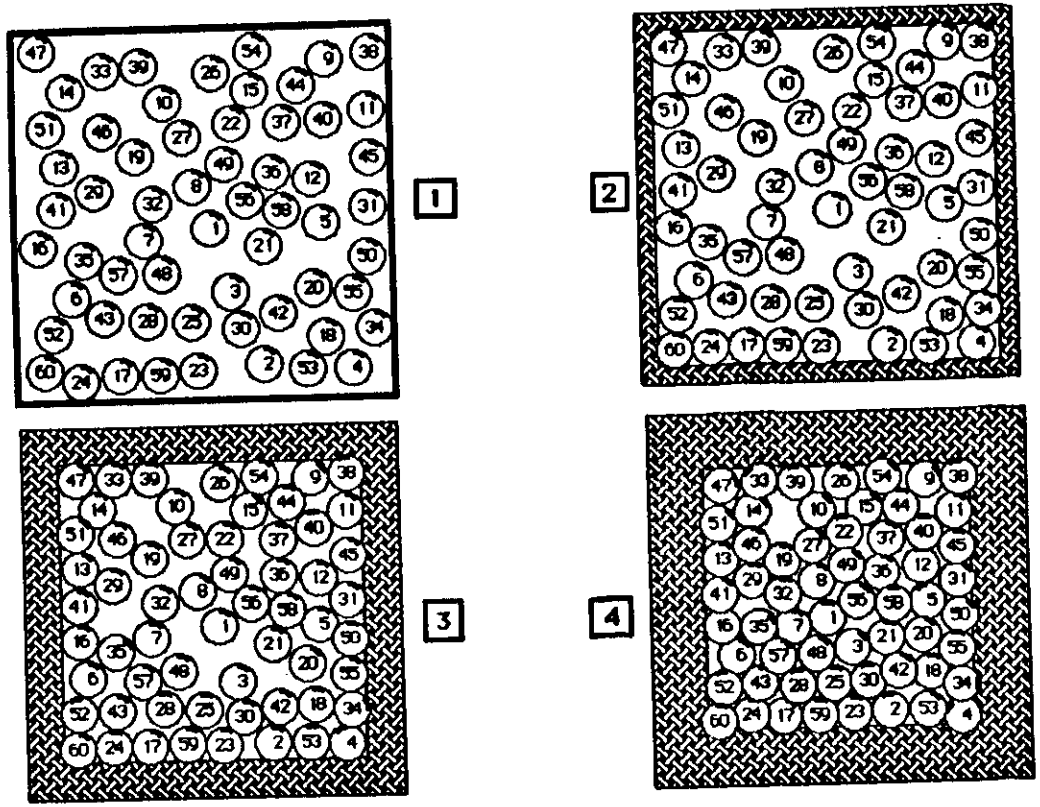
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