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WORKSHOP
GLOBAL GEOPHYSICAL INFORMATICS WITH APPLICATIONS TO RESEARCH IN EARTHQUAKE PREDICTIONS AND REDUCTION OF SEISMIC RISK

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HIERARCHICAL MODEL OF DEFECT DEVELOPMENT AND SEISMICISTY

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Abstract.

The multiscale model of the defect development kinetics is studied. Some critical effects are found. The simulation produces the phenomena that make it possible to predict the time of "large" defect's appearance.

In this work we study kinetics of defect development (i.e. seismic process). According to modern ideas geophysical medium is treated as hierarchical discrete structure, consisting of some singularities (blocks). The sizes of blocks vary in the large scale range [3].

We propose and solve hierarchical system of kinetic equations of the model of defect development, which are similar to equations, obtained in [4]. This method is dynamic analogy of renorm-group approach, popular in modern physics. The features of hierarchical kinetics will be used for prediction of the moments "large" defect generation. This prediction will use certain function of the number of "middle" defects.

Our model of defect development is based on the following physical assumptions.A defect is area, where capacity to relax stresses is essentially increased. The process of defect development is multiscales and automodel on different levels. Defects appear accidentally and for defects of the same scale level independently. Some space combinations of defects form defect of the next scale level. Defects are "healed" (disappeared) with some intensity, depending on their sizes.

THE MODEL. We consider a tree with degree n of each vertex (fig. 1). Each vertex may be in two states 0 and 1. Vertices in state 1 will be called defects. Time is discrete. In the initial moment all vertices are in state 0. Generation of defects on the lower (zero) level has the same intensity α_0 for all vertices. A vertex of level i becomes a defect if it covers $\geq k$ defects on the level $(i-1)$. The time of existing of each defect on the level i is $t = C_0 C^i$, for some constant C_0 and C. Then this vertex

becomes 0 (nondefect) and we say that the vertex is "healed". Another version of "healing" is random and independent transition $1 \to 0$ with probability $\beta_i = t_i^{-1}$. It may be considered similary to the basic version and leads to the analogous results).

EQUATIONS. Let $p_i(t)$ be the density of defects on i level; $n_i(t)$ the density of "new" (generating in the moment t) defects; $q_i(t)$ the density of old defects in the moment t, and $\alpha_i(t)$ the probability of the transition $0 \to 1$ on the level i. Clearly, we have

$$p_i(t) = n_i(t) + q_i(t) \qquad (1)$$

$$q(t) = \sum_{j=t-t_i+1}^{t-1} n_i(j)$$

The simple calculations, based on the fact that adefect of level $(i+1)$is generated by at least k defects of i level, gives the recursion

$$\alpha_i(t) = \frac{\sum_{l=0}^{k-1} \binom{n}{l} q_i^l (1-q_i)^{n-l} \left(\sum_{j=k-l}^{n-l} \binom{n-l}{j} \alpha_i^j (1-\alpha_i)^{n-j-l} \right)}{\sum_{l=0}^{k-1} \binom{n}{l} q_i^l(t) (1-q_i(t))^{n-l}} \qquad (2)$$

As new defects may be generated only in zero vertices, we get

$$n_i(t) = (1-q_i(t))\alpha_i(t). \qquad (3)$$

The equations (1)-(3) form hierarchical system of kinetic equations of defect development. Later on we assume that $p_i(0)=0$ for all i.

THE STATIONARY REGIME. According to [2] each of the functions $p_i(t), q_i(t), \alpha_i(t), n_i(t)$ has finite limit (resp. $p_i(\infty), q_i(\infty), \alpha_i(\infty), n_i(\infty)$) as $t \to \infty$ and the rate of the convergence is exponential. We study the behavior of $\alpha_i(\infty)$ and $p_i(\infty)$ as functions of the intensity α_0 of defect generation on the lower level. It is shown [2], that there is α_0^{cr} such that for $\alpha_0 < \alpha_0^{cr}$ $\alpha_i(\infty) \to 0$ as $i \to \infty$, and for $\alpha_0 > \alpha_0^{cr}$ $\alpha_i(\infty) \to 1$ as $i \to \infty$.

The numerical solution of the system confirms this theoretical results but also allows to make some more precise statements (and to calculate α_0^{cr}). If $\alpha_0 < \alpha_0^{cr}$ and difference $\alpha_0 - \alpha_0^{cr}$ is small, $p_i(\infty)$ is almost constant in some interval of i (and then it rapidly decreases to 0). This interval increases when α_0 tends to α_0^{cr} . For i from this interval $\alpha_i(\infty)$ decreases linearly on i (then it begins to decrease more rapidly). The most interesting is dependenceof lg $n_i(\infty)$ on i (fig. 2), corresponding to the frequency law for earthquakes. The linear part of this graph is naturally associated with Gutenberg - Richter's law.

The case $\alpha_0 > \alpha_0^{cr}$ is more difficult. We again consider dependence of lg $n_i(\infty)$ on i when α_0 is near α_0^{cr} (fig. 3). We may distinguish three intervals of values of i (three intervals of scales). In small scales (as in the case $\alpha_0 < \alpha_0^{cr}$) we see linear dependence. For the second interval of scales graph becomes more slanting. For the third group of scales the slope is restored. The length of the initial linear part depends monotonically on $\alpha_0 - \alpha_0^{cr}$. The lower scale levels do not react on the transition of α_0 over the critical value α_0^{cr}.In the case when $\alpha_0$is close to α_0^{cr}, $p_i(\infty)$ is almost constant, but then rapidly enough (on i) increases to 1. Pattern, "destroyed" in large scales, may contain considerable part of "nondefect" volume.

THE TRANSIENT PROCESS. In the case $\alpha_0 < \alpha_0^{cr}$ the density of defects on all levels is stabilised, and the time of stabilization increases as the scale increases (fig. 4). The case $\alpha_0 > \alpha_0^{cr}$ is especially interesting for kinetics. In the case the transient process may be regarded as destruction (in the case of patterns) or as strong earthquake with its cloud of forshocks and aftershocks.

On the figure 5 it is shown that for all levels i, $i \geq i_0$, the densities $p_i(t)$ exceed some threshold in the same moment t_0 (intersect in the same point). It may be shown that the value of the threshold is a real root lying in segment [0,1] of the equation

$$P = \sum_{k}^{n} \binom{n}{j} P^j (1-P)^{n-j}$$

and depends only on n and k, but not on α_0. Observation of accumulation of defects on some level $i > i_0$, gives situation with critical concentration of defects up to the moment of destruction as in criterion of destruction of Zhurkov .

The most important is the next observation.In the neighborhood of t_0 p_i has "a jump", which is increasing with the increasing of i. So in the moment t_o we have the global instability ("destraction") of the pattern . Computation for different $\alpha_0 < \alpha_0^{cr}$ shows (fig. 6) the following dependence t_0 on "overcriticality" $\Delta\alpha = \alpha_0 - \alpha_0^{cr}$. $t_0 \simeq B_0(\Delta\alpha_0)^{-\gamma}$, for some constant β_0 and γ , depending on k and n. So the time of existen of the pattern is regulated by "overcriticality" of defect generation on the lower level. The number of "quiet" levels also depends on "overcriticality" $\Delta\alpha$: it increases with decreasing of $\Delta\alpha$.

Graph of summary activity $S(t) = \sum_{i=i_0}^{i_m} n_i(t)$ (fig. 7) allows us tomake the conclusion that the forshock activity increases before the catastrophe. The activity after the catastrophe resembles aftershocks, but its decreasing is more rapid than in Omory's law. The ratio of "forshock" and "aftershock" activity, evidently, is equal to P/(1-P), where P is a root of the equation (4). For fixed n the the ratio decreases when k decreases.

THE FLUCTUATION CASE. Evidently, the kinetics of defect generation is essentially determined by random fluctuation of the density and the intensity around their mean values. So we randomize the initial system, changing α_{i-1} in equations (2) by random realisations of Puasson's distribution with corresponding intensities. We also change the intensity of defect generation on the lower level as function of time. Earlier we assumed α_0=const, but the real situation, is, of course, more complicated and α_0 depends on changeable balance between accumulation and dissipation of energy. Now we assume that tectonic motion provides uniform increasing of α_0, and defect generation, dissipating elastic energy, leads to decreasing of α_0, and the influence of defect increases with increasing of size. We use the next formula

$$\alpha_0 t)_{\bar{0}}\alpha(t) = d - \varkappa \sum_i E_i N(t), \qquad (5)$$

where d is "the rate of deformation"; $N_i(t)$ is the number of defects on level i, generated in the moment t; E_i is "the energy", dissipated when one defect of level i appeared, determining by the formula $E_i = n^{\sigma i}$; $\varkappa, \sigma$ are constant. Computational experiments with the fluctuation case of the model show some similarity of kinetics of defect development with seismic process. In available realisations quasistationarity on large time interval combines with essentially changeability on relatively small intervals between the largest acts of defect generation. It reminds known phenomenon of "seismic circle".

In our experiments the size distribution of acts of defect generation is well approximated by linear dependence (in logarithmic scale) (fig. 8). Such dependence is typical for wide range of "deformation rate". When d increases, interval of linearity becomes more short and "bend down" taks place for smaller value of

E_i. It is physically quite naturally that when the rate of contribution of energy in the system becomes less, the number of large events decreases.

In connection with precursory activity, revealing in the case α_0=const, we tried to find precursors in the fluctuation case. In the capacity of the precursor we choose "the bend up" of frequancy law in the middle range of energy (scales). We research connection of the number of defects on level i N_i with energy E_i in the moving interval of time. We calculate the best approximation of

$$\lg N_i(E) = a + b \lg E_i + A E^{\alpha}_i$$

The first two terms in the right part on (6) corresponds to standard dependence such as in Gutenberg - Richter's law. The third term defines "the bend" for large energies. The negative value of A is "the bend down ". The positive value of A is "the bend up". We supposed that positive A characterizes instable situation with increasing part of large defects.

Consideration of the dependence A(t) for some intermediate interval of E_i shows essential correlation of the pósitive values of A with the moments of large events. It is naturally they are not taken into account when we define A(t). It is interesting that when the rate of deformation is small the largest possible events as well as essential positive values of A(t) do not occur.

We assume that our experiments naturally explain certain successes in prediction of time of the strongest earthquakes. Algorithms from [1] are based on increasing of activity in scaling levels, preceding the level of events, being predicted. It is in good agreement with the idea of "bend up" of frequency law.

computations, which were kindly made by V.G.Kosobokov, shows that well known precursory algorithm M8[1] quite successfully predicts the largest events of the model catalogs.

THE BASIC EFFECTS. The case with the permanent intensity.

1. There is the critical level α_0^{cr} of the intensity of defect generation on the lower level, such that the transition over it leads to destruction.

2. In the neibourhood of α_0^{cr} dependence on size of the number of generated defects, which is similar to the Gutenberg-Richter's law, is observed.

3. In the "overcritical" situation the stationary regime on the lower scale levels practically doesn't differ from "noncritical" one. (The pattern,destroyed in large scales, may contain the considerable part of "nondefect" volume in the small scales.)

4. The overcritical regime is characterized by single catastrophe with increasing forshocks (for levels which is more than some one).

5. Decreasing activity of aftershock type follows the catastrophe .

6. The number of "quiet" scales and time of waiting of the catastrophe decrease when "overcriticality" increases.

7. In the overcritical regime frequency law is more slanting in the intermediate interval of scales (it is in good agreement with the behavior of known precursors [1]).

8. For single levels analogy of the concentration criterion of destruction by Zhurkov takes place.

9. "Destruction" of more "solid" patterns takes place when ratio of the number of forshocks to the number of aftershocks is

larger.

The fluctuation case.

1. The regime of defect's generation is quasistationary with elements of "the seismic circle".

2. The dependence of the size on the number is similary to Gutenberg - Richter's law in wide range of deformation rates.

3. "Bend up" of frequency law precedes the largest acts of defect generation.

4. Precursory effects do not take place when deformation rates are relatively small.

CONCLUSION.

The hierarchical system of kinetic equations allows (though rough) to consider both the stationary regime of defect generation and kinetics of setting "long-range order". The progress in this field (particularly in the earthquake prediction) we connect in the first place with taking into account space variation of parameters. In its present form our model may be considered only as the kernal of the future effective theory, but it too allows to get important conclusions.

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Fig. 1

A tree with degree of vertices equal 3.

Fig. 2.

The graph of dependence lg $n_i(\infty)$ on i for $\alpha_0<\alpha_0^{cr}$.

Fig. 3.

The graph of dependence lg $n_i(\infty)$ on i for $\alpha_0>\alpha_0^{cr}$.

Fig. 4.

Dependence of density of defects on time for different levels when $\alpha_0<\alpha_0^{cr}$.

Fig. 5.

Dependence of density of defects on time for different levels when $\alpha_0>\alpha_0^{cr}$.

Fig. 6.

The dependence of t_0 on $\Delta\alpha=\alpha_0-\alpha_0^{cr}$.

Fig. 7.

The dependense of activity S(t) on time as $\alpha_0<\alpha_0^{cr}$.

Fig. 8.

The dependence A(t). Arrows show the moments of the largest events.

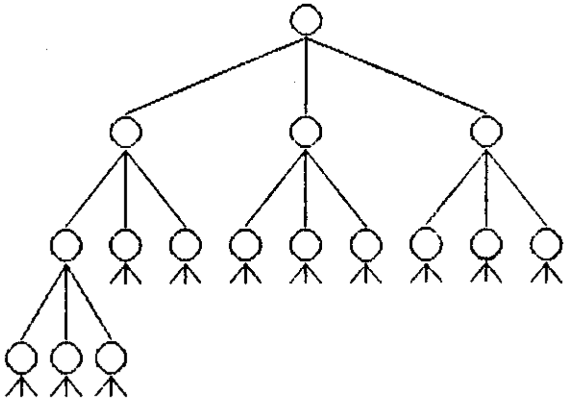

Fig. 1

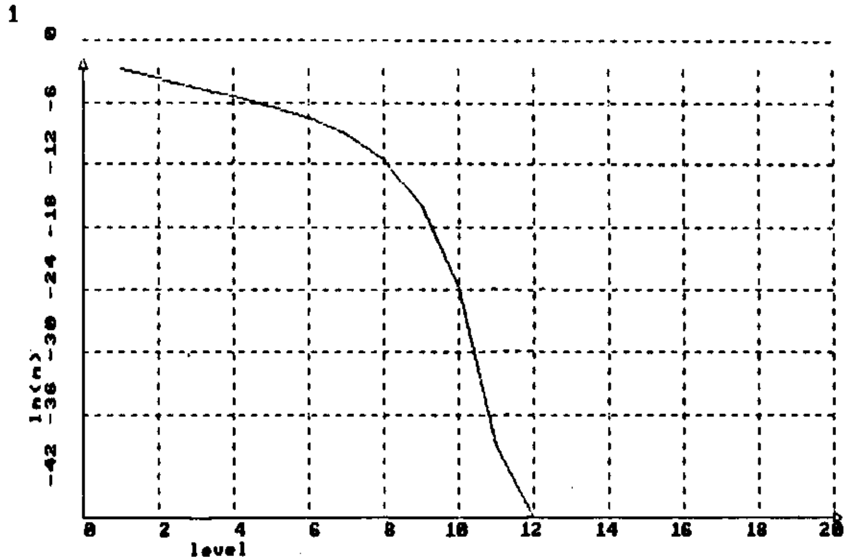

Fig. 2

1

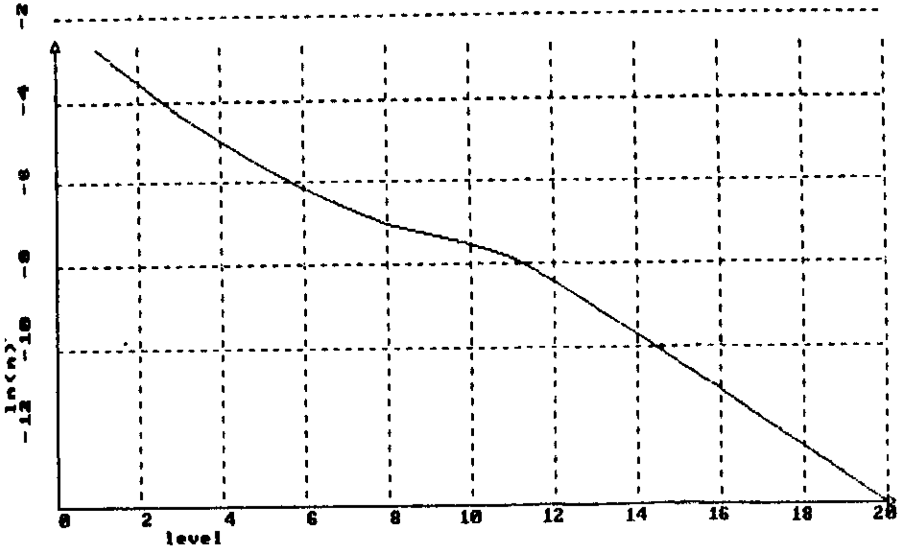
-2
-4
-6
-8
-10
-12
ln(n)
0
2
4
6
8
10
12
14
16
18
20
level

Fig. 3

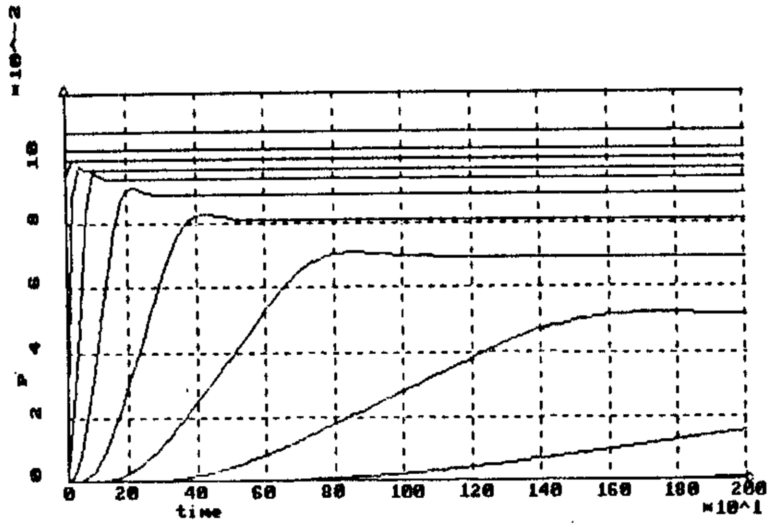
*10^-2
10
8
6
4
2
0
P
0
20
40
60
80
100
120
140
160
180
200
time
*10^1

Fig. 4

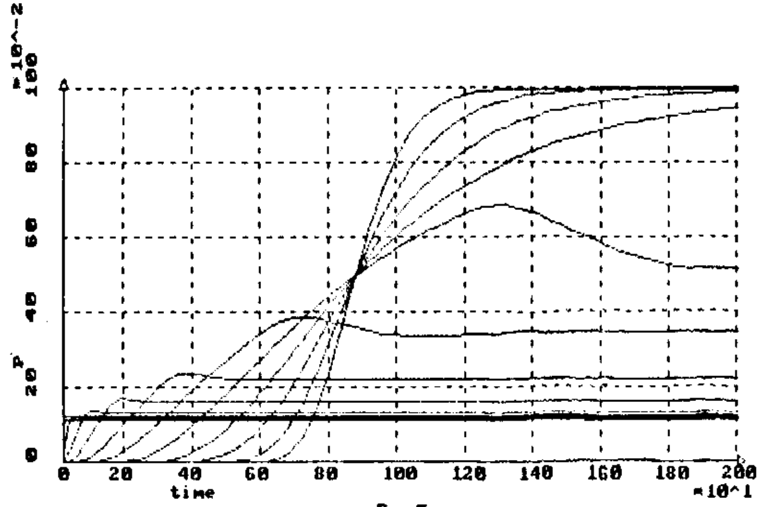

Fig. 5

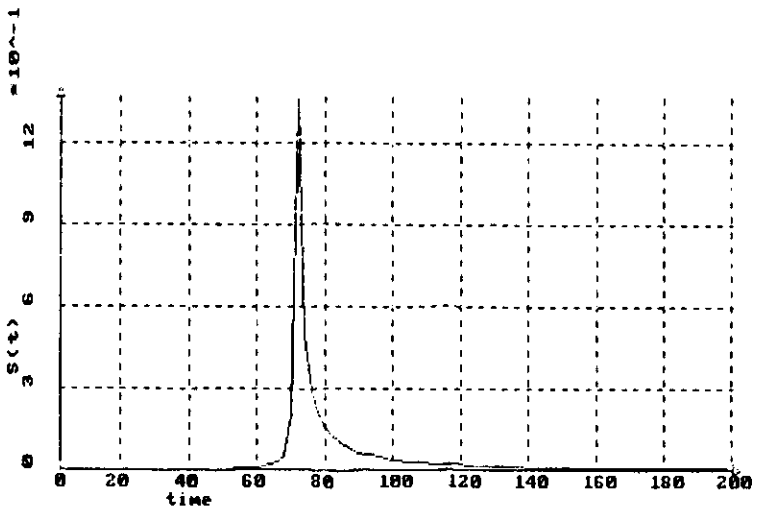

Fig. 7

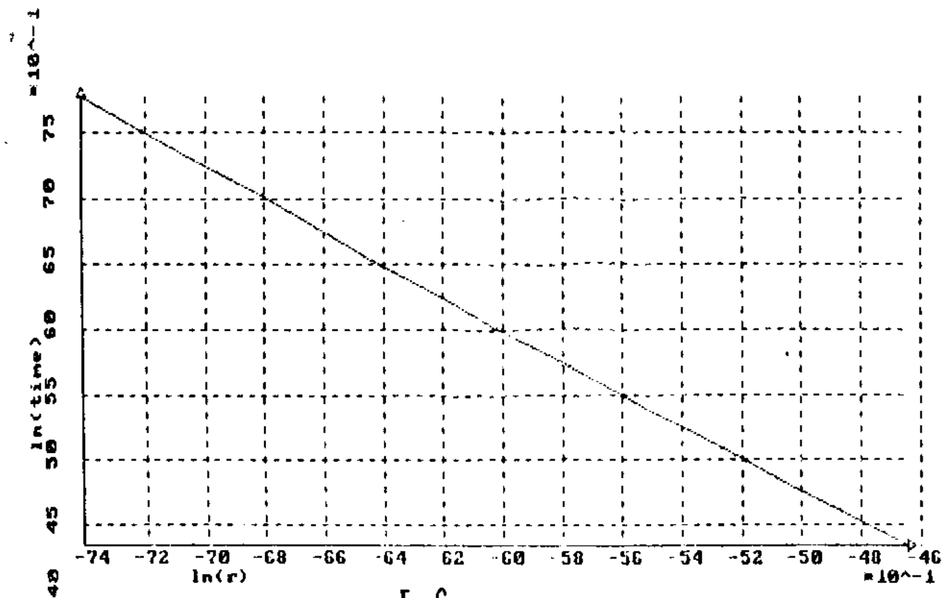

Fig. 6

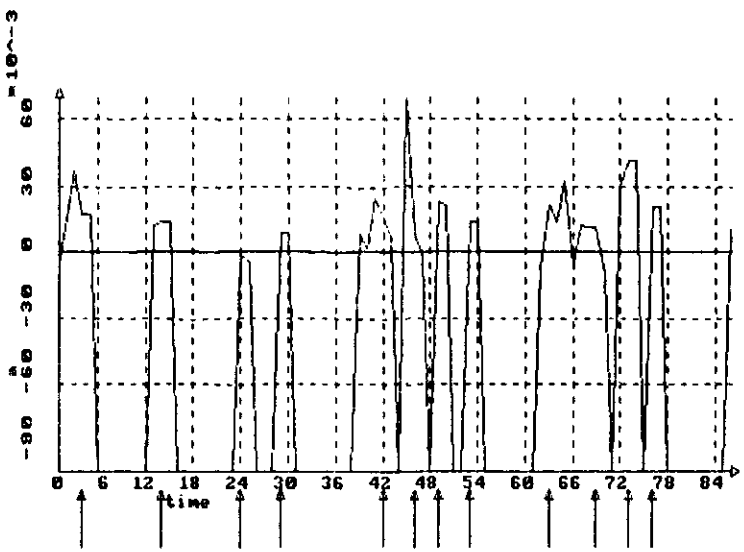

Fig. 8