INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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WORKSHOP
GLOBAL GEOPHYSICAL INFORMATICS WITH APPLICATIONS TO RESEARCH IN EARTHQUAKE PREDICTIONS AND REDUCTION OF SEISMIC RISK

(15 November - 16 December 1988)

THE FIGURE OF THE EARTH

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THEORIE
DE LA
FIGURE
DE LA TERRE,
Tirée des Principes de l'Hydrostatique;
Par M. CLAIRAUT, de l'Académie Royale des
Sciences, & de la Société Royale de Londres.

A PARIS,
Chez DURAND, Libraire, rue Saint-Jacques,
à Saint Landry & au Griffon.

MDCCXLIII.

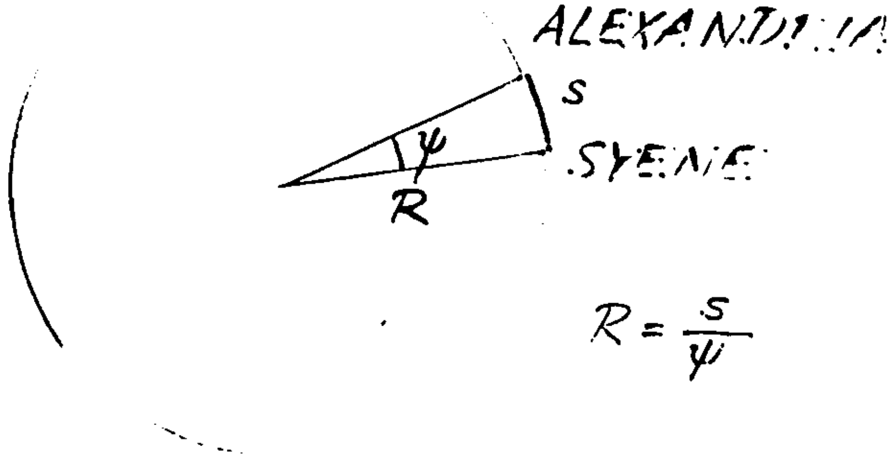
Н. П. ГРУШИНСКИЙ

ТЕОРИЯ ФИГУРЫ ЗЕМЛИ

*Допущено Министерством
высшего и среднего специального образования РСФСР
в качестве учебника для студентов университетов*

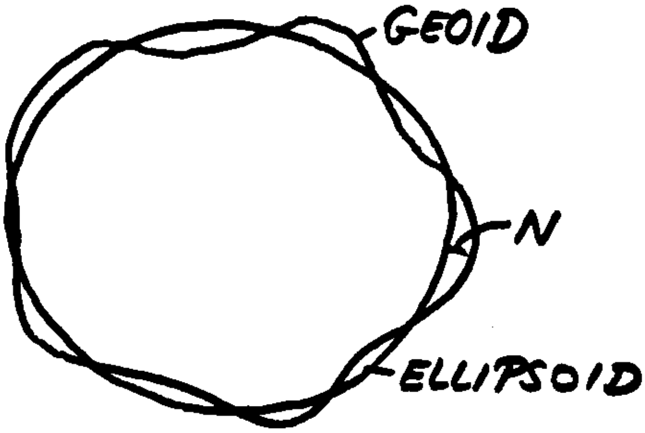
ГОСУДАРСТВЕННОЕ ИЗДАТЕЛЬСТВО
ФИЗИКО-МАТЕМАТИЧЕСКОЙ ЛИТЕРАТУРЫ
МОСКВА 1963

ERATOSTHENES



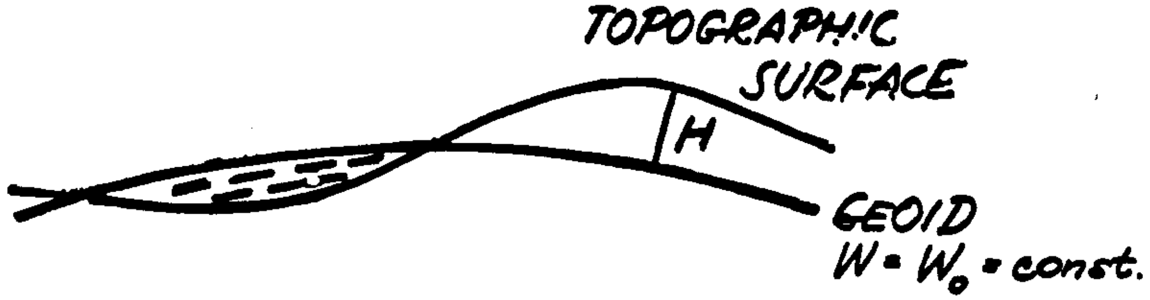
$$R = \frac{s}{\psi}$$

GEOID AND REFERENCE ELLIPSOID



N...GEOIDAL HEIGHT

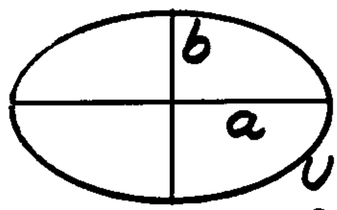
$$-100\,m < N < 100\,m$$



H ... TOPOGRAPHIC HEIGHT

2

THE EARTH ELLIPSOID



$U = U_0 = \text{const.}$
level ellipsoid

M ... MASS

$f = \dfrac{a-b}{a}$ GEOM. FLATTENING

$J_2 = \dfrac{C-A}{Ma^2}$ DYN.

$a = 6378\,137\,m$

$GM = 3986\,005 \times 10^{-8}\,m^3 s^{-2}$

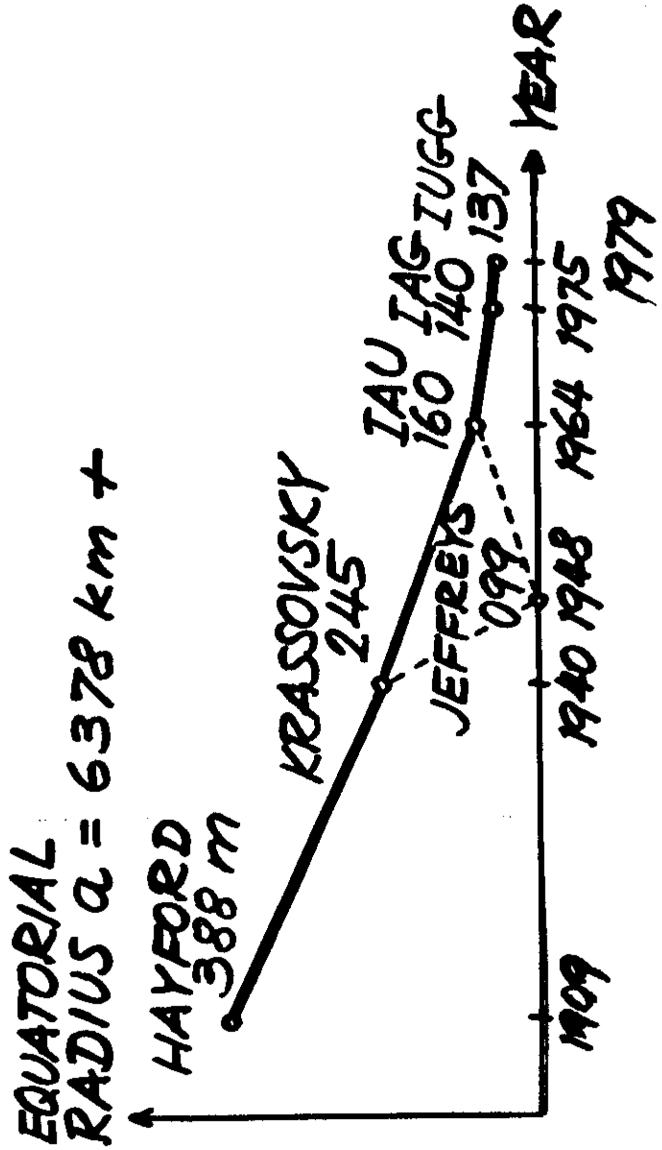
$J_2 = 0.001\,082\,63$

$\gamma_e = 9.780\,33\,m\,s^{-2}$

$1/f = 298.257$

CONSTANTES ELLIPSOÏDALES

		a	1/F
1799	COMM. POIDS ET MESURES	6375 653	334
1810	DELAMBRE	6376 985	308.64
1840	BESSEL	6377 397	299.15
1866	CLARKE	6378 206	294.98
1880	CLARKE	6378 245	293.5
1909	HAYFORD	6378 388	297.0
1910	HAYFORD	6378 062	298.2
1942	KRASSOVSKY	6378 245	298.3
1948	JEFFREYS	6378 097	297.28
1964	U.A.I.	6378 160	298.25
1983	A.I.G.	6378 136 (±1)	298.26



SYSTEMES GEODESIQUES DE REFERENCE

CONSTANTES DE BASE

	1967	1980
a	$= 6378\ 160\ m$	$6378\ 137$
GM	$= 398\ 603 \times 10^{9}\ m^{3}s^{-2}$	$398\ 600.5$
J_2	$= 0.001\ 0827$	$0.001\ 08263$
ω	$= 0.000\ 072\ 92115\ s^{-1}$	IDENTIQUE

CONSTANTES DEDUITES

	1967	1980
f^{-1}	$= 298.247$	298.257
γ_e	$= 9.780\ 318$	$9.780\ 327$

CONSTANTES FONDAMENTALES

VITESSE DE LA LUMIERE

$$c = (299\ 792\ 458 \pm 1)\ ms^{-1}$$

CONSTANTE NEWTONIENNE DE LA GRAV.

1975	$G = (6672 \pm 4) \times 10^{-14}\ m^{3}s^{-2}kg^{-1}$
1979	6672 ± 4
1983	6673 ± 1

CONSTANTE GRAVIFIQUE DE LA TERRE

1975	$GM = (3986\ 005 \pm 3) \times 10^{8}\ m^{3}s^{-2}$
1979	004.7 ± 0.5
1983	4.4 ± 0.1

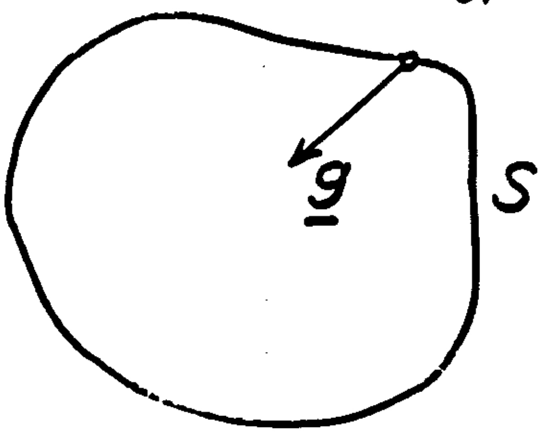
RAYON EQUATORIAL DE LA TERRE

1975	$a = (6378\ 140 \pm 5)\ m$
1979	137 ± 2
1983	136 ± 1

PESANTEUR EQUATORIALE

1975	$\gamma_e = (978\ 032 \pm 1) \times 10^{-5}\ ms^{-2}$
1979	033 ± 1
1983	032 ± 1

BOUNDARY-VALUE PROBLEM
OF MOLODENSKY



given: $W, \underline{g}$
determine: S

Principle: $\underline{g} = F(S, W)$

$$\downarrow$$

$$S = \Phi(\underline{g}, W)$$

GEODETIC BOUNDARY-VALUE PROBLEM

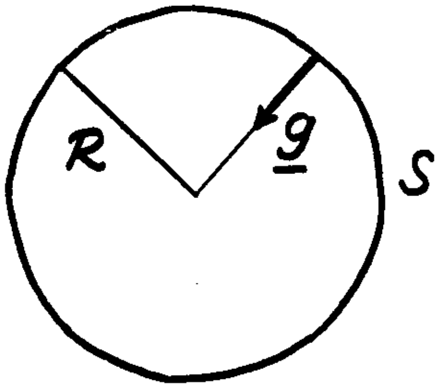
ELLIPSOID	CLAIRAUT	1743
GEOID	STOKES	1849
EARTH'S SURFACE	MOLODENSKY	1945

STOKES $N = c\iint_{\sigma} \Delta g\, S(\psi)\, d\sigma$

EXAMPLE:

SPHERE

$W = \frac{GM}{R}$, $g = \frac{GM}{R^2}$



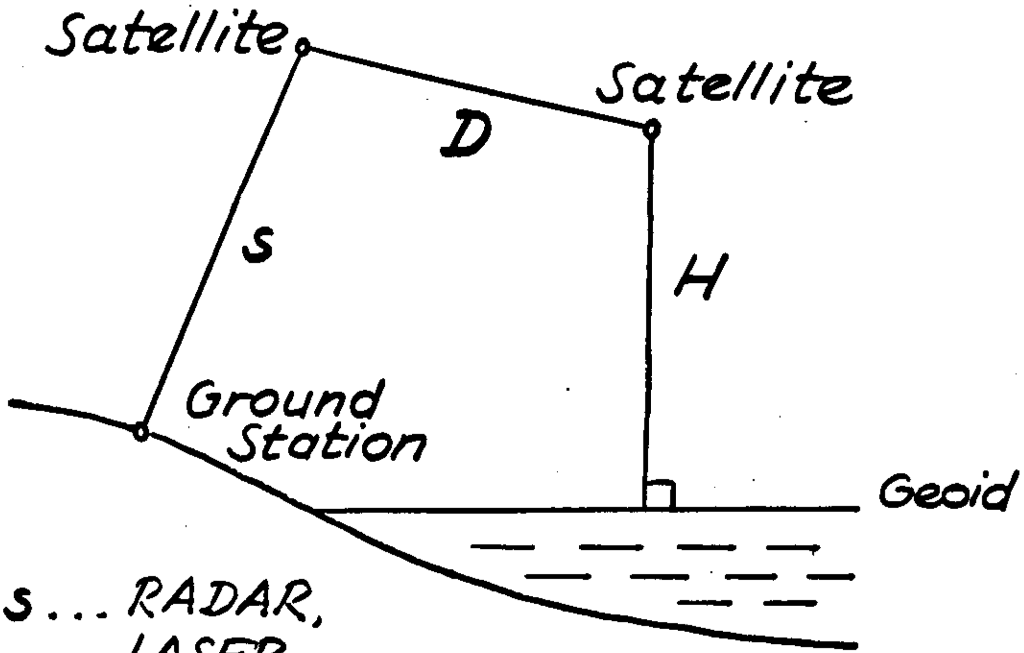
SPHERE

$$g = \frac{W}{R}$$
$$\downarrow$$
$$R = \frac{W}{g}$$

GENERAL

$$\underline{g} = F(S, W)$$
$$\downarrow$$
$$S = \Phi(\underline{g}, W)$$

SATELLITE TECHNIQUES



s ... RADAR, LASER

ds/dt ... DOPPLER

H ... ALTIMETRY

dD/dt ... SATELLITE-TO-SAT. TRACKING

SATELLITE TECHNIQUES

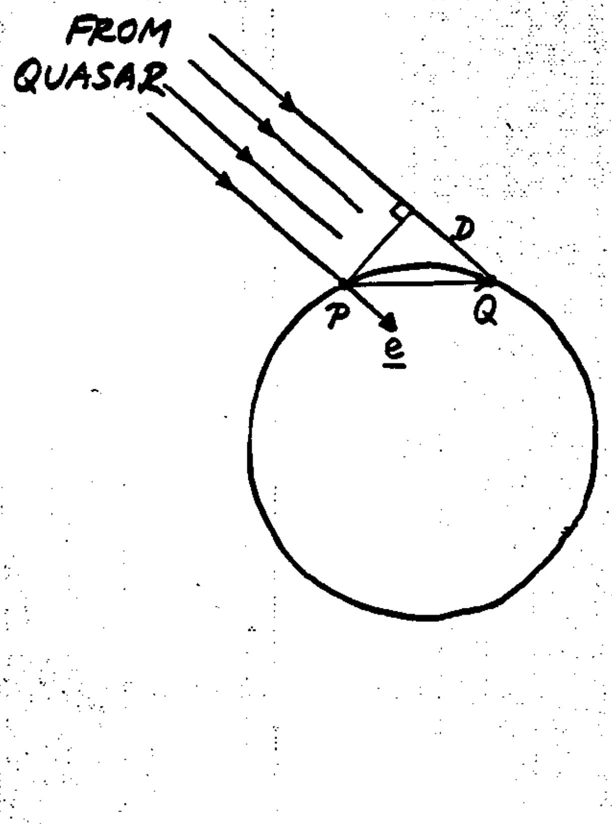
ALTIMETRY	GEOS-3 (1975-1979)	0.5 m
	SEASAT (3 MO. IN 1978)	0.2 m
DOPPLER	TRANSIT/TRANET	1 m
	NAVSTAR/GPS (1985)	0.3 m
LASER	LAGEOS	0.1 m
	STARLETTE	

GRAVITY FIELD

GRAVSAT A (1986) SST

GRAVSAT B (1990) GRADIOMETRY

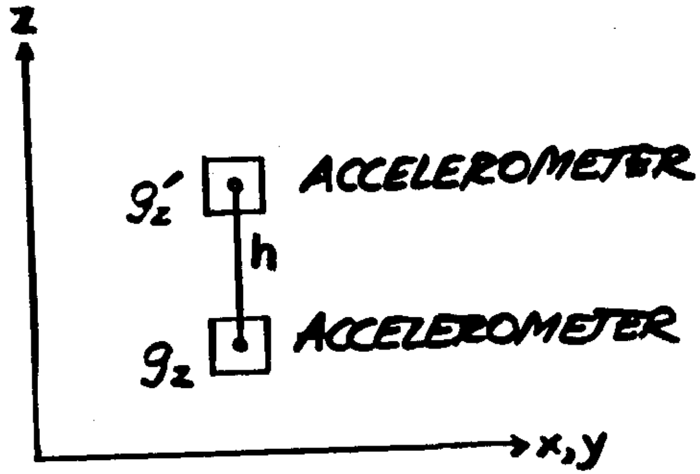
VERY-LONG-BASELINE INTERFEROMETRY



$$\vec{PQ} \cdot \underline{e} = (\underline{x}_Q - \underline{x}_P) \cdot \underline{e} = D$$

$\underline{e}$ ... unit vector

GRADIOMETER AS DIFFERENTIAL ACCELEROMETER



$$W_{zz} = \frac{\partial^2 W}{\partial z^2} = \frac{\partial g_z}{\partial z}$$

$$W_{zz} = \frac{\partial g_z}{\partial z} \doteq \frac{\Delta g_z}{\Delta z} = \frac{g_z' - g_z}{h}$$

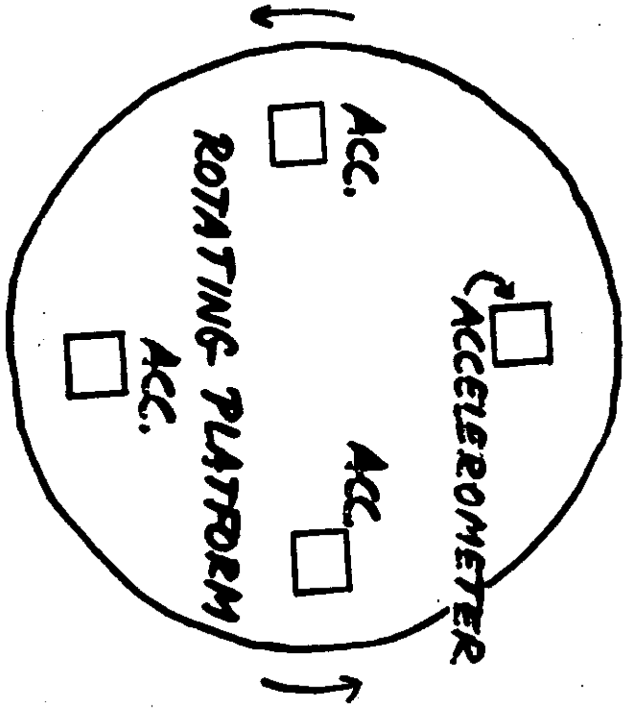
SECOND-ORDER GRADIENTS

EÖTVÖS TENSOR

$$E = \begin{bmatrix} W_{xx} & W_{xy} & W_{xz} \\ W_{yx} & W_{yy} & W_{yz} \\ W_{zx} & W_{zy} & W_{zz} \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix}$$

$$W_{12} = W_{xy} = \frac{\partial^2 W}{\partial x \partial y}$$

BELL AEROSPACE DIV.



DATA COMBINATION PROBLEM

$l_i = f_i(X, W)$

$$\left.\begin{array}{lr} \text{parameter vector} & X \ldots\ldots p \\ \text{potential} & W \ldots\ldots \infty \end{array}\right\} \text{unknowns}$$

number of observations .... q

LINEARIZATION

$A\,\delta X + B\,\delta W = L$

LINEARIZED PROBLEM

$$AX + BT = L$$

$$\boxed{MY = L}$$

$$m_{11} y_1 + m_{12} y_2 + m_{13} y_3 + \cdots = l_1$$
$$m_{21} y_1 + m_{22} y_2 + m_{23} y_3 + \cdots = l_2$$
$$\vdots$$
$$m_{q1} y_1 + m_{q2} y_2 + m_{q3} y_3 + \cdots = l_q$$

$$\boxed{\underset{q \times \infty}{M}}$$

SOLUTION: $\boxed{Y = M^{-1} L \ ?}$

VARIATIONAL PROBLEM

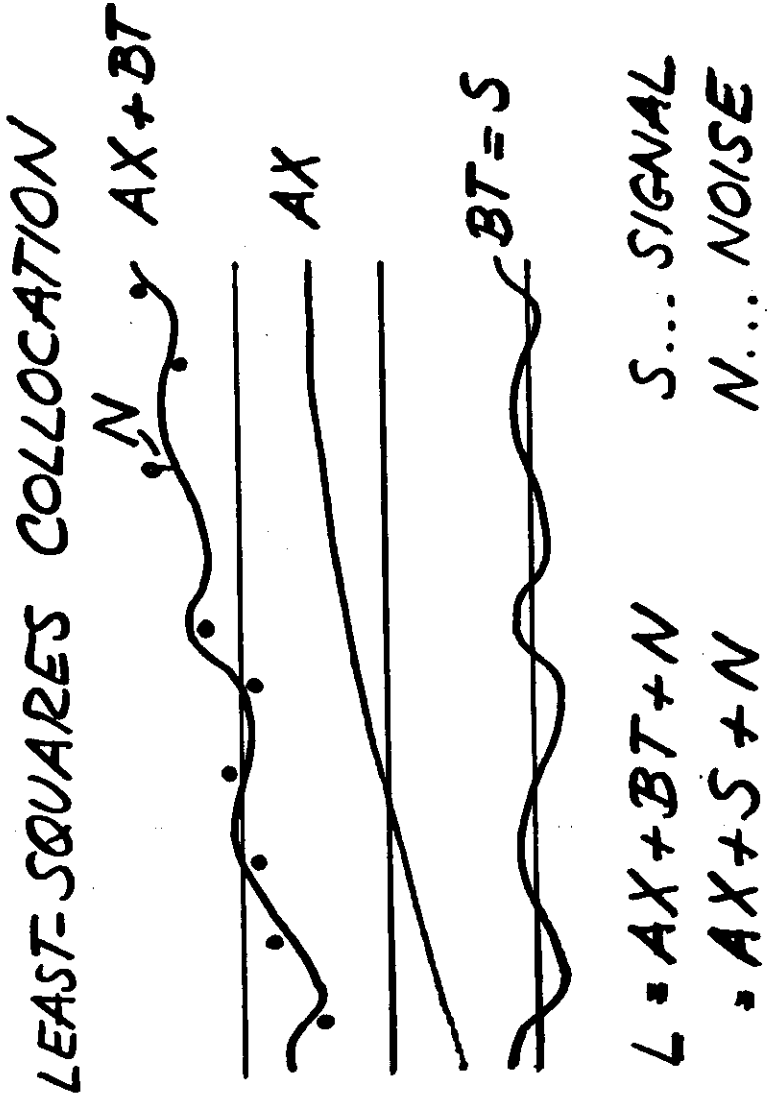
$$L = S + N = BT + N \qquad (X = 0)$$

$$\alpha \|T\|^2 + \beta \|N\|^2 = \text{MINIMUM}$$

FUNCTION NORM — ERROR NORM

SOLUTION: KOLMOGOROV-WIENER

$$T_p = \begin{bmatrix} C_{p1} & C_{p2} & \cdots & C_{pq} \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1q} \\ C_{21} & C_{22} & \cdots & C_{2q} \\ \vdots & & & \vdots \\ C_{q1} & C_{q2} & \cdots & C_{qq} \end{bmatrix}^{-1} \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_q \end{bmatrix}$$



FUTURE PROBLEMS

TEMPORAL VARIATIONS:
CONTINENTAL DRIFT
PLATE TECTONICS

EARTH ROTATION
IRREGULARITIES

PRECISE AND FAST POSITIONING
GLOBAL POSITIONING SYSTEM (GPS)

DETAILED GRAVITY FIELD

RELATIVISTIC EFFECTS

ISOSTASY

MOUNTAINS RISEN LIKE A FERMENTING DOUGH

BOŠKOVIĆ 1755

J.H. PRATT 1854

G.B. AIRY 1855

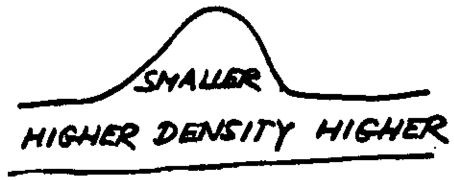
J.F. HAYFORD 1909 INT. ELLIPSOID

W. A. HEISKANEN 1928 INT. GRAVITY FORMULA

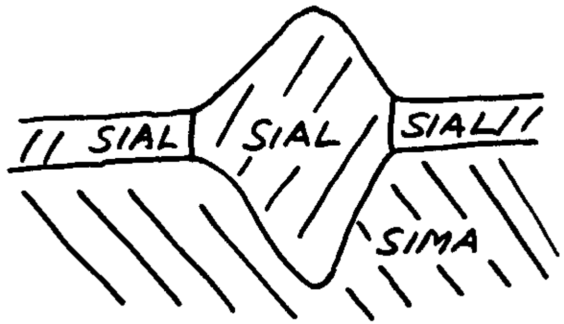
H. SÜNKEL 1987 GEOID IN AUSTRIA

2 MODELS FOR ISOSTASY

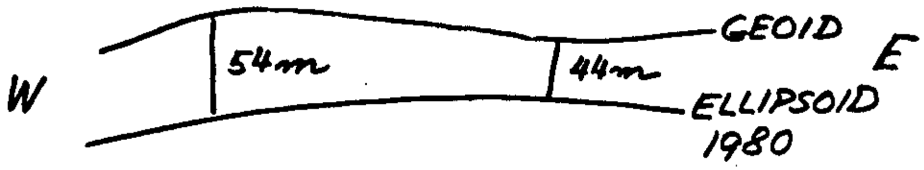
BOŠKOVIĆ-PRATT



AIRY



GEOID IN AUSTRIA



$44 m < N < 54 m$

TOPOGRAPHY + ISOSTASY: MAIN PART

RESIDUAL CORRECTIONS FROM
GEODETIC MEASUREMENTS
(LATITUDE, LONGITUDE, GRAVITY)!

$-1.3 m < \delta N < +1.6 m$

BY LEAST-SQUARES COLLOCATION