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WORKSHOP
GLOBAL GEOPHYSICAL INFORMATICS WITH APPLICATIONS TO RESEARCH IN EARTHQUAKE PREDICTIONS AND REDUCTION OF SEISMIC RISK

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BLOCK MODEL OF EARTHQUAKE SEQUENCE

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Block model of earthquake sequence

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Introduction. Mathematical models of lithosphere dynamics are tools for the study of the earthquake preparation process and are especially useful in earthquake prediction studies. Available data often do not constrain the statistical significance of premonitory patterns determined empirically before a few large events. An adequate model should indicate the physical basis of such premonitory patterns, and suggest new premonitory patterns that might exist in real catalogs.

We describe here a model that produces an artificial catalog of seismicity. The properties of this catalog can be compared with and contrasted to the properties of real seismicity catalogs. By this means it is possible to determine which properties of an earthquake sequence can be derived from simple assumptions about the processes that occur in the lithosphere. We consider both the instability of the process and the possibility that strong earthquakes might be predicted.

Earthquake sequences in real catalogs have some general properties that appear in spite of different geological structure and level of seismicity in differing seismic regions. The sequence of earthquakes is apparently stationary; no noticeable trend has been discerned in the level of seismicity during the 100 or so years of detailed studies of world wide seismicity. There is a considerable stochastic component in the earthquake sequence; the sequence of main shocks, earthquakes that remain in a catalog after aftershocks are excluded, is close to Poissonian. On this stationary stochastic background different regular patterns appear. The best known of them is linear frequency of occurrence law. Another type of regular behavior is migration of earthquake epicenters along geological structures.

Earthquakes often appear in clusters. One common clustering pattern is a main shock followed by a series of its aftershocks characterized by Omori's law. The aftershocks can also produce their own aftershock series with the same properties. Foreshock activity is not so clearly expressed as aftershock sequences since the number of foreshocks of strong earthquakes is usually small or zero while aftershock series of such earthquakes can contain hundreds of events. Finally, there are "swarms" or series of earthquakes that are impossible to divide into main shock, foreshocks and aftershocks because they have similar magnitudes.

The seismic cycle implies characteristic time intervals between the strongest earthquakes in a given seismic region and different characteristics of seismicity in the periods before a strong earthquake, after a strong earthquake, and in the period between consecutive strong earthquakes. This is not a cycle in usual sense of this word since the periods between the strongest earthquakes are not equal and can deviate considerably from the average characteristic period. Changes of characteristics of seismicity during seismic cycle are unclear and can probably only define the time of a forthcoming strong earthquake within 20-50 years. For more accurate prediction of strong earthquakes different types of premonitory seismicity patterns, such as abnormal clustering, activation-quiescence, strong variation of seismic activity in time, have been found.

Although there is no adequate theory of the seismo-tectonic process, different properties of the lithosphere, such as spatial heterogeneity, hierarchical block structure, different types of nonlinear rheology, gravitational and thermodynamic processes, physico-chemical and phase transitions, fluid migration and stress corrosion are probably relevant to these properties of seismicity. The qualitative stability of the properties of earthquake sequences in different seismic regions suggests that the lithosphere can be modelled as a large dissipative system that does not essentially depend on the particular details of the specific processes active in a geologic system.

Dissipative nonlinear systems appear in many scientific and engineering problems [1], and have common general properties. They are usually unstable so that small variations in initial data or slight external disturbances cause considerable changes in behavior of the system after a finite period. This instability leads to apparently stochastic behavior of comparatively simple deterministic systems, and makes it practically impossible, using the information on the current state of the system, to predict its state after some characteristic time period. This stochastic character of a nonlinear system does not exclude the possibility of prediction but defines an upper limit for valid prediction. For small times the system keeps information on its initial state whereas for larger periods the information is lost and behavior becomes unpredictable.

These systems display different types of self-organization that arise in the process of evolution of the system. The important characteristic of self-organization is an attractor of the system. If all the phase space trajectories of the system eventually move closer to a subset of the phase space we call this

subset an attractor. An attractor could be a complicated structure such as a Kantor set but its dimensionality is usually less than that of the full phase space. This often permits adequate description of multi- or infinite-dimensional systems by a comparatively small number of parameters. Unfortunately, we do not have explicit constitutive equations for the lithosphere as a nonlinear system, the characteristic regularities of earthquake sequences are different from standard patterns of dynamical systems [1], and we do not even know what kind of nonlinear system the lithosphere might be. Nevertheless, the artificial catalog produced by the model displays many of the known properties of earthquake sequences, including premonitory seismicity patterns, as well as some typical features of dissipative nonlinear systems.

The model exploits the hierarchical block structure of the lithosphere[4]. Blocks of the lithosphere are connected by comparatively thin, weak, less consolidated boundary layers, such as transition zones, lineaments, tectonic faults. In the seismo-tectonic process major deformation and most earthquakes occur in such boundary layers. The boundary layers themselves have block structure and can be considered as systems of blocks of lesser scale.

In the model a seismically active region is represented as a system of absolutely rigid blocks divided by infinitely thin boundary layers. Relative displacement of all blocks is supposed to be infinitely small relative to their geometric size. The system of blocks moves as a consequence of prescribed motion of boundary blocks. As the blocks are rigid, all deformation takes place in the boundary layers. The corresponding stresses depend on the value of relative displacement of blocks. This dependence is assumed to be linear elastic until strength thresholds are exceeded. At that point this boundary layer breaks, stress drops, and these breaks are considered as earthquakes.

The presence of different time scales is an essential property of the model. The boundary blocks move with a constant rate in "slow", or tectonic time. The slowness of this time means that the whole system of blocks should be in a quasistatic equilibrium state in which the sum of all the forces and moments acting on each block is zero. After a break the system of blocks moves to a new equilibrium state in "fast" time, while the position of the boundary blocks is unchanged. During this motion some other boundary layers can be broken. When the system reaches a new equilibrium state elastic interaction and strength are restored and slow time resumes. This healing is necessary to ensure the process is stationary.

An important assumption is contained in the statement that boundary blocks move at a constant rate and that earthquakes occur under essentially constant displacement boundary conditions. The model assumes that the stresses that drive plates do not directly affect seismotectonic zones but that rather the seismo-tectonic zones are controled by the constant motion generated by these plate driving stresses. If we were to drop this assumption and treat the model as having constant stress boundary conditions earthquake ruptures in the model would never reach equilibrium.

As slow time evolves an artificial catalog of earthquakes is generated. Each event is characterized by its time of occurrence, source or the pattern of breaks, and magnitude defined as the logarithm of the energy released by the system of blocks between the old and the new equilibrium states.

Description of the model. The seismically active medium is represented in the model by a system of rigid, two-dimensional poly-

gonal blocks whose position is determined by the position of a point in the block and rotation angle about this point (x, y, φ). All relative displacements of the blocks are supposed to be infinitely small in comparison to their geometric size. The blocks are connected by infinitely thin and linear elastic boundary layers.

The elastic interaction is defined as follows. If z is common to two blocks, and δz is the vector of relative displacement of these blocks at the point z, we define δz_t and δz_n as tangent and normal component of δz. Positive δz_t corresponds to the right shear, a negative sign corresponds to the left shear. Positive δz_n correspond s to extension, a negative sign corresponds to compression. Tangent and normal stresses σ_t and σ_n at the point z are defined as

$$\sigma_t = K_t \, \delta z_t, \quad \sigma_n = K_n \, \delta z_n. \qquad (1)$$

K_t and K_n are shear and compressional elastic constants of the boundary layer. All the boundary layers are divided into small segments and stresses are calculated only at the centers of such segments.

The system of blocks moves according to the prescribed motion of boundary blocks. This motion is supposed to be slow, so that at each moment the system of blocks is in quasistatic equilibrium state. This means that sum of all forces and moments acting on each block is zero. Here the force F_i acting on i-th block is the integral of stress over the boundary of this block, and moment M_i is the integral of the moments of the stresses. Since the relative displacements of all the blocks are infinitely small the equilibrium equations

$$F_i = 0 \text{ and } M_i = 0 \qquad (2)$$

are linear in x_j, y_j, and φ_j. This results in 3N linear equations for 3N variables, where N is the number of blocks.

The elastic energy of the system of blocks is the integral of

$$dE = (\delta z_t\ \sigma_t + \delta z_n\ \sigma_n)/2$$

over all the boundaries. The energy E is quadratic in x_j, y_j, and φ_j. It is positive definite if there is at least one boundary block and the system of blocks is connected. The solution of (2) coincides with minimum of energy E. So the system (2) is not degenerate.

The formulas (1) for elastic interaction are valid only when stresses are below certain strength thresholds $\sigma_n = C$ and $|\sigma_t| = -A\ \sigma_n + B$. The first corresponds to tensile strength and the second corresponds to "dry friction", or the Byerlee law [2]. If one of the thresholds is exceeded, a break occurs and stress drops.

Three possible types of failure, tension fracture, right shear, and left shear, are possible in the model. A tension fracture appears if $\sigma_n > 0$ at the moment of the break. After this $\sigma_n = 0$ and $\sigma_t = 0$. A right (left) shear failure appears if $\sigma_n \le 0$, and $\sigma_t > 0$ ($\sigma_t < 0$) at the moment of the break. After this $\sigma_n = K_n\ \delta z_n$ is defined in the same way as for elastic interaction, and $\sigma_t = -a\ \sigma_n$ ($\sigma_t = a\ \sigma_n$) is defined according to a dynamic friction law, $0 \le a \le A$.

This change of the stresses implies a change of the equilibrium state which can be computed on the assumption that no other failures occur. We assume at this point that the system of blocks starts moving towards this new equilibrium state in "fast" time. This means that the position of the boundary blocks does not change during this motion. During this motion it is possible that strength thresholds may be exceeded elsewhere. Once this occurs a new equilibrium state is computed and the motion continues in a new direction towards a this equilibrium state until the system of blocks either stabilizes or another failure reorients the fast timew evolution of the system.

Once the system stabilizes elastic interaction is restored with shear slip value Δ taken into account by the stress values

$$\sigma_t = K_t(\delta z_t - \Delta), \quad \sigma_n = K_n\ \delta z_n. \qquad (1')$$

The value of Δ is defined by the requirement that the stresses do not change in the process of healing:

$$\sigma_t = K_t(\delta z_t - \Delta) = a\ \sigma_n.$$

Correspondingly, expression for elastic energy should be rewritten as

$$dE = ((\delta z_t - \Delta)\ \sigma_t + \delta z_n\ \sigma_n)/2.$$

A tension fracture heals only when the normal component of relative displacement equals 0 in the process of motion. The value Δ of shear slip is defined in this case by

$$\sigma_t = K_t(\delta z_t - \Delta) = 0$$

at the moment of healing. After the healing the system of blocks continues to move in "slow" time, until the next break.

This process of transition of the system of blocks from one equilibrium state to another is considered an earthquake in our model. An earthquake is characterized by its time moment in "slow" time, its source composed of all the broken boundaries, the first of which is the epicenter, and the elastic energy released by the system of blocks. The sequence of earthquakes produced by the model forms an artificial catalog.

Results of calculations. A 3-layer system of 24 blocks including 4 boundary blocks (Fig. 1a) was taken for the calculations. The boundaries were divided into edges of the length 0.25, so for a typical block its horizontal boundary was divided into 8 edges, and its vertical boundary was divided into 4 edges. Parameters of the model defining interaction and failure conditions were taken the same for all the edges:

K_t = 0.6, K_n = 1., A = 0.6, B = 0.5, C = 0.5, a = 0.

The upper and lower boundary blocks are moving in opposite directions, as it is shown by arrows on Fig. 1a, with constant rate v = 1.5, and the right and left boundary blocks are rotating with constant rate -1.0, in agreement with the motion of the first two boundary blocks. All the boundary blocks are initially shifted towards the center of the picture to create hydrostatic pressure. The value of this shift is 5.0. Initial values of inelastic slip A are taken to be 0 for all edges. Its energy-frequency relation (Fig. 2a) is close to linear for the values of energy release between 0.25 and 25. Decay of the graph below these values is connected with minimal possible size of fracture, and decay above these values is connected with the size of the system. Several regular patterns can be found in the system, such as quiescence, foreshocks, migration of epicenters. There are practically no aftershocks in the catalog. It could be expected, as the model has no time delay mechanism that may produce aftershock activity. A bigger 5-layer model of 54 blocks (Fig. 1b) with the same parameters as the 3-layer model but v=2.5 was also considered. The earthquake sequences in both models appear to have similar properties. For example, energy-frequency relation for the 5-layer model (Fig. 2b) is similar to that for the 3-layer model.

Instability. Dependence on initial data was studied for the 3-layer model in the following way. First, a segment of an artificial catalog was calculated for the period of time T between 0 and 100 to exclude influence of zero initial data for T=0. After this the value of inelastic slip A for one of the edges was changed into A+ε, where ε varied from 0.1 to 10^{-6}, and calculation continued. Fig. 3 presents plots of the total energy of the system of blocks as a function of time for ε=0.01 and for undisturbed variant. The two plots are practically the same for $T < T_{cr} \approx 25$, then they begin to diverge, and finally the two catalogs are absolutely different. The same behaviour was found for other variants, T_{cr} varying from 15 to 150. This behaviour is typical for unstable nonlinear systems, and imposes a theoretical restriction on the possibility of prediction for the time intervals larger than these characteristic values of T_{cr}.

Premonitory patterns. Algorithm CN for intermediate-term prediction of strong earthquakes [6] has been applied to the artificial catalog generated by the model. This algorithm uses parameters of seismic regime that were found empirically for the California catalog for prediction of strong earthquakes with $M \geq 6.4$. It was then successfully applied in other regions [7].

Parameters of seismicity are presented as function of time, defined on a sequence of main shocks in the region within a sliding time window.

Level of seismic activity

N - number of main shocks with the magnitude above some threshold

Σ - the total energy of main shocks

G - the ratio of numbers of main shocks in two magnitude range

Seismic queiscence

q - "deficiency" of the seismic activity

Variation of seismicity

K - Difference between numbers of main shocks at two successive time-intervals

Spatial concentration

S_{max} - the average area of fractures at foci

Z_{max} - the ratio of the average radius of fractures at foci to the average distance between them

Clustering in time and space

B_{max} - maximal number of aftershocks

Some combinations of these functions for the time periods before strong erthquake - periods D and the periods of time far from strong erthquake - periods N had been found in [6] for California and Nevada catalogs. These combinations are called attributes D and N respectively.

To apply algorithm CN to the artificial catalogs we have to transfer conventional units of energy release and time into magnitude and time that correspond to a real catalog. To do this, magnitude is defined as

$$M = \alpha \log(\Delta E) + \gamma,$$

where ΔE is elastic energy release, and the coefficients $\alpha = -1$ and $\gamma = 5$ are chosen so that the slope of the magnitude-frequency distribution is close to 1, and the gap in this distribution is at $M = 6.3$, the same as in California catalog. Time scale 2 (one unit of time in artificial catalog corresponds to 2 years) is chosen so that earthquakes with $M \geq 6.4$ appear, on the average, once in 7 years, as it is in California.

Algorithm CN. Time of Increased Probability (TIP) of appearance of strong erthquake is announced in the region at the moment t if:

1. $A(t) = n_1 - n_2 \geq \underline{A}$,

2. $\sigma(t) \geq \underline{\Sigma}$.

Here n_1 and n_2 are numbers of attributes D and N, $\sigma(t)$ estimates the total fracture area for three preceding years. The Tip lasts a year. The TIP may be cancelled: either if a strong earthquake with $M \geq M_{\Sigma}$ ($\sigma(t) \geq \underline{\Sigma}$) occurred, or swarm of earthquakes with total energy more then $\underline{\Sigma}$ occured (this swarm may include the earthquakes with $M \geq M_0$). Consecutive TIPs may overlap and prolong each other. If no strong earthquake occured during the TIP it is called a false alarm; strong earthquake which was not preceded by a TIP is a failure to predict. The total duration of TIPs is called alarm time .

To apply this algorithm to the artificial catalogs, generated by 3-layer and 5-layer model, we have to exclude all attributes containing the function B_{max}, because the artificial catalog does not include aftershocks. All the rest parameters were not changed.

First we studied initial 52 years of the artificial catalog, generated by 3-layer model, the same duration as in California [6].

The values $A(t), \sigma(t)$ were computed for the discrete moments of time with 2 months step. The thresholds $\underline{A} = 3, \underline{\Sigma} = 2.6$ are chosen to get best results of prediction. After this we have applied this rule to examine the rest 668 years of the artificial catalog.

The same rule for TIP has been applied then to the artificial catalog, generated by 5-layer model.

Results of TIP diagnostics. Result of prediction for two artificial catalogs are shown in Table 1 (the first line of the table corresponds to the three layer model, the second one corresponds to five layer model). TIPs cover 30% of total time and precede 47% of strong erthquakes.

It is rather difficult to estimate the significance of prediction, because the alarm periods depend on the moments of strong earthquakes due to the condition (2) in the definition of algorithm CN.

Table 1

Results of TIP diagnostics for artificial catalogs.

N	Time interval for diagnostics(in year)	Number of strong earthquakes		Duration of TIP	
		total	inside TIP	years	%
1	720	92	43	218.2	31.7
2	720	89	42	202.8	29.5

The results of prediction for the artificial catalogs have been compared with the results of the retrospective prediction for real catalogs in several regions [6,7,8]. For this purpose the artificial catalogs have been divided into time intervals compatible with duration of the real catalogs. The results are presented in Table 2. Quality of prediction for real and artificial catalogs is almost the same. The smaller number of failures-to-predict in the real catalogs can be explained by the difference in adjusting parameters of the algorithm for these two cases. In the real catalogs discretisation thresholds, magnitude M_0 of strong

earthquakes and level Σ of energy release were redefined for each region. In the artificial catalogs these parameters were found for the initial 52 year period and fixed for the other segments of the catalogs. It is possible that the number of failures to predict in the real catalogs will increase when we shall start forward prediction.

The results of prediction of strong earthquakes in the artificial catalogs using CN algorithm suggest that our model reflects some real regularities of the earthquakes preparation process. This result is also important for verification of the CN algorithm itself: for the first time it was applied here to a catalog long enough to allow statistical verification of the results of prediction.

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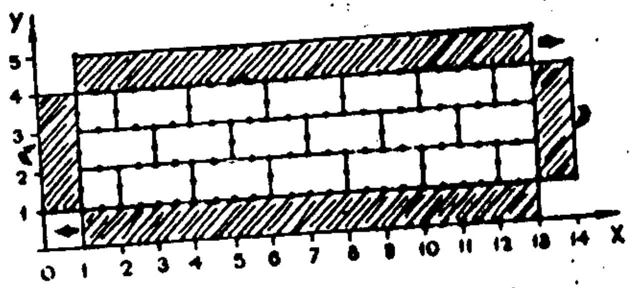
Table 2

Results of prediction for artificial catalog and real catalogs of seismic regions of USSR.

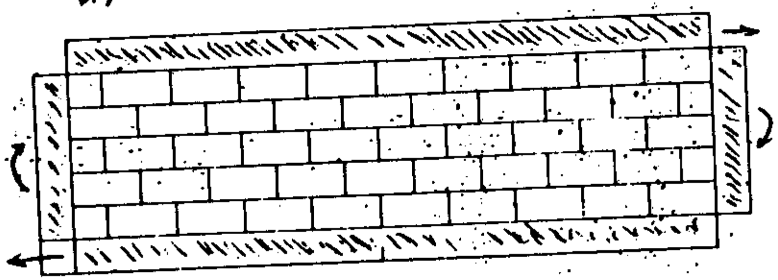
artificial catalog			real catalogs		
number of earthquakes	number of successes	alarm time %	number of earthquakes	number of successes	alarm time %
5	3	25	4	3	23
5	2	36	3	2	11
8	3	21	1	1	16
6	3	32	3	2	17
6	3	27	3	2	34
4	1	39	2	2	25
5	1	34	4	4	36
5	2	40	3	2	31
9	5	19	6	6	51
4	3	40	2	2	19
5	2	26	5	3	18
7	3	40	3	3	25
4	4	32	6	5	27
5	4	27	9	9	49
7	3	21			
7	1	16			

References

1. Haken H., 1983, Advanced synergetics, Springer-Verlag, 356p.
2. Byerlee J.D., 1978, Friction of rocks. Pure and Appl. Geophys. 116, pp. 615-626.
3. Burridge R., and L. Knopoff, 1967, Model and theoretical seismicity, Bull. Soc. Seismol. Amer. 57, pp. 341-371.
4. Cundall P.A., 1971, A computer model for simulating progressive, large-scale movements in blocky rock systems. Symp. Internat. Soc. Rock Mechanics, Nancy, France, 12p.
5. Gabrielov A.M., V.I. Keilis-Borok, T.A. Levshina, and V.A. Shaposhnikov, 1986, Block model for dynamics of the lithosphere. Computational seismology Iss. 19, pp. 168-178, Moscow, Nauka
6. C.A. Allen, V.I. Keilis-Borok, I.M. Rotvain, K. Hatten, 1986, A set of long-term seismological precursors; California and some other regions. Computational seismology Iss. 19, pp. 23-37, Moscow, Nauka
7. O.E. Dmitrieva, V.I. Keilis-Borok, V.G. Kosobokov et al, 1987, Identification of the periods of incresed probability of strong earthquake in seismoactive regions of USSR and others countries. Computational seismology Iss. 20, pp. 99-110, Moscow, Nauka
8. O.E. Dmitrieva, I.M. Rotwain, V.I. Keilis-Borok, M. De Becker. Premonitory seismicity patterns in a platform region (Ardennes - Rhenish and Brabant massives, lower Rhine graben) (in press)



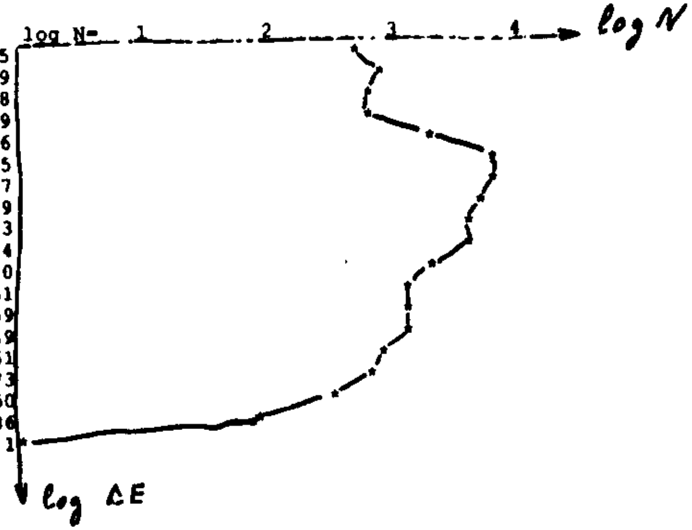
a)



b)

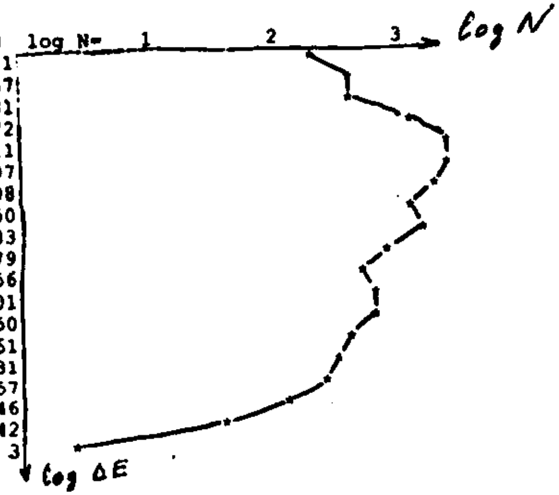
Fig. 1 System of blocks a) 3-layer model, b) 5-layer model

	de	N
2	0.025	625
3	0.040	979
4	0.063	668
5	0.100	729
6	0.158	2266
7	0.251	6795
8	0.398	7307
9	0.631	6089
10	1.000	4843
11	1.585	4074
12	2.512	2280
13	3.981	1561
14	6.310	1559
15	10.000	1549
16	15.849	961
17	25.119	673
18	39.811	350
19	63.096	86
20	100.000	1



a)

	de	N
2	0.025	241
3	0.040	457
4	0.063	481
5	0.100	1272
6	0.158	2811
7	0.251	2797
8	0.398	2108
9	0.631	1450
10	1.000	1633
11	1.585	879
12	2.512	556
13	3.981	701
14	6.310	650
15	10.000	461
16	15.849	331
17	25.119	257
18	39.811	146
19	63.096	42
20	100.000	3



b)

Fig. 2. Energy / frequency distribution

a) 3-layer model, b) 5-layer model

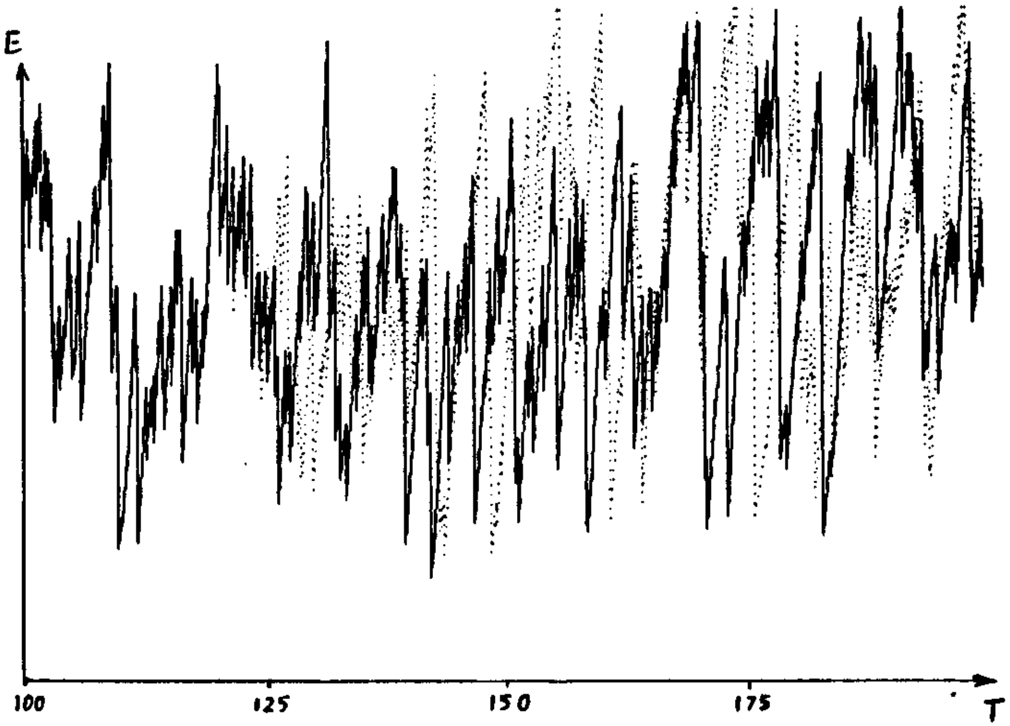


Fig. 3. Energy as function of time (100 ≤ T ≤ 200):

——— - undisturbed system

...... - disturbance 10^{-2} at T=100.