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WORKSHOP
GLOBAL GEOPHYSICAL INFORMATICS WITH APPLICATIONS TO RESEARCH IN EARTHQUAKE PREDICTIONS AND REDUCTION OF SEISMIC RISK

(15 November - 16 December 1988)

COMPLEXITY OF EARTHQUAKE PRECURSORS
- The Dimension Reduction Model of Earthquake Precursors

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Foreword

Like making predictions in other scientific fields, prediction of earthquakes is now facing difficulties.

The problem of making predictions is confronted with in other scientific disciplines. As an example at hand, according to Newton's mechanics, solar and lunar eclipses can be predicted on the basis of a number of given initial values, like predicting the motions of other celestial bodies. This is a kind of deterministic prediction based on established physical laws. Inspired by the success in predicting the motions of the celestial bodies, many people tend to think that all predictions can be made deterministically in principle, and that we can predict the future of everything just according to their past and present situations. Nevertheless, in order to reach this goal, we need sufficient observational informations.

Such conventional ideas are challenged by problems arising from the following respects:

a) The theoretical research is attacked by the development of modern physics. Many studies in geo–sciences are based on deterministic physical theories. At the turn of the century, the physicists mostly continued their studies in a number of classical research topics. Most of them admitted that the basic laws of the universe (including the geo–sciences) were deterministic and reversible. They believed that errors in predicting the future based on such deterministic and reversible laws were caused by factors connected with man, i.e. by the complexity arisen from our ignorance or from our failure to control the corresponding variables. We are now in the end of this century, and through our cerebration, more and more people believe that the numerous fundamental processes which had brought the nature into shape are at the very beginning non–reversible and random, and that the existing deterministic and reversible laws describing the basic interactions in the nature can not tell us the true and panoramic features of the nature. The gap between the deterministic and the probabilistic descriptions in physics is now diminishing and converging. Such deep–going changes require us to reconsider our studies on the prediction problems.

b) Problems also come from our observational practices. Another scientific method in making predictions is to make statistic analogies. Through observing and studying various phenomena emerged prior to a certain earthquake, we assume that they are "precursors" of that event. Afterwards, whenever we detect such precursory phenomena, we might be able to forecast that another shock is on the way. During the past 20 years, scientists both in and outside China had invested tremendous amount of manpower as well as huge sums of money on such work, and had gained abundant data together with rich experience and, of course, lessons. The most important problem among our gained knowledge is that we have, up to the present, not found a particular "precursor" which emerges before every earthquake, and that no precursor has been found to be surely followed by the occurrence of an earthquake. The nature is seemingly mocking the arduous and earnest work of the scientists. After all, what is a seismic precursor? This is a problem confronting scientists all over the world. The setbacks and challenges encountered in searching for the seismic precursors have made some scientists reconsider the traditional guidelines in science: Is it because that we have not yet found such ideal precursors just because we have not time long enough for our observation, or because we haven't exerted ourselves hard enough? Or is it because we have some problems in our scientific guidelines, i.e. we have problems in our prediction methods? In other words, perhaps our previously–assumed precursors did not exist at all in the first place, or the seismic precursors are just too sophisticated, and we have just set up an over–simplified target to search for.

On the eve of any breakthrough in a branch of science, such characteristics is generally observed: If we try to explain the natural phenomena by the established theoretical framework of science and using the conventional observational methods, the problems we find are generally more than solved ones. In earthquake prediction, we are just facing such a situation.

Natural phenomena can be classified into simple and complicated ones, while science starts its development from the study of simple phenomena. Any achievement obtained in the study of simple phenomena will not only enhance people's respect to the Goddess of Science, but also add to the treasures of mankind because of the methodology formed when solving those simple phenomena. On the other hand, when science develops into a stage to solve more complicated phenomena, would these methodologies be effective tools or restraints to people's minds? All these are acute as well as important problems confronting the scientists.

The cybernetics, system theories and information theory founded in the 1960's are primarily used in the discussion of engineering and technical systems and remarkable achievements have been obtained through the discussions. Later, the dissipative structuring theory, synergetics and catastrophy theory came into shape one after the other. These are used in the discussion of the huge natural systems

which are far more complicated than the engineering and technical systems. Today, "exploring the complexity" has become the common goal for various branches of science.

This paper is aimed at exploring the complexity of seismic precursors. Using the phenomena discovered in recent years, the author intends to emphatically illustrate the complicated feature of these phenomena. It is also the author's intention to show the contradictions which might be encountered when simple methods are used to tackle these sophisticated phenomena. The paper also discusses the methods which can be used to solve these contradictions and the possible perspectives of the study. Hopefully, such discussions would call for more cerebrations and discussions in our seismological studies.

I. The Complexity of Seismic Precursors

Facts from experiments aimed at observing the precursors occurred prior to rock failures in the laboratory are cited here to demonstrate the general features of the complexity of seismic precursors.

During our experiment using marble specimen for uniaxial compressions, we use laser holography method to measure the deformation field on the whole surface of the specimen during the process of the rock failure, so as to search for the precursors before the failure occurs at the said specimen. For a convenient illustration, we choose on the specimen points A, B, C and D and analyse their situation of deformation (Fig.1) . In the early stage of the deformation at point C, deformation increases along with the increase of stress. Just before the specimen ruptures, deformation increases drastically. Therefore, so far as point C is concerned, the drastic increase of deformation can be regarded as the precursor of rock failure. At point B, however, the precursor of rock failure is represented by abrupt decrease of deformation. The fact that completely different precursors are observed respectively at points C and B fully shows that precursors present very complicated spatial distributions. We may ask what is actually the precursor of rock failure? As revealed by the experiments, it would never be possible to find the same precursors of rock failure at all points, namely, we can not expect all the points to show accelerated deformation or vice versa before the rupture occurs. It can also be observed from Fig. 1 that synchronized changes can be detected at points A,B,C and D in the period of low stress, which suggests uniform deformation is taking place within the rock specimen. Approaching the rupture point, remarkable differences appear in the changes at respective points, some of which show accelerated deformation, some show attenuated deformation, some even remain unchanged. We can hardly tell which one among the 3 phenomena (accelerated, attenuated or no deformation) should be taken as the precursor of rock failure. Nevertheless, we can tell that the increase of differences in the spatial distribution of deformation is the short–term precursor of rock failure. The phenomena shown in Fig.1 is obtained from experiment of rock deformation, yet such a result brings about overall significances. In our earlier experiment looking for precursors of rock failures, we practically made the same assumption that the physical properties of the rock media are homogeneously distributed during the entire deformation process until the rupture takes place. Therefore, all the observational results obtained so far tend to reflect averaged values of certain physical properties. In this case, it is understandable that details like the inhomogeneous spatial distribution of rock properties are ignored. Unfortunately, we find the valuable precursory information of rock failure mostly lies in these ignored details. With the development of technical equipments since the 1980's, major achievements obtained in the study of precursors are concentrated in the study of such details like the inhomogeneous spatial distribution of deformations. Both in and outside China, similar results have not only been got in the measurement of deformation of rocks shown in Fig.1, but also in the measurement of physical values like the velocity of elastic waves and the attenuation in the rock specimens. In the theoretical and laboratory study of the localization of rock deformations, similar results are also obtained.

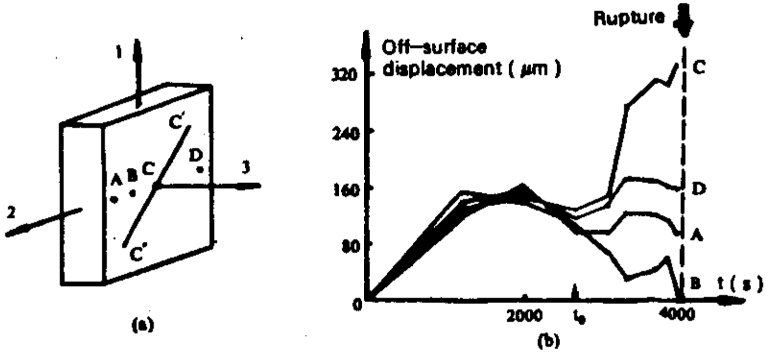


Fig.1 a) The off-surface displacement occurred during deformation of the rock samples observed using the laser holography technique. A,B,C and D denote 4 points on the surface of the specimen. Their locations corresponding to the final rupture plane C'C' are plotted in this figure. b) The off-surface displacement respectively at points A,B,C and D (corresponding to the vertical deformation of the ground surface dependent on time t of the deformation). Note that when rupturing is approached, the 4 points present remarkably different situations. Increase of such difference in deformation at the respective points might bethe precursors of the rupturing, whereas deformation at each of the points can hardly be taken as an overall precursor.

A set of holograms by the laser interferometry technique in an rock deformation experiment using marble samples are shown in Fig.2. The holograms are arranged in the order of stress increases. In the pictures, the fringes represent the isopleths of the off-surface displacement, possibly similar to the contours on the topographic

maps. As can be seen from Fig.2, very few fringes are observed at the initial stage of deformation, and the entile sample shows uniform deformation. Along with the stress increases, the process in which deformation concentrates at a certain locality is initiated. We observe not only increases of the fringes, but also changes in the pattern of the fringes. The deformation pattern on the entile surface of the specimen becomes more and more complicated. In order to figuratively illustrate the changes in spatial distribution of different physical properties during the deformation process of rock samples, we borrow two special terms from the Chinese Beijing opera denoting the types of facial make-ups: "Bailian" (white facial make-up) to represent the simple patterns whereas "Hualian" (colored facial make-up) to represent the more complicated patterns.

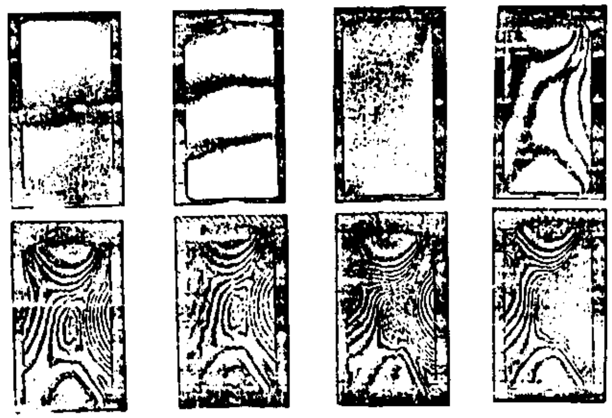


Fig.2 Laser interferograms taken during the deformation process of the marble sample. σ_u stands for the strength of the sample, whereas σ for the stress under which the pictures are taken. The fringes show the isopleths of the off-surface displacements of the tested sample. As can be seen from the pictures, the distribution of deformation becomes more and more complicated along with the increase of stress σ / σ_u , and the patterns become more and more complicated when time progressively draws closer to the rupture.

It is worth noting that the above descriptions only reflect one of the rock-rupturing tests. In many other experiments done repeatedly, we find such pre-rupturing deformation patterns are just non-repeatable no matter how carefully we prepare the test samples. For example, we cut one sample after another from the same piece of competent mother rock, getting each sample into exactly the same geometrical size so as no differences are detectable even using the most precise measures. We use exactly the same environment for the experiments and even use different rocks to prepare the samples, the pre-rupturing deformation patterns still do not repeat the pictures taken in the above-illustrated test. Nevertheless, one common feature is observed, i.e. different rock samples all show their deformation distribution from simple into more and more complicated patterns along with stress increases.

To sum up the above discussions, if we take one point on a certain rock sample as a seismic station, and the pre–rupturing deformation changes observed at that point as the seismic precursors, we can see the following in the light of the afore–described experiment: a) Dissimilar precursors are observed at different points before the sample ruptures, or in other words, the phenomena observed at one point are hard to be found at another. b) The pre–rupturing precursors on one rock sample mostly differ from those observed on another. Nevertheless, if we take all rock samples into consideration, the pre–rupturing precursors of them all show one common feature: the spatial distribution of the deformation develops from simple into more and more complicated patterns. Such a common feature can well be taken as the precursor of rock failure. The implication of such a precursor is based on the observation of more complicated phenomena, and should therefore be distinguished from the implications of the traditional seismic precursors.

Fig. 3, 4 and 5 furnish the data of in situ observation on wellwater level and ground deformation before the occurrence of natural earthquakes. These cases exactly explain for the above–mentioned features.

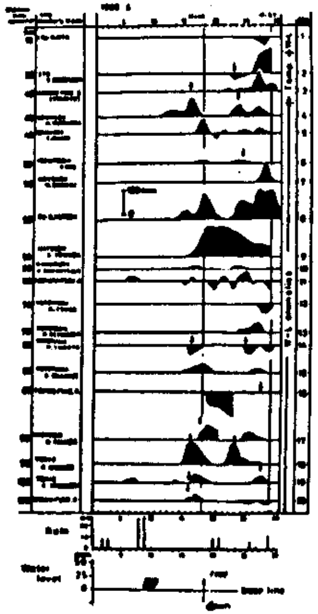


Fig.3 Water level variations of 20 wells presenting anomalies in an area about 120 km around the epicenter of the June 29, 1980 Izu peninsula, Japan earthquake of *M* 6.7. Note that when drawing closer to the occurrence of the shock, all wells show drastic abnormal variations in their water levels, some of which rose, and some dropped. Actually, no two wells showed the same kind of variations. The variation of water levels forms spatially an extraordinarily complicated pattern.

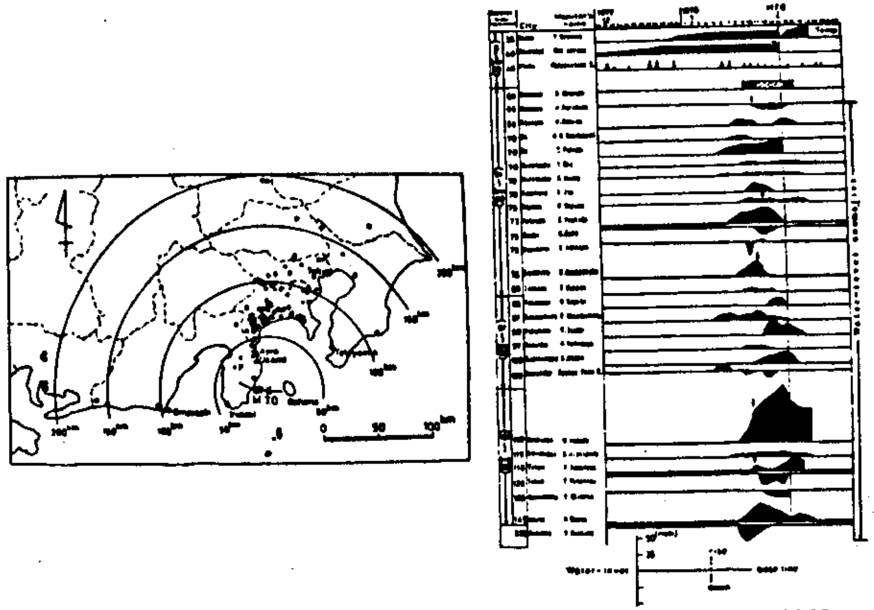


Fig.4 Variations of ground water level observed before the Jan. 14, 1978 Oshima, Japan earthquake of *M* 7.0.

a) The plane distribution of wells used for the ground water level observations. All the empty circles denote the observational wells, while the solid ones denote those wells which showed pre–shock anomalous variations. In the map, "X" stands for the epicenter of the shock. Equidistance circles respectively for 50 km, 100 km, 150 km and 200 km from the epicenter are also plotted in the map.

b) Abnormal orientation changes of water levels observed at respective wells. The dark areas denote the differences between the water level and the values extrapolated from the previous variation tendencies. The arrows denote the time at which the observers report the anomalies to the authorities. The dissimilar modes of variations observed at different wells at unequal focal distances are shown in this figure. Note that when coming closer to the shock, water levels of some wells rose while those of others dropped, showing a most complicated pattern.

The complexity of the spatial distribution of the seismic precursors can also be expressed by the order in the systems. But what is order? Taking a school for example, when the students are having free activities on the playground, they scatter everywhere. In the light of macroscopic statistics, the density distribution of students on the playground is nearly uniform. This corresponds to the disorder mode. But when these students are required to perform group callisthenics, they no longer show uniform distribution on the playground no matter how the pattern of the callisthenics is altered. This corresponds to an order mode. The development process of the seismic precursors is very much similar to such situations. The process in which the complicated patterns are formed practically corresponds to the one in which the system switches from the disorder into the order mode.

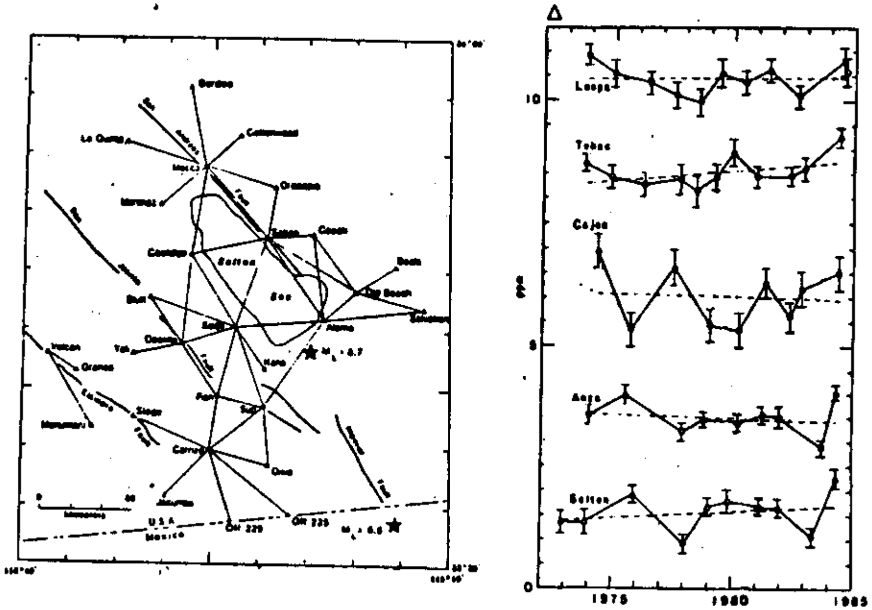


Fig.5 Precursors of ground deformation prior to the M 6.6 event occurred in 1979 at the Imperial Valley along the border areas between the USA and Mexico.

a) The star stands for the epicenter of the M 6.6 event. A ground deformation observatory network formed by a triangulation array was emplaced to the north of the epicenter.

b) 5 observatories in the network had detected the time—dependent variations of unit area stress (Δ) . In the figure, the dotted lines denote the occurrence time of the event. As can be seen from the figure, ground deformation registered at respective stations shows remarkably great differences. Some observatories present dilatations while others contraction, showing a very complicated pattern.

In the succeeding contexts, we intend to make a deeper discussion into these problems. The key problems being discussed will be concentrated on the following: How should the complexity of the seismic precursors be described? What are the factors responsible for these complexity? Let's start the discussion from the first problem.

Ⅱ. The Self—similar Phenomenon in the Seismic Activities

We begin our discussion with the description of the complexity of seismic precursors. Many phenomena in nature possess layered structures, in which there exists a special category, namely, when we properly enlarge or reduce the geometrical size of the structure, the structure itself remains unchanged. We define such a structure as the self—similar one. The geometrical objects having self—similar features are called the fractals. Using such structuring characteristics, many complicated phenomena can be described.

By properly altering the sizes, the characteristics of the fractals are most clearly seen. If we have a geometrical object with d dimensions, we can enlarge it along each of its independent directions to l times that of the original. As a result, we obtain N objects out of the original one. The relation between these 3 numbers can be expressed by $l^d = N$. We now take the logarithm of both sides of the equation:

$$d = \frac{\ln N}{\ln l} \tag{1}$$

in the equation, d thus defined is referred to as the fractal dimension. In simple geometric patterns like a cube, we get $d=3$ from the above formula. This agrees with the concept of the topological dimensions. As to complicated geometric patterns, the characteristic number d used to describe the self—similar structures is not necessarily an integer. For such complicated patterns, we do not enlarge their geometric sizes to l times that of their originals, but reduce the measurement unit to ε. By continuously reducing ε, we can make accurate measurements. The factors N and l in formula 1 used to define the fractal dimension should be changed into $N(\varepsilon)$ and ε. It is also necessary to note whether the limit still exists when ε is continuously reduced.

$$d = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln \varepsilon} \tag{2}$$

The above formula shows another definition of the fractal dimension d[①].

One of the simplest examples of the self—similar structures is the problem of the length of the coastal lines. In geometry, a straight line is the simplest pattern. The

① The following 3 definitions of the fractal dimensions are commonly seen:

1) The volumetric or Hausdorff dimension:

If $N(\varepsilon)$ is the minimum number of small balls with diameter ε enough to cover a point set, the volumetric dimension of the point set can thus be defined:

$$d = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln \varepsilon}$$

2) The information dimension:

When defining the volumetric dimension, only the number of the needed balls is taken into consideration, whereas the number of points covered by each of the small balls are not classified. Hence the information dimension d_1 is proposed.

$$d_1 = \lim_{\varepsilon \to 0} \frac{\ln \sum P_i \ln(\frac{1}{P_i})}{\ln \varepsilon}$$

the probability $N(\varepsilon)$ of a point covered by the i-th ball is expressed.

3) The correlation dimension d_2:

$$d_2 = \lim_{\varepsilon \to 0} \frac{\ln C(\varepsilon)}{\ln \varepsilon}$$

where $C(\varepsilon) = \frac{1}{N^2} \sum_i \sum_j H(\varepsilon - |y_i - y_j|)$

y_i, $i=1$, 2 ,... N is the solution sequence of the system.

$$H(\varepsilon - |y_i - y_j|) = \begin{cases} 1 & (\varepsilon - |y_i - y_j| \geq 0) \\ 0 & (\varepsilon - |y_i - y_j| < 0) \end{cases}$$

coastal lines are a sinuous line which presents curvatures of considerable sizes, and still smaller curvatures are contained in each of the curves. Using different rulers to measure the length of the coastal lines, we get different lengths. Obviously, "length" can not be used as a good quantitative characteristics to reflect such a complicated curve like the coastal lines. If we use another ruler with length L to measure the coastal lines, we find the following relations between p and L:

$$p \propto L^{1-d} \tag{3}$$

This shows the coastal lines are self–similar①. The fractal dimension is thus a more pertinent concept to make quantitative descriptions of the complicated coastal lines as compared with the straight line.

A more complicated example can be cited from the description of the fracture surface of a piece of rock or the plane of a natural fault. Such a fracture surface is not a simple plane, but a complicated, rough and uneven boundary. In the science dealing with materials, when the fracture surface of a piece of metal material is ground by a small portion, we get a number of isolated platforms which are similar with the islands in the ocean. According to the size of the material being ground gradually, the total area A of the platforms and their total circumference L also follow the self–similar relations:

$$\ln A = a + b \ln L$$

where d denotes the fractal dimension of such a fractal. Experimental measurements using similar methods show that the fault and the nodal planes in the rocks are also fractals. Therefore, fractal dimension is the most pertinent quantitative description of such complicated fracture planes as compared with the simple planes

A still more complicated situation is the earthquake activities. Such earthquake activities also portray a self–similar structure. This characteristics can be demonstrated through the analysis of the seismic energy and the temporal and spatial distribution of seismic activities.

1. Energy fractal

The relation between earthquake magnitude and frequency can be found as the following:

$$\lg N = a - bM \tag{4}$$

This is the well–known Gutenberg–Richter formula which is the most classical empirical one in seismology. In the formula, M stands for earthquake magnitude, N for the total number of earthquakes greater than magnitude M. a is a constant and finally, b is a parameter of the b–value used to express the proportional relations between strong and small events, and is thus an important parameter expressing the structural features of a group of earthquake activities.

① Using p as a ruler to measure the coastal lines, we use the ruler only 1 time. But when we use L as the ruler, the length of the ruler is enlarged by $l = \frac{1}{L}$ times. Therefore, $N = \frac{p}{L} = (\frac{1}{L})^d$. Hence formula (3).

The relation between the seismic energy E and the magnitude M can be expressed by:

$$E = 11.8 + 1.5M \tag{5}$$

We assume that the seismic energy is proportional to the cubic size of the focal region, i.e. the seismic energy is converted from the strain energy stored in the focal body, and that d is the fractal dimension of the self–similar system relevant with the length L of the seismic fault, then we have

$$nL_n^d = L_d \tag{6}$$

From formula 5, $\lg L_n^3 = 1.5M_n + C_1$ then we have

$$L_n = C10^{M_n/2}$$

Substituting into formula 6:

$$n10^{dM_n/2} = C^{-d}L^d$$

By taking the logarithms, we have:

$$\lg n = \lg C^{-d}L^d - \frac{d}{2}M_n$$

Comparing with formula 4, we get:

$$b = \frac{d}{2} \tag{7}$$

As can be seen from formula 7, and in the light of energy analysis, seismic activities present self–similar features, and there is a very simple relation between the structural fractal dimension d and the b–value of the shocks.

2. The temporal fractal of the earthquakes

The distribution of seismic events occurred in a given area on the time axis is actually an assemblage of a large number of points which are discontinuous with unequal density on different part of the time axis. We divide the time period since the historic records were started into sub–periods having durations of ε, and respectively make statistics on the distribution of events of $M > M_0$ events. Then we use $N(\varepsilon)$ to register the total number of sub–periods in which there are earthquake occurrences and plot the diagram for $\lg(\frac{1}{\varepsilon}) \sim \lg(\varepsilon)$, we obtain the fractal dimension of the time fractal of the earthquakes:

$$d = \frac{\lg N(\varepsilon)}{\lg(\frac{1}{\varepsilon})}$$

3. The spatial fractal of earthquakes

Taking the focal body as one point, we can tentatively divide the seismic region into sub–areas of equal sizes, the non–dimensional area of each sub–area is denoted by . We make statistic work on each of the sub–areas in order to find out the number $N(\varepsilon)$ of the ones having zero values. In the diagram $\lg(\frac{1}{\varepsilon}) \sim \lg(\varepsilon)$, we can locate the linear areas, the slope of which is the fractal dimension. This shows that the spatial distribution of earthquakes is also fractal.

It is worth mentioning that no matter temporally or spatially, the linear areas in the $\lg(\frac{1}{\varepsilon}) \sim \lg(\varepsilon)$ diagram exist only in a certain segment. This segment defines the range of a scaleless area in which the self-similar phenomena exist. In our discussion of the fractals and the fractal dimensions, this range should not be exceeded, because the non-scale law holds only in such a range.

There are also other empirical formulas expressing the fractal structures of earthquakes, for example, $\lg M_0 \propto 1.5 \lg S$, in which M_0 denotes the seismic moment, S denotes the area of the earthquake-generating fault plane. Although the distribution of seismic precursors is extremely complicated, we are lucky to find that the earthquakes themselves present the overall characteristics of self-similarity and fractal. Such a regularity extensively exists not only in the macroscopic earthquake occurrences, but also in the microscopic fractures. The occurrence of earthquakes shows broken and inhomogeneous fractures not only temporally but also spatially and thus possess fractal characteristics. The seismic events fill up only a small part in the 4-dimensional time and space, leaving many empty cavities and pores. The existence of the fractal structures is a common feature in the complicated system of earthquake preparation. Therefore, the fractal dimension can be used to demonstrate the regularity in this sophisticated phenomenon.

Ⅲ. The Dimension Reduction Characteristics of Earthquake Precursors

Returning to our study of the seismic precursors, one may ask what should after all be taken as the characteristics of seismic precursors?

When a sheet of paper is torn, a failure line (straignt or curved) will be formed. When a rock sample is fractured, a failure surface (plane or curved) will appear. The failures tend to show the following features of dimension reduction: failure in a 2-dimensional body results in a one-dimensional failure line, whereas that in a 3-dimensional body produces a 2-dimensional fracture surface. In other words, failure always occurs within a limited volume in a body. All the above-mentioned phenomena denote the outcome of the fracture.

We are now confronted with such questions: What is the situation before the failure takes place? Does the spatial distribution of various failure precursors present the tendency of dimension reduction? Are there local variations in the respective properties of rocks? What are the regularities of such variations if they really exist? How can those characteristics of dimension reduction be applied into our prediction of the failures? Obviously, all these constitute the most fundamental and important problems for our understanding of the rupture mechanics as well as the discussion of fracture forecasts.

The experimental result from the measurements of the rock deformation and wave velocity of the rocks samples or the analysis on the microscopic samplings

shows that the number of fractal dimensions is continuously reducing during the rupture process of the rock samples despite that dissimilar fractal features are observed at respective times. In other words, the distribution of precursors, no matter spatial or temporal, shows with no exceptions features of dimension reduction. This is most clearly seen in the measurement of the b-values. In the previous section, we illustrated the following relation between the number of fractal dimensions and the b-values: $b = \frac{d}{2}$. No matter in the observed natural earthquakes or in the laboratory experiment of acoustic emissions of rock samples, the b-values decrease with no exceptions before the rupture takes place.

Recently, Kong Fanchen analysed the active linear structures of China. He found the active structures are also fractal, the number of fractal dimensions of which is related with the complexity of the structures. The size of the fractal dimension numbers controls the displacement and the rate of motion of the active structures as well as the features of seismicity. Larger fractal dimension number suggests more complicated texture of the active structures.

The self-similar process of faultings is sure to produce fractal structures. During the deformation process, such a process of strain localization occurs within the rocks: When the exterior load increases, there first occurs homogeneous deformation and the micro-fractures distribute evenly over the whole space. It is obvious that $d = 3$ at this stage. In the light of thermodynamics, the system has the greatest entropy and shows the most orderless structure. When the deformation reaches a certain extent, clusters of micro-fractures begin to emerge in the interior of the rocks. Such a process of micro-fracture concentration, or the formationing process of the dissipative structures in which a stable inhomogeneous state is bifurcated out of the non-stable homogeneous state, is a self-similarone corresponding to the structural orders of the earthquake-generating areas. This process has resulted in the fractal distribution of the acoustic emissions or the microseisms which reflects the system has switched from a non-order state into an order one. This is not hard to understand, as the degree of symmetry of the system in which $d = 3$ is surely greater than that of the system in which $d = 2$, and hence its entropy is also greater, whereas its order is smaller. The phenomenon of the "white face" changing into the "coloured face" during the deformation process can either be expressed by the variation of the fractal dimensions in the parameter distribution, or by the variation of orders in the system.

Such being the case, what should be the relationship between the order of the system and its number of structural dimensions? We first make the following simple deduction:

In the light of the Boltzmann formula $S = k \lg W$ in which W denotes the number of freedoms of the system, the thermodynamic entropy has certain correlation with the information entropy of the system in some restricted situations. Therefore, we can roughly calculate the thermodynamic entropy from the information entropy of the system. The entropy S of the system can be expressed by:

$$S = -k\int_{\Omega} P(r)\lg P(r)d\sum$$

where k stands for the Boltzmann constant and $P(r)$ for the distribution density of events at point r in the space. For a cylindrical coordinate system relevant only with r, Fig. 6a has the following relations:

$$S = -2\pi k\int_{0}^{A} P(r)\lg P(r)dr$$

As we know, in the equal–probabilistic distribution, S will surely take its maximum value. When the deformation just begins, $P(r) = \frac{1}{2\pi A}$ (see Fig. 6b) . Therefore, $S = k\lg W$ is the maximum value.

When the rock sample enters its inhomogeneous deformation stage, its inner parameters tend to show fractal distributions, i.e. $P(r) = r^{-d}$. In the formula, $d = D_T - D$ where D_T denotes the topologic dimensions in the occupied space and . With a view to eliminate the unreasonable singularity at $r = 0$, it is assumed the probabilistic distribution takes the following form (Fig. 6c) :

$$P(r) = \begin{cases} \beta r^{-d} & r_0 \leq r \leq A \\ \alpha & r \leq r_0 \end{cases}$$

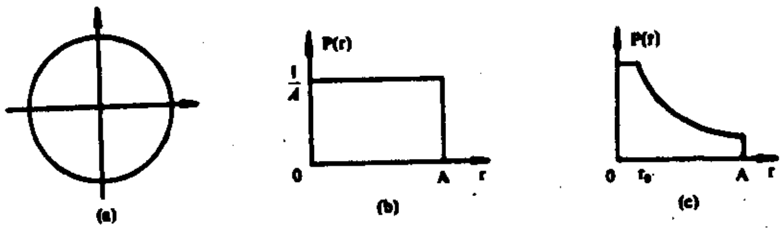


Fig. 6 Diagram used in calculating the entropy.

where r_0 is physically defined as the nucleus–forming radius of a certain physical value, e.g. the displacement. In the light of the continuous and normalizing conditions, we have:

$$\alpha = \beta r_0^{-d}$$

$$\alpha r_0 + \beta\frac{A^{1-d} - r_0^{1-d}}{1-d} = \frac{1}{2\pi}$$

Then entropy S is:

$$S = -2\pi k\int_{0}^{r_0} \alpha\lg\alpha dr - 2\pi k\int_{r_0}^{A} \beta r^{-d}(\lg\beta - d\lg r)dr$$

$$= -2\pi k r_0\alpha\lg\alpha - 2\pi k\beta\lg\beta\frac{A^{1-d}}{1-d} + 2\pi k\beta d\frac{A^{1-d}\lg A}{1-d} - 2\pi k\beta d\frac{A^{1-d}}{(1-d)^2}$$

When $\frac{A}{r} << 1$, the above formula can be simplified as:

$$\alpha = \frac{1-d}{2\pi r_0}\left(\frac{A}{r_0}\right)^{d-1}$$

$$\beta = \frac{1-d}{2\pi}A^{d-1}$$

Substituting into the previous formula, we have:

$$S = k(1-d)^2\lg\frac{A}{r_0} + k(1-d)\lg\frac{2\pi r_0}{1-d} + (1-d)k\lg A + k\lg\frac{1-d}{2\pi} - k\frac{d}{1-d} + C$$

where C is a constant used for corrections. Considering when $D = 3$, $S = k\lg A$. We have:

$$S = 2k(D-2)^2\lg\frac{A}{r_0} + k\lg r_0(D-2)2 - k(3-D)\lg\frac{2\pi}{D-2} - k\frac{3-D}{D-2}$$

where the characteristic measure of r_0 is 10^{-3}m and A is approximately 10^5. This testifies the dimension reduction process occurred in the deformation of the rock samples is represented by the decrease of entropy and the increase of order of the system.

Ⅳ. Exploring the Complexity of Seismic Precursors

Different phenomena showing the complexity of the seismic precursors have already been introduced in the above chapters. In this chapter, we intend to make a preliminary discussion on the reasons responsible for such a complexity.

Let's start from the meteorological situations, because this is a case which involves the existence of the complexity and is thus dealt with in numerous studies. In 1963, Lorenz, a meteorologist from the MIT, USA, had found that complicated situations similar with those in seismic precursors also exist in the weather forecastings. In order to illustrate the difficulties in making long–term weather forecasts, he had, using the dynamics equations, proved the constant solution of the atmospheric motions is a non–periodic one which is later called the chaos solution. The equation governing the atmospheric circulations is a partial differential one. Using the Fourier transformation approach, Lorenz took the first 3 orders and proposed that the non–dimensional coefficients x, y and z of those first 3 orders have decided the mode of the atmospheric motions. These coefficients follow the differential equations:

$$\begin{cases} \frac{dx}{dt} = -10x + 10y \\ \frac{dy}{dt} = 28x - y - xz \\ \frac{dz}{dt} = -\frac{8}{3}z + xy \end{cases}$$

At a certain time t, x, y and z satisfying the above equations constitute one status point in the phase space (x,y,z) . When time t changes, the status points in the phase space will join one another into a status locus. We now take a closer look at the shape of the status locus in the phase space of the Lorenz equation.

Considering the status locus in the phase space xyz defined by the above equation group, an ellipsoid can be found, e.g.:

$$14x^2+5y^2+5(z-56)^2=18000$$

with the advancement of time, all the status loci can only fall into an area enclosed by such an ellipsoid disregarding their initial modes. The Lorenz equation has 3 fixed points, namely, the solutions when $\frac{dx}{dt}=\frac{dy}{dt}=\frac{dz}{dt}=0$: (0, 0), ($\pm 6\sqrt{2}$, $\pm 6\sqrt{2}$,27). Among them, point (0,0) stands for the stationary mode, whereas the latter 2 points stand for regular circulations. Nevertheless, stability analysis made on those 3 points shows that they are all non–stable, and are thus modes that can only be approached but never reached. Through testification made so far, people also find that no periodic solution can be obtained for the Lorenz equation, which shows that the status loci can never form closed loops. Therefore, the loci of the constant solution in the space xyz are formed by a number of infinitely–long and infinitely–numerous winding curves, just like a ball of thread. All those loci distribute in the confined areas of the phase space and they never intersect each other at any point (Fig. 7) .

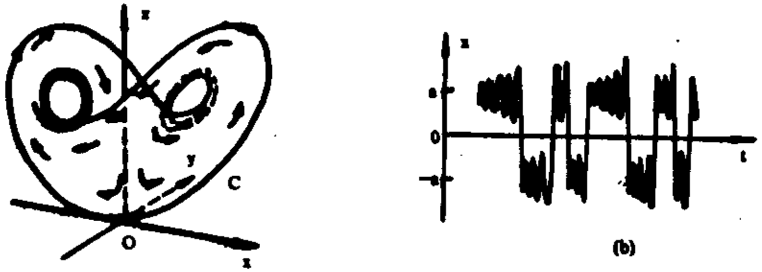
From the physical point of view, the characteristics of the solutions of such a non–linear dynamic equation can be analysed as the following:

1. The status loci of the atmospheric motions tend finally to reach their limit assemblage whose number of dimensions is smaller than that of the initial phase space. Such an assemblage is referred to as an attractor. Therefore, in most of the areas in the phase space, the system is not returnable.

The evolution in which the high dimension phase space contracts into the low dimension attractor is actually a process to converge the number of freedoms. In this process, great number of patterns with quicker motions and smaller sizes would be consumed and the number of the long–term effective freedoms which determine the system will be decreased. Many freedoms become irrelevant variables in the evolution process and finally only the few freedoms supporting the attractors would be left.

The attractors in the Lorenz equation can be classified into the strange ones. As we know, if the system at time t tends to be in a constant mode irrelevant with time, i.e. the fixed point in the phase space, it is a zero–dimension attractor. If only one periodic motion is left in the system at time $t\to\infty$, it is a one–dimension attractor corresponding to the limit loop formed by a continuous curve in the phase space.The zero– and one–dimensional attractors all belong to indifferent attractors. The attractor in the Lorenz equation is somehow different compared with the above–mentioned ones. It belongs to the strange attractor. The "strangeness" of those strange attractors can be illustrated as follows: a) Being the subsets in the

phasespace, the strange attractors often have fractal structures and non–integral dimensions; b) The motions on the strange attractors are very sensitive to the initial conditions. Even very small changes on the part of the areas entering the strange attractors would result in completely different locus of the motions. Furthermore, it is very difficult to predict the location of a status point at the next time just from its position at any given time t.



(a) (b)

Fig.7 a) Diagram showing the time–dependent variation mode of the Lorenz equation in the phase space xyz. The loci of all ultimate mode form a subset in the phase space, and is referred to as the Lorenz strange attractor. This attractor does not change with the evolution of the system. The shape of such an attractor can be imagined as the following: We bend a long piece of wire into the shape of curve C in the figure and then cover the whole frame with a thin film of soapsuds. Then we pierce the film to get 2 holes at the fixed points ($\pm 6\sqrt{2}$, $\pm 6\sqrt{2}$,27) .We further imagine that the soapsud film is consisted of infinitely–numerous layers of lobes just like the one illustrated, and all the lobes are joined to one another only at the wire frame. Such a strange attractor are filled with infinitely numerous status loci.
b) The $x-t$ curves corresponding to each status locus in which $a=6\sqrt{2}$. One of the status parameters is shown in this figure in which the variation of x is very much the same with the smooth random process. Such a random process is produced by the deterministic Lorenz equation. With the advancement of time, the system show a kind of spontaneous, constant, prolonged and irregular movement, or in other words, chaotic motions.

2. The dynamic system of the atmospheric motions is composed of two parts, namely, its status and dynamic characteristics. The status characteristics refers to the basic situations of the system, or a point at the center of the phase space, whereas the dynamic characteristics is the law illustrating the time–dependent variations of the status. Such a variation process can be described by the locus in the phase space. The Lorenz equation is a deterministic one which is composed of only several simple coefficients, and not any random terms are included into it. If we take a look at its solutions, the locus in the attractor circles on infinitely, showing a kind of random behaviour. Such a random nature is a fundamental characteristics which would not disappear no matter how much information we can collect. The random nature thus formed is more and more extensively termed as "chaos". The word

"chaos" is one among the oldest ever existed in Chinese vocabulary. Citing Chinese methodology for example, the creator of the universe Pan Gu is said in China's legendary tales to be born in the chaotic state between the heaven and the earth. "Chaos" denotes a genuine and original natural state. The word is also used to describe a state most extensively existing in nature.

Analysis of the Lorenz equation shows such a chaotic behaviour originates from the sensitivity of the deterministic system depending on the initial conditions. In the light of the quantum-mechanics, the initial measurement is forever non-deterministic. With the lapse of time, such a non-deterministic feature will increase in the order of the geometric or arithmetic progressions. Such a chaotic phenomenon will make the non-deterministic feature grow so rapidly that any deterministic prediction becomes totally impossible. In physics, chaotic phenomena can be regarded as synonymous to complexity.

3. After the emergence of the chaotic phenomena, deterministic predictions of the system become extremely difficult. Nevertheless, chaos also show its own regularity which paves the way for the establishment of the conceptional basis for solving the prediction problems. Firstly, the status of the system does not distribute in the whole phase space with the advancement of time. In stead, it contracts into a subset in the phase space. This is fully demonstrated in that the evolution of system is neither deterministic nor random. The mean value of a physical quantity on the strange attractor is stable and non-sensitive to the initial values. Secondly, the distribution of the status of the system in a subset like the strange attractor has its deterministic-probabilistic values. The discovery of such new behaviours tends gradually to fill up the gap between the deterministic and the probabilistic descriptions in physics.

In the light of the above-mentioned Lorenz equations, we can see that the complexity is associated with the non-linear mechanic equation.

The time-dependent variation process of the dynamic system may occur either in the continuous time, or in the scattering time. The former is termed as "flow" whereas the latter "projection". Another example for the scattering time can be cited as the following:

Up to the present, it is hard to get an equation group in the dynamics to illustrate the seismic process. Yet it is possible to use an approach similar to the Rizhchinko method to demonstrate the features of earthquake predictions using the energy equation of earthquakes as an example. A focal region $\Omega(x_i)$ ($i=1,2,3$, definition remains unchanged in the succeeding contexts) can be taken for instance. For the media outside that region, $\omega_0(x_i)$ denotes the average input energy exerted into the space by the exterior media over a long time, $\omega_C(x_i)$ denotes the energy released by the media during the earthquake, $\varepsilon(x_i, t)$ stands for the density of the strain energy of the media, and finally, ε_m refers to the maximum energy density that can be built up in the media, or the energy strength. On the basis of energy

balance, we have

$$\frac{\partial\varepsilon}{\partial t}=\omega_0-\omega_C$$

It is possible to consider that when the strain energy ε in the media is larger and the built-up energy is closer to the media's strength ε_m, $\omega_C(x_i)$ would also be greater. Hence we have:

$$\omega_C=K\varepsilon\left(\frac{\varepsilon}{\varepsilon_m}\right)=K\frac{\varepsilon^2}{\varepsilon_m}$$

where K stands for the proportional constant. Combining the above two formulas and rewrite them into the iterative equation, we have:

$$E_{n+1}=1-\mu E_n^2 \tag{8}$$

where $\mu=\frac{K\omega_0}{\varepsilon_m}+\frac{3}{4}$, $E=\frac{4K\varepsilon+2\varepsilon_m}{4K\omega_0+3\varepsilon_m}$, in which E_m denotes the strain energy in the media before the n-th event, and refers to the variations of the strain energy taking place in the media both before and after the n-th event. Equation 7 is identical in form with the insect population equation in ecology and is thus a non-linear dynamic equation. If we confine the parameter in formula 7 within the range (0,2), then the formula can be considered to be a non-linear image starting from line segment $I=(-1,1)$ to itself. So long as is chosen from I, will also fall within I. Through calculation, we find the iterative equation 7 shows many abrupt changes when parameter μ goes from 0 to greater values (see Fig. 8) .

a) When $0<\mu<0.75$, if an initial value is chosen randomly within line segment I, the iterative equation tends to reach rapidly a fixed point E^*. It suggests physically that the strain energy in the media would remain unchanged within such a parameter range: $E_n=E_{n+1}=E^*$.

b) When $0.75<\mu<1.25$, after a short iteration, value E_n would alternately appear in the two modes E_1^* and E_2^*:

$$E_1^*=1-\mu E_2^{*2}$$

$$E_2^*=1-\mu E_1^{*2}$$

This is a 2-point period.

c) After $\mu>1.25$, the above-listed 2-point period loses its stability. With the increase of the μ-value, there appears stable 4-point, 8-point or even 2^n-point periods. The corresponding stable range becomes narrower and narrower. This is called the multi-period bifurcation. When $\mu=1.40115....$ the infinite period is reached. At this time, if E_n is given, the E_{n+1} thus iterated within a specified range takes random values. The above-mentioned characteristics emphatically suggests that the random behaviour has appeared in the simple and deterministic system which is restricted by the energy equation.

As a matter of fact, such situation is often found when the system becomes just a little more complicated. Nussbaum has simulated the seismic process using a model composed of 2 sliding blocks. The equation illustrating the system is deterministic yet the motions of the two blocks present random features under specific parameter conditions (Fig. 9) . Like the equation of the seismic energy discussed in the afore—going paragraphs, the simple and deterministic system would also present random behaviours. The solution of the above—mentioned energy equation can be cited as an example of the chaotic problems, a number of whose characteristics can also be found in our practical earthquake problems. For instance, the self—similar structures in the energy equation near the bifurcation point can also be observed in the self—similar structures between the strong the small events of natural earthquakes.

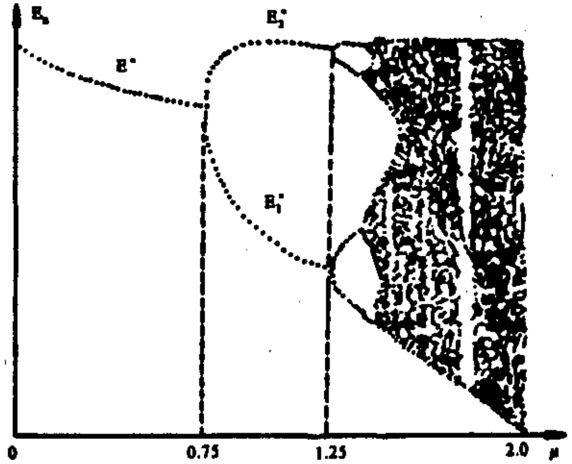


Fig.8 The relation between the solution of the iterative equation 7 of seismic events and the parameter μ.
When $0 < \mu < 0.75$, the iteration process rapidly tends to reach the fixed point E^*, suggesting no variations are taking place on the part of the strain energy in the media.
When $0.75 < \mu < 1.25$, the E^* value alternately presents two modes E_1^* and E_2^*, each of the projections corresponds to the deterministic variations in the strain energy. At $\mu = 0.75$, the equation presents a two—period bifurcation.
After $\mu > 1.25$, there may occur 4—point, 8—point ... 2^n—point periods. When E_n is given, the E_{n+1} thus iterated may take random values and the system presents chaotic features.

We now come back to the former example of energy equation. When parameter takes the specified values, the solution of the system is deterministic. When parameter takes other values, however, the solutions of the equation present random (non—deterministic) features. Nevertheless, the range of the randomness as well as the random probabilistic distribution of the solutions are all confined by the deterministic equation.

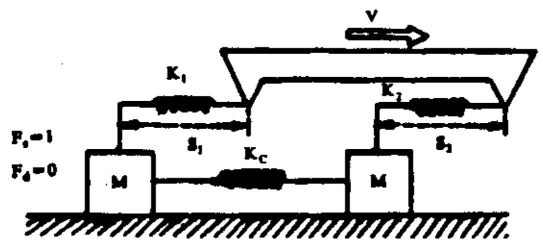


Fig.9 A simple seismic model consisted of 2 sliding blocks. Note the choosing of the values for the static friction coefficient and for the dynamic friction coefficient. The system follows the non—linear dynamic equation. Under given parameter conditions, the motion of the 2 sliding blocks may also show random behaviours.

From this, we can see that the laws in physics are not all deterministic, yet the random phenomena are also based on the physical principles. The simple empirical extrapolation is not built on physical basis, and thus presents a number of limitations. Therefore, the mutual supplementation between the deterministic and the non—deterministic characteristics might provide a better approach for tackling the earthquake prediction problems. It will exert a significant influence on the guidelines, capability evaluation and method of the entile prediction work. Our work in this respect is just at the beginning, and therefore a lot more has to be done in the future.

Finally, we want to quote one paragraph from Prigogin's treatise Exploring Complexity as the summary of this paper: "...No matter we devote ourselves in what branch of science, we can not escape from such a feeling: We are now living in an age of great changes. At this critical moment, science itself is also undergoing a process of theoretical revolution... If a physicist was asked several years before about what did he know and what he didn't understand, he was sure to answer: The real problems exist only in the frontiers of the universe, occurring in the orders of the basic particles and the cosmology. On the other hand, he would claim that he had already fully understood the basic laws concerning the macroscopic orders. It seemed to him that the classical sciences were already fully satisfactory and that he had already gained a penetrating perspective into the fields covered by the deterministic and the reversible laws. Today, an ever—growing minority of scientists are beginning to be skeptical on such an optimistic point of view. It is right in the macroscopic orders that a lot of fundamental questions are far from being answered... If we look ahead of us today, we will find evolutions, varieties and unstabilities. We have long been aware of the fact that we are now living in a complicated world, in which we can find not only deterministic and reversible features, but also random and non—reversible features. If we take a look at the deterministic phenomena such as the motion of the frictionless simple pendulum or the orbit of the moon circling around our earth, we get to know that the motion of the frictionless simple pendulum is reversible. This is because identical functions are present in the equation describing its motions not only in the past, but also in the future. Nevertheless, we find other non–reversible processes, such as the diffusion and chemical reactions. In the latter examples, specified temporal orientation is existent, and with the elapse of time, the system will become homogeneous and uniform. We have to thank for the existence of the random processes, because they have helped us to keep away from the absurd point of view, i.e. the conviction that the rich and colourful natural phenomena are staged according to a fixed repertoire, just like the ticking of the Big Ben... Ours is an era marked with the mutual collisions by and convergences of diversified concepts and methods, among which, and most important of all, the key factors of non–linearity and unstability have endowed matters with very high sensitivity and long–ranged orders and have thus evolved the multi–typed self–organizing modes. The emergence of the self–organizing phenomena as a new scientific discipline enables us to imagine how the complexity has come into being in nature, and to understand to what extent can it be studied".