INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONE: 2240-1
CABLE: CENTRATOM - TELEX 460392-1

H4.SMR 346/18

SCHOOL ON
NON-ACCELERATOR PHYSICS
25 April - 6 May 1988

GRAVITATIONAL WAVES (1 & 2)

by

G. Pizzella
Universita degli Studi "La Sapienza"
Rome

GRAVITATIONAL WAVES

- Introduction
- Generation of g.w.
- Possible astrophysical sources
- Resonant and non resonant antennas
- Energy absorbed by the resonant antennas
- Noise in a resonant antenna
- Experiments in the world with resonant and non resonant antennas
- Comparison between the two antennas
- The Rome experiment
- The transducer
- Measurements
- Preliminary Rome-Stanford coincidences

GRAVITATIONAL WAVES

O. Heaviside 1893
(Electromagnetic theory)

H.A. Lorentz 1900

H. Poincaré 1905
(Circolo Matematico di Palermo)

A. Einstein 22 June 1916
31 January 1918

G. Ricci Curbastro (1853-1925)

T. Levi-Civita (1873-1941)
University of Padova

Any metric theory of gravity, which incorporates Lorentz invariance in its field equations, forsee the propagation of gravitational waves $ds^2 = g_{ik} dx^i dx^k$

Theories alternative to G.R.

- Scalar-tensor theory
- vector-tensor theory
- Rosen's bimetric theory
- Rastall's theory
- BSLL theory
- stratified theories

C.M. Will: "Theory and Experiment in Gravitational Physics"

Cambridge Univ. Press
London 1981

Adimensional coupling constants

Interaction

strong $\dfrac{g^2}{\hbar c} \sim 1$

electromagnetic $\dfrac{e^2}{\hbar c} = \alpha \sim \dfrac{1}{137}$

weak $\dfrac{G_\mu m_p^2}{(\hbar c)^3} \sim 10^{-5}$

gravitational $\dfrac{G m_p^2}{\hbar c} \sim 6 \times 10^{-39}$

Gravitational wave experiments

1960-1970 J. Weber

1970-1975 Various room temperature experiments

1971 → beginning II generation resonant g.w. antennas

1975 → beginning the development of laser interferometry

Plans, at least, until 1995

15 experimental groups

$$10^{-30} \text{ joule} \simeq 10^{-11} \text{ eV}$$

Energy of a large body

~ 2000 kg

General Relativity

Einstein eq. in vacuum $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$

The metric tensor $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

Weak field approx. $\begin{cases} g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu} \\ |h_{\mu\nu}| << 1 \end{cases}$

The wave equation $\frac{\partial^2 h_{\mu\nu}}{\partial x^\alpha \partial x_\alpha} \equiv \Box h_{\mu\nu} = 0$

Plane waves propagating along x $h_{\mu\nu} = h_{\mu\nu}(t - x/c)$

light velocity

Gravitational waves

Transverse-Traceless gauge
"TT gauge"

Only 4 components of $h_{\mu\nu}$ are different from zero

$$\left.\begin{aligned} h_{32} = h_{23} &= h_\times \\ h_{22} = -h_{33} &= h_+ \end{aligned}\right\} \text{for a wave propagating along } x^1$$

- Two polarization states

- Transverse waves

Do gravitational waves carry energy?

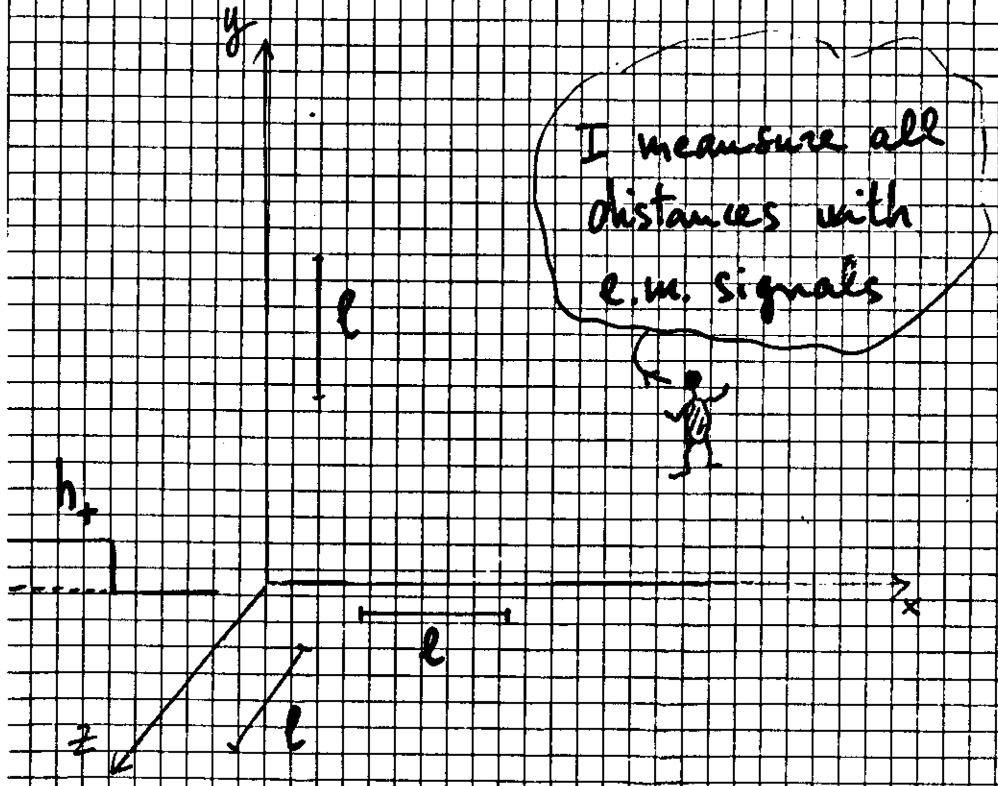
$$W = \frac{c^3}{16\pi G}\,\dot{h}^2 \quad \left[\frac{\text{joule}}{m^2\, s}\right]$$

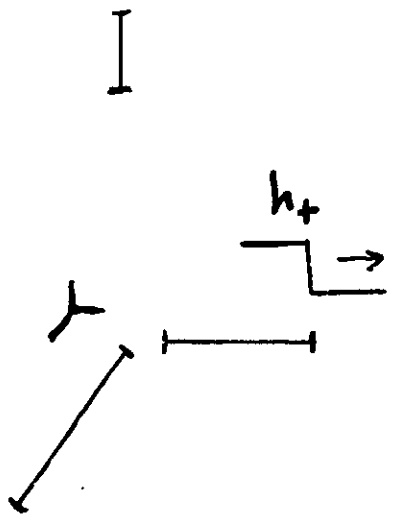
$$= 8\cdot 10^{33}\,\dot{h}^2$$

For $W = 1400\ \frac{\text{joule}}{m^2\, s}$ (solar e.m. on the Earth)

and $\omega = 5000$ rad/s

$$h \sim 10^{-19}$$





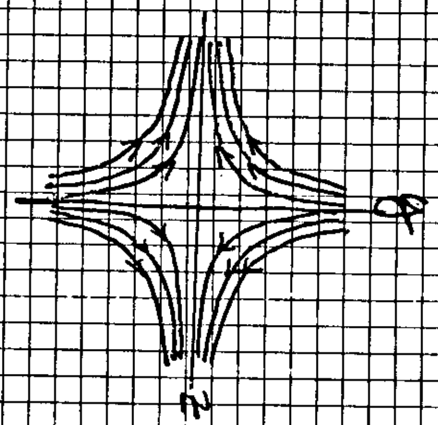
$$\frac{\delta \ell}{\ell} = 0 \qquad \ell \,//\, x$$

$$\frac{\delta \ell}{\ell} = -\frac{1}{2} h_+ \qquad \ell \,//\, y$$

$$\frac{\delta \ell}{\ell} = +\frac{1}{2} h_+ \qquad \ell \,//\, z$$

We change now point of view:

we pass from Einstein geometro-dynamical description to normal dynamics based on the concept of forces:



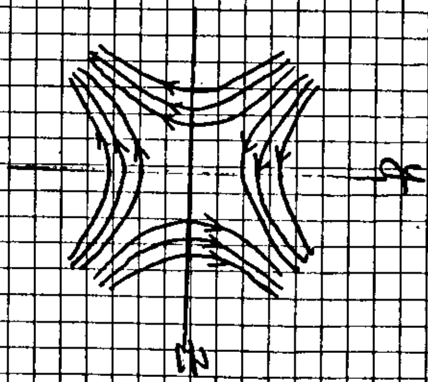


Figure 2 Scheme for a laser interferometer.

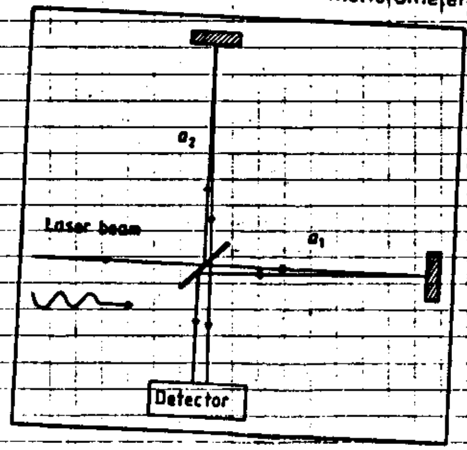
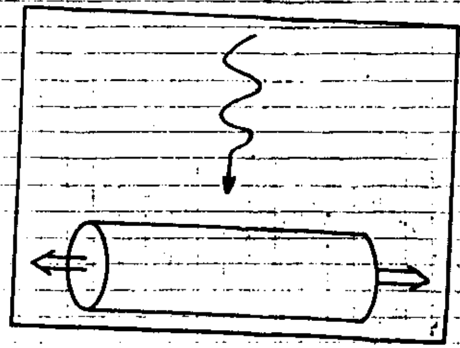


Figure 3 Interaction of a metal cylinder with a gravitational wave



Generation of gravitational waves

$$I = \frac{G}{36\pi c^5 R_0^2}\left[\left(\frac{\dddot{D}_{yy}-\dddot{D}_{zz}}{2}\right)^2 + \dddot{D}_{yz}^2\right] \frac{\text{Joule}}{m^2 s}$$

$D_{\alpha\beta}$ = mass quadrupole tensor

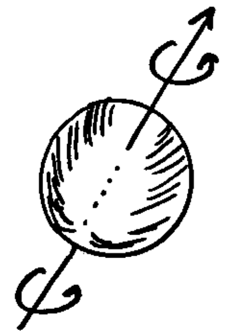
R_0 = distance from source

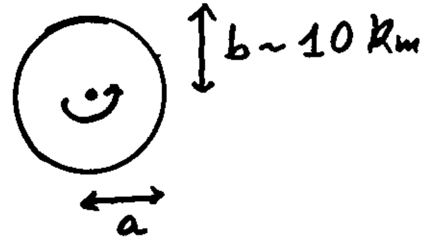
$$\frac{dE}{dt} = \frac{G}{45c^5}\,\dddot{D}_{\alpha\beta}^2 \quad \frac{\text{Joule}}{s}$$

$$\frac{G}{45c^5} = 6.1\times10^{-55}\ m^{-2}\, s^{3}\, kg^{-1}$$

Einstein: "... in all thinkable cases it must have a practically vanishing value".

Pulsar





$$W = \frac{288}{45}\,\frac{G}{c^5}\, I^2 \omega^6 \left(\frac{a-b}{ab}\right)^2 \text{ watt}$$

1937+214
the fast pulsar

2.5 kpc

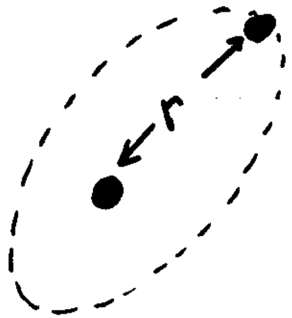
$\nu_{gw} = 1283{,}856542$ Hz

$W = 2\times10^{29}$ watt

$h_0 \simeq 3\times10^{-27}$ $\quad |a-b| \simeq 100\,\mu m$

on the Earth

Binary system



$$W = \frac{32}{5}\frac{G}{c^5}\left(\frac{m_1 m_2}{m_1+m_2}\right)^2 r^4 \omega^6$$

PSR 1913+16 (Taylor) $T = 7^h 45^m 7^s$

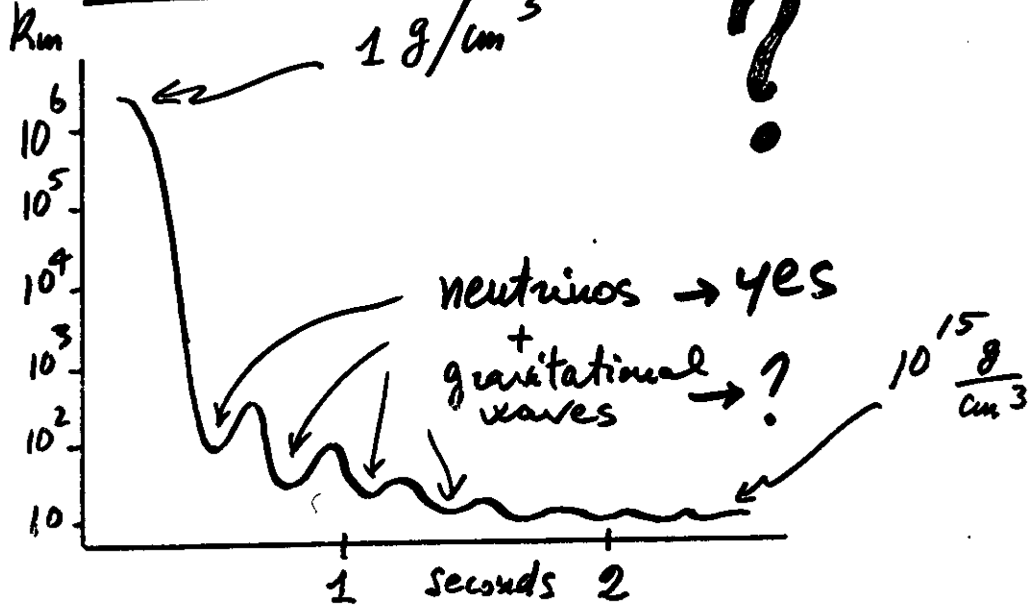
5 kpc

$W \simeq 6.4\times10^{23}$ watt

$h_0 \simeq 1\times10^{-22}$

Supernovae

?



$M \simeq 8\,M_\odot \rightarrow 1.44\,M_\odot$

~ detected

e.m. waves	$\simeq$	$0.002\ M_\odot$
cosmic rays	=	" "
neutrinos	=	$0.2\ M_\odot \simeq 3\cdot10^{53}$ erg
g. w	=	$(0.01 \rightarrow 0)\,M_\odot$? ?

Duration of g. w. bursts $\tau_g \sim \frac{1}{2}$ ms

Resonant antennas

$$v = \sqrt{\frac{Y}{\rho}} =$$

5400 $\frac{m}{s}$ at 4.2 K

equivalent to many simple oscillators with resonance angular frequencies

$$\omega_k = (2k+1)\,\frac{\pi v}{L}$$

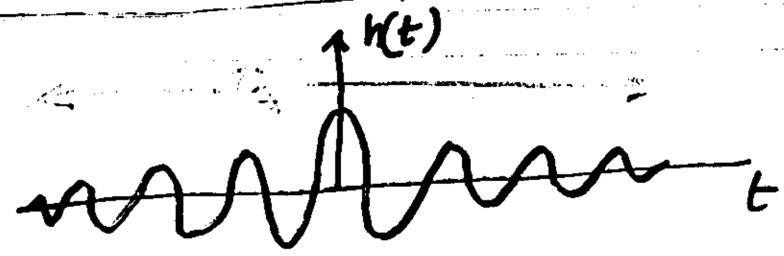
$$\omega_0 = \frac{\pi v}{L} \sim 2\pi\,(1 \div 2\ \mathrm{kHz})$$

$$m = \frac{M}{2}$$

$$\ddot{\xi} + 2\beta_1 \dot{\xi} + \omega_0^2 \xi = \frac{2}{\pi^2} L\, \ddot{h}$$

$$2\beta_1 = \omega_0 / Q$$

Special wave form



$$h(t) = h_0\, e^{-\frac{|t|}{\tau_g/2}} \cos\omega_0 t$$

If $\tau_g >> 2\pi/\omega_0$

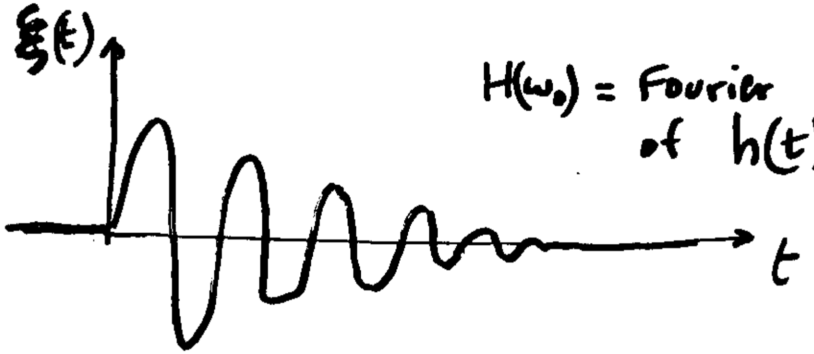
$$H(\omega_0) = \frac{h_0 \tau_g}{2}$$

$$\xi(t) = -\frac{L}{\pi^2}\, h_0 \tau_g \omega_0\, e^{-\beta_1 t} \sin\omega_0 t$$

In general for short bursts

$$\xi(t) = -\frac{2L}{\pi^2}\, H(\omega_0)\, \omega_0\, e^{-\beta_1 t} \sin\omega_0 t$$

$H(\omega_0)$ is the Fourier transform of $h(t)$ at the resonance



$H(\omega_0)$ = Fourier transform of $h(t)$ at ω_0

$\xi(t)$ is the displacement of the bar end faces in absence of noise

$$\xi(t) = -\frac{2L}{\pi^2} H(\omega_0)\,\omega_0\, e^{-\beta_1 t} \sin\omega_0 t$$

Expected h values from gravitational collapses

$$\frac{M^* c^2}{4\pi R^2 \tau_g} = \left[\frac{\text{joule}}{m^2 s}\right] = \frac{c^3}{32\pi G}\,\omega^2 h^2$$

R = distance of the source

τ_g = duration of g.w. emission

$M^* c^2$ = energy of g.w.

Center of Galaxy

8.5 Kpc

$$h = 4\times10^{-17}\sqrt{\frac{M^*}{M_\odot}\,\frac{1/\tau_g}{1000\ Hz}}$$

Virgo cluster

19 Mpc

$$h = 1.8\times10^{-20}\sqrt{\frac{M^*}{M_\odot}\,\frac{1/\tau_g}{1000\ Hz}}$$

Our goal $h \sim 10^{-21}$

$(M \sim 10^{-2} M_\odot)$

spectral energy density

$$f(\omega) = \frac{c^3}{16\pi G} |\omega H(\omega)|^2 \quad \left[\frac{\text{joule}}{m^2\, Hz} \right]$$

$$H(\omega) \approx \frac{h}{1/\tau_g} \quad \left[Hz^{-1} \right]$$

the cross section

$$\Sigma = \frac{16}{\pi} \left(\frac{v}{c}\right)^2 \frac{G}{c} M \quad \left[m^2\, Hz \right]$$

$$\left(\Sigma = 8.4 \times 10^{-25}\ m^2\, Hz \right)$$

Rome – CERN

the energy absorbed by the antenna

$$E = \Sigma \cdot f(\omega) \quad [\text{joule}]$$

Signal in the g.w. antenna

Center of Galaxy $1 M_\odot$

$$\Delta E \sim 9 \times 10^{-24}\ \text{Joule}$$
$$\simeq 6.5\ \text{Kelvin}$$

Virgo cluster $1 M_\odot$

$$\Delta E \sim 1.3 \times 10^{-6}\ K$$
$$\approx 10^{-29}\ \text{Joule}$$
$$\approx 10^{-10}\ eV$$

LMC $1 M_\odot$

$$\Delta E \simeq \frac{6.5}{25}\ K \simeq 0.25\ K$$

NOISE

Thermal noise $\sim kT = 1.38 \times 10^{-23}$ joule at 1 K

With optimum data filtering

$$NOISE = \frac{2kT}{\beta Q} + 2kT_n$$

T = antenna temperature

β = fraction of energy available in the transducer

Q = merit factor

T_n = electronic amplifier noise

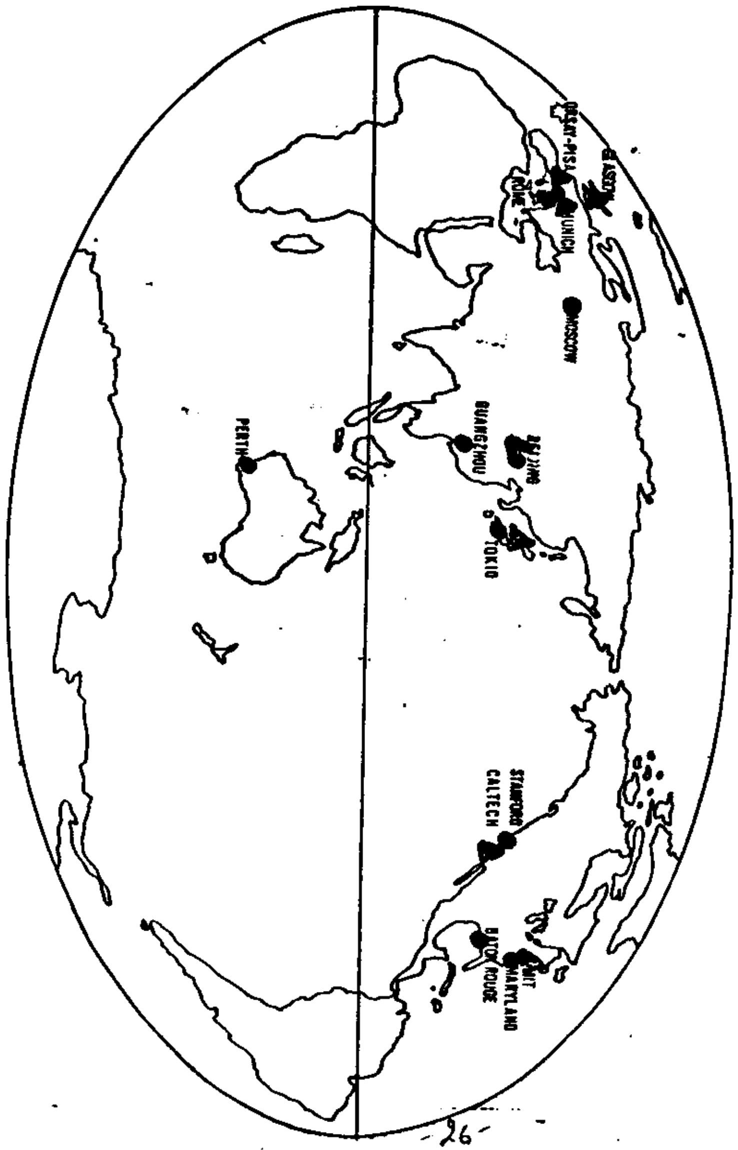
			Today
T	$\longrightarrow$	0.03 K	4.2 K
β	$=$	10^{-2}	10^{-2}
Q	$=$	10^{7}	$7 \cdot 10^{6}$
T_n	$\longrightarrow$	10^{-7} K	10^{-3} K

$$NOISE \approx 8 \times 10^{-30} + 2.8 \times 10^{-30} \text{ joule}$$

$$\approx 8 \times 10^{-7} \text{ K}$$

Signal $\sim 1.3 \times 10^{-6}$ K

$\left(1\, M_\odot \text{ from Virgo}\right)$



⑧ ▲ interferometers

● resonant antennas

NON RESONANT DETECTORS ①

$$\Delta \ell(t) = \frac{1}{2} \ell h_+(t)$$

Shot noise $\quad \delta \ell = \sqrt{\frac{\hbar c \lambda \Delta\nu}{\pi P}}$

Sensitivity $\quad h_+ \simeq \frac{2\Delta \ell}{\ell} = 2\sqrt{\frac{\hbar c \lambda \Delta\nu}{\pi P \ell^2}}$

$P = 1$ watt

$\lambda = 0.6\,\mu m$

$\Delta\nu = 1000$ Hz

in order to have $h_+ \simeq 1.4 \times 10^{-20}$

$\ell = 300$ Km

N reflections $\quad \ell = N \ell_0$

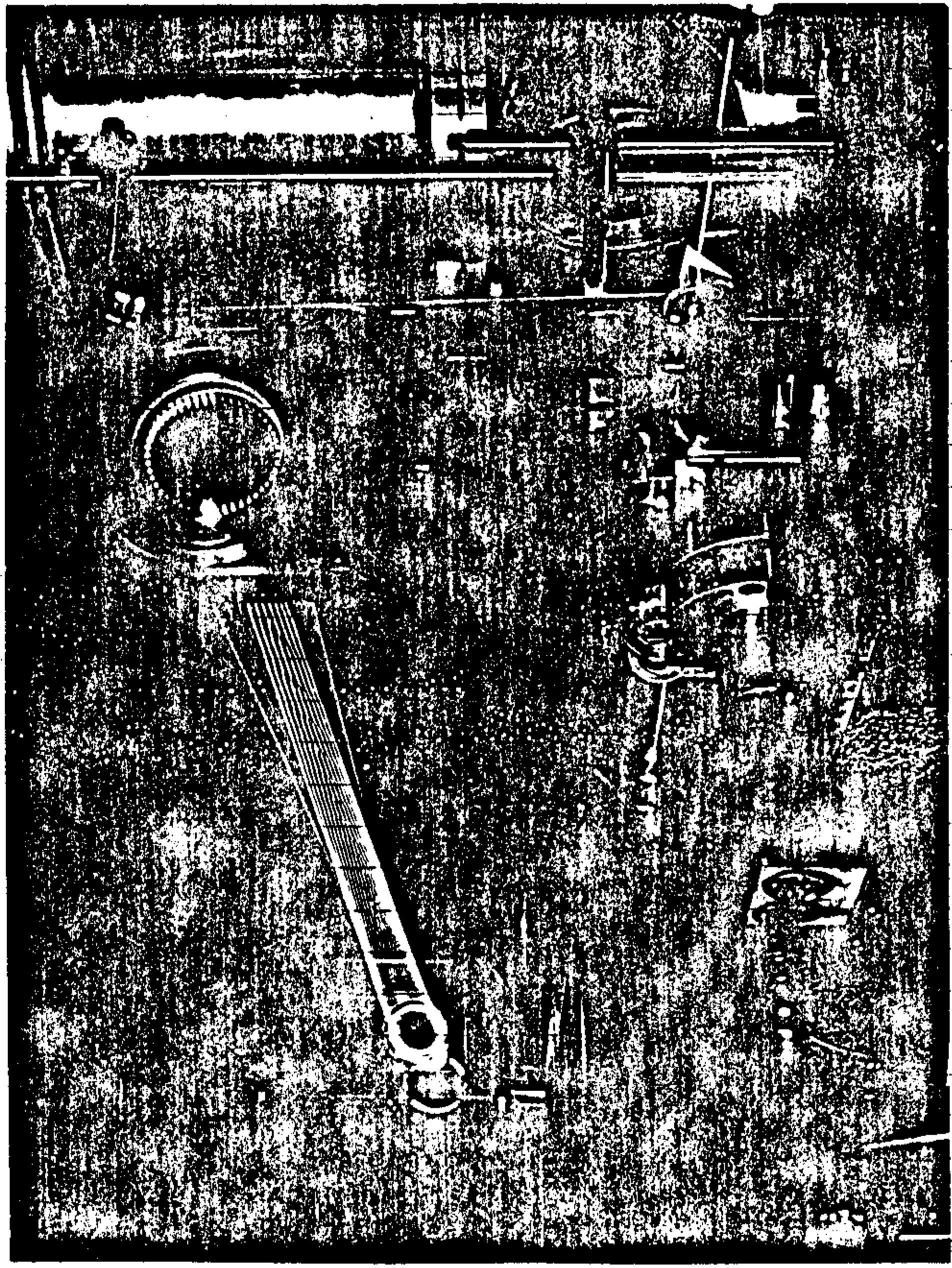
$N = 100$

$\ell_0 = 3$ Km

time spend by light in the interferometer

$\frac{300\,Km}{} = 10^{-3}\,s \cdot \Delta\nu = 1000$ Hz

$h_0 \simeq 2 \cdot 10^{-21}$ 1% H_0 in V.C.	Signal	Noise
Interferometers	$\Delta \ell \simeq \frac{1}{2} \ell h_0$ $\ell = 100 \cdot 10$ Km $\boxed{\Delta \ell = 10^{-15}\ m}$	Shot noise $\delta \ell = \sqrt{\frac{\hbar c \lambda \Delta\nu}{\pi P}}$ $\Delta\nu = 1000$ Hz $P = 1$ W $\lambda = 0.6\,\mu m$ $\boxed{\delta \ell = 2.5 \cdot 10^{-15}\ m}$
Resonant antennas	$\eta_0 \cong \frac{\ell}{2} e^{-\beta t} h_0 \tau_g \omega_0$ $t = 0$ $\ell = 3$ m $\tau_g = 10^{-3}$ s $\omega_0 = 5000$ rad/s $\boxed{\eta_0 \cong 1.5 \times 10^{-20}\ m}$	Electronic + brownian noise with optimum filter $\frac{1}{4} M \omega_0^2 \eta_0^2 = 2 K T_n$ $\eta_0 = \sqrt{\frac{8 K T_n}{M \omega_0^2}}$ $T_n = 10^{-6}$ K $M = 2300$ Kg $\omega_0 = 5000$ rad/s $\boxed{\eta_0 \cong 4.4 \times 10^{-20}\ m}$



- 29 -

IL NUOVO CIMENTO   VOL. 9 C, N. 4   Luglio-Agosto 1986

Preliminary Results on the Operation of a 2270 kg Cryogenic Gravitational-Wave Antenna with a Resonant Capacitive Transducer and a d.c. SQUID Amplifier.

E. AMALDI, C. COSMELLI, G. V. PALLOTTINO, G. PIZZELLA
P. RAPAGNANI and F. RICCI

Dipartimento di Fisica dell'Università «La Sapienza» - Roma
Istituto Nazionale di Fisica Nucleare - Sezione di Roma

P. BONIFAZI and M. G. CASTELLANO

Istituto Nazionale di Fisica Nucleare - Sezione di Roma
Istituto di Fisica dello Spazio Interplanetario del C.N.R. - Frascati (Roma)

P. CARELLI and V. FOGLIETTI

Istituto Nazionale di Fisica Nucleare - Sezione di Roma
Istituto di Elettronica dello Stato Solido del C.N.R. - Roma

G. CAVALLARI

C.E.R.N., European Organization for Nuclear Research - Geneva

E. COCCIA and I. MODENA

Dipartimento di Fisica dell'Università «Tor Vergata» - Roma
Istituto Nazionale di Fisica Nucleare - Sezione di Roma

R. HABEL

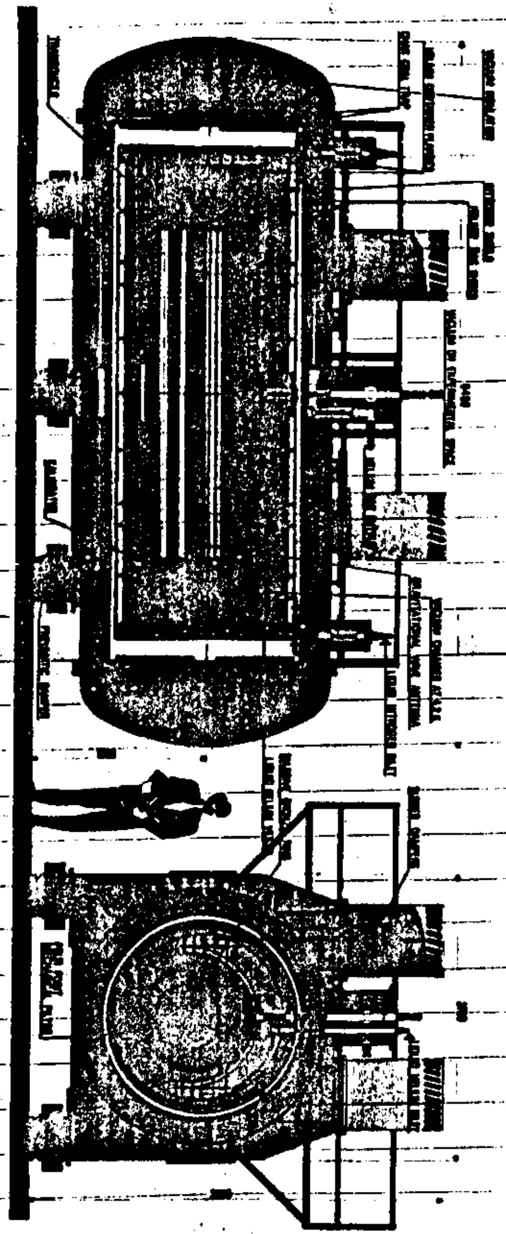
Istituto Nazionale di Fisica Nucleare - Sezione di Roma
ENEA, Centro Ricerche Energia - Frascati (Roma)

(ricevuto il 12 Maggio 1986)

Summary. — In November 1985 the gravitational-wave antenna of the Rome group, installed at CERN, has started operating. It consists of a 5056 aluminium cilinder 3 m long, 2270 kg heavy, cooled at 4.2 K. The antenna vibrations are detected by means of a resonant capacitive

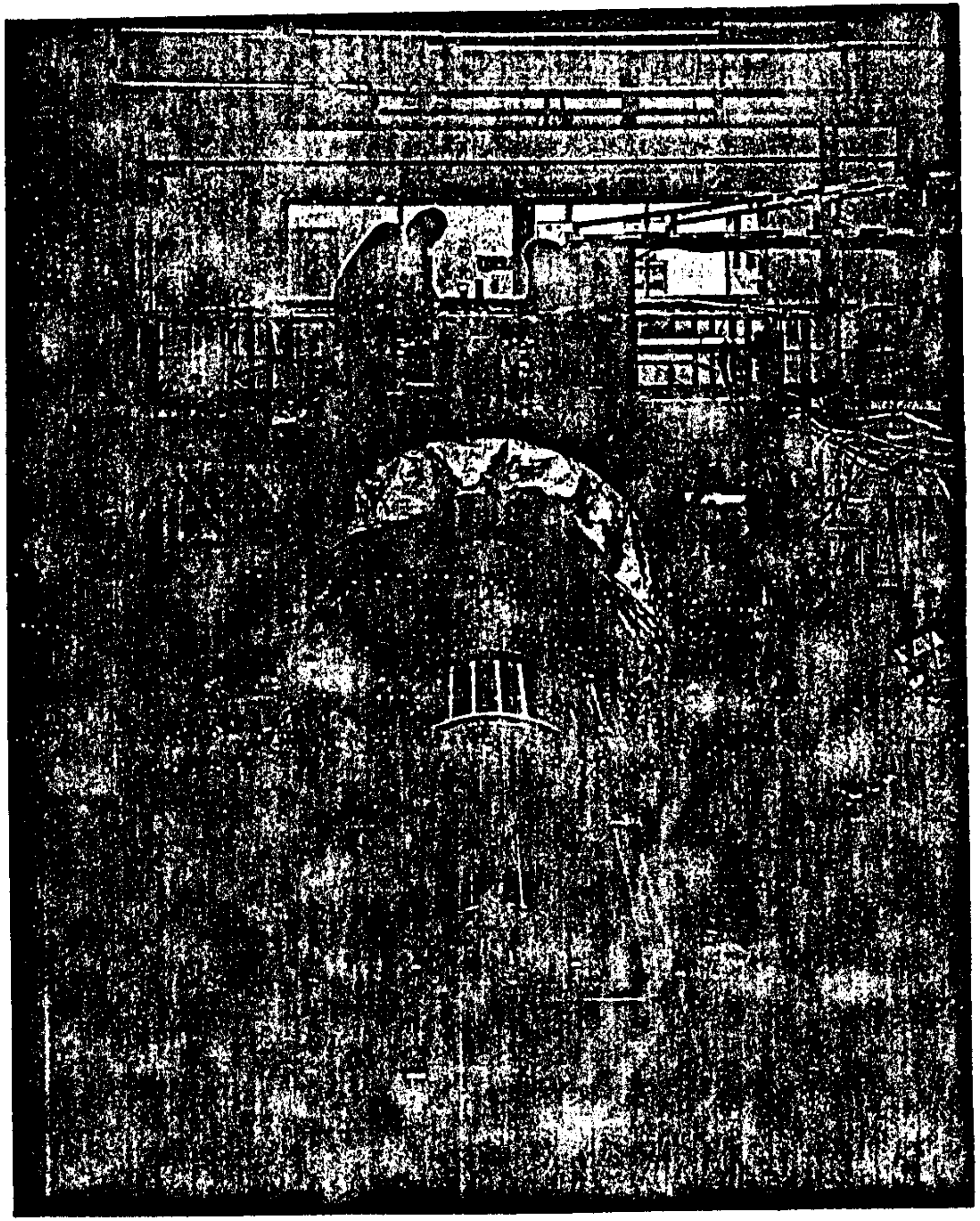
- 30 -

(12)

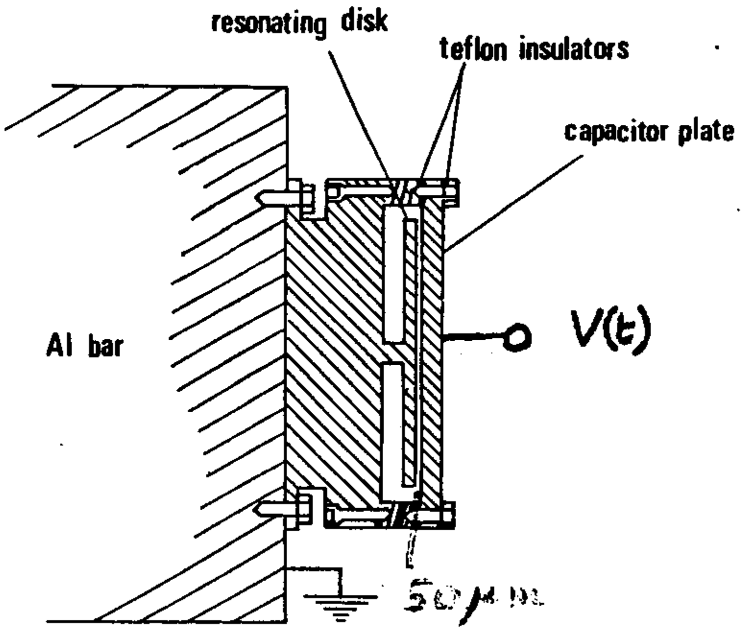


(18)

X666-11-86



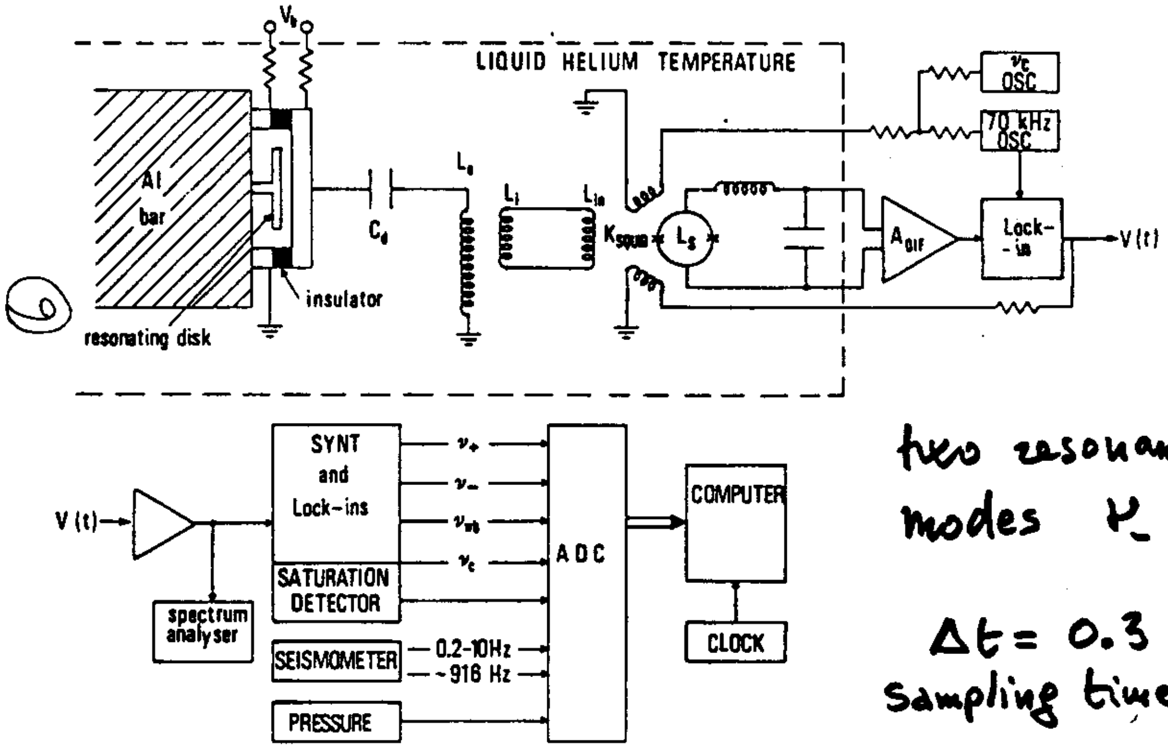
Resonant transducer



$$V(t) = \frac{Q}{C(t)} = \alpha \cdot \xi_t(t)$$

ξ_t is the transducer displacement

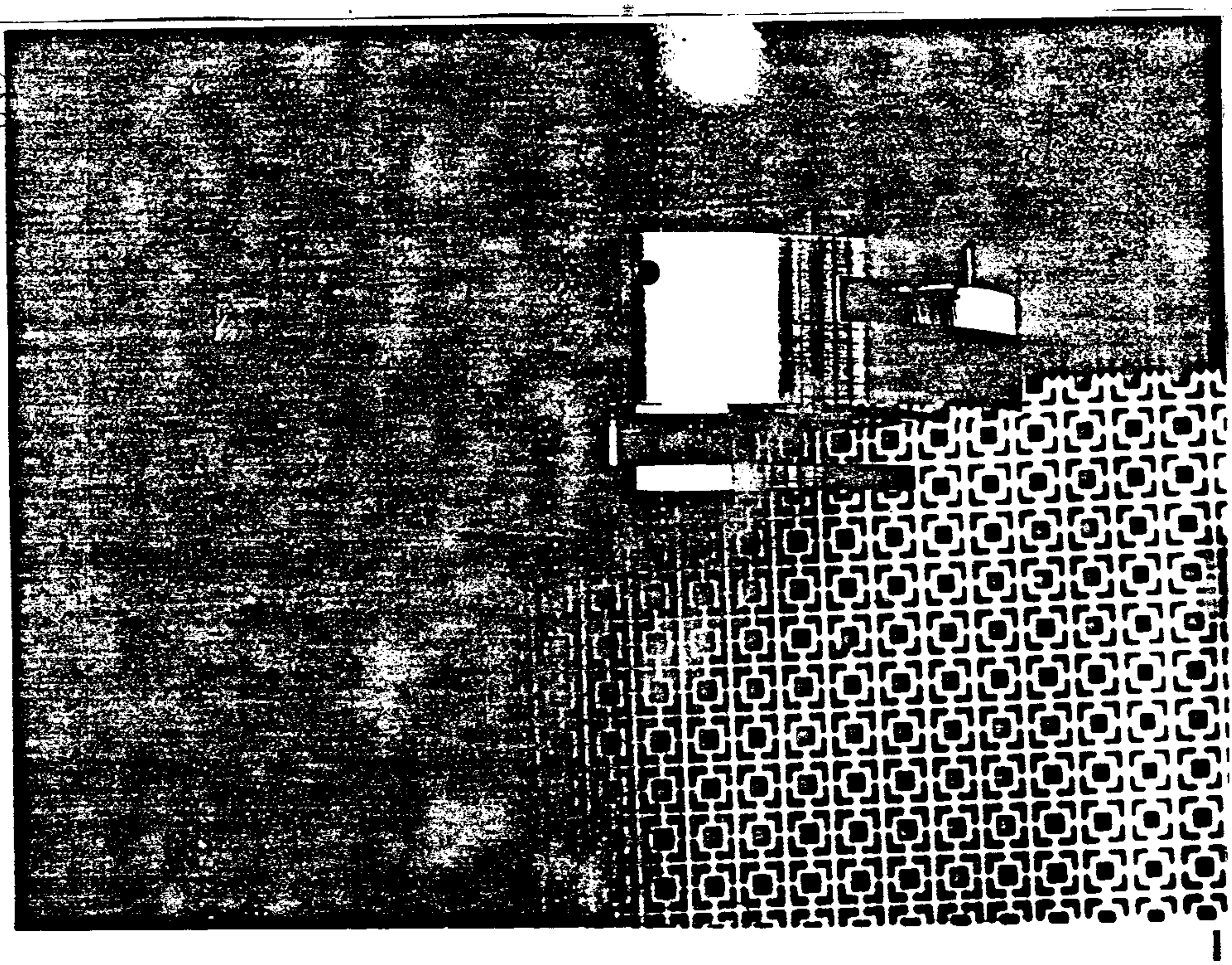
6

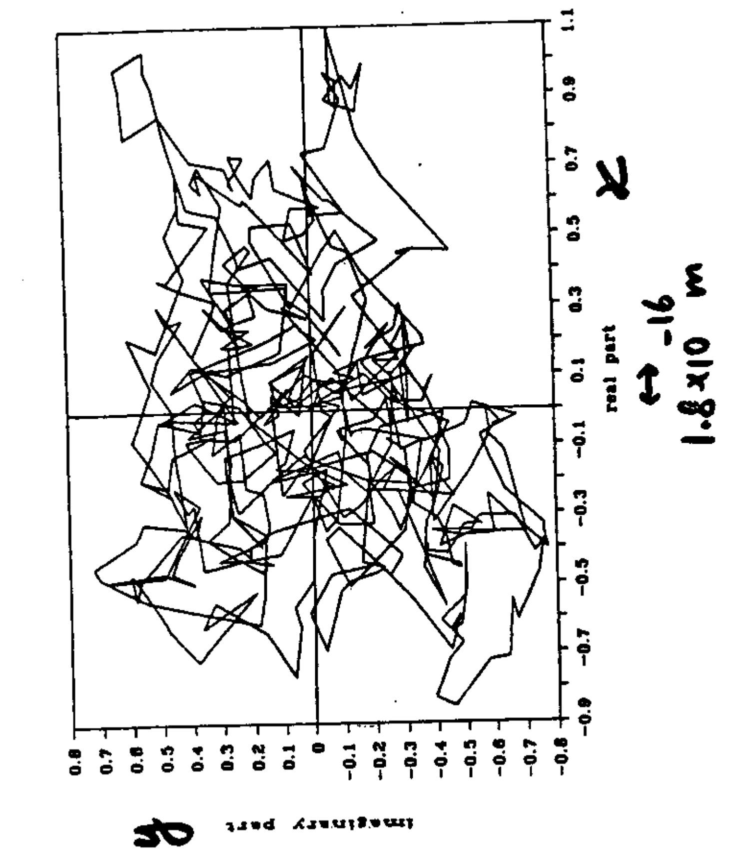


two resonance modes ν_-, ν_+

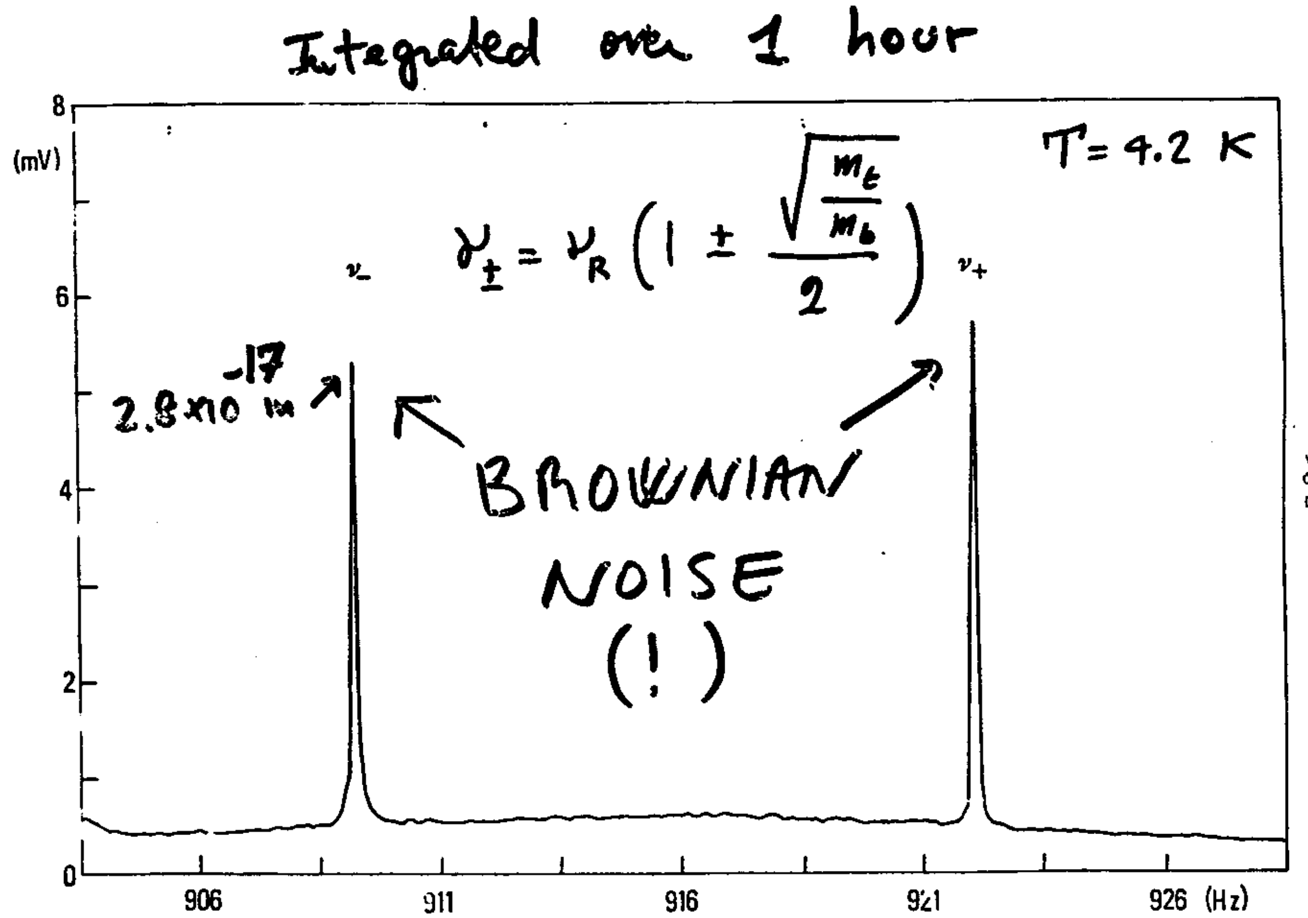
$\Delta t = 0.3$ s
sampling time interval

(17)



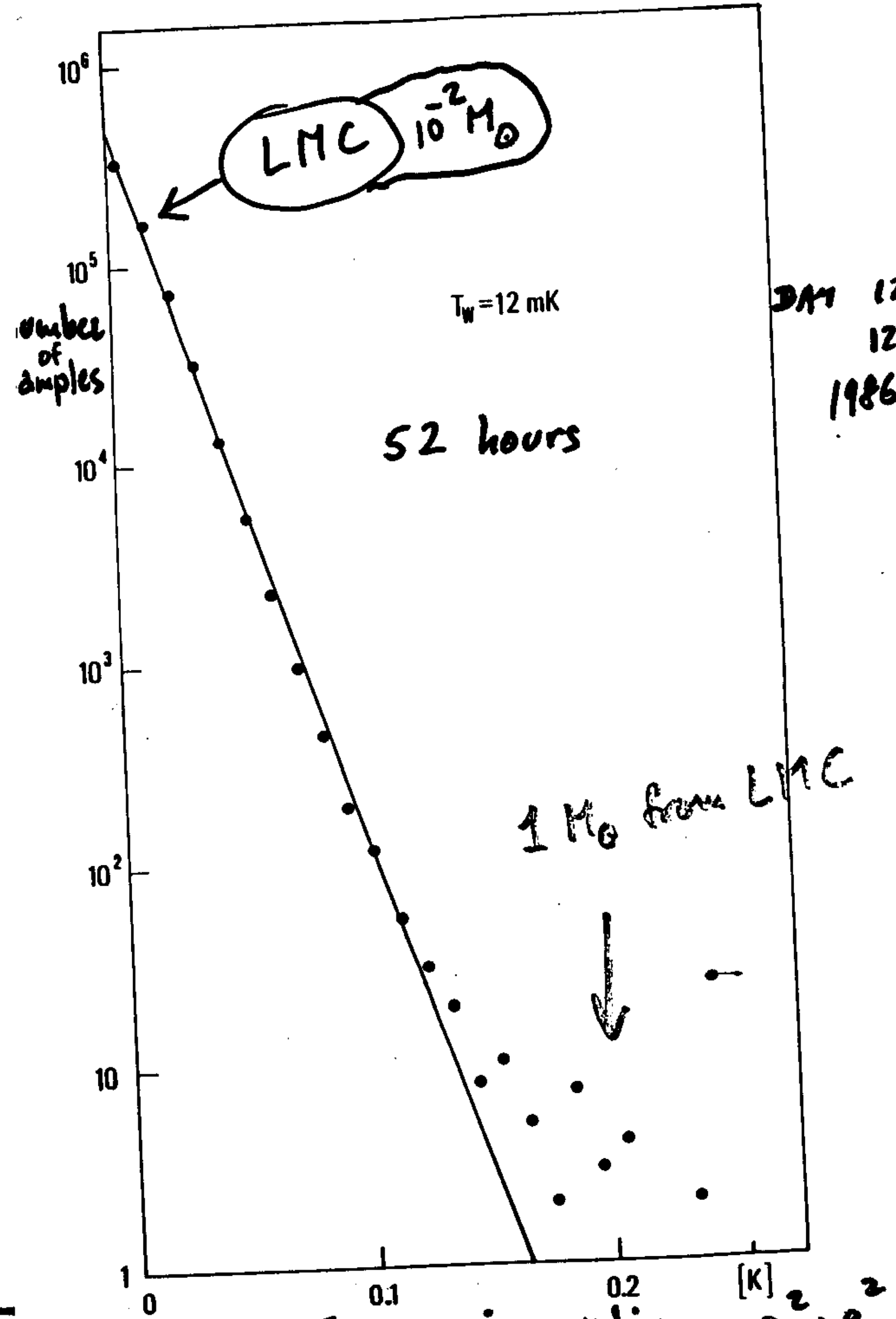


imaginary part
real part
y
x
$\leftrightarrow 1.8 \times 10^{-16}$ m



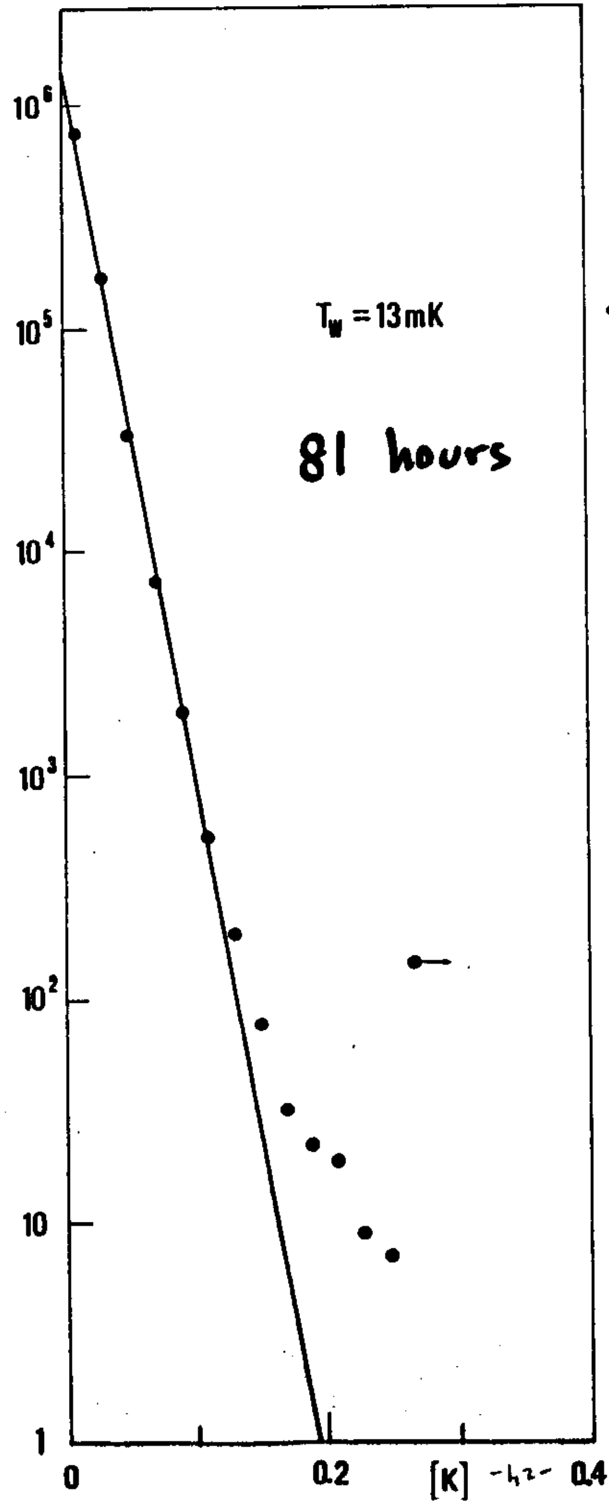
Integrated over 1 hour
(mV)
(Hz)
T = 4.2 K
ν_-
ν_+
$\nu_{\pm} = \nu_R \left(1 \pm \frac{\sqrt{\frac{m_t}{m_b}}}{2}\right)$
2.8×10^{-17} m
BROWNIAN
NOISE
(!)

Fig. 11

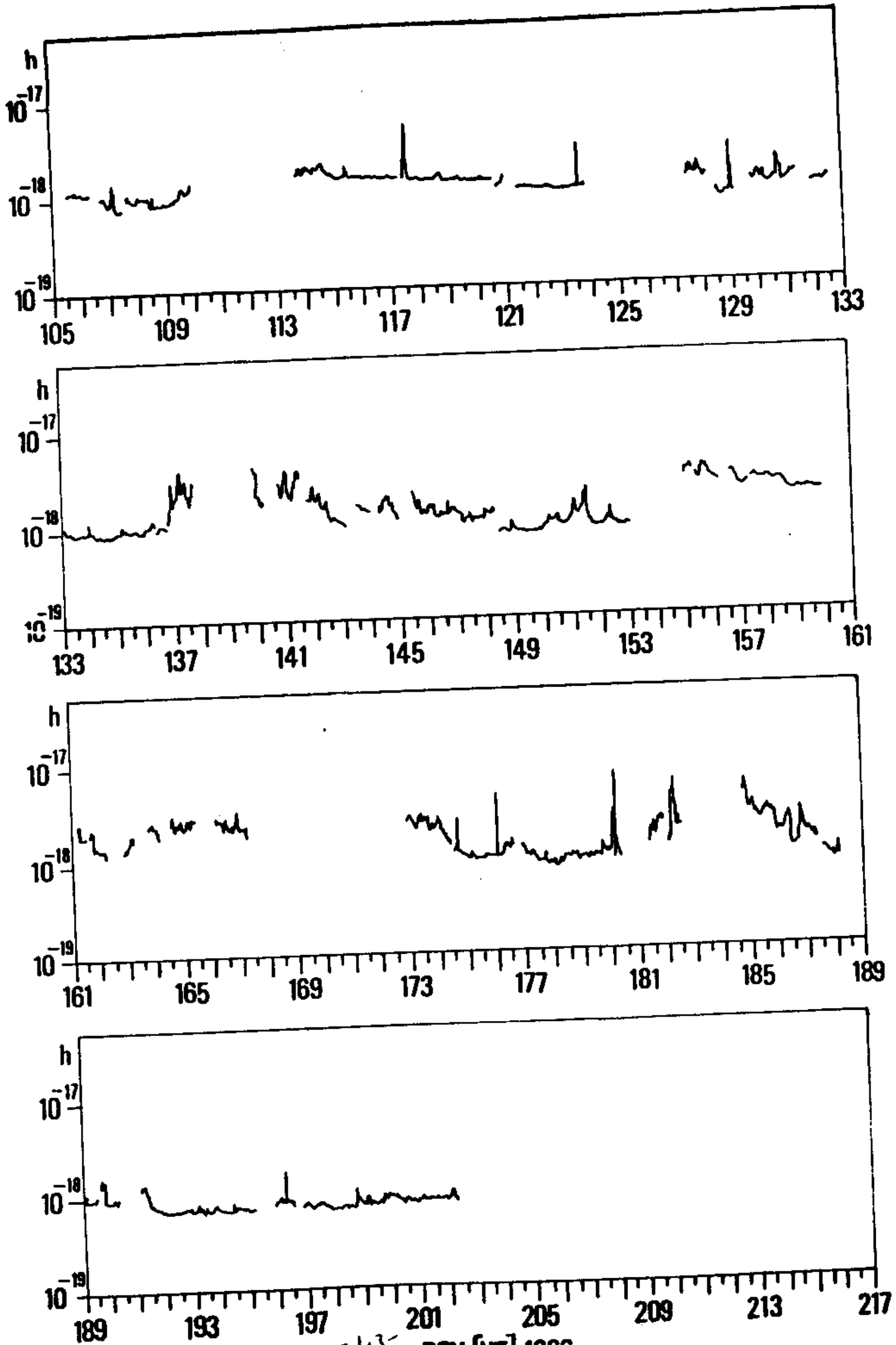


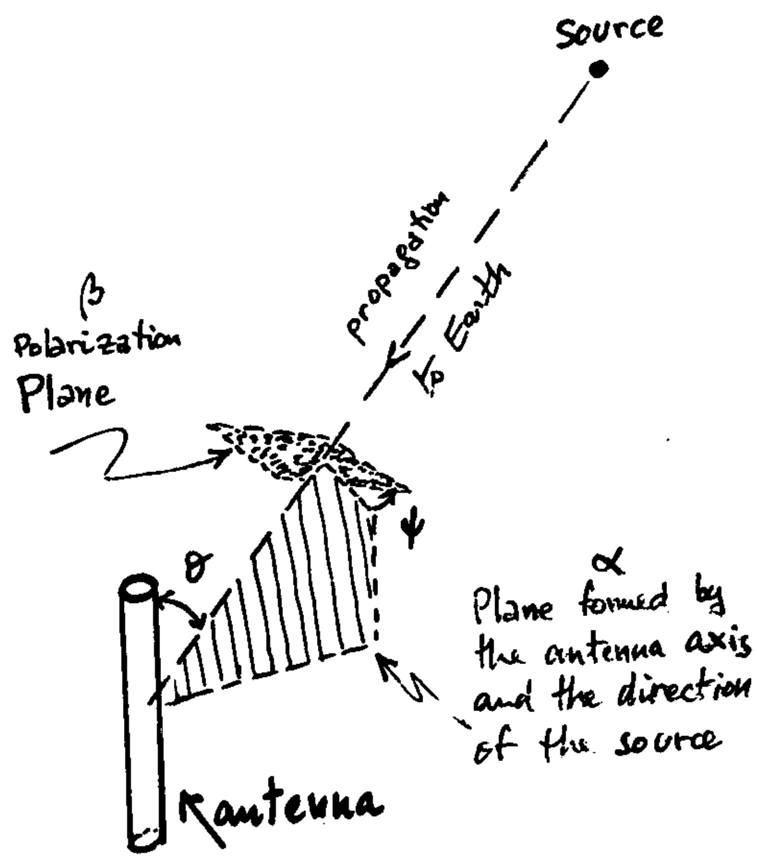
DAY 121 10
123 14
1986

Energy innovation = $\xi_x^2 + \xi_y^2$



DAY 132 23
136 06
1986





$$\Sigma_{\chi}(\theta,\psi) = \Sigma \sin^4\theta \left(\frac{1-\chi}{2} + \chi \cos^2 2\psi \right)$$

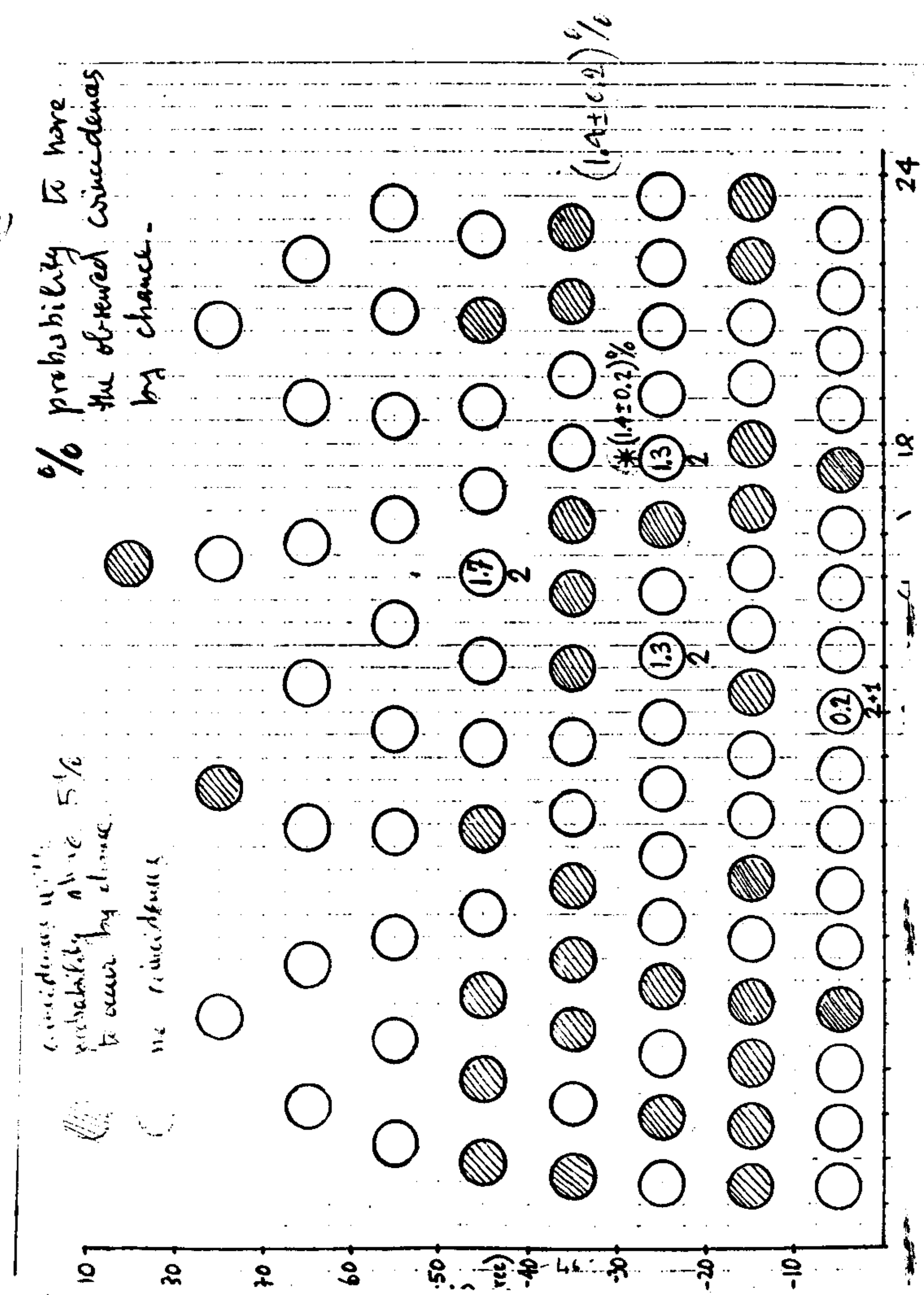
χ = fraction of polarized wave

ψ = angle between one of the wave asimptotic lines of force and the plane α

Galactic normalization

$$\Delta E(t)_{normalized} = \Delta E(t)_{measured} / \sin^4 \vartheta(t)$$

$\vartheta(t)$ = angle at the time t of measurement between the antenna axis and the direction of the Galactic Center.



imposing the signals in the two antennas be equal after normalization