INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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SCHOOL ON
NON-ACCELERATOR PHYSICS
25 April - 6 May 1988

NEUTRINO OSCILLATIONS

by

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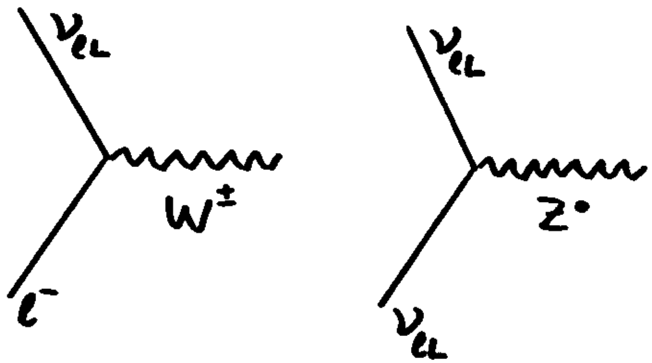
Neutrino Oscillations

Book: F. Boehm and P. Vogel,
Physics of Massive Neutrinos,
Cambridge University Press

ν's are created and detected in weak CC and NC processes.

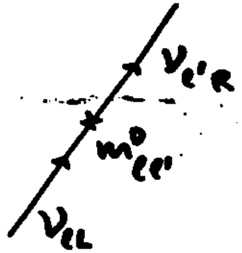
Weak eigenstates (flavor):

$$\nu_{\ell L}\,,\ \nu^c_{\ell R} \qquad \ell = e, \mu, \tau, \ldots$$



Dirac Masses

$$\mathcal{L}_m^D = \sum_{\ell\ell'} m^D_{\ell\ell'} \, \bar{\nu}_{\ell L} \nu_{\ell' R} + h.c.$$



$\Delta L = 0 \qquad (L = L_e + L_\mu + L_\tau)$

$m^D_{\ell\ell'}$ diagonal : $\Delta L_e, \Delta L_\mu, \Delta L_\tau = 0$

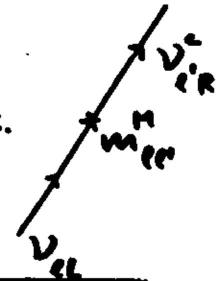
$m^D_{\ell\ell'}$ non diagonal: $\Delta L_e, \Delta L_\mu, \Delta L_\tau \neq 0$:

$$\nu_\ell = \sum_{i=1}^{N} U_{\ell i} \nu_i$$
$$M \nu_i = m_i \nu_i$$

N = number of flavors

Majorana Masses

$$\mathcal{L}_m^M = \sum_{\ell\ell'} m^L_{\ell\ell'} \bar{\nu}^c_{\ell R} \nu_{\ell' L} + m^R_{\ell\ell'} \bar{\nu}^c_{\ell L} \nu_{\ell' R} + h.c.$$



with $\nu^c_R = (\nu_L)^c \qquad \Delta L = 2$

$$\nu_{\ell L} = \sum_{i=1}^{N} U^L_{\ell i} \varphi_{iL} \qquad \nu_{\ell R} = \sum_{i=N+1}^{2N} U^R_{\ell i} \varphi_{iR}$$
$$M_i \varphi_i = m_i \varphi_i \qquad \varphi_i^c = \varphi_i \ \text{(Majorana)}$$

Majorana + Dirac

$$\nu_{\ell L} = \sum_{i=1}^{2N} U^L_{\ell i} \varphi_i \qquad \nu_{\ell R} = \sum_{i=1}^{2N} U^R_{\ell i} \varphi_{iR}$$

What happens if $m_\nu \neq 0$?

I. Cosmology : $0 \leq m_\nu \leq 200$ eV or $m_\nu > 2$ GeV

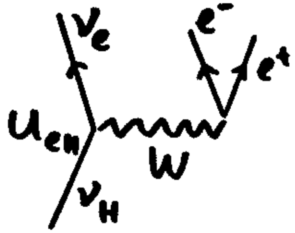
II. Kinematics in weak decays (M or D)

$$^3H \to {}^3He + e^- + \bar{\nu}_e : \quad m_{\bar{\nu}_e} < 18 \text{ eV}$$

III Time of flight (M or D):

SN 1987A : $m_\nu < 20$ eV

IV ν decay (M or D):



V $\Delta L = 2$, $0\nu\beta\beta$ decay (M only)

VI Oscillations (M or D)

III Oscillations (Vacuum) Pontecorvo

D, M: $\nu_\ell = \sum_{i=1}^{N} U_{\ell i}\,\nu_i$ N flavors

$$M\nu_i = m_i \nu_i$$

To simplify:

$$\begin{pmatrix}\nu_e\\ \nu_\ell\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

ν created as ν_e at $x=0$ with energy E_ν:

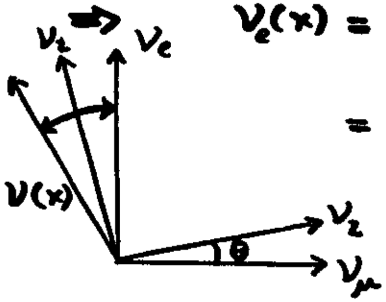
$$\nu_e(0) = \nu_e = c\,\nu_1 + s\,\nu_2$$

$$p_i = \sqrt{E_\nu^2 - m_i^2} \cong E_\nu - \frac{m_i^2}{2E_\nu} \qquad E_\nu \gg m_i$$

Propagation in space:

$$-i\frac{d\psi}{dx} = p\,\psi(x)$$

If ψ eigenfunction of p: $\psi(x) = e^{ipx}\psi(0)$



$$\nu_e(x) = c\cdot e^{ip_1x}\,\nu_1 + s\,e^{ip_2x}\,\nu_2$$

$$= e^{ip_1x}\left(c\,\nu_1 + s\,e^{i\frac{\Delta m^2}{2E_\nu}x}\,\nu_2\right)$$

$$\Delta m^2 = m_1^2 - m_2^2$$

$$\langle \nu_e | \nu_e(x)\rangle = e^{ip_1x}\left(c + s\cdot e^{i\frac{\Delta m^2}{2E_\nu}x}\right)$$

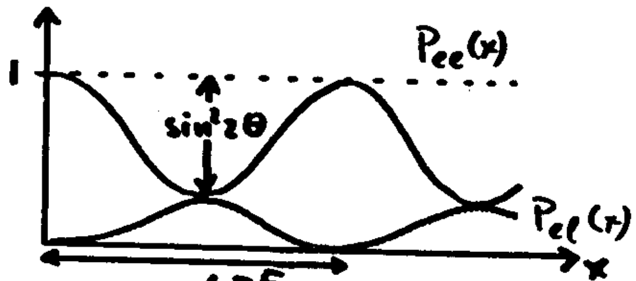
$$P_{ee}(x) = |\langle \nu_e | \nu_e(x)\rangle|^2$$

$$= c^4 + s^4 + 2c^2s^2\cos\frac{\Delta m^2}{2E_\nu}x$$

$$\Rightarrow \boxed{P_{ee}(x) = 1 - \tfrac{1}{2}\sin^2 2\theta\left(1-\cos\frac{2.5\,\Delta m^2}{E_\nu}x\right)}$$

$\Delta m^2\,(eV^2)$, $E_\nu\,(MeV)$, $x\,(m)$

$$\boxed{P_{\ell e}(x) = 1 - P_{ee}(x) = \tfrac{1}{2}\sin^2 2\theta\left(1-\cos\frac{2.5\,\Delta m^2}{E_\nu}x\right)}$$



$$L_{osc} = \frac{4\pi E_\nu}{\Delta m^2}$$

General: $P_{\ell'\ell}(x) = \sum_{i,k} U^{+}_{\ell' i} U_{\ell i} U_{\ell' k} U^{+}_{\ell k} \exp\left(i\frac{\Delta^2_{ik}}{2E_\nu}x\right)$

$$\Delta^2_{ik} = m_i^2 - m_k^2$$

M or D: Flavor oscillations $\nu_{\ell L} \longleftrightarrow \nu_{\ell' L}$

M and D: in addition matter-antimatter

$\nu_{\ell L} \longleftrightarrow \bar{\nu}_{\ell L}$ (sterile)

Experiments

1) Disappearance $\nu_\ell \longleftrightarrow x$.

Look for the probability of finding ν_ℓ in a beam consisting originally of ν_ℓ:

- Samples many channels at once, including those where x is undetectable
- Not well suited for small mixings

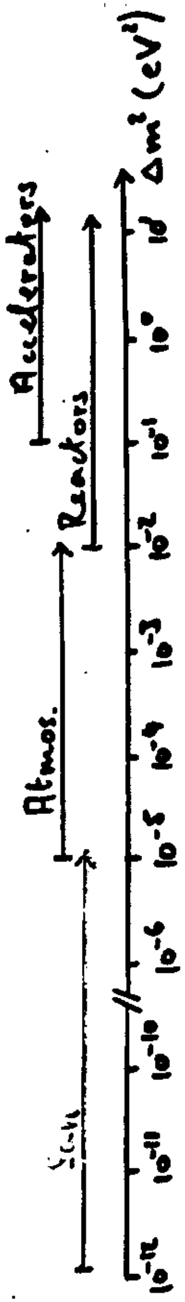
2) Appearance $\nu_\ell \longleftrightarrow \nu_{\ell' \neq \ell}$

Look for $\nu_{\ell'}$ in a beam consisting originally of ν_ℓ

- Only possible if $\nu_{\ell'}$ detectable
- Well suited for small mixings

Table 2. Neutrino sources and typical numbers.

	Type	E_ν	Flux at detector site φ_ν ($cm^{-2}s^{-1}$)	Distance to the detector L (m)	Δ^2_{min} (eV^2)
Sun	ν_e	0–0.4 MeV	10^{10}–10^{11}	1.5×10^{11}	10^{-12}
		0.4–14 MeV	10^{6}–10^{9}		3×10^{-11}
Atmosphere	$\nu_\mu, \bar\nu_\mu$; $\nu_e, \bar\nu_e$	0–1 TeV	1	10^{4}–10^{7}	10^{-5}
Reactor	$\bar\nu_e$	0–8 MeV	10^{13}	10–100	10^{-2}
LAMPF (beam stop)	$\nu_\mu, \bar\nu_\mu$; ν_e	30–50 MeV	10^{7}	10	2
CERN-PS, BNL-AGS	$\nu_\mu, \bar\nu_\mu$	0.5–3 GeV	10^{4}–10^{7}	50–1000	0.3
CERN-SPS, FNAL	$\nu_\mu, \bar\nu_\mu$	5–200 GeV	10^{3}–10^{5}	1000	10



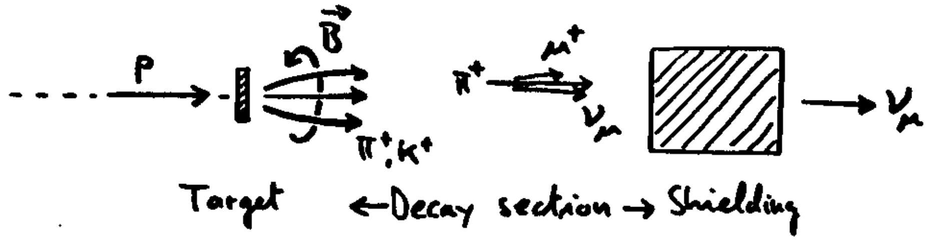
Accelerators

1) ν beam (CERN, Fermilab, BNL)

Decay of K, π in flight:

$$\pi \to \mu \nu_\mu$$
$$K \to \mu \nu_\mu$$
$$K \to \pi e \nu_e$$

Wide Band Beam:



2) π decay at rest (Los Alamos, SNS-RAL)

$$\pi^+ \to \mu^+ \nu_\mu \qquad (\tau = 26\,ns)$$
$$\hookrightarrow e^+ + \nu_e + \bar{\nu}_\mu \qquad (\tau = 2.2\,\mu s)$$

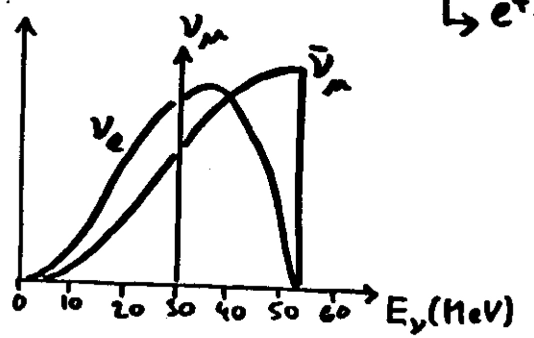
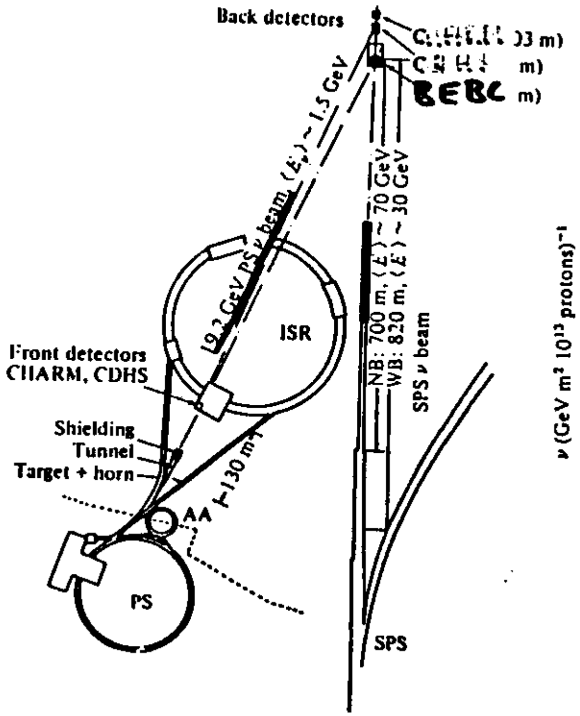
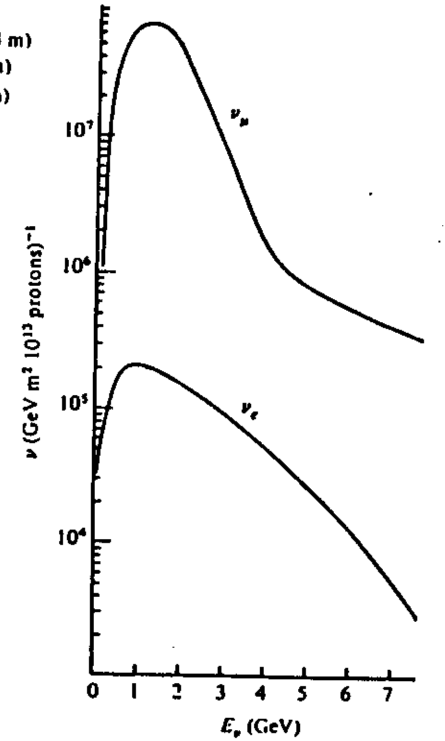


Figure 5.13. Layout of experimental area at CERN (from Bobisut 85).

Figure 5.14. Expected ν_μ and ν_e fluxes at the BEBC detector at CERN (from Bobisut 85).





19.2 GeV beam

$\nu_\mu \leftrightarrow \nu_e$

1) BEBC, CERN-PS : $R_{mes} = \dfrac{\nu_e N \to e^- X}{\nu_\mu N \to \mu^- X} = \dfrac{2}{344}$

compare with $R_{calc} = 4.2 \cdot 10^{-3}$

2) E-734, BNL: 418 $\nu_e n \to e^- p$ seen

$R_{mes} = \dfrac{\nu_e n \to e^- p}{\nu_\mu n \to \mu^- p}$ agrees with R_{calc}

3) PS-191, CERN-PS: $R_{mes} = \dfrac{e \text{ events}}{\mu \text{ events}} = (2 \pm 0.9)\%$

but : $R_{calc} = (0.7 \pm 0.2)\%$!

$\nu_\mu \leftrightarrow \nu_\tau$, $\nu_e \leftrightarrow \nu_\tau$

4) Canada - FNAL - Japan - Korea, FNAL WBB
Emulsion, can resolve τ vertex
no τ seen

$\nu_\mu \leftrightarrow x$

5) CDHS, CHARM, CERN-PS, bare beam (no foc.)
one detector at 100 m, one at 850 m
measured rates scale with $\frac{1}{x^2}$

6) CCFR, FNAL-NBB ($E_{\pi,K} = 100 - 250$ GeV)
one detector at 700 m, one at 1100 m
measured rates scale as expected

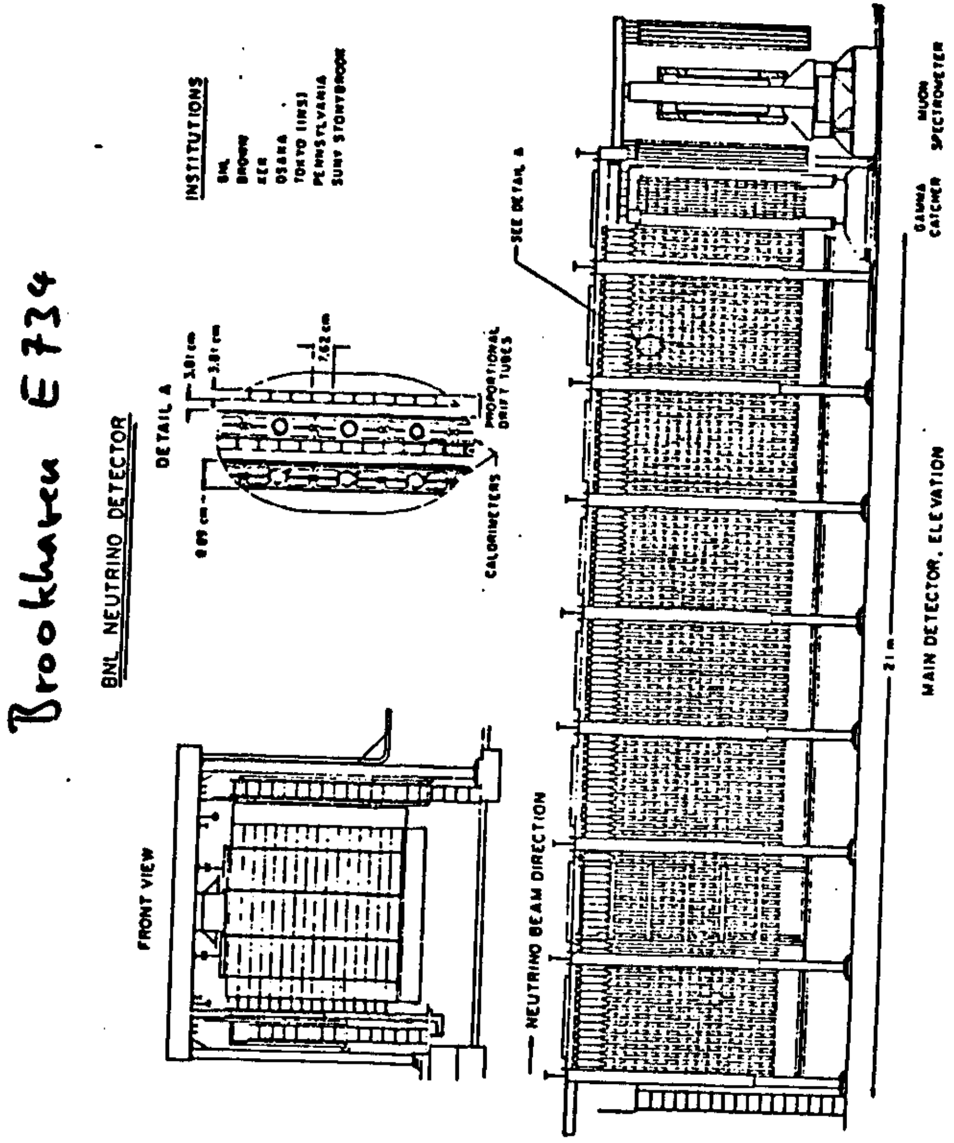


Figure 3. A schematic of the E734 detector.

E 734 - Brookhaven

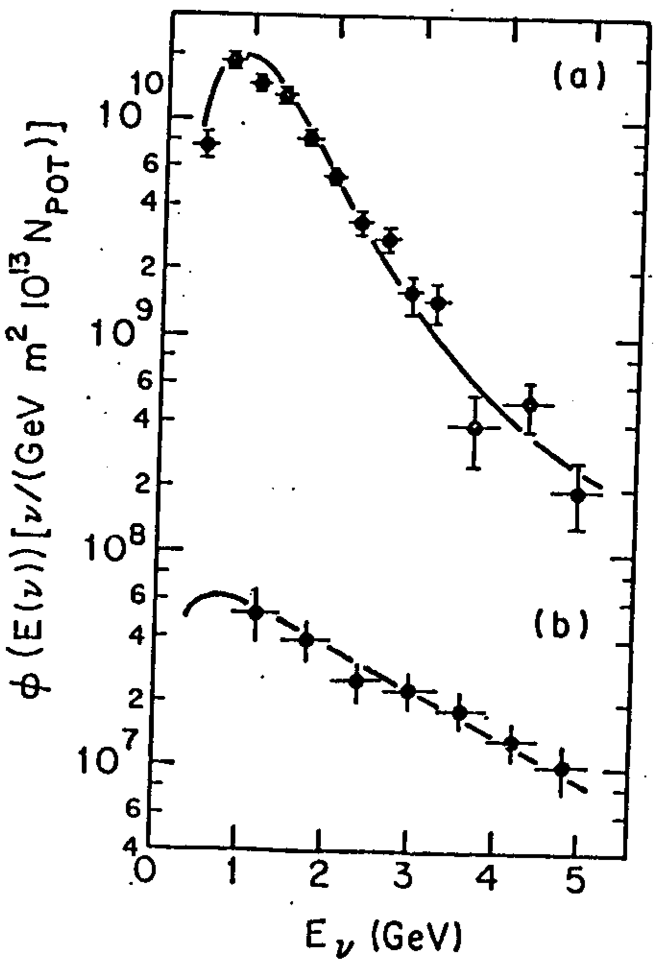
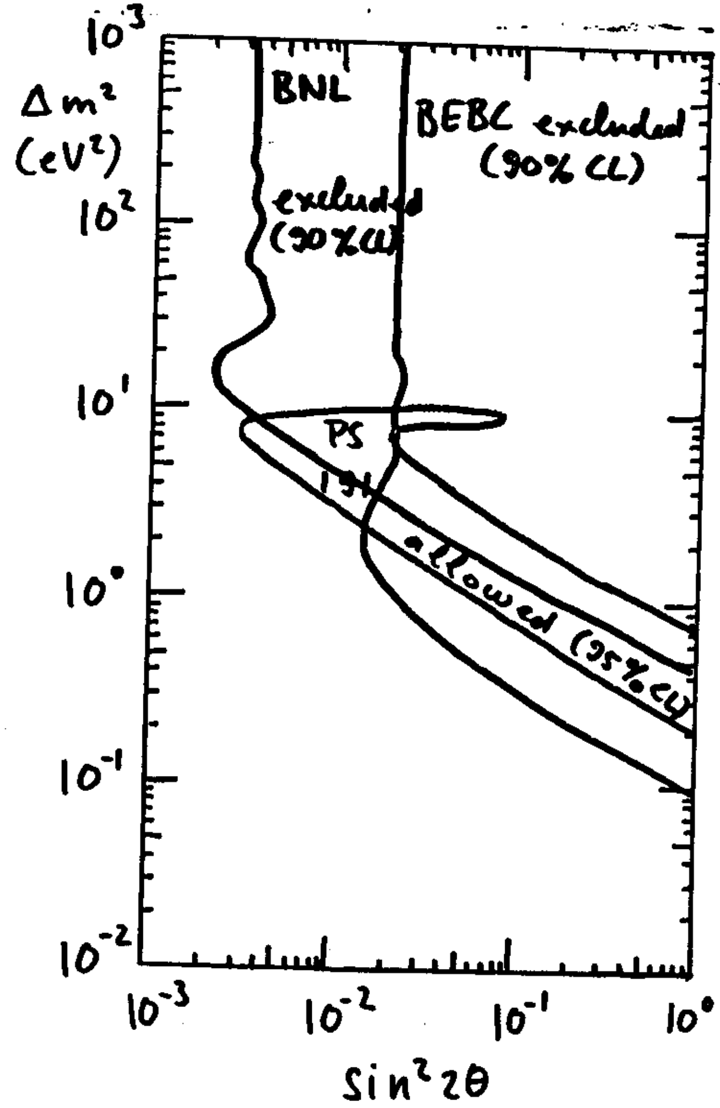
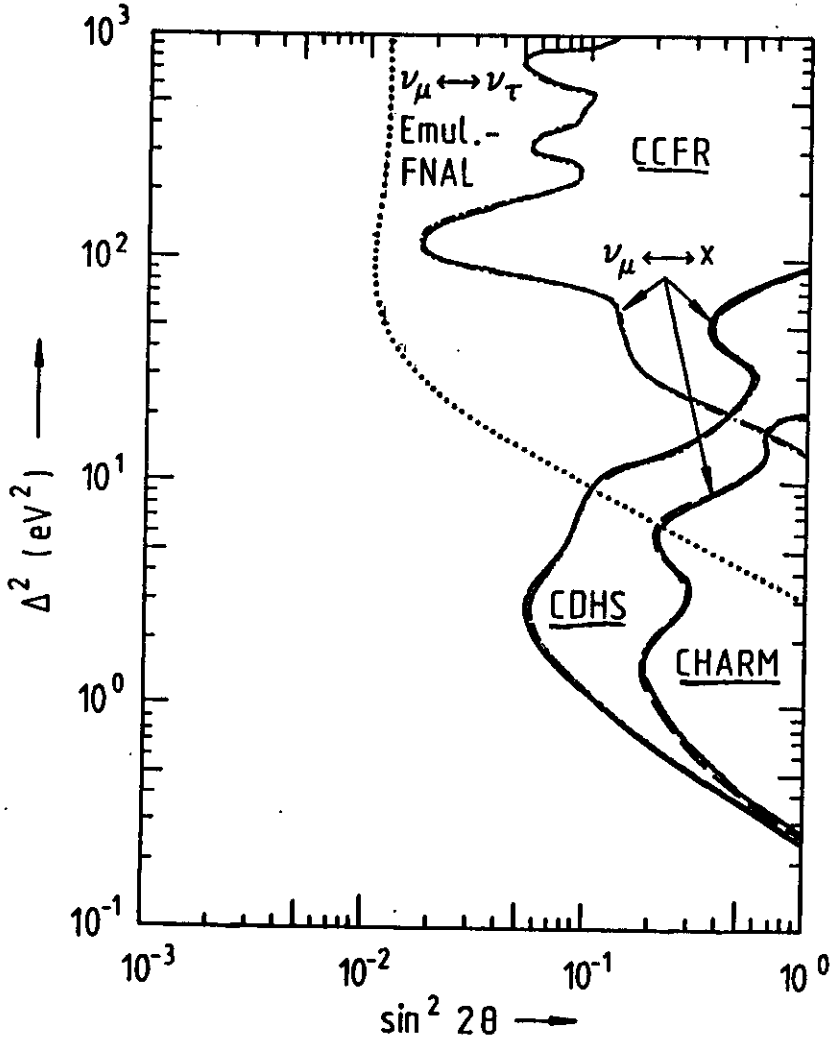


FIG. 2. (a) $\Phi(E(\nu_\mu))$ obtained from data set (i) of $\nu_\mu n \to \mu^- p$ events. (b) $\Phi(E(\nu_e))$ obtained from data set (ii) of $\nu_e n \to e^- p$ events. The solid curves are calculated from a neutrino beam program (Ref. 7). Errors shown are statistical only. The experimental resolution in E_ν is dominated by Fermi motion of the nucleon in the target nucleus and is roughly ±25% at $E_\nu = 1$ GeV.



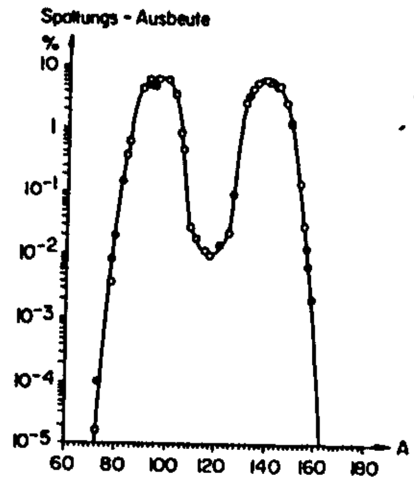
$\nu_\mu \leftrightarrow \nu_e$

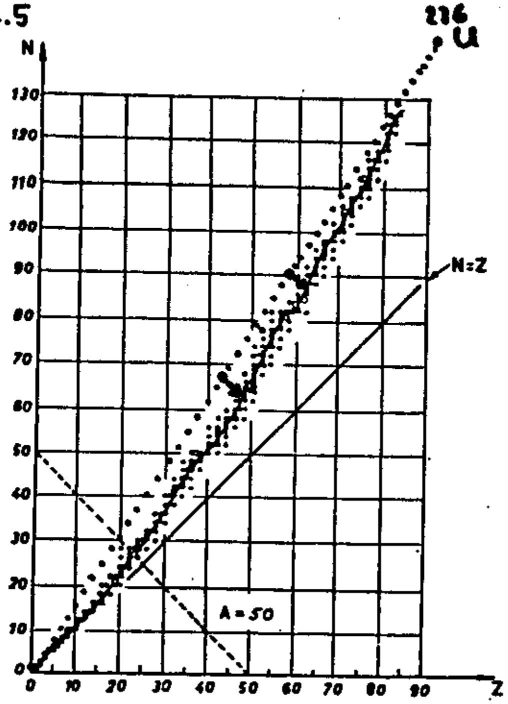


Reactors

Fission: $n + {}^{235}U \rightarrow (A_1, Z_1) + (A_2, Z_2) + k\, n$

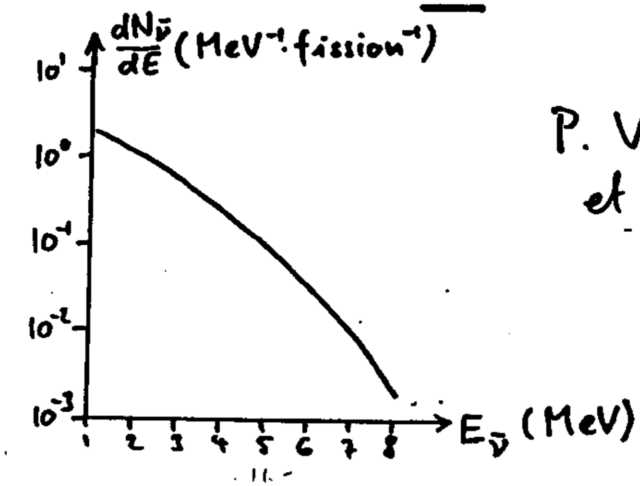
$\bar{k} \cong 2.5$





Followed by ~5 β decays of fission fragments:

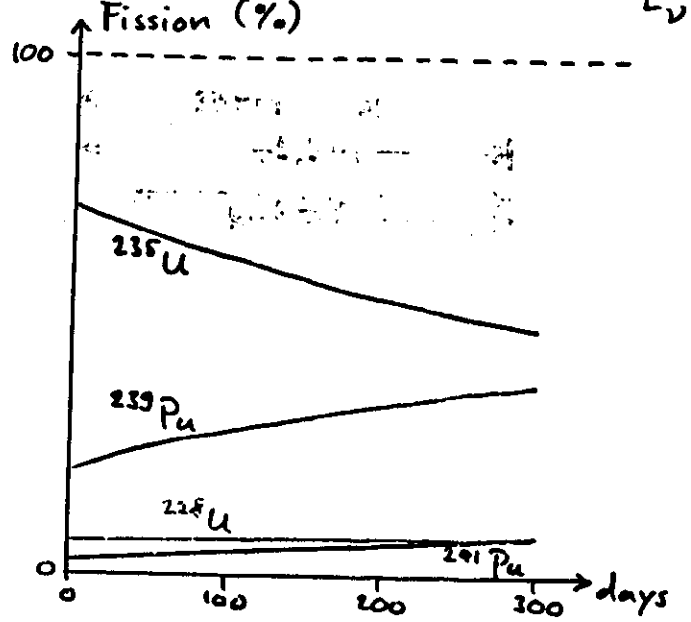
$$(A, Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}_e$$



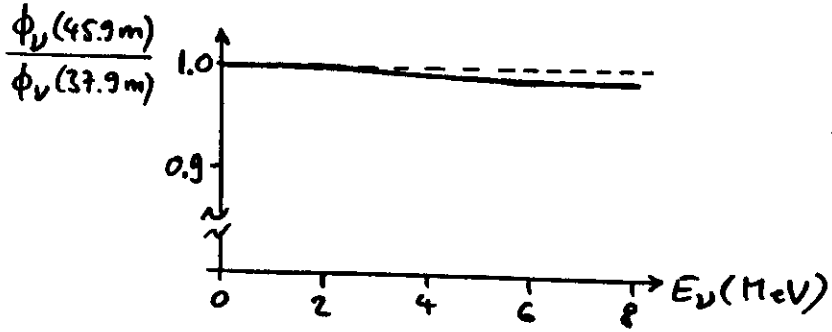
P. Vogel et al.

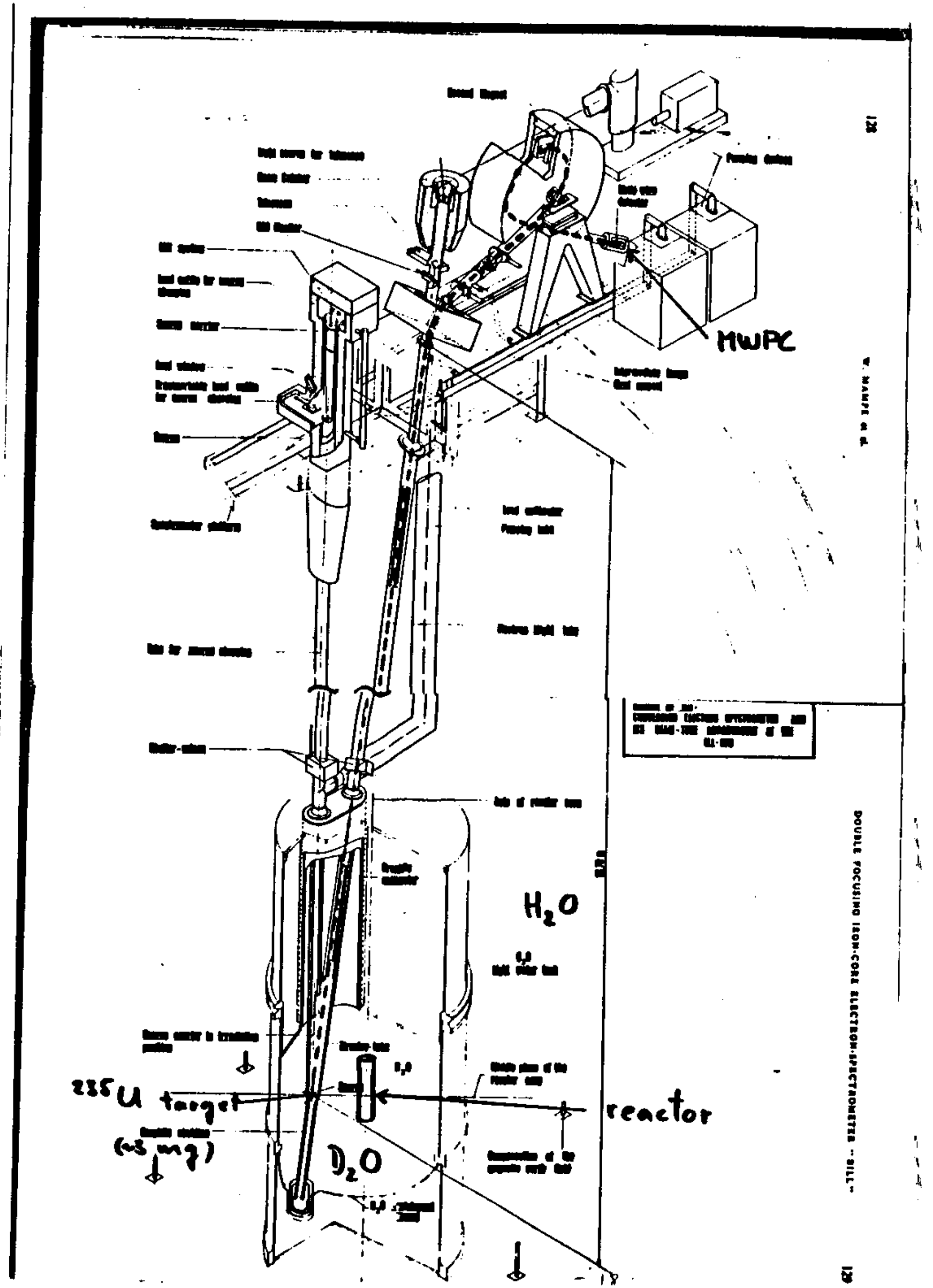
Goesgen, 2806 MWth PWR ~ $5\cdot10^{20}\ \bar{\nu}_e\ s^{-1}$

$E_\nu = 0 \div 8$ MeV



	37.9 m	45.9 m	64.7 m
^{235}U	62.5 %	59.1 %	55.2 %
^{238}U	6.7 %	6.8 %	7.0 %
^{239}Pu	26.7 %	29.3 %	32.1 %
^{241}Pu	4.1 %	4.8 %	5.6 %

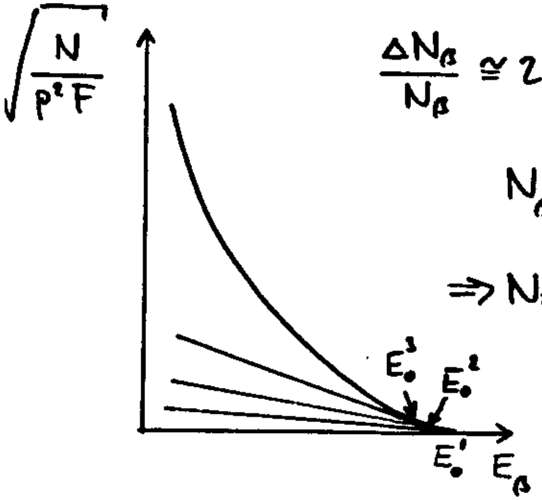




The $\bar{\nu}_e$ spectrum at d=0m

1) ^{235}U, ^{239}Pu (~90%)

β spectra of fission fragments measured on-line at ILL reactor using β-spectrometer BILL:



$$\frac{\Delta N_\beta}{N_\beta} \cong 2\%$$

$$N_\beta(E) \cong \sum a_i N_\beta(E, E_0^i, \bar{Z}^i)$$

$$\Rightarrow N_{\bar{\nu}_e}(E) \cong \sum a_i N_\beta(E_0^i - E, E_0^i, \bar{Z}^i)$$

$$\frac{\Delta N_{\bar{\nu}}}{N_{\bar{\nu}}} = \pm 3\%$$

2) ^{238}U, ^{241}Pu (~10%)

Use $N_\nu^{calc}(E)$ from P. Vogel ($\frac{\Delta N_\nu}{N_\nu} \cong 20\%$) and renormalize:

$$N_{\bar{\nu}_e}(E) \cong N_{\bar{\nu}_e}^{calc}(E) \left(\frac{N_\nu(E)}{N_\nu^{calc}(E)} \Big|_{235,239} \right).$$

$\Rightarrow$ **PWR: $\frac{\Delta N_\nu}{N_\nu} \cong 3.7\%$**

Experiments:

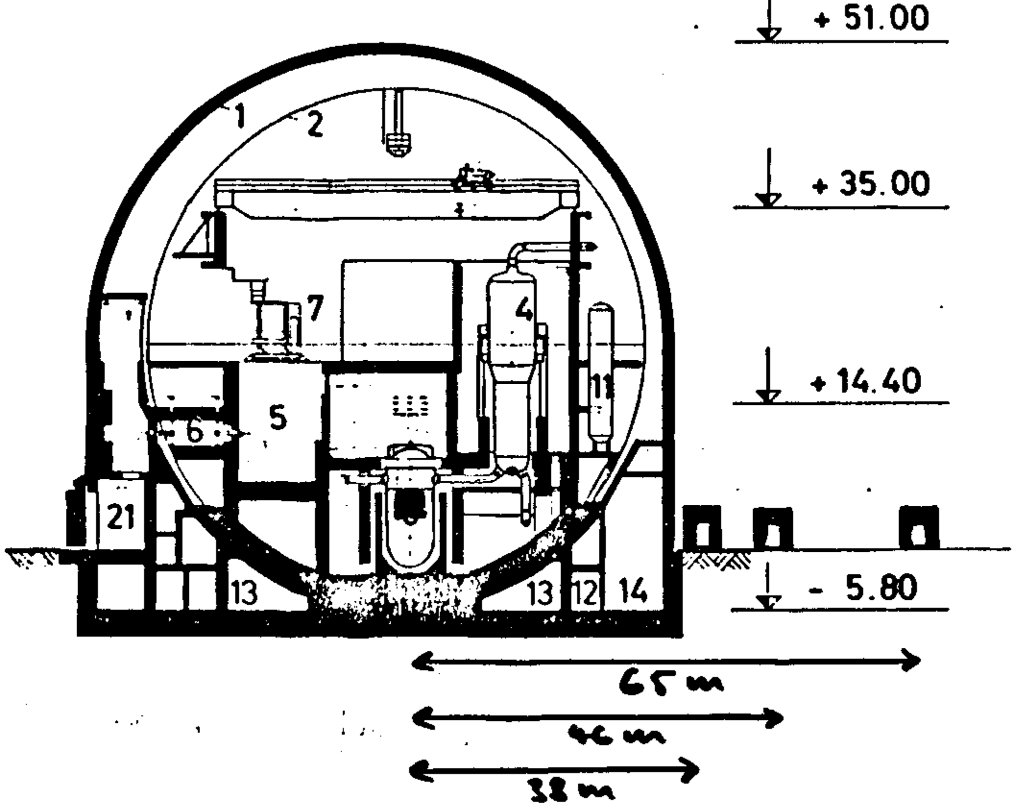
1) Gösgen, PWR, 38 m, 46 m, 65 m

2) Bugey, PWR, 13.6 m, 18.3 m

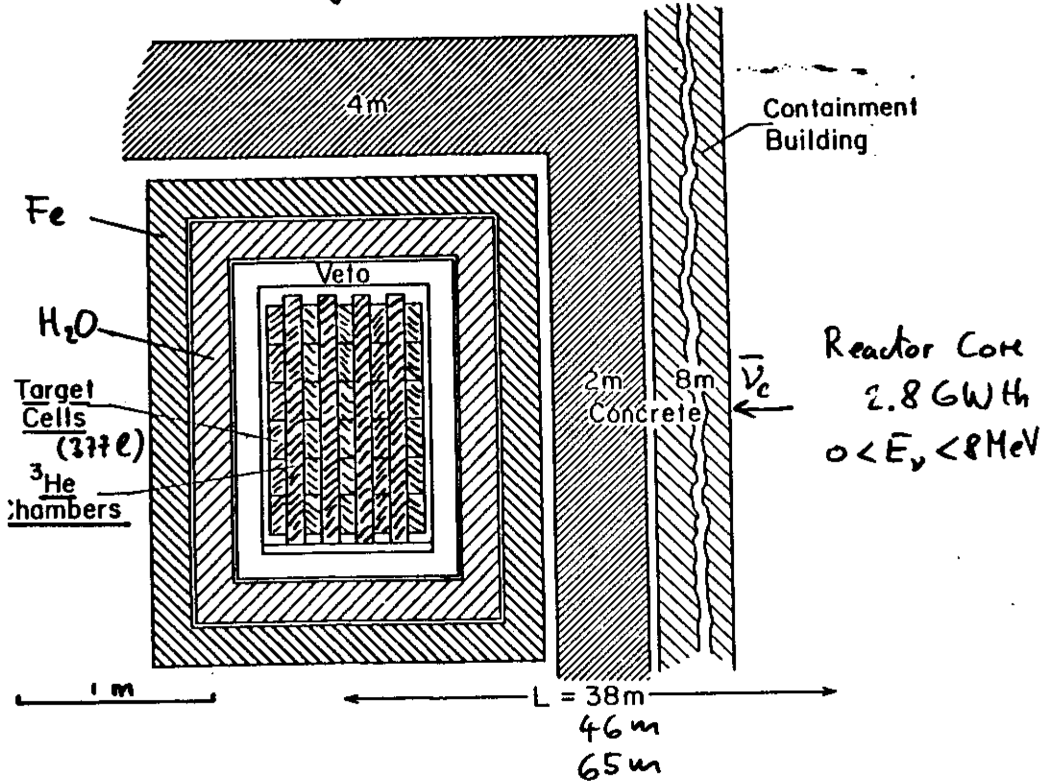
3) Savannah River, Pu, 18.3 m

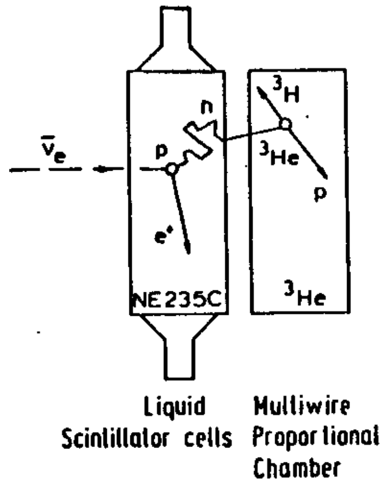
4) Rovno, PWR, 18.5, 25 m

Gösgen



Gösgen, $\bar{\nu}_e \leftrightarrow x$



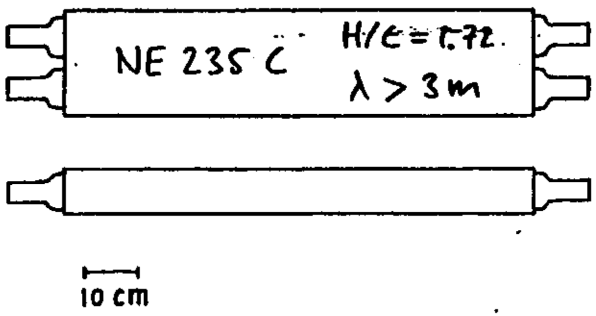


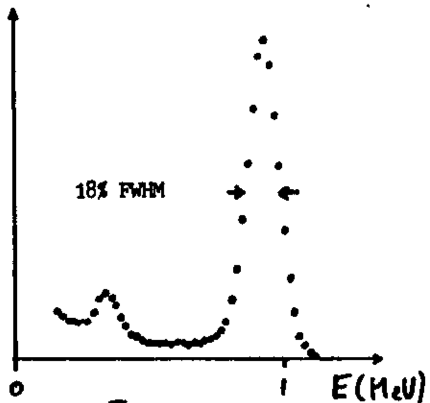
$$E_\nu \cong E_{e^+}^{kin} + 1.8\,\text{MeV}$$

$$\sigma(E_e) = \sigma_0 \cdot p_e E_e$$

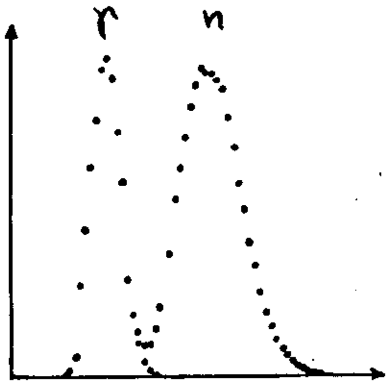
$$\sigma_0 = 9.4 \cdot 10^{-44}\,\text{cm}^{-2} \cdot \text{MeV}^{2}$$

Scintillators





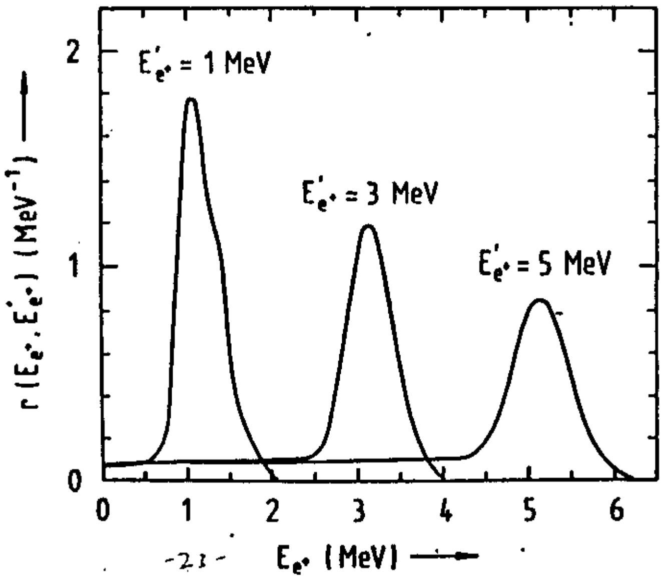
Energy



PSD

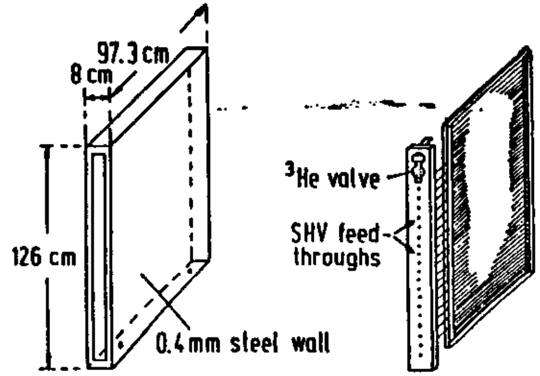
MC response function:

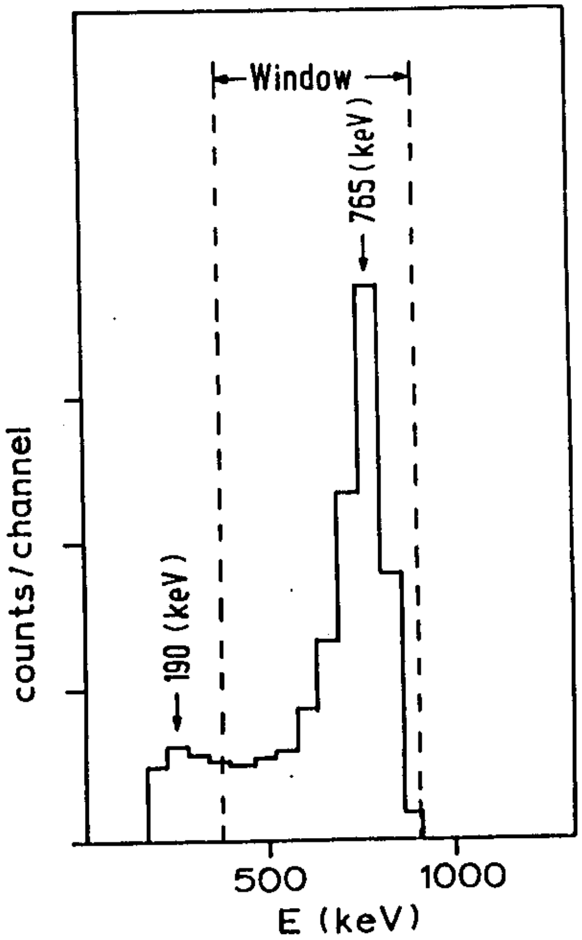
1) Range effect
2) Bremsstrahlung
3) Annihilation



^{3}He MWPC

95% ^{3}He + 5% CO_2
(1 atm)





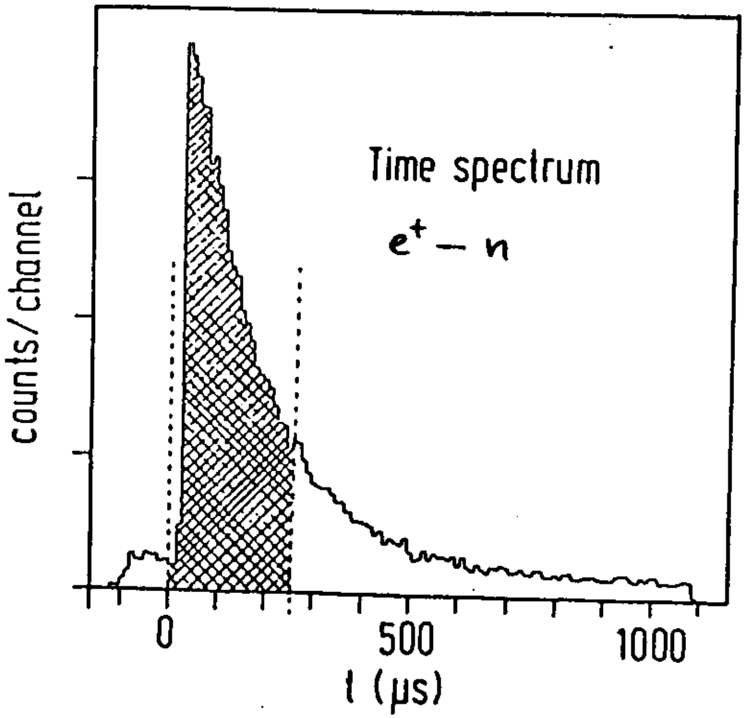


FIG. 10. Distribution of time intervals between neutrino-induced events in a scintillator cell and a wire chamber. The shaded area corresponds to the employed time window of 250 μsec.

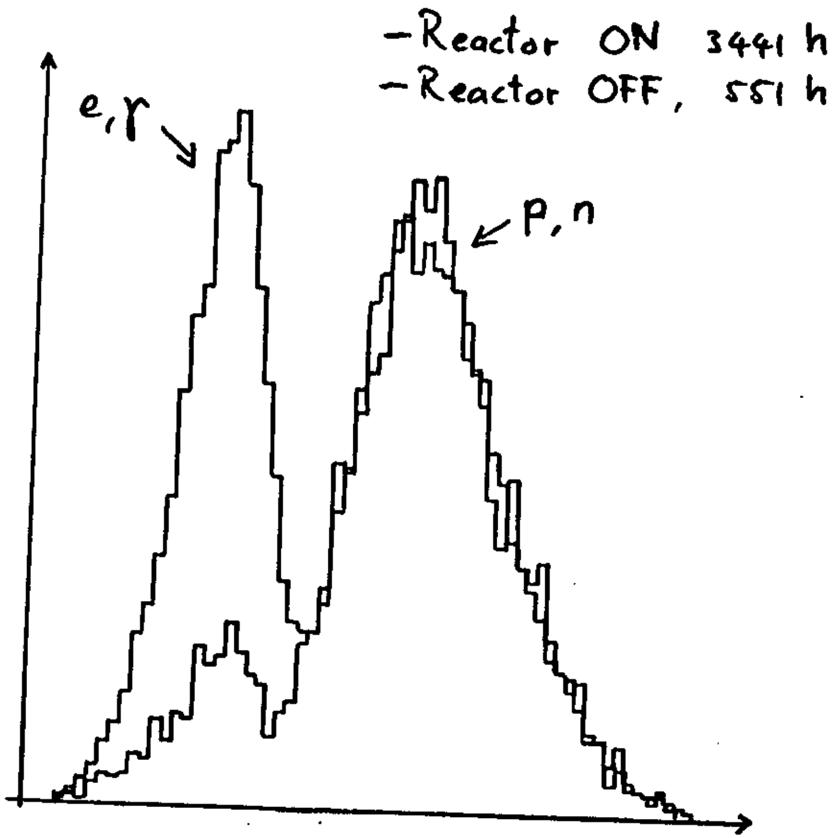
Acceptance:

$\varepsilon_n = 0.210 \pm 0.008$

$\varepsilon_{dt} = 0.879 \pm 0.003$

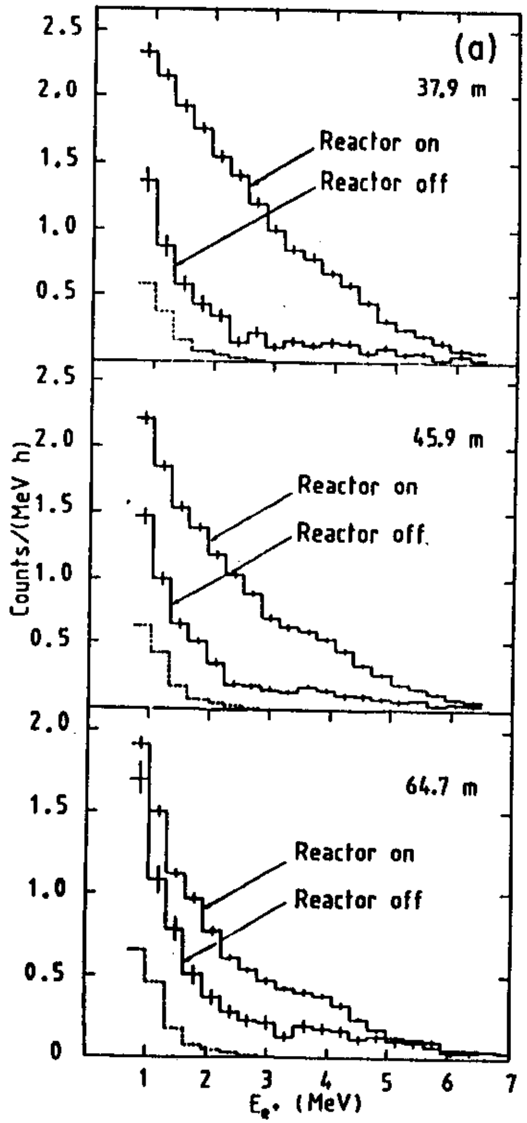
$\varepsilon_{pos} = 0.900 \pm 0.012$

$\varepsilon_{tot} = 0.166 \pm 0.007$

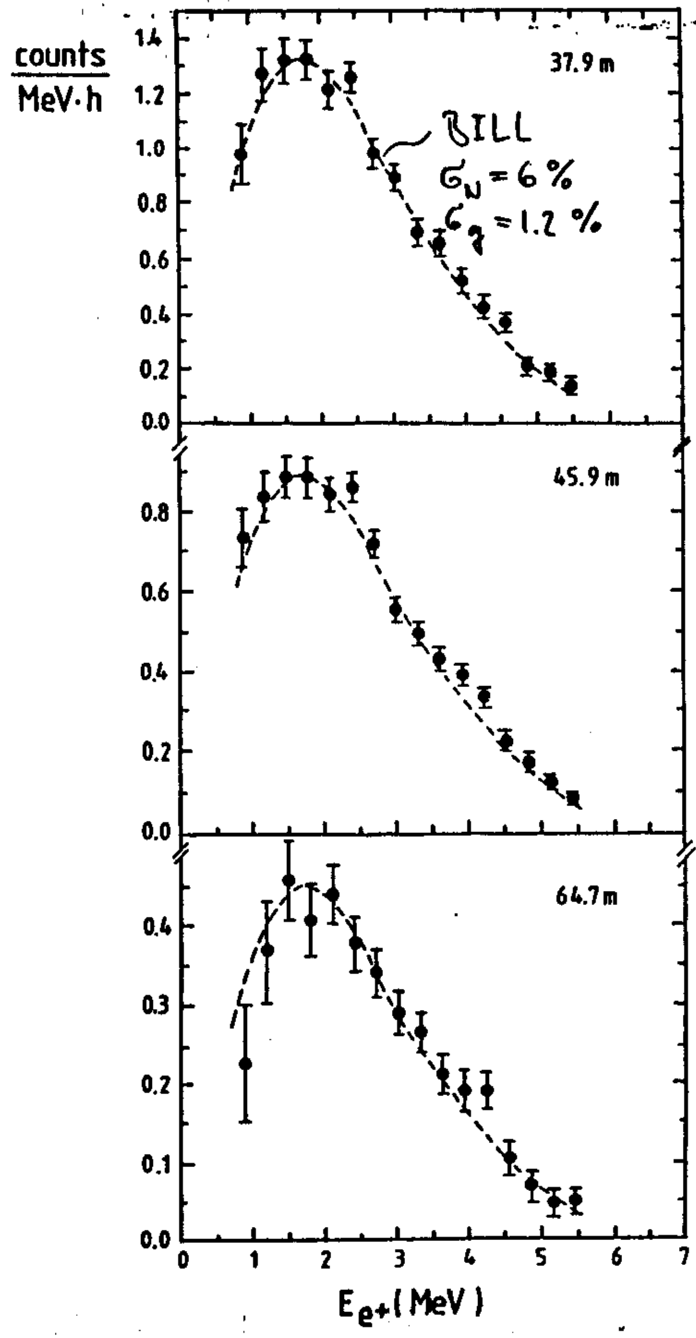


Pulse shape spectrum

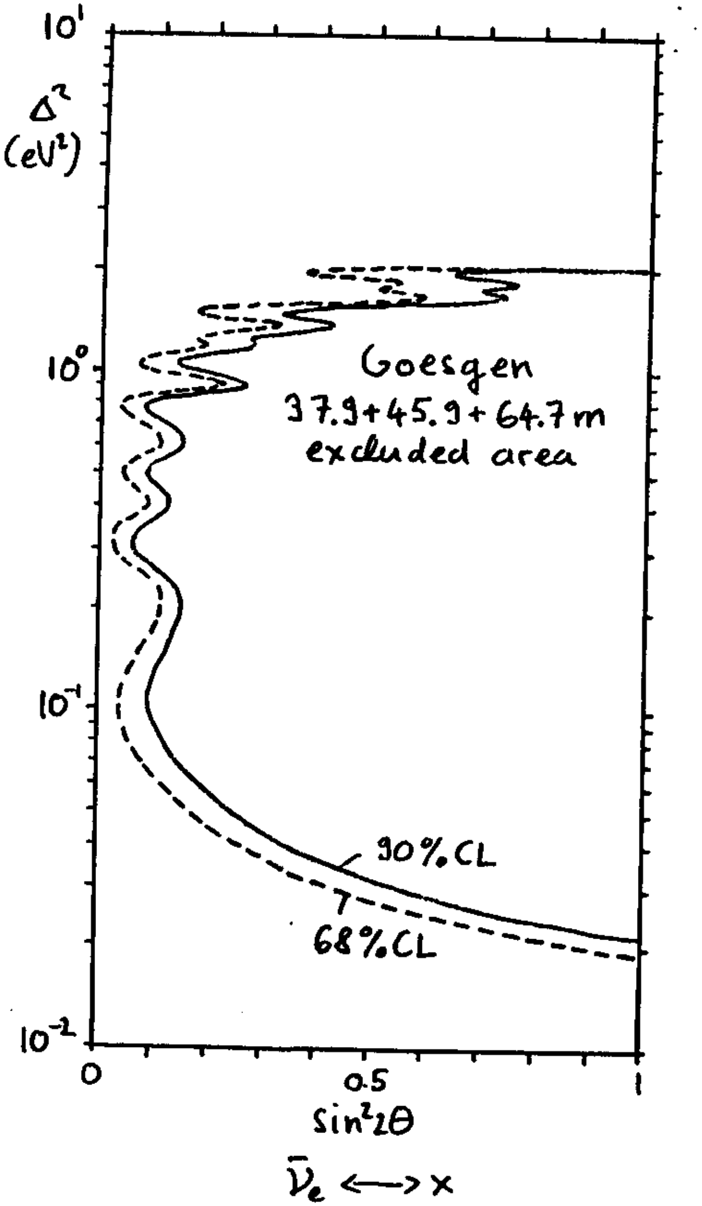
Gösgen

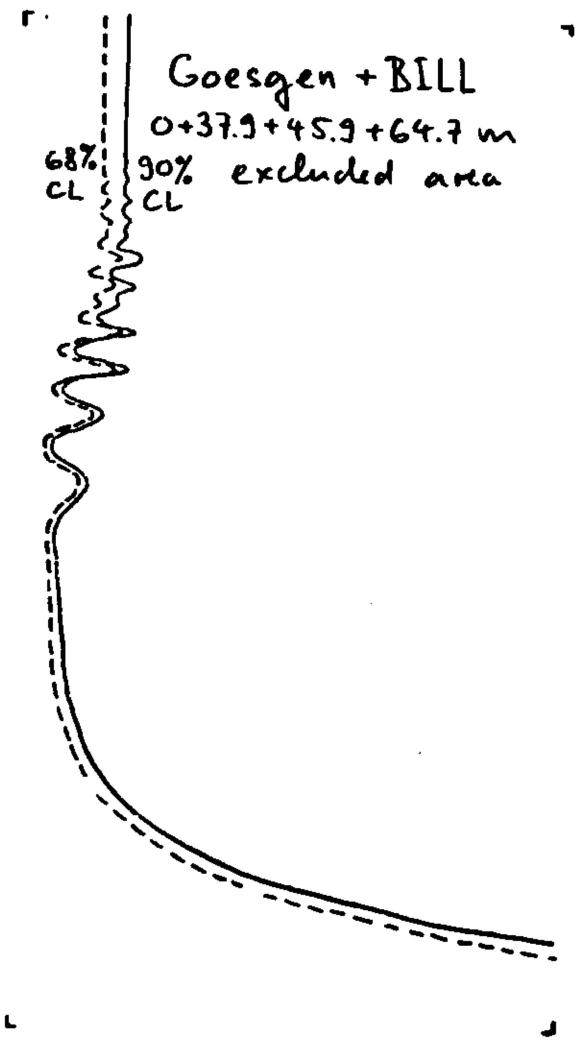


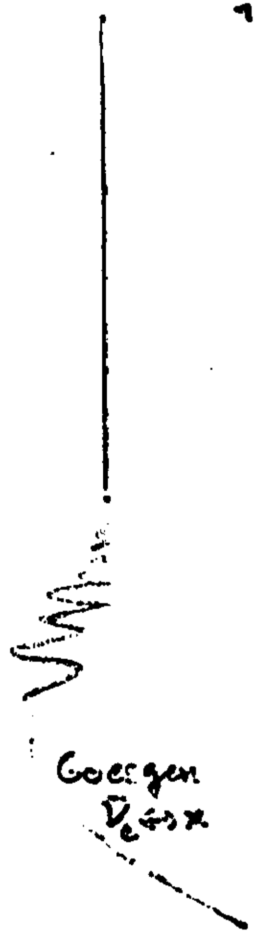
Gösgen



$$\frac{\sum Y_{exp}}{\sum Y_{No\,osc}} = 1.022 \pm 0.062$$


10^1
Δ^2
(eV²)
10^0
10^{-1}
10^{-2}
Goesgen
37.9+45.9+64.7 m
excluded area
90% CL
68% CL
0
0.5
1
$\sin^2 2\theta$
$\bar{\nu}_e \longleftrightarrow x$


Goesgen + BILL
0+37.9+45.9+64.7 m
68% CL
90% CL
excluded area


Goergen