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SCHOOL ON NON-ACCELERATOR PHYSICS
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STANDARD MODEL AND BEYOND (11)

by

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4. Inclusion of STRONG INTERACTIONS in the SM

Before going to possible extensions of the SM, we recall the main features of strong interactions.

The tables of hadrons show spectra of hundreds of states with increasing mass and spin ($\frac{1}{2}$, $\frac{3}{2}$, ... $\frac{11}{2}$, ... for baryons; 0, 1, .. 4, ... for mesons).

They show a peculiar regularity: both baryons and mesons can be grouped in multiplets*, but:

baryons appear only in SINGLETS, OCTETS and DECUPLETS

mesons only in SINGLETS and OCTETS

These are the lowest representations of SU(3) (symmetry proposed by Gell-Mann and Ne'eman in 1960)

FIRST PUZZLE

Why only 1, 8 and 10 (and not e.g. 3, 6, 15, 27... of SU(3)) ?

To solve this puzzle, Gell-Mann and Zweig (1964) introduced the hypothesis of <u>QUARKS</u>:

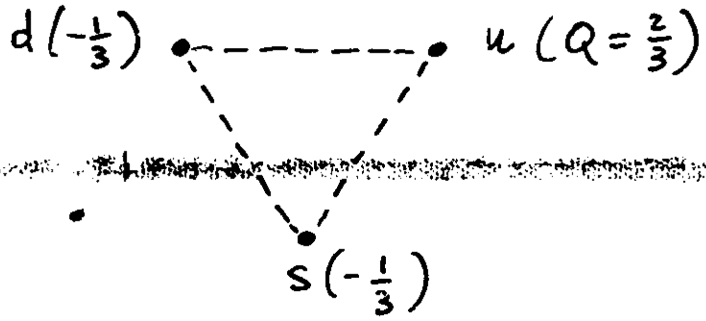
<u>baryons</u> are made of 3 quarks

$$3 \times 3 \times 3 = 1 + 8 + 10$$

<u>mesons</u> = quark-antiquark pairs

$$3 \times \bar{3} = 1 + 8$$

This hypothesis implies that quarks are peculiar particles, with spin ½ and fractional charges



SECOND PUZZLE

How can there be 3 quarks (fermions) in the same state?

E.g. $\Delta^{++} \ (s = 3/2,\ s_z = 3/2) = (u^{\uparrow} u^{\uparrow} u^{\uparrow})$

Hypothesis of colour

Each quark possesses extra quantum numbers :

$$SU(3)_f \qquad (\text{"flavour"} : u, d, s)$$

$$SU(3)_c \qquad (\text{"colour"} : u, u\cdot, u)$$

For a given type of quark, there are in fact 3 different ones ; e.g.

$$\Delta^{++} = (u^{\uparrow} u^{\uparrow} u^{\uparrow})$$

Note

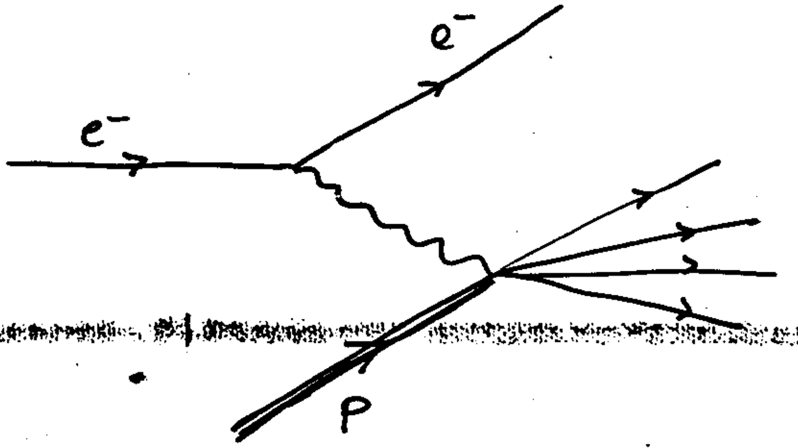
The flavour group was extended from $SU(3)_f$ to $SU(4)_f$ when the 4th quark (c for "charm") was discovered in 1974.

The number of flavours became 5 with the discovery of a new type of quarks (b for "bottom") in 1977 , and hopefully should become at least 6.

EVIDENCE FOR QUARKS ?

No experimental evidence for free quarks, but indirect evidence of quarks (with $S = 1/2$ and $Q = 2/3, -1/3$) inside nucleons.

This evidence came from the deep inelastic scattering of 20 GeV electrons by protons, at SLAC (early 70's)



Structure of nucleons investigated with high mom. transfer: they appear as made of point-like constituents.

EVIDENCE FOR COLOUR ?

1) $e^- e^+ \to$ hadrons

$$R = \frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} = 2\left(\frac{N}{3}\right)$$

(below charm threshold)

Experiments in agreement with $N=3$
($N = \#$ of colours)

2) $$\pi^0 \to \gamma + \gamma$$

$$\Gamma_{\pi^0 \to \gamma\gamma} = 7.9 \left(\frac{N}{3}\right)^2 \ \text{eV}$$

$$\left(\Gamma_{\pi^0 \to \gamma\gamma}\right)_{exp} = 7.95 \pm 0.55 \ \text{eV} \quad \to \underline{N = 3}$$

THIRD PUZZLE

Why there are no free quarks?

Once more symmetry gives a solution: the $SU(3)_c$ symmetry is promoted to local gauge symmetry.

One obtains a field theory: quantum chromodynamics (QCD), with 8 vector gauge fields (corresponding to the 8 generators of $SU(3)$) which are massless:

GLUONS: they mediate the interactions among quarks.

Main difference between QCD and QED:

QED : photons are neutral. Vacuum polarization produces a screening of the electric charge.
The charge appears smaller at large distances, and it increases at short distances (high mom. transfer).
The (neutral) photons do not modify this behaviour.

QCD : gluons carry colour as quarks do, so that they interact directly among themselves.

This fact changes dramatically the situation: the "effective" coupling α_{QCD} decreases by decreasing the distance (ASYMPTOTIC FREEDOM) while it increases beyond any limit by going to larger and larger distances (CONFINEMENT)

In conclusion, also strong int.s besides electroweak are described by a GAUGE FIELD THEORY. All together the Standard Model is based on the gauge group

$$G_S \equiv \boxed{SU(3)_C \times SU(2)_L \times U(1)_Y}$$

unbroken — broken to $U(1)_{QED}$

Before going beyond the "Standard Theory", we have to look at the present picture of the matter constituents.

	SINGLETS	TRIPLETS			of $SU(3)_c$
DOUBLETS	ν_e	u	u	u	1st GENERATION
	e^-	d	d	d	(light)
SINGLETS of SU(2)	e^+	$\bar{u}, \bar{d}$	$\bar{u}, \bar{d}$	$\bar{u}, \bar{d}$	

ν_μ	c	c	c	2nd GENERATION
μ^-	s	s	s	(heavier)
μ^+	$\bar{c}, \bar{s}$	$\bar{c}, \bar{s}$	$\bar{c}, \bar{s}$	

ν_τ	t	t	t	3rd GENERATION
τ^-	b	b	b	(much
τ^+	$\bar{t}, \bar{b}$	$\bar{t}, \bar{b}$	$\bar{t}, \bar{b}$	heavier)

Why to go beyond the SM?

Notwithstanding its phenomenological success

- it fits all weak interaction date
- it gives e consistent framework for strong interactions
- no experimentel hint for going beyond,

the SM is not satisfactory from the theoretical point of view.

In fact, it contains a great deal of arbitraryness:

- No reason for the assignment of the quantum numbers of quarks and leptons (I, Y), except that of reproducing the right structure of currents, and values of Q.

- Number of arbitrary parameters is rather large; some of them not yet determined experimentally. In the minimal version:

3 coupling constants $g_1, g_2, g_3 = g_s$

1 breaking scale v_0

1 mass of Higgs H

6 quark masses

3 charged lepton masses

3 mixing angles (KM matrix)

1 CP-violating phase

1 strong CP-violating parameter $\bar{\theta}$

19

Extra parameters in extended versions: neutrino masses and relative mixing angles and phases.

These points suggest to go beyond the standard theory: an attractive possibility consists in going to GRAND UNIFICATION.

5. GRAND UNIFICATION

How to go to grand unification

The main motivation for grand unified theories (GUTs) are the unsolved problems of the standard theory:

1) S.T. does not provide a complete unification (g_1, g_2, g_3), while we would like a single coupling constant

2) Quark and lepton charges remain unrelated, while we would like to explain why $Q_d = \frac{1}{3} Q_e$, i.e. why $Q_p = -Q_e$ holds to extreme accuracy

3) We would like to relate the low-energy parameters of S.T., obtaining e.g. mass relations, and thus reducing the degree of arbitrariness.

In order to satisfy 1) and 2), we look for a "simple" group G (with only one independent g) that contains Y (and then Q) among its generators: then the relative value of Q within a given multiplet (representation of G) are fixed.

The group G must satisfy the following <u>conditions</u>:

- → G must contain $G_S = SU(3)_c \times SU(2)_L \times U(1)_Y$ as a <u>subgroup</u>, to keep good features at M_W scale. Then its rank : <u>$r \geq 4$</u>.

- → Due to chiral nature of G_S (f_L are doublets and f_R are singlets), G must have <u>complex representations</u> with respect to G_S (representations conjugate to one another not equivalent), where to accomodate f_L and f_R.

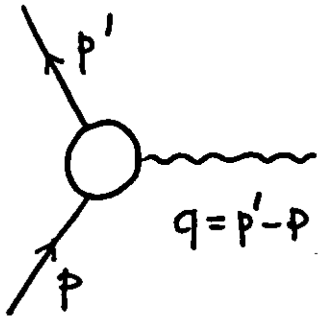
- → <u>Renormalizability</u> of the theory requires representations which are free from triangle anomalies.

These conditions single out <u>SU(5)</u> (Georgi + Glashow 1974) as the <u>simplest choice</u> ; in fact, it is the <u>only</u> possible choice for <u>$r = 4$</u>.
Another attractive choice is <u>SO(10)</u> for <u>$r = 5$</u> (Georgi '74; Fritzsch + Minkowski '75) _ For higher rank there are other possibilities, e.g. <u>E_6</u> (Gürsey et al. '75)
<u>Unification of colour & flavour</u> was first proposed by Pati & Salam (1973) with $G = (SU(4))^3$.

Before going to specific GUTs, we discuss the main feature of grand unification, i.e. how such <u>different</u> coupling consts g_1, g_2, g_3 can be <u>forced to coincide</u>.

The clue for understanding grand unification is that the "effective" coupling constants are not really constant (as the "bare" ones appearing in the Lagrangian), but depend on the scale at which the (renormalized) vertex functions are normalized. (Georgi, Quinn, Weinberg '74)

The evolution of the effective coupling $g(\mu^2)$, when $q^2 = \mu^2$ is the external momentum, can be expressed by the renormalization group equation



$$\frac{d\, g(\mu^2)}{d \ln \mu} = \beta(g)$$

The function $\beta(g)$ can be evaluated pertubatively for small values of g:

$$\beta(g) = \frac{1}{4\pi} b g^3 + O(g^5)$$

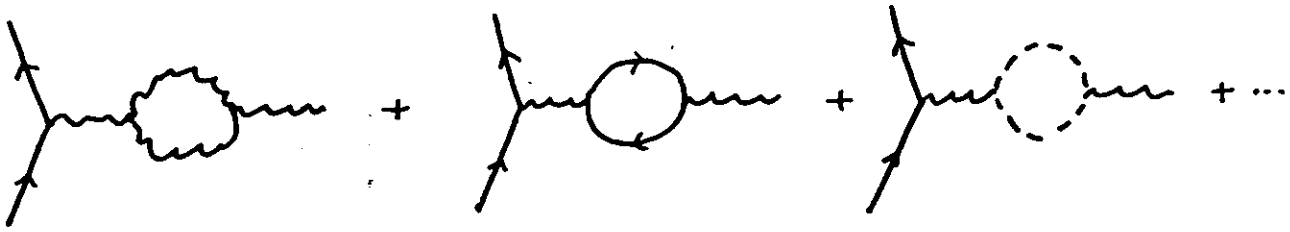
b is a constant depending only on the gauge group and multiplet representations.

One gets to lowest order:

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M)} + \frac{b}{4\pi} \ln \frac{M^2}{\mu^2}$$

If $b<0$, $g^2(\mu)$ decreases with increasing $\mu < M$: <u>asymptotic</u> <u>freedom</u>.

Evaluation of b_1, b_2, b_3 (corresponding to $g_i(\mu)$, $i=1,2,3$) from the graphs



gives

$$b_1 = + \frac{1}{4\pi}\left\{\frac{4}{3} N_g + \frac{1}{10} N_s\right\}$$

$$b_2 = - \frac{1}{4\pi}\left\{\frac{22}{3} - \frac{4}{3} N_g - \frac{1}{6} N_s\right\}$$

$$b_3 = - \frac{1}{4\pi}\left\{11 - \frac{4}{3} N_g\right\}$$

where: N_g = no. of fermion generations ($N_g \geqslant 3$)

N_s = no. of Higgs doublets ($N_s = 1$ in minimal model)

Unless $N_g > 5$, both g_2 and g_3 <u>decrease with increasing μ</u>; moreover

$$|b_3| > |b_2| > b_1$$

so that <u>strong interactions</u> are in fact <u>stronger than electroweak</u> (since there are <u>3</u> colours).

The requirement of grand unification at a scale M

$$\underline{g_1(M) = g_2(M) = g_3(M) = g_{GUT}},$$

determines the $\underline{\text{unifying scale } M}$ if one knows the values of $\underline{\text{two}}$ couplings at some scale μ.

It is convenient to use

$$\alpha_s \equiv \alpha_3 = \frac{g_3^2}{4\pi} \qquad \text{and} \qquad \alpha = \frac{e^2}{4\pi}$$

where :

$$\frac{1}{\alpha} = \frac{1}{\alpha_2} + \frac{5}{3}\frac{1}{\alpha_1}$$

↳ different normalization of g_1

One gets

$$\boxed{\frac{1}{\alpha(\mu)} - \frac{8}{3}\frac{1}{\alpha_s(\mu)} = \frac{11}{2\pi}\ln\frac{M^2}{\mu^2}}$$

Usually $\alpha_s(\mu)$ is expressed in terms of the $\underline{\text{QCD parameter}}$ $\Lambda \simeq 0.2$ GeV

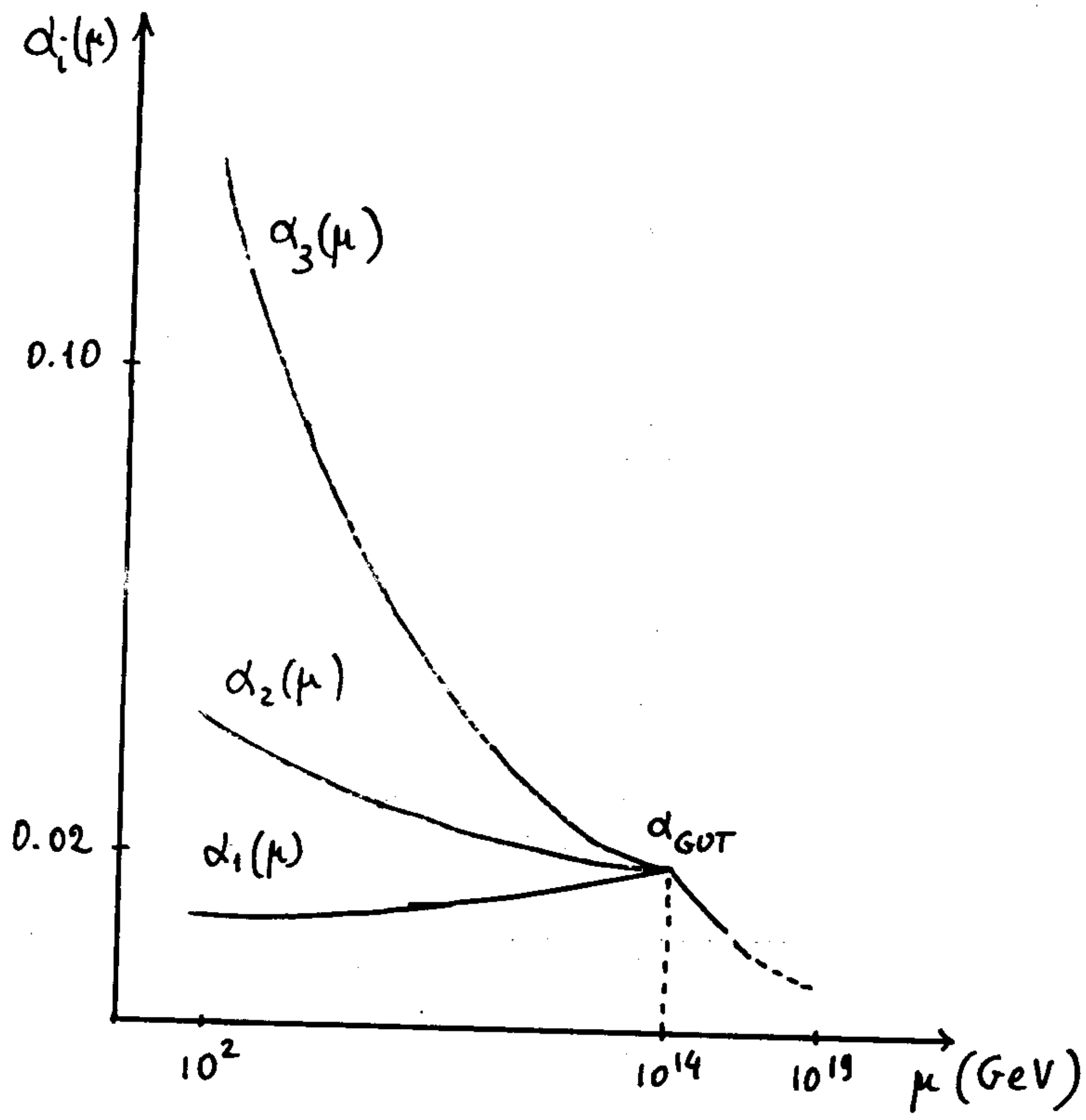
$$\alpha_3(\mu) = \frac{1}{b_3 \ln \Lambda^2/\mu^2}$$

Taking $\mu \approx M_W$ and $\alpha_s(M_W) \approx 0.1$:

$$M \simeq O(10^{14}\ \text{GeV})$$

$$\alpha_{GUT} \simeq 0.023$$

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M)} + b_i \ln\frac{M^2}{\mu^2}$$



Note that the rate of approach to Grand Unification is very slow:
from 10^2 GeV of Electroweak unification, one has to reach $\sim 10^{14}$ GeV!

The SU(5) way

For a more quantitative discussion we go to a specific model, based on the group $G = SU(5)$.

a) Fermion sector

Each generation of fermion can be fitted nicely in the representation (anomaly free)

$$15 = \bar{5} + 10$$

which is decomposed with respect to the subgroup G_S according to ($G_S = SU(3) \times SU(2) \times U(1)_Y$)

$$\bar{5} = (1,2)_{-1} + (\bar{3},1)_{2/3}$$

$$10 = (1,1)_2 + (\bar{3},1)_{-4/3} + (3,2)_{1/3}$$

E.g. for the first generation

$$\bar{5}: \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L + (d_1^c\, d_2^c\, d_3^c)_L$$

$$10: \quad e_L^+ + (u_1^c\, u_2^c\, u_3^c)_L + \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix}_L .$$

Since the electric charge is a generator of $SU(5)$, it is traceless :

$$Q = I_3 + \tfrac{1}{2} Y \quad , \quad \mathrm{Tr}\, Q = 0$$

For the representation $\bar{5}$:

$$3Q_{d^c} + Q_e = 0 \longrightarrow \boxed{Q_d = \frac{1}{3} Q_e}$$

Since there are <u>3 colours</u>, the d-charge is ⅓ of the charge of the electron (charge quantization). Since u and d are in a <u>I-doublet</u>

$$Q_u - Q_d = 1 \longrightarrow Q_u = \frac{2}{3}$$

and

$$\boxed{Q_P = -Q_e}$$

Old mistery explained!

b) <u>Gauge fields</u>

The gauge bosons must be assigned to the <u>adjoint representation</u> 24 (one for each generator). From

$$5 \times \bar{5} = 1 + 24$$

one obtains the decomposition

$$24 = \underset{\substack{\downarrow \\ \text{gluons} \\ (SU(3)_c)}}{(8,1)_0} + \underbrace{(1,1)_0 + (1,3)_0}_{\substack{\downarrow \\ A^i_\mu,\ B_\mu \\ SU(2)_L \times U(1)_Y}} + \underbrace{(3,2)_{-5/3} + (\bar{3},2)_{5/3}}_{\substack{\downarrow \\ \text{lepto-quarks}}}$$

(same quantum no. of qq, $\bar{q}\bar{q}$ and $q\ell$, $\bar{q}\bar{\ell}$).

Denote the leptoquarks by

$$\begin{pmatrix} X & X & X \\ Y & Y & Y \end{pmatrix} \qquad \begin{array}{l} Q = 4/3 \\ Q = 1/3 \end{array}$$

and similarly for their antiparticles.
They can mediate <u>PROTON DECAY</u>, and they must be <u>superheavy</u>.

How can X, Y become superheavy and $W^\pm, Z^0$ acquire a mass ~ 100 GeV?

One requires a <u>two-step breaking</u>

$$G \xrightarrow[M_X]{} G_S \xrightarrow[M_W]{} G_0$$

The minimal set of <u>scalar multiplets</u> which can accomplish the job consists of:

$$\Phi_{24} \quad \text{and} \quad \varphi_5$$

The scalar potential presents a minimum at the v.e.v.

$$\langle \Phi_{24} \rangle_0 \sim (1,1)_0 \sim V_0$$

The vacuum symmetry is indeed $G_S = SU(3) \times SU(2) \times U(1)$.
The <u>12 leptoquarks X, Y acquire mass</u> of order V_0 (one needs $V_0 \sim 10^{14}$ GeV), while the <u>8 gluons</u> + <u>4 electroweak</u> gauge fields remain still <u>massless</u>.

The second step of breaking can be generated by the φ_5 in the Higgs potential

$$\langle\varphi_5\rangle_0 \sim (1,2)_0 \approx \frac{1}{\sqrt{2}} v_0 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

(φ_5 has the same rôle of the Higgs doublet in the SM)

Now $W^{\pm}$ and Z^0 acquire mass of order v_0 ($v_0 \sim 100$ GeV).

The situation is summarized by

Gauge group	No. of gauge fields	Higgs
$G \equiv SU(5)$	24	$\Phi_{24} \to \langle 1,1\rangle_0 \neq 0$ 12 (X,Y) massive
$\downarrow$		
$G_S \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$	12	$\varphi_5 \to \langle 1,2\rangle_0 \neq 0$ $(W^{\pm}, Z^0)$ massive
$\downarrow$		
$G_0 \equiv SU(3)_c \times U(1)_Q$	9	

Symmetry predictions

Due to the higher symmetry, $\sin^2\theta_W$ is no longer an arbitrary parameter as in the SM, but it has a fixed value.

Recall that

$$\sin^2\theta_W = \frac{e^2}{g_2^2} = \frac{g_{1S}^2}{g_{1S}^2 + g_2^2} = \frac{\alpha_{1S}}{\alpha_{1S}+\alpha_2}$$

Note that $g_{1S} \equiv g_1$ in SM has a different normalization from g_1 in $SU(5)$

$$\alpha_{1S} = \frac{3}{5}\alpha_1$$

At the GUT scale M_X :

$$\boxed{\sin^2\theta_W = \frac{3\alpha_1}{3\alpha_1 + 5\alpha_2} = \frac{3}{8}}$$

This value is scaled down going to lower scales $\mu < M_X$

$$\sin^2\theta_W = \frac{3\alpha_1(\mu)}{3\alpha_1(\mu)+5\alpha_2(\mu)} = \frac{3}{8} - \frac{55}{48\pi}\alpha \ln\frac{M_X^2}{\mu^2}$$

Taking into account the variation of $\alpha_1(\mu)$ and $\alpha_2(\mu)$, and including also higher order corrections, one gets at the scale M_W :

$$\sin^2\vartheta_W(M_W) = 0.214 \pm 0.004 \qquad (SU(5))$$

which appears too low with respect to the value obtained from the global fit of low-energy NC data and UA1 + UA2 data:

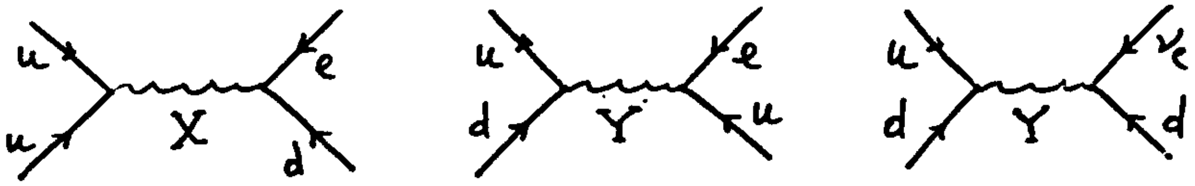
$$\sin^2\vartheta_W = 0.230 \pm 0.005 .$$

Proton decay

SU(5) symmetry which puts quarks and leptons in the same multiplet obviously implies violation of B and L. However, in the minimal model B−L is conserved. This is because B−L is related to a global symmetry, which is left unbroken by $\langle \varphi_5 \rangle \neq 0$.

Proton can decay, but B−L conservation implies that it decays into antileptons ($e^+, \bar{\nu}_e, \ldots$), but not into leptons ($e^-, \nu_e, \ldots$).

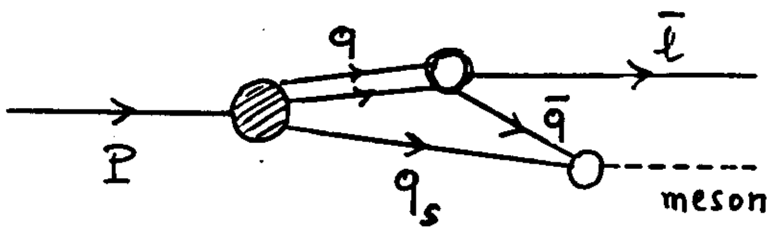
In order to discuss decay of proton (and bound neutron) in SU(5), one has to consider all possible graphs with X and Y exchange of the type



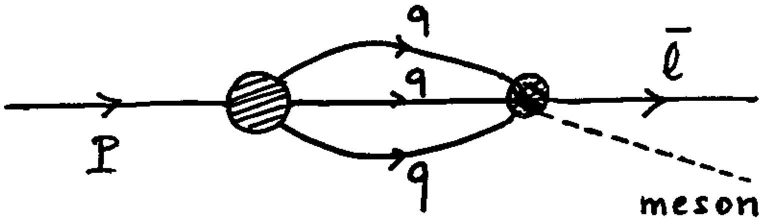
The effective – gauge boson exchanged – interaction (of dimension 6) is (neglecting generation mixing)

$$H = \frac{g^2}{2M_X^2}\,\epsilon_{ijk}\Big\{(\overline{u^c_{Lk}}\,\gamma_\mu\, u_{Lj})\Big[(\overline{e^c_R}\,\gamma^\mu d_{Ri}) + 2\,(\overline{e^c_L}\,\gamma^\mu d_{Li})\Big] - (\overline{u^c_{Lk}}\,\gamma_\mu\, d_{Lj})(\overline{\nu^c_R}\,\gamma^\mu d_{Ri}) + \mathrm{h.c.}\Big\}$$

Proton decay can occur in terms of the above graphs through the mechanism



with a "spectator" quark, but also through a "fusion" mechanism



Dimensional arguments give for the proton lifetime

$$\tau_p = (\text{factors}) \cdot \frac{1}{\alpha_G^2(M_X)} \; \frac{M_X^4}{m_p^5}$$

However, a more accurate evaluation of τ_p (and of branching ratios) need an estimate of the following corrections:

a) wave functions of the initial state (quarks bound in the proton) and final states.
Various approximations have been used (non-relativistic SU(6) model, or bag model) leading to somewhat different results.

b) <u>enhancement factor</u> A which originates from renormalization corrections. One finds

$$\alpha(M_X) \approx 0.024 \quad , \quad A \approx 2.9$$

c) <u>generation mixing</u>, which gives rise to decay into strange mesons (simplest choice is the Cabibbo low-energy mixing)

However, the main uncertainty is due to the estimate of M_X which is related, through $\alpha_s(\mu)$, to the scale parameter of strong interactions: $\Lambda = 150^{+150}_{-75}$ MeV.

One obtains: $M_X = (1 \text{ to } 4) \times 10^{14}$ GeV

$$\tau_p = 2 \times 10^{29 \pm 1.7} \text{ years}$$

and

$$\frac{1}{\Gamma(P \to e^+ \pi^0)} = \underline{4 \times 10^{29 \pm 1.7} \text{ years}}$$

This is to be compared with the experimental limit (IMB):

$$\frac{1}{\Gamma(P \to e^+ \pi^0)} > 10^{32} \text{ years} ,$$

which implies: $M_X \gtrsim 10^{15}$ GeV.

In conclusion, both SU(5) predictions of the parameter $\sin^2\vartheta_W$ and of the proton lifetime τ_p are not in agreement with experimental data.

However, this does not mean that GRAND UNIFICATION is ruled out, but only the minimal SU(5) model is in trouble.

One can build extended versions, either by enlarging the Higgs sector, or better by enlarging the gauge group, which give predictions in agreement with data.

Interesting examples based on the gauge group $SO(10) \supset SU(5) \times U(1)$.

$\boxed{SO(10)}$

1. All fermions of 1 generation in 1 irr. repr. of the group

$$16 = 1 + \bar{5} + 10$$

(the extra singlet state corresponds to the right-handed $\nu_R = \nu_L^c$ counterpart of ν_L)

2. <u>Left-right symmetry</u> at grand unification: while <u>parity</u> violation is intrinsic in $SU(5)$, it appears as effect of SSB of LR symmetry at lower scales.

3. Small but nonvanishing <u>mass for ν_L</u> (which is massless in minimal $SU(5)$) and large mass for ν_R
E.g.
$$m_{\nu_R} \times 10^6 \, GeV \quad ; \quad m_{\nu_L} \approx \frac{m_e^2}{M_{\nu_R}} \sim 1 \div 10^2 \, eV$$

4. <u>Multi-step breaking</u>, with intermediate symmetries. E.g.:

$$G \xrightarrow[M_X]{} G' \xrightarrow[M_{X'}]{} G_S \xrightarrow[M_W]{} G_0$$

$$G' = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_S = SU(3)_c \times SU(2)_L \times U(1)_Y$$

This modifies the <u>behaviour</u> of <u>the gauge couplings</u> with energy scale: the effect is to obtain large values for M_X and longer proton timelife; also larger values for $\sin^2\theta_W$.

On the other hand, going to non-minimal models, one has to pay the price of some arbitrariness (especially in the Higgs sector), so that these models are <u>less</u> <u>predictive</u>.

An alternative route to go beyond minimal model is <u>SUPERSYMMETRY</u>.

It gives also a solution to a problem which is present in all Grand Unified model, i.e. that of <u>hierarchy</u>.

E.g. in SU(5), the problem is how to stabilize the large splitting between the components of the Higgs

$$5 = (3,1) + (1,2)$$

since the colour triplet $\underline{3}$ should be superheavy ($\sim 10^{12} \sim 10^{14}$ GeV) to avoid rapid proton decay, while the isospin doublet $\underline{2}$ should produce the electroweak breaking at the scale $M_W \sim 10^2$ GeV.

[For a recent solution based on a "flipped" $SU(5) \times U(1)$: Nanopoulos lectures]

6. SUPERSYMMETRY

Motivations and general features

Supersymmetry, i.e. symmetry between bosons and fermions, is studied intensively since ~15 years, mainly for its interesting formal properties, and for the possibility of including gravity.

Application of supersymmetry to grand-unification (SUSY-GUTs) is interesting, since it is generally believed that it may be relevant, not only at superhigh scales of the order of $M_P \approx 10^{19}$ GeV, but already at moderate energies comparable with $M_W \approx 10^2$ GeV. The main motivations are:

a) supersymmetric field theories have remarkable convergence properties - Quadratic renormalization of parameters is absent, so that hierarchy problem may be solved

b) going from global to local supersymmetry, i.e. supergravity, one introduces also gravitational interaction in particle physics.

Contrary to ordinary GUTs, <u>Susy-GUTs</u> contein at least a <u>generator Q</u> which transforms fermions F into bosons B, and viceverse:

$$Q|F\rangle = |B\rangle \,, \quad Q|B\rangle = |F\rangle$$

Q is called spinorial charge, since it transforms as a 2-component (Weyl) spinor

$$[Q_\alpha, P_\mu] = 0 \qquad \rightarrow Q \text{ is conserved}$$

$$[Q_\alpha, M_{\mu\nu}] = i(\sigma_{\mu\nu})_{\alpha\beta} Q_\beta \quad \rightarrow Q \text{ transforms as spinor.}$$

The components Q_1, Q_2 satisfy <u>anticommutation</u> relations:

$$\{Q_\alpha, Q_\beta^*\} = 2(\sigma^\mu)_{\alpha\beta} P_\mu$$

$$\{Q_\alpha, Q_\beta\} = \{Q_\alpha^*, Q_\beta^*\} = 0$$

In general there can be <u>N independent charges</u>, but shall restrict to <u>N=1 global sypersymmetry</u>.

Gauge theories with supersymmetry contain <u>two types</u> of (<u>massless</u>) <u>supermultiplets</u> (with equal number of bosonic and fermionic degrees of freedom):

1) <u>SCALAR</u> (chiral), conteining a complex scalar and a left-handed spinor.
2) <u>VECTOR</u>, conteining a vector field and a massless spinor field.

<u>SCALAR</u> supermultiplet

$$\Phi = \begin{pmatrix} \tilde{\varphi} \\ \varphi \end{pmatrix} \begin{array}{l} \rightarrow h = -\frac{1}{2} \text{ spinor} \\ \rightarrow \text{complex scalar} \end{array}$$

<u>VECTOR</u> supermultiplet

$$\mathcal{V}_\mu = \begin{pmatrix} V \\ \tilde{v} \end{pmatrix} \begin{array}{l} \rightarrow h = \pm 1 \\ \rightarrow h = \pm 1/2 \end{array}$$

<u>Notice</u>: supersymmetric partners are denoted by $\tilde{\varphi}$, $\tilde{v}$ (we neglect "<u>auxiliary fields</u>")

Φ contains <u>left-handed spinor</u> (helicity $h=-1/2$)

Φ^* " <u>right-handed spinor</u> ($h=+1/2$)

Specifically, a <u>SUSY version of the standard theory</u> based on

$$\underset{\substack{\downarrow \\ \text{gauge symmetry}}}{G_S} \times (N=1 \text{ global supersymmetry})$$

contains <u>4</u> <u>gauge</u> <u>vector</u> <u>supermultiplets</u>

$$\mathcal{A}^{\pm,0} = \begin{pmatrix} A^{\pm,0} \\ \tilde{a}^{\pm,0} \end{pmatrix} \begin{array}{l} \rightarrow \text{gauge fields related to } I_i \\ \rightarrow \text{"gauginos"} \end{array}$$

$$\mathcal{B}^0 = \begin{pmatrix} B \\ \tilde{b} \end{pmatrix} \begin{array}{l} \rightarrow \text{gauge field related to } Y \\ \rightarrow \text{"gaugino"} \end{array}$$

The matter sector consists of as many supermultiplets as quark and lepton iso-doublets (q_L, ℓ_L) and iso-singlets (q_R, ℓ_R). They are of the type

$$Q = \begin{pmatrix} q \\ \tilde{q} \end{pmatrix} \begin{matrix} \rightarrow \text{quark} \\ \rightarrow \text{scalar partner} \\ \text{(or s-quark)} \end{matrix} \qquad L = \begin{pmatrix} \ell \\ \tilde{\ell} \end{pmatrix} \begin{matrix} \rightarrow \text{lepton} \\ \rightarrow \text{scalar partner} \\ \text{(or s-lepton)} \end{matrix}$$

Moreover, one must include two distinct Higgs supermultiplets (to provide mass both to up- and down-type quarks, and the required degrees of freedom)

$$H_1^{(0,-)} = \begin{pmatrix} \tilde{h}_1 \\ h_1 \end{pmatrix}, \quad H_2^{(+,0)} = \begin{pmatrix} \tilde{h}_2 \\ h_2 \end{pmatrix} \begin{matrix} \rightarrow \text{Higgsino} \\ \rightarrow \text{scalar Higgs} \end{matrix}$$

Since no susy-partners of quarks and leptons have been observed in their mass range, presumably they are much heavier, so that supersymmetry must be broken, to break degeneracy.

We notice that supersymmetry invariance implies that:
number of bosonic degrees of freedom =
no. of fermionic ones

Note (at page 55)

Bounds on superparticle masses

i) From e^+e^- collisions :

$m(\tilde{e}) \geqslant 22$ GeV

(similar bounds for $\tilde{\mu}$, $\tilde{\tau}$)

$m(X^{\pm}) \geqslant 22.5$ GeV

($X^{\pm}$ = charged gauginos)

ii) From $\bar{p}p$ collisions :

$m(\tilde{q}) \gtrsim 45$ GeV

$m(\tilde{g}) \geqslant 53$ GeV

[a stable neutralino is assumed to be lighter than 25 GeV]

The above feature is the <u>key ingredient</u> to solve the hierarchy problem.

In ordinary SM, scalar particles like the Higgs boson H get corrections to their masses:

$$\delta m_H^2 \approx g^2 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \approx O\left(\frac{\alpha}{\pi}\right)\Lambda^2$$

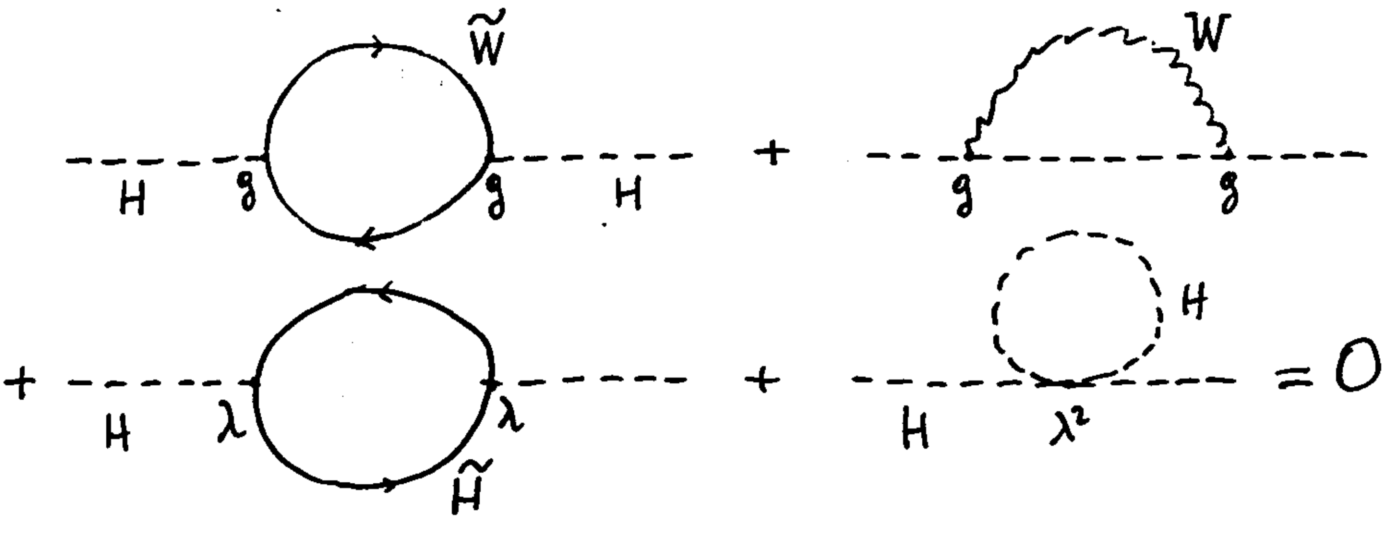
In order to get $\dfrac{\delta m_H^2}{m_H^2} < 1$

one should have a "natural" scale for the cut-off Λ:

$\Lambda \lessapprox 1$ TeV.

If no physical scale is present $\simeq 1$ TeV, one should go to $M_P \approx 10^{18}$ GeV ; or in GUTs to $M_X \approx 10^{15}$ GeV !

In the <u>susy version of SM</u>, quadratic divergences are cancelled :



If one assumes exact supersymmetry, the above contributions cancel exactly; in fact, susy must be a <u>broken symmetry</u> since the boson-fermion degeneracy does not appear in Nature:

$$m_B \neq m_f$$

$$\begin{aligned} \delta m_H^2 &\simeq O\left(\frac{\alpha}{\pi}\right)\left\{\Lambda^2 + O(m_B^2)\right\} \\ &\quad - O\left(\frac{\alpha}{\pi}\right)\left\{\Lambda^2 + O(m_f^2)\right\} \\ &\simeq O\left(\frac{\alpha}{\pi}\right)\left(m_B^2 - m_f^2\right) \end{aligned}$$

$$\frac{\delta m_H^2}{m_H^2} \lesssim 1 \qquad \rightarrow \qquad |m_B^2 - m_f^2| \lesssim 1\,\text{TeV}^2$$

If supersymmetry is combined with grand unification, one gets <u>supersymmetric versions</u> of GUTs (<u>susy-GUTs</u>) which present advantages with respect ordinary GUTs.

Supersymmetric SU(5)

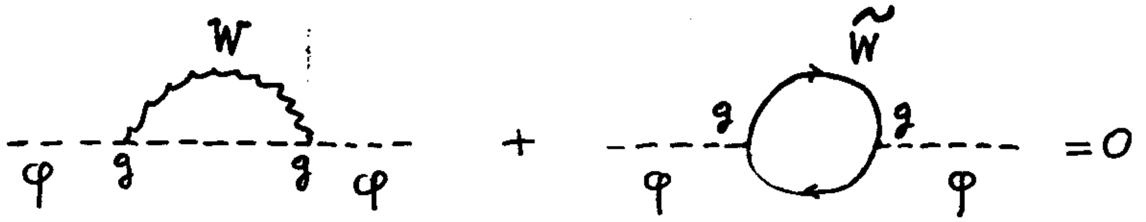
In order to compare the predictions of GUTs and susy-GUTs, we go to the specific case of susy-SU(5), i.e. a theory which is based on the gauge group SU(5) and has a global ($N=1$) supersymmetry.

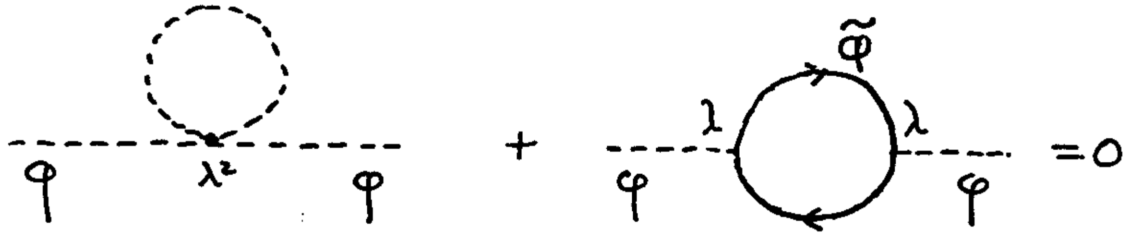
Each multiplet of minimal "ordinary" SU(5) is now replaced by a supermultiplet which contains the appropriate susy-partners (gauginos in $\underset{\sim}{24}$, s-quarks and s-leptons in $\underset{\sim}{\bar{5}} + \underset{\sim}{10}$,...).

Two independent Higgs supermultiplets transforming as $\underset{\sim}{5}$ and $\underset{\sim}{\bar{5}}$ are needed in the place of φ_5 : $H_1(5)$ and $H_2(\bar{5})$ $(H_2 \neq H_1^*)$.

While the breaking of gauge SU(5) occurs at superhigh scale M_X, the "effective" breaking of supersymm. is supposed to occur at a scale $M_S \simeq M_W$.

a) The first point which differentiates "ordinary" from "susy" versions of SU(5) is the fine tunings of the parameters in the Higgs potential.
While in ordinary SU(5) the fine-tuning at the tree level is completely destroyed by the appearance of quadratic divergences at higher order, in the susy-version, the boson-fermion symmetry produces a cancellation of divergences





The ratio of the masses of the $\varphi(1,2)$ and $\varphi(3,1)$ components of φ_S :

$$\frac{m_H}{M_{H_3}} \approx \frac{v_0}{V_0} \sim 10^{-13}$$

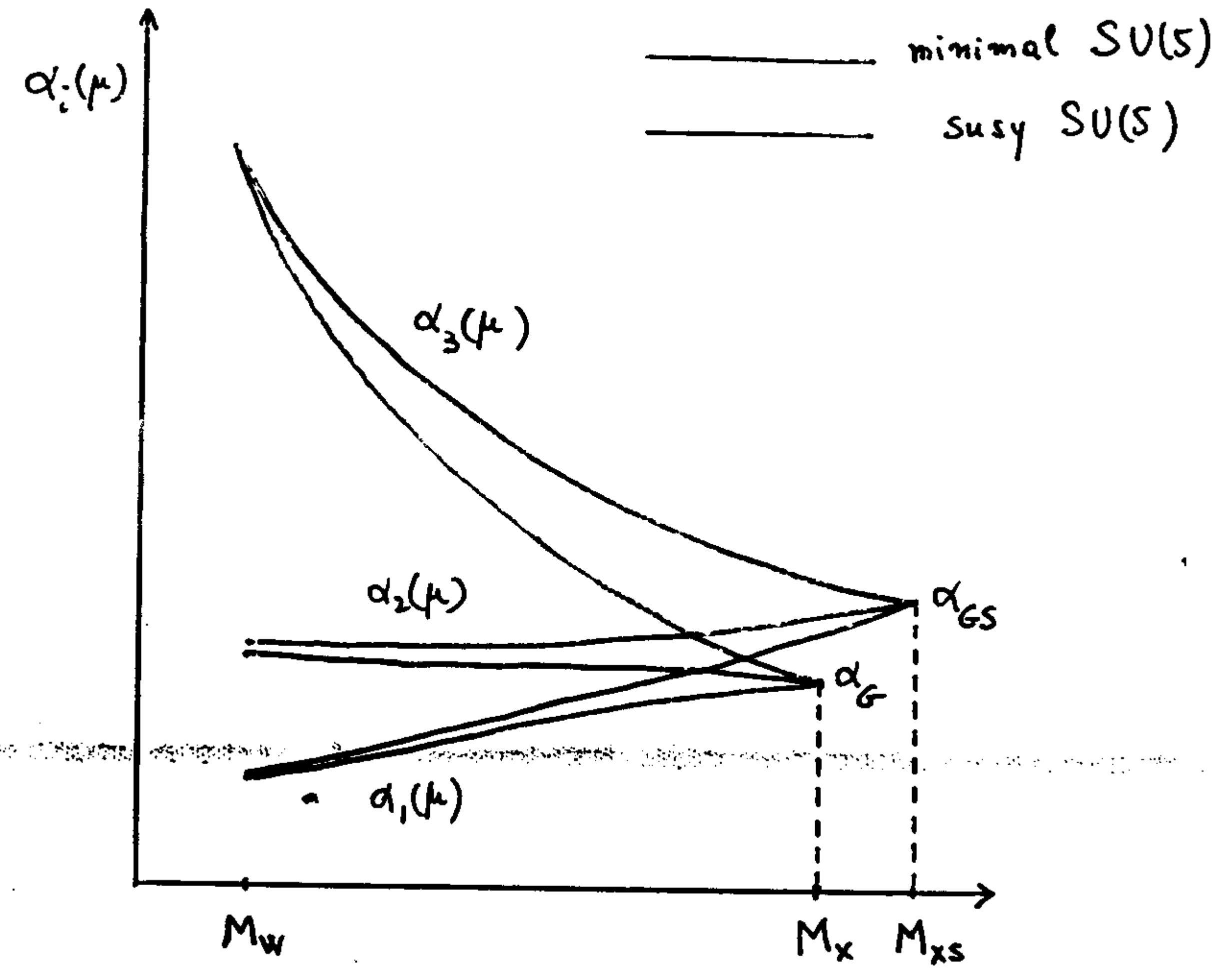
is not modified by renormalization.

b) The second point is the modification of the behaviour of the couplings $\alpha_i(\mu)$, due to the presence of gauginos, higgsinos and scalar partners of fermions

The b_i - coefficients which determine the rate of approach to unification are (aside from a factor $\frac{1}{4\pi}$):

	susy- SU(5)	SU(5)
b_1	$2N_g + \frac{3}{10}N_s$	$\frac{4}{3}N_g + \frac{1}{10}N_s$
b_2	$-6 + 2N_g + \frac{1}{2}N_s$	$-\frac{22}{3} + \frac{4}{3}N_g + \frac{1}{6}N_s$
b_3	$-9 + 2N_g$	$-11 + \frac{4}{3}N_g$

Notice that the presence of gauginos (and Higgsinos) decrease $|b_3|$ and changes the sign of b_2.



The GU scale $M_X \simeq 2\times 10^{14}$ GeV is replaced in the susy version by the <u>larger value $M_{XS} \simeq 4\times 10^{15}$ GeV</u> ($N_H = 2$). This would correspond naively to
$\tau_p \simeq 3\times 10^{35}$ yr.
Compare also $\alpha_G \simeq \frac{1}{42}$ with $\alpha_{GS} \simeq \frac{1}{24}$

c) The changes in the behaviour of the $\alpha_i(\mu)$ produce also a change in the value predicted for $\sin^2\theta_W(M_W) = 3\alpha_1/(3\alpha_1+5\alpha_2)$.

Comparison between predictions of "ordinary" SU(5) and susy-SU(5) :

SU(5)

$$\sin^2\theta_W(M_W) = 0.214 \pm 0.004$$

$$M_X = (2.0^{+2.1}_{-1.0}) \times 10^{14}\ \mathrm{GeV}$$

susy-SU(5)

$$\sin^2\theta_W(M_W) = (0.237 \pm 0.004) + - \frac{4}{15}\frac{\alpha(M_W)}{\pi}\ln\left(\frac{M_S}{M_W}\right)$$

$$M_X = (2 \div 5) \times 10^{15}\ \mathrm{GeV}$$

From experiments

$$\sin^2\theta_W = 0.230 \pm 0.005$$

$$M_X > 7 \times 10^{14}\ \mathrm{GeV}$$

Note on supersymmetry breaking

Of course, one has to implement a susy-model (susy version of SM or susy-GUT) with a suitable breaking mechanism.

In principle, one can introduce explicit breaking terms in the Lagrangian ("soft" terms)

or produce spontaneous susy breaking.

The first is rather unsatisfactory;

the second can be generated by v.e.v. of a suitable scalar component either in

a chiral supermultiplet, or in

a vector supermultiplet.

Moreover, one could generate susy breaking:

i) at a scale $M_S \simeq m_S$ (to preserve supersymm. above the electroweak scale, to solve hierarchy)

ii) at a very high scale (e.g. $M_S \simeq 10^{10}$ GeV), but "effective" breaking scale for low-energy physics $\sim M_W$.

It appears to be rather difficult, even if not impossible, to build a realistic susy-model with spontaneous susy breaking which satisfies all the phenomenological requirements.

From the theoretical point of view, it is more satisfactory to promote supersymmetry from a global (rigid) symmetry to a local symmetry, like the other gauge symmetries.

Supergravity

A very interesting feature of local supersymmetry is that it requires the introduction of the gravitational interactions: this is a consistent requirement for those mechanisms which contemplate a susy breaking at a very high scale.

In order to give a hint to this point, let us recall again the case of QED:

$$\mathcal{L}_0 = \frac{1}{2}\, i\, \bar{\Psi}\, \gamma^\mu \partial_\mu \Psi$$

The above Lagrangian $\mathcal{L}_0$ can be made supersymmetric by adding the contribution of the scalar partner ϕ of ψ

$$\mathcal{L}_0 = \frac{1}{2} i \bar{\psi} \gamma^\mu \partial_\mu \psi + \partial^\mu \phi^* \partial_\mu \phi$$

Susy transf.

ξ = constant spinor

$$\begin{cases} \delta\phi = \xi\psi \\ \delta\psi = -i\gamma^\mu \xi \partial_\mu \phi \end{cases} \qquad (*)$$

The requirement of the invariance under the transf. (*) in the local case $(\xi \rightarrow \xi(x))$ implies an additional term in the Lagrangian

$$\mathcal{L}_0 \rightarrow \mathcal{L} = \mathcal{L}_0 + \underbrace{\mathcal{L}_N}$$

$$\mathcal{L}_N = -k \bar{\psi}_\mu \gamma^\mu \cancel{\partial} \phi^+) \psi$$

which contains a (massless) spin $3/2$ field ψ_μ :

$$\delta \psi_\mu = \frac{1}{k} \partial_\mu \xi$$

But this is not the end in the present case!

The variation of $\mathcal{L}_N$ with respect to $\delta\psi$ gives

$$\delta \mathcal{L}_N \simeq k \bar{\psi}_\mu \gamma_\nu T^{\nu\mu} \xi$$

which can be cancelled only by adding the term

$$\mathcal{L}_G = -g_{\mu\nu} T^{\mu\nu} \quad , \quad \delta g_{\mu\nu} = k \bar{\Psi}_\mu \gamma_\nu \xi$$

$\mathcal{L}_G$ contains a (massless) spin 2 field $g_{\mu\nu}$ (inclusion of a kinetic term gives rise to Einstein's eq.)

All together

$$\underbrace{\mathcal{L}_{inv} = \mathcal{L}_0 + \mathcal{L}_N + \mathcal{L}_G}$$

$\mathcal{L}_N$ → contains the GRAVITINO ($S=3/2$)

$\mathcal{L}_G$ → contains the GRAVITON ($S=2$)

Susy-partners of the same multiplet

Supergravity can be coupled to Yang-Mills gauge fields; thus one gets local extensions of susy-models.

It is interesting to point out that spontaneous breaking of local supersymmetry take place with a "superhiggs mechanism":

the would-be spin-$\frac{1}{2}$ "goldstino" is eaten

by the spin-$3/2$ gravitino which becomes massive (the goldstino provides the 2 missing degrees of freedom).
The gravitino mass

$$\boxed{m_{3/2} \approx \frac{M_S^2}{M_P}}$$

is related to the effective breaking scale M_S (the breaking can be generated by the V.E.V. of a gauge-singlet field at the scale M_P ; this field is not observable)

In realistic models $m_{3/2}$ can be of the order of m_W :

- it gives a measure of the mass splitting between susy partners
- it introduces soft-breaking terms in the scalar potential

The result is that, in the low-energy limit, one obtains an effective Lagrangian with coincides with that of global susy, with the addition of soft susy-breaking term in the potential.

7. CONCLUSIONS

We have analysed the main features of the SM, and outlined the following steps in going beyond the SM

$$
\begin{tikzcd}
\boxed{\text{SM}} \arrow[d, "\text{simple gauge group}"] \\
\boxed{\text{GUT}} \arrow[d, "\text{global supersymmetry}"] \\
\boxed{\text{susy-GUT}} \arrow[d, "\text{local supersymmetry}"] \\
\boxed{\text{supergravity}}
\end{tikzcd}
$$

In each step, one gains some scores toward the solution of the main theoretical problems of the SM. In fact, some progress has been made along the way.
Gravitational interactions have also been included, but one big problem remains unsolved: supergravity, like gravity, is not a renormalizable theory.

The next step is to go to SUPERSTRINGS.

However, at this point, we are in front of a rather puzzling situation.

Even if the SM is not satisfactory from the theoretical point of view, it is perfectly consistent with all experimental data. In fact, none of the theoretical speculations which lead beyond the SM has yet received any experimental support.

Solution of this dilemma can only come from new experimental data: either from the detection of a new particle threshold at high energy colliders, or from the appearance of some small discrepancy in high-energy precision experiments. But accelerators can investigate energy regions just above the electroweak scale.

Evidence of new kinds of interactions and states at grand unification scale or at intermediate scales can be obtained in the searches of non-accelerator physics, in large detector in underground laboratory, even through feeble, but distinctive, signals.