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SMR/348 - 2

EXPERIMENTAL WORKHSOP ON
HIGH TEMPERATURE SUPERCONDUCTORS
(11 - 22 April 1988)

FLUX QUANTISATION, WEAK-LINK RINGS AND SQUIDS

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FLUX QUANTISATION, WEAK-LINK RINGS + SQUIDs

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1. FLUX QUANTISATION

* $\Psi(r)$ complex, many-body wave-function

$$|\psi| e^{i\phi(r)} \quad : \quad n_s = |\psi|^2$$

* Example - current carrying wire

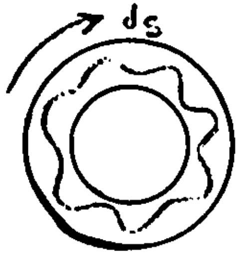
$\rightarrow j_s$ $\quad$ n_s pairs $\rightarrow v_s$

$$\psi = n_s^{1/2} e^{-i\underline{k}.\underline{r}}$$

$$j_s = -\frac{i\hbar e^*}{2m^*}\int \{\psi^* \nabla\psi\} d^3r \; + c.c$$

$$= n_s e^* \frac{\hbar k}{m^*} = n_s e^* v_s$$

* A ring



* Wave function single-valued
↓
Only special values of circulating currents
↓
Quantised flux generated.

①

Quantum mechanics in presence of B-field

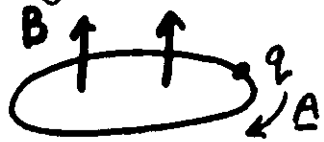
$$\vec{P} \rightarrow \vec{P} - qA$$

q: effective charge (2e for BCS)

A: Vector potential

$\therefore \; i\hbar\nabla \rightarrow i\hbar\nabla + qA$

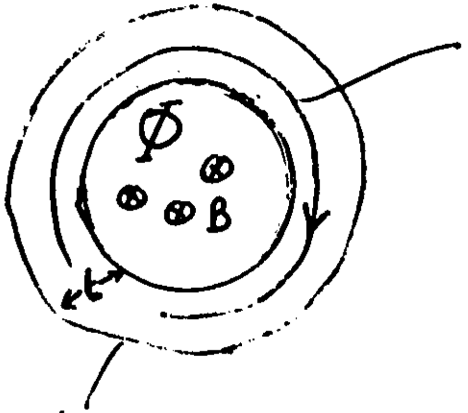
Vector potential A takes into account forces on charges from e.m.f.s generated in rings



$$\oint \underline{A}.\underline{dl} = \int_{area} \underline{B}.\underline{dS} = \Phi$$

$$\Phi = B \times Area$$

$$\text{emf induced} = \frac{d\Phi}{dt} \rightarrow \text{accelerates charges}$$



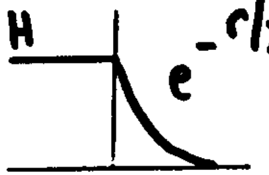
$j_o = 0$ well inside ring

S/C Ring

$t \gg \lambda$

magnetic penetration ②

Type-I s/c - currents only flow within a distance λ of surface

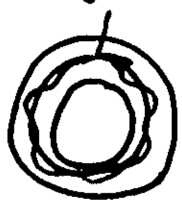


$$j_c = \text{curl } H$$

$$j_s(r) = \frac{e^*}{2m^*}\int \psi^*(-i\hbar\nabla\psi - q\underline{A})\psi \, d^3r + c.c.$$

$$\psi = n_s^{1/2} e^{i\phi(r)}$$

$\oint .\underline{dl}$



$$j_s(r) = \frac{e^*}{2m^*} n_s \left\{\hbar \frac{d\phi}{dl} - qA\right\} + cc$$

$$\oint j_s \cdot \underline{dl} = \frac{n_s e^*}{m^*}\left\{\hbar \oint \frac{\overset{\rightarrow d\phi}{d\phi}}{\cancel{dl}}\cancel{dl} - q\oint A.dl\right\}$$

$\|$ 0 — well inside ring $j_s = 0$

$\downarrow$ $n2\pi$ — single-valued

$\downarrow$ $\int B ds = \Phi$ flux.

Hence $q\int B.ds = \hbar n 2\pi$

or $\Phi = n\frac{2\pi\hbar}{q} = n\frac{h}{q}$

$q = 2e$ (BCS)

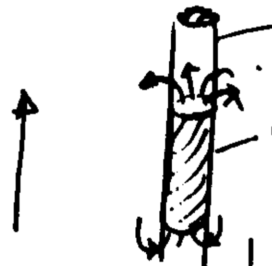
Thick ring $\phi = \pm\phi_0, \pm 2\phi_0$ etc

$$\phi_0 = \frac{h}{2e} = 2.07 \text{ gauss cm}^{-2} \times 10^{-7}$$
$$= 2.07 \times 10^{-15}\ \text{T m}^{-2}$$

Earth's field ~ 0.5 gauss

Φ_0 ~ in ring 4×10^{-7} cm^2 ≈ 10μ diam ring

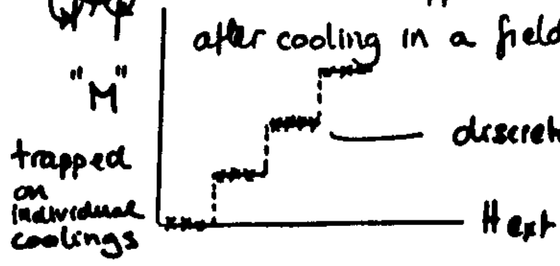
First measured ~ 25 years ago {Fairbanks, Deaver



Quartz fibre ~ 10μ diam.

evaporated s/c. – flux-trapping tube

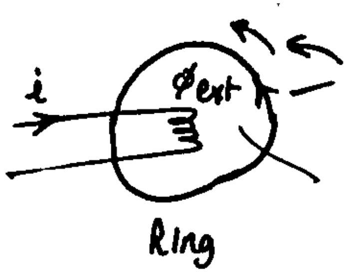
Measure "trapped" magnetisation after cooling in a field



Conventional s/c – once ϕ is trapped, chance of changing state by thermal activation alone > age universe for T < 5mK below T_c

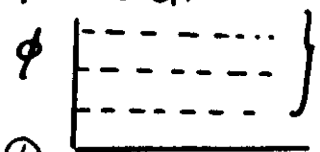
FLUX INSIDE A S/C RING REMAINS CONSTANT

Rigid – single-valued wavefunction.



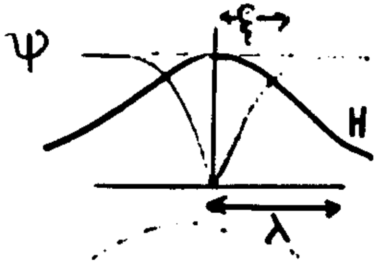
induced currents maintain const. Φ

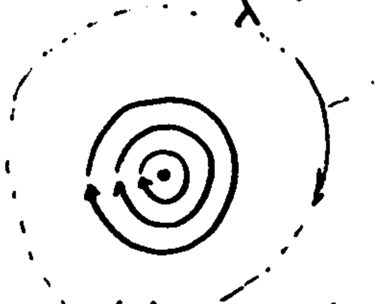
$\phi = \phi_{ext} - Li$ constant



FLUX INSIDE TYPE-II S/C

1. Isolated ϕ-line





circulating currents

$r \to 0 \quad \psi = r e^{i\phi}$

$r \to \infty \quad \to \psi_0 e^{i\phi}$

$\oint j \cdot dl \to 0$ as $r \to \infty$

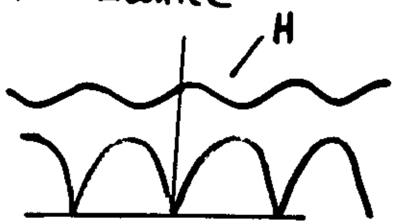
Again, Φ inside contour at ∞ $= M\phi_0$

$= \phi_0$ for energy considerations

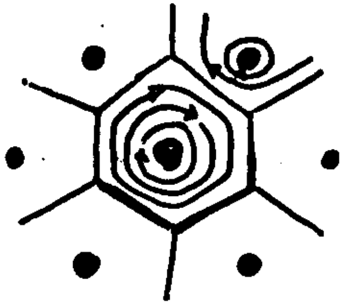
(energy $\propto m^2$)

FLUX LINES

2. Lattice



$\psi \to 0$ centre of circulating currents



hexagonal lattice

Circulating currents $\to 0$ on green planes

$\oint j\,dl = 0$

Φ / unit cell $= \phi_0$

density $= B/\phi_0$

spacing $\sim \left(\frac{\phi_0}{B}\right)^{1/2}$

$\sim \xi \left(\frac{B_{c2}}{B}\right)^{1/2}$

(5)

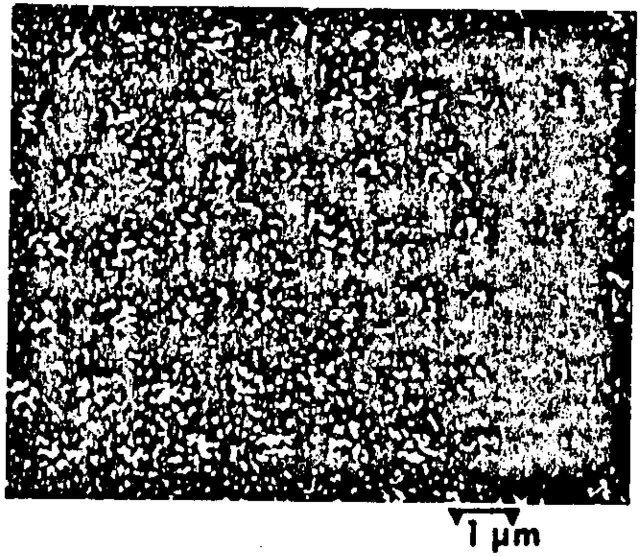


FIG. 2. Flux spots in a $YBa_2Cu_3O_7$ sample decorated after cooling in a field of 13 G.

Distorted hexagonal lattice

spacing $\sim \xi \left(\frac{B_{c2}}{B}\right)^{1/2}$

(6)

CEG

Flux quantisation in ceramic superconductors

March '87 Saclay (France) AC Josephsen

$$hf = \underline{\underline{2e}}\,V$$

pairs

April '87 Birmingham – $\phi_0 \rightarrow h/\underline{\underline{2e}}$

* Also first to demonstrate quantum nature of s/c flow – not $R < 10^{-15}$ but really $R = 0$ – like electrons round an atom – quantum states.

$$\phi_0 = \frac{h}{\underline{2e}} \times \{0.97 \pm .04\}$$

2e suggests pairing of electrons

BCS

Eliminated multi-pairing models

Now known to be consistent with RVB

Flux quantization in a high-T_c superconductor Nature 326, 855('87)

C. E. Gough*, M. S. Colclough*, E. M. Forgan*, R. G. Jordan*, M. Keene*, C. M. Muirhead*, A. I. M. Rae*, N. Thomas*, J. S. Abell† & S. Sutton†

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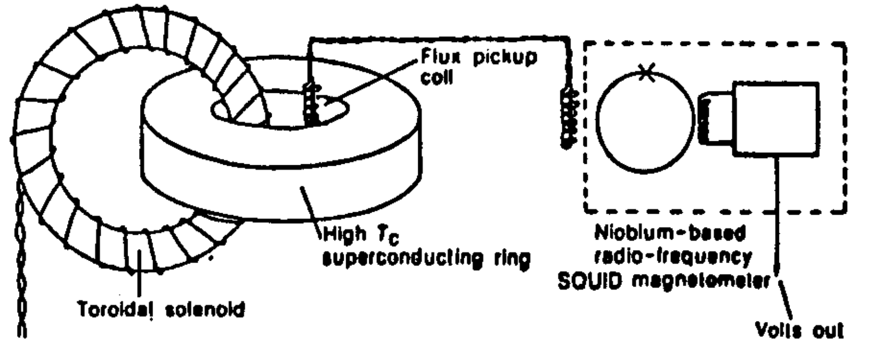
PISA 8th April

Nature 30th April.

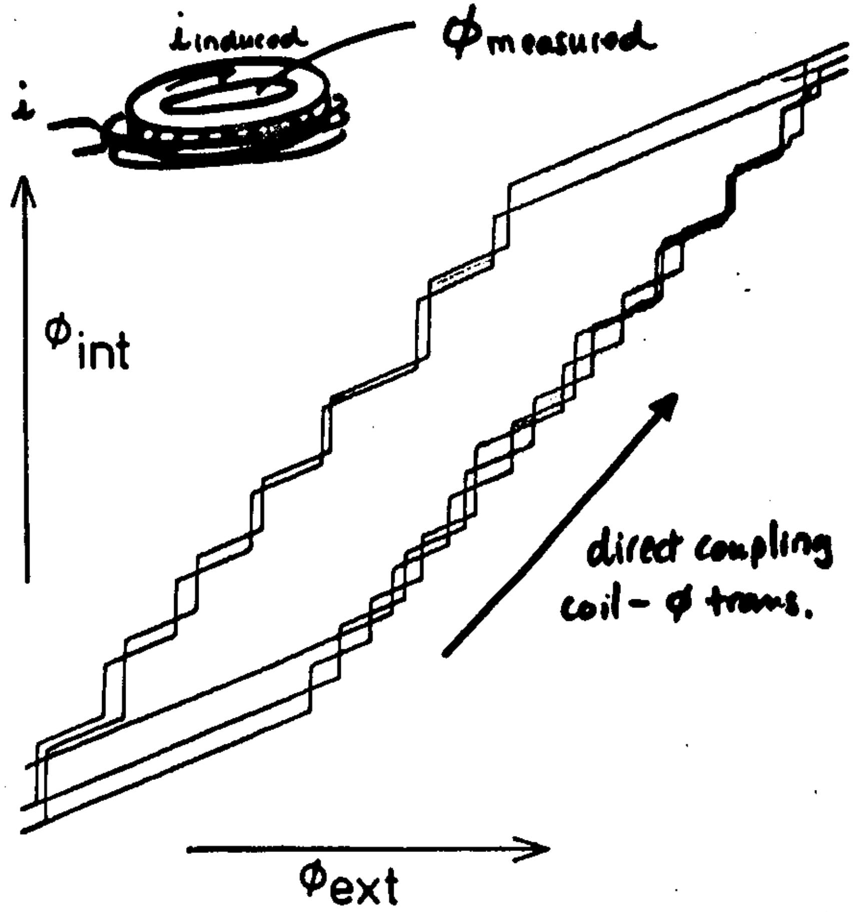


Schematic of Birmingham flux-quantization experiment (courtesy of C. Gough).

Esteve et al SACLAY $hf = 2e$

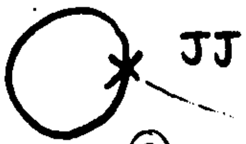
APS March.

$Y_{1.2}Ba_{0.8}CuO_4$ (Wu et al)



Like a conventional SQUID

Zimmerman



Ring provides loop and weak link

Ring $\uparrow$ $\phi = n\phi_0$

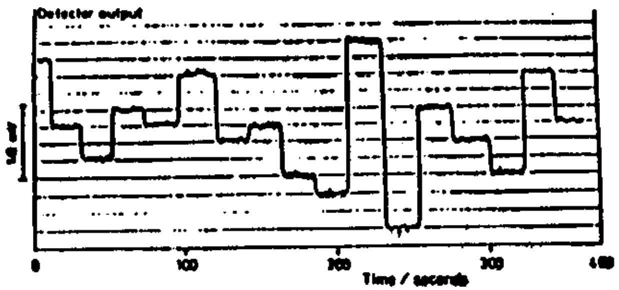
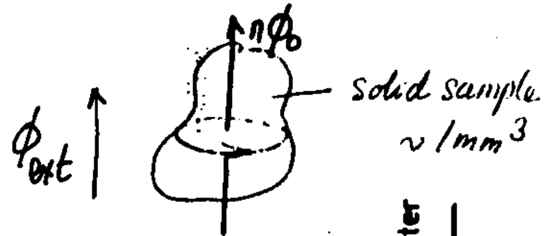


Fig. 1 Excitation of quantised flux transitions in a ring of YBCO.



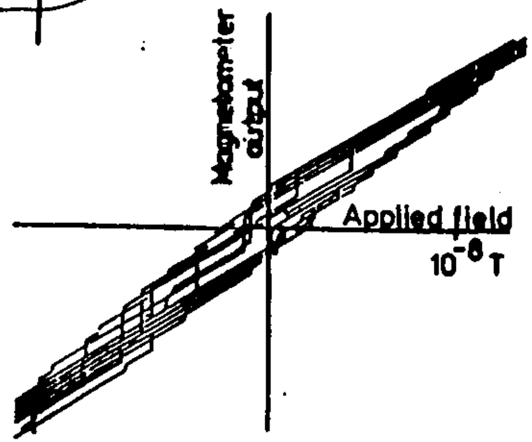
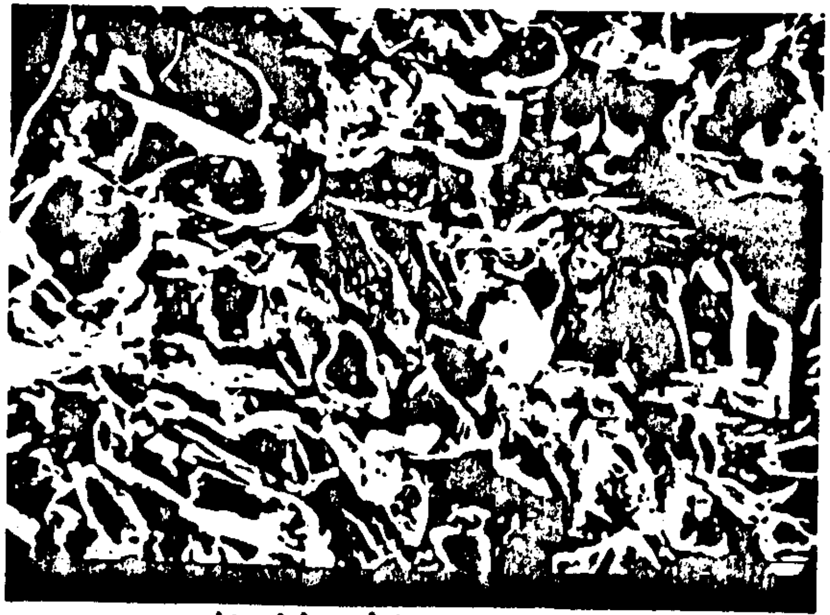
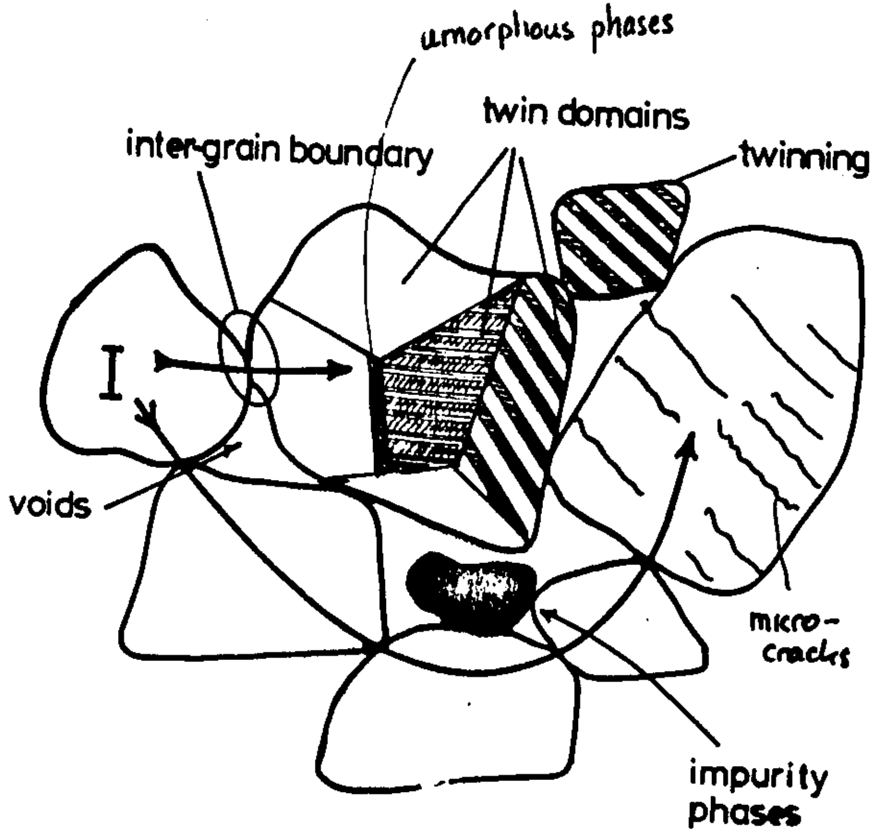


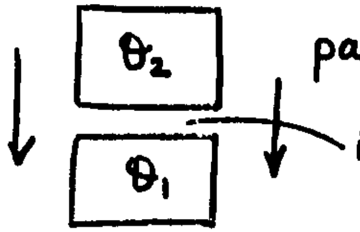
Fig. 3 Magnetisation of a weakly-superconducting sintered sample of bulk YBCO.



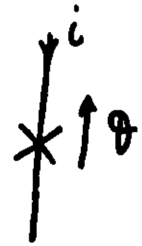
10 μ.
Sintered powder



WEAK LINKS   JOSEPHSON JUNCTIONS



pair tunnelling



JOSEPHSON

DC $\quad i = i_c \sin(\theta_2 - \theta_1) = i_c \sin\theta$

AC $\quad V = \dfrac{\phi_0}{2\pi} \dfrac{d\theta}{dt}$

$$\frac{di}{dt} = i_c \cos\theta \frac{d\theta}{dt} \leftarrow V\frac{2\pi}{\phi_0}$$

$$\therefore \quad V = \frac{\phi_0}{2\pi} \frac{1}{i_c \cos\theta} \frac{di}{dt}$$

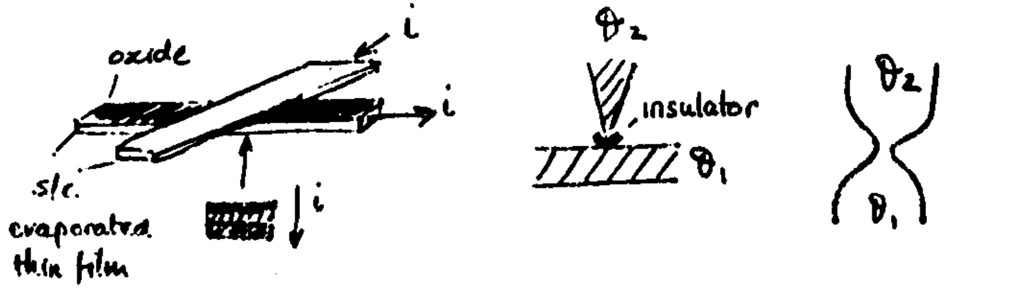
$$= L_J \frac{di}{dt}$$

$\equiv$

$L_J = \dfrac{\phi_0}{2\pi} \dfrac{1}{i_c \cos\theta}$

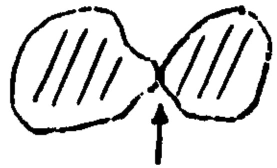
WEAK-LINK $\equiv$ NON-LINEAR INDUCTANCE

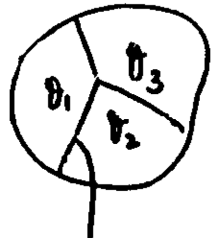
TYPES OF WEAK LINK



JUNCTION   POINT CONTACT   MICRO-BRIDGE

HIGH - T_c CERAMIC S/C   grains





intergrain contacts

domain boundaries

twin boundaries

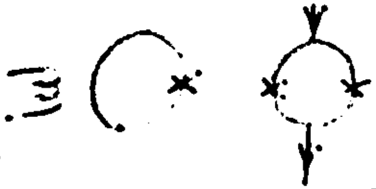
metallic

s/c - insulator - s/c

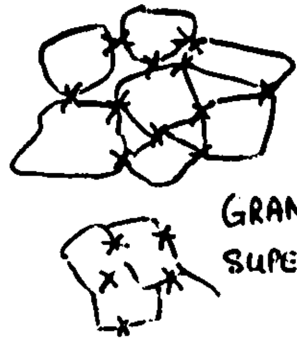
s/c - not so good s/c - s/c

microbridge

WEAK - LINK RINGS



$\equiv$



GRANULAR SUPERCONDUCTORS

SQUID DEVICES