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EXPERIMENTAL WORKSHOP ON
HIGH TEMPERATURE SUPERCONDUCTORS
(11 - 22 April 1988)

SUPPLEMENTARY NOTES ON GRANULAR SUPERCONDUCTORS - MAGNETISATION AND SUSCEPTIBILITY FROM THE BEAN MODEL

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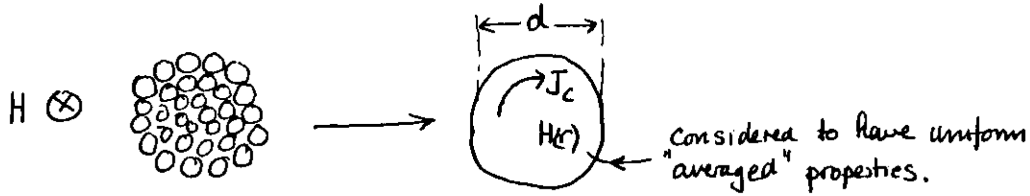
Supplementary notes on granular superconductors – magnetisation and susceptibility from the Bean model..

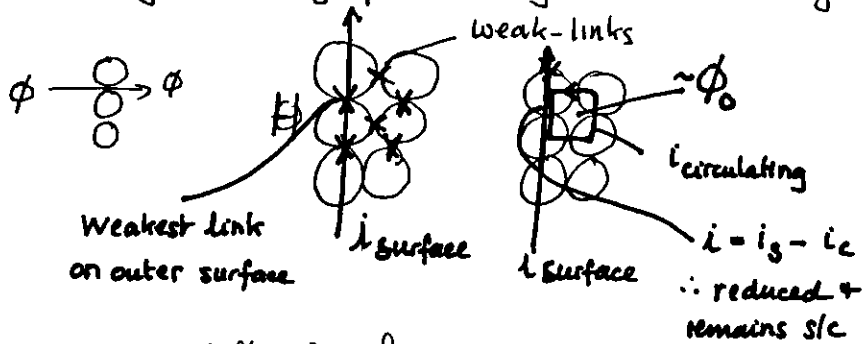


To evaluate magnetic properties we assume the Bean model with a field dependent $J_c(B)$ determined by the strength of the weak links between the grains. For example, if we assume a simple model of regular sized grains, d in diameter, each with a critical current i_c, $J_c = i_c/d^2$. For 10μ grains and $J_c \sim 10^3\,\mathrm{A\,cm^{-2}}$ at 77K (best quality sintered YBCO), $i_c = 10^3 \times (10^{-3})^2 = 1\,\mathrm{mA}$ between grains (or, of course, within the grain as we cannot tell whether the weakest-links are between or within the grains in a specific sample).

On initially increasing the field from zero, the outer-surface grains will support a supercurrent preventing field penetration into the bulk of the sample. The largest current that can be supported between the grains is i_c ($\sim 1\,\mathrm{mA}$, for best samples at 77K). This will correspond to an external applied field of $H = \frac{1}{d}\frac{i_c}{10}$ gauss. ($H = n\,i$ where $n = 1$ turn/unit length (the outer conducting grains) and the factor 10 converts to practical units of Amps and cm). There are $\frac{1}{d}$ grains/unit length on the outer surface so that $i = i_c/d$. Typically, therefore, for a very well sintered ceramic YBCO sample at 77K field will be completely excluded giving a 100% Meissner shielding current up to $H = 10^{-3}/10^{-3} \times 10 = \frac{1}{10}$ gauss.

To detect such shielding currents flowing entirely on the surface measurements must be made in less than the earth's field with very small modulating fields in susceptibility measurements.

Once this field is exceeded by either the static applied field or the modulating a.c. field used in susceptibility measurements flux will penetrate the bulk. This will involve flux entering individ[ual] voids between grains setting up circulating currents between grain[s].

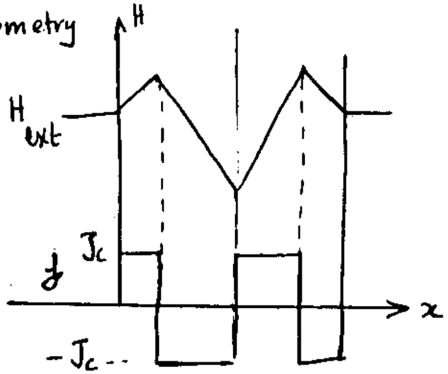


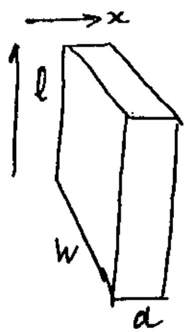
The penetration of flux into the sample will take place irregularly on a microscopic scale (e.g. that of the individual grain[s]) the flux entering through those weak-links that are weaker than the rest. However, on a macroscopic scale $\gg d$ the flux penetration may be taken to be uniform.

Under these circumstances the field inside the sample will be determined by the critical current density J_c, which determines the largest field gradient that can be supported inside the superconductor via Maxwell's equation curl $H = J_s$, which reduces to $\frac{dH}{dx} = J_s$ in one-dimension.

To determine the irreversible magnetic properties of a granular superconductor we assume that everywhere the field has penetrated the sample the field gradient is at its maximum increasing or decreasing value determined by J_c

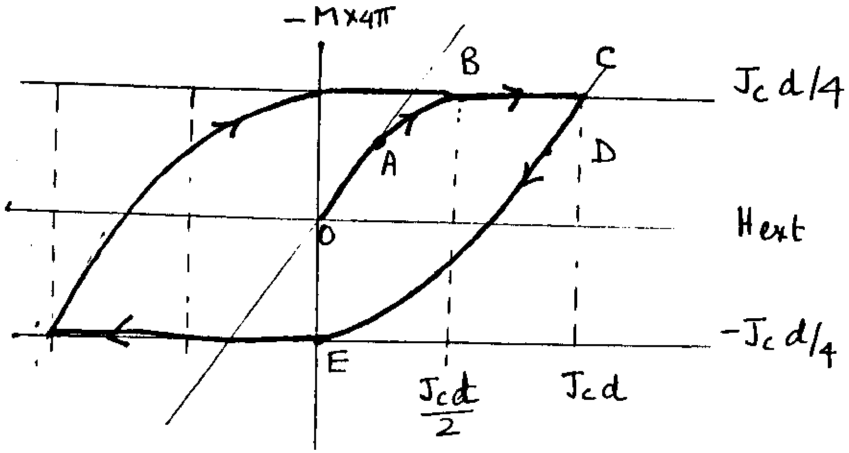
Slab geometry



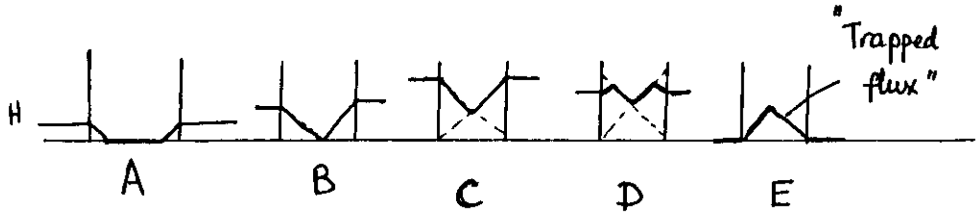


The above diagram illustrates the field-profile ~~p~~ within a flat slab ($d \ll w, l$) predicted by the "critical state" model. The bulk supercurrents flowing on the central ~~inside of~~ region of the slab were established while the field was increasing to a value larger than the ~~ext~~ external field shown. On decreasing, from a large field value the field profile on the inside is initially unchanged but the shielding currents on the outside regions are reversed to their maximum value of supercurrent, reversing also the field gradient.

The following diagram illustrates the magnetisation curves obtained for this simple model. We consider refinements later.



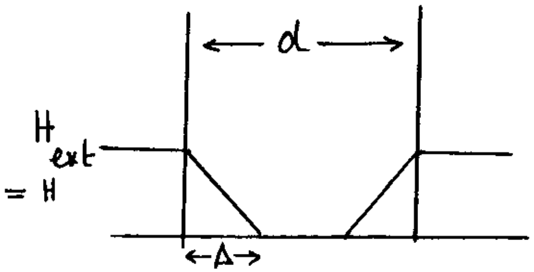
The field profiles at various points of the irreversible magnetisation loops are sketched below



Calculation of shape of hysteresis curve for slab geometry

Increasing field

Consider initial section OAB.



$\mathrm{curl}\, H = J_c$

$\frac{dH}{dx} = \pm J_c$

linear slope

The field penetrates a distance $\Delta = H/J_c$ into slab from both sides. Therefore the "average" field inside the slab $\tilde{H} = \left(\frac{H \times \Delta}{2}\right) \times 2 \times \frac{1}{d} = H^2/J_c d$

Hence the magnetisation $4\pi M = \tilde{H} - H = \frac{H^2}{J_c d} - H$.

Note that $\left(\frac{dM}{dH}\right)_{H=0} 4\pi = -1$ (100% diamagnetic slope)

$\left(\frac{\partial M}{dH}\right) = 0$ when $H = J_c d/2$, where

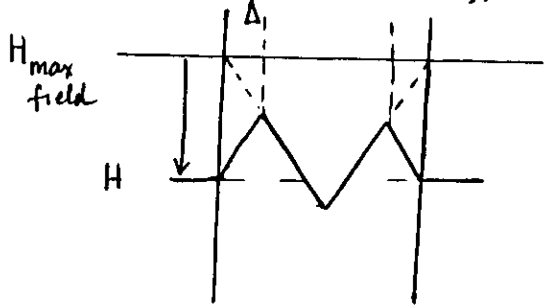
$4\pi \tilde{M}_{max} = -J_c d/4$

This accounts for the magnetisation shown on page 3

Decreasing field

Maximum hysteresis is achieved when H increases to $J_c d$ or above.

On decreasing H the field gradient at the surface reverses over a distance $\Delta = (H_{max} - H)/2J_c$



This leads to a reduction in flux (or average field) in the sample given by

$$\widetilde{\Delta H} = \left(\frac{H_m - H}{2}\right)\left(\frac{H_m - H}{2J_c}\right) \times \frac{2}{d}$$

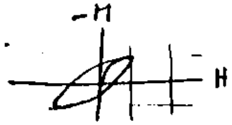
So that $4\pi \Delta M = (H_m - H) - (H_m - H)^2 / 2J_c d$

Note that this has an initial diamagnetic slope $4\pi \frac{dM}{dH} = -1$ and the quadratically varying term is ½ that evaluated on the initial section of the magnetisation curve.

ΔM reaches a maximum when $(H_m - H) = J_c d$ at a value $4\pi M = + J_c d/4$ – the maximum value of trapped flux.

If $H_{max} < J_c d$, the magnetisation in the reverse direction will still approach the above value but it will not be reached until $H < 0$.

eg →



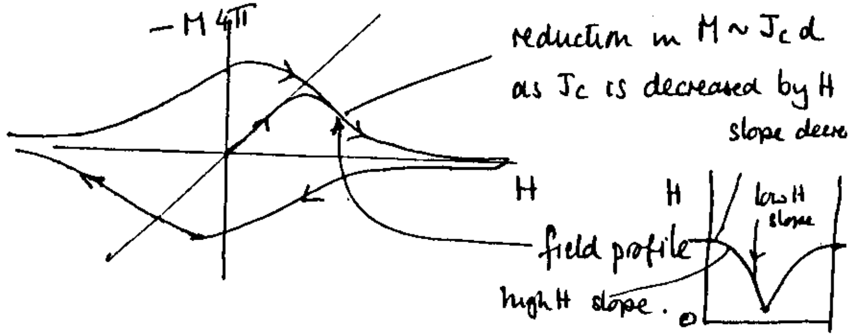
Evaluation of J_c from magnetisation loops

* Measurement of $4\pi\Delta M_{max} = \frac{J_c d}{4}$ $\left(= \frac{J_c d}{40}\right.$ in Amps and cm units$\left.\right)$ would provide a simple method for determining J_c for slab geometry. Similarly, the field at which the maximum magnetisation is achieved $\frac{J_c d}{2}$.

Similar curves can be derived for cylindrical and spherical geometries, which differ largely only in a small geometric factor of order unity. See review article by Campbell and Evetts in Reviews of Modern Physics (?) – also published as a monograph & also the original papers by Bean in Phys Rev Letters and J. Appl Physics.

* In a ceramic granular s/c, the critical current is strongly dependent on magnetic field – which will vary within the s/c. As a crude approximation $J_c \sim 1000 \times \exp(-\frac{H}{10})$, where H is in gauss, for a well-sintered YBCO sample at 77K.

If we assume such a field dependence, a magnetisation similar to that shown below would be derived

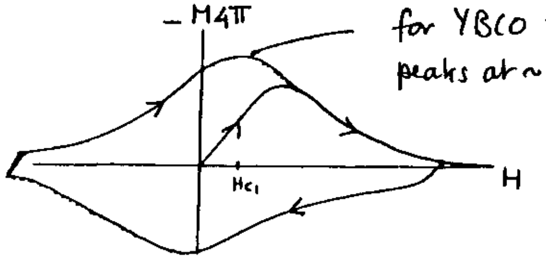


To correlate with experiment a field dependence for $J_c(H)$ must be assumed and the magnetisation computed (numerically)

* <u>Diamagnetism of powder grains</u>

Each grain will have a typical type-II irreversible magnetism determined by critical currents (flux pinning) within the grains

typical



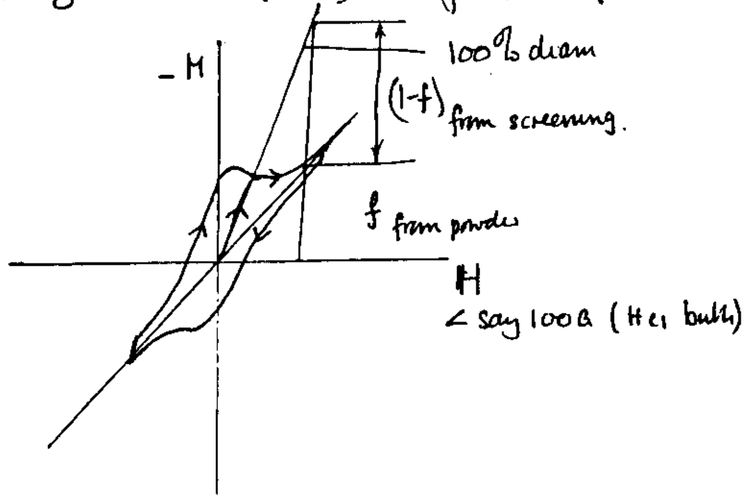
At peak $M_{MAX} \sim \frac{200}{4\pi} \simeq \frac{J_c d}{40}$ at 77K

Giving an order of magnitude estimate for $J_c \geqslant 10^5 - 10^6 \, A\,cm^{-2}$ within grains.

At low fields $\lesssim$ 100 gauss the grains will be almost perfectly diamagnetic at 77K and will exclude flux from their volume. Therefore, the flux calculated earlier from the fields will be an overestimate since the total flux will be reduced be the volume available for flux penetration between ~~voids~~ grains. Assume a filling factor for the powder f for a given local field H the flux will be given by $(1-f)H$ and the magnetisation from screening currents will be reduced accordingly. Hence

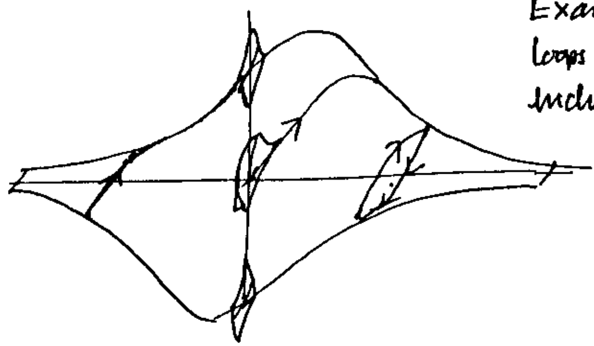
$$4\pi M = (1-f)4\pi M_{screening} + f M_{powders}$$

The resultant magnetisation is then the sum of the two magnetisation curves (both of which have a diamagnetic slope of $-\frac{1}{4\pi}$ at zero initial field). So for small fields



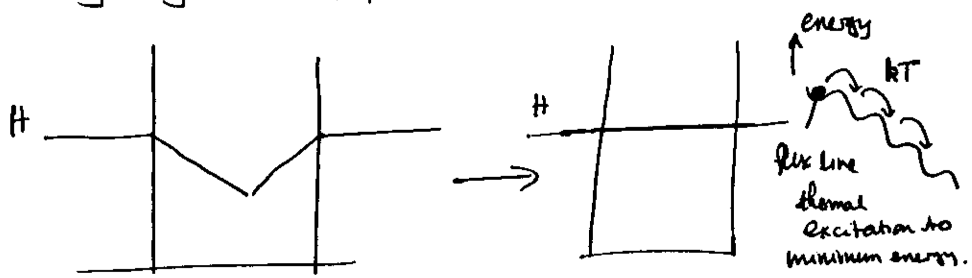
By comparing the initial magnetisation slope to the background slope the "filling" factor f can, in principle, be ~~calculated~~ estimated. The above low field-feature will only be seen at 77K for $|H| < 10$ gauss but can be seen in several repeated measurements at 4.2K, where little comment is usually made

Examples of magnetisation loops with minor loops included



Thermal creep

The critical state model assumes that local pinning of flux inside a grain or by the weak links between grains prevents the flux reaching its thermodynamically equilibrium state, which for the simple granular model with $f = 0$ would correspond to H inside being everywhere H_{ext}.



critical current model

TD equilibrium.

The energetics of weak-link rings involve the Josephson energy and inductances of rings between grains. Therefore the microstructure is involved. But, at high temperatures for constant applied field the flux distribution will slowly relax by thermal excitation to the TD equilibrium state where the supercurrents flowing $\rightarrow 0$.

This problem was treated by Anderson (P R Letters) many years ago. It predicts a slow change to TD state with $\Delta M \propto \log t$ ~ time. Thermal creep.