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EXPERIMENTAL WORKHSOP ON HIGH TEMPERATURE SUPERCONDUCTORS (11 - 22 April 1988)

JOSEPHSON AND RELATED MACROSCOPIC QUANTUM PHENOMENA - I

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JOSEPHSON EFFECT

- ★ PHASE OF THE ORDER PARAMETER
- ★ FLUX QUANTIZATION
- ★ DC & AC JOSEPHSON EFFECTS
- ★ JOSEPHSON INTERFERENCE IN MAGNETIC FIELDS

PHYSICS OF CuO BASED CERAMIC SUPERCONDUCTORS

- ★ SUMMARY OF IMPORTANT & ANAMOLOUS EXPERIMENTAL RESULTS
- ★ ELECTRONIC STRUCTURE
- ★ MOTT INSULATOR & ORIGIN OF ANTIERROMAGNETISM
- ★ QUANTUM PARAMAGNET OR RESONATING VALENCE BOND STATE
- ★ RVB CONDUCTOR
- ★ NOVEL QUASI-PARTICLES & SUPERCONDUCTIVITY
- ★ UNDERSTANDING EXPERIMENTAL RESULTS BASED ON RVB THEORY.

PHASE OF THE ORDER PARAMETER IN SUPERCONDUCTIVITY

one particle plane wave state: $e^{i\vec{k}\cdot\vec{r}}$: $\phi(\vec{r}) = \vec{k}\cdot\vec{r}$

$$\vec{\nabla}_r \phi(\vec{r}) = \vec{k} \rightarrow \text{momentum}$$

$$e^{iE_k t}\, e^{i\vec{k}\cdot\vec{r}} = e^{i\phi(\vec{r},t)} \qquad \phi(\vec{r},t) = E_k t + \vec{k}\cdot\vec{r}$$

$$\partial_t \phi = E_k \rightarrow \text{Energy}$$

$$\vec{j}(\vec{r}) = \frac{e}{m}\frac{\hbar}{2i}\left(\psi^* \vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)$$

If $\psi(\vec{r}) = e^{i\phi(\vec{r})}\sqrt{\rho(\vec{r})}$, $\quad \vec{j}(\vec{r}) = \vec{\nabla}\phi(\vec{r})\ \psi^*(\vec{r})\psi(\vec{r})$

How does a macroscopic phase appears in Superconductivity ?

$$\psi(\vec{r}_1, \vec{r}_2 \cdots \vec{r}_N) = e^{i\phi(\vec{r}_1, \vec{r}_2 \cdots \vec{r}_N)} \sqrt{\rho_N(\vec{r}_1 \ldots \vec{r}_N)}$$

Bosons: $\phi(\vec{r}_1 \ldots \vec{r}_N) = \text{const}, \phi_0$

Simple excited (CURRENT CARRYING) state: $\phi(\vec{r}_1 \ldots \vec{r}_N) = \sum_i \phi(\vec{r}_i)$

Current density

$$\vec{J}(\vec{r}) = \rho_1(\vec{r})\, \vec{\nabla}\phi(\vec{r})$$

Fermion..

$$\psi(\vec{r}_1 \cdots \vec{r}_N) = A\left[\chi(\vec{r}_1-\vec{r}_2)\,\chi(\vec{r}_3-\vec{r}_4) \cdots \chi(\vec{r}_{N-1}-\vec{r}_N)\right]$$

$$= e^{i\phi_F(\vec{r}_1 \ldots \vec{r}_N)}\, \rho(\vec{r}_1 \ldots \vec{r}_N)$$

$\phi_F(\vec{r}_1 \ldots \vec{r}_N)$ is VERY COMPLICATED

Since ρ is positive, $\phi_F(\vec{r}_1, \vec{r}_2 \ldots \vec{r}_N) = \phi_F(\vec{r}_2, \vec{r}_1, \vec{r}_3 \ldots \vec{r}_N) - \pi$

Current carrying states:

$$\psi(r_1 \ldots r_N) = A\left[\chi(\vec{r}_1-\vec{r}_2)\, e^{i\phi\left(\frac{r_1+r_2}{2}\right)}\, \chi(r_3-r_4)\, e^{i\phi\left(\frac{r_3+r_4}{2}\right)} \cdots \chi(r_{N-1}-r_N)\, e^{i\phi\left(\frac{r_{N-1}+r_N}{2}\right)}\right]$$

MEISSNER EFFECT & LONDON RIGIDITY AMOUNTS TO THE STATEMENT THAT THE MODIFICATION OF THE GROUND STATE WAVE FUNCTION OCCURS ONLY THROUGH CHANGE IN ϕ AND THAT TOO AT THE BOUNDARY OF THE SAMPLE OVER THE LONDON PENETRATION DEPTH.

ANDERSONS PSEUDO SPIN FORMALISM AND PHASE OF THE ORDER PARAMETER

$$H_{BCS} = \sum_k (\epsilon_k - \mu)\, c_k^\dagger c_k - \sum V_{kk'}\, c_{k'}^\dagger c_{-k'}^\dagger\, c_{-k}\, c_k$$

The subspace containing $k, -k$ pair wise empty or pair wise occupied are important

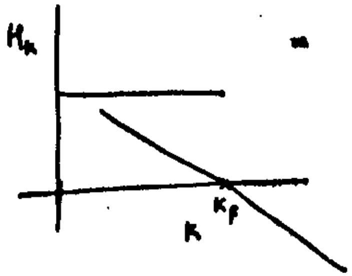
$$|1_k, 1_{-k}\rangle \longrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \equiv \alpha_k$$

$$|0_k, 0_{-k}\rangle \longrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \equiv \beta_k$$

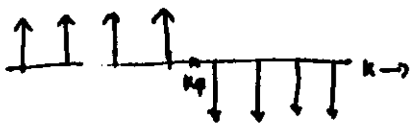
$$c_{-k}\, c_k\, |1_k, 1_{-k}\rangle = |0_k, 0_{-k}\rangle \Rightarrow \underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_{S_k^-} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

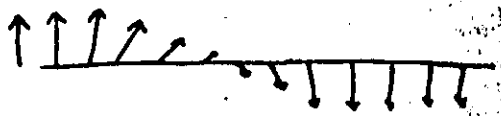
$$H_{BCS} = \sum \overbrace{(\epsilon_k - \mu)}^{-H_k}\, S_k^z - \sum V_{kk'}\, S_k^+ S_{k'}^-$$

$$= -\sum H_k S_k^z - \sum V_{kk'} \left(S_k^x S_{k'}^x + S_k^y S_{k'}^y\right)$$



V=0 (FREE ELECTRON GAS)





S_x and S_y component of the pseudo spin

The rotational symmetry is spontaneously broken in the x-y plane.

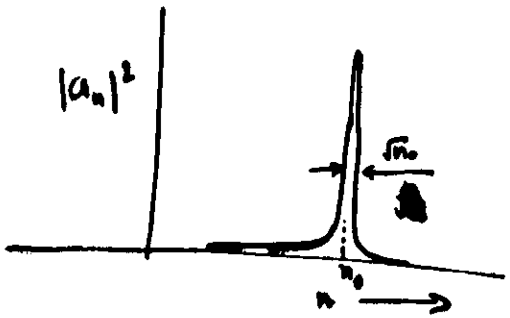
The ground state wave function:

$$|G\rangle = \prod_k \left(|u_k|\alpha_k + |v_k| e^{i\theta_0}\beta_k\right)$$

$$\alpha_k = \begin{pmatrix}1\\0\end{pmatrix}, \quad \beta_k = \begin{pmatrix}0\\1\end{pmatrix}$$

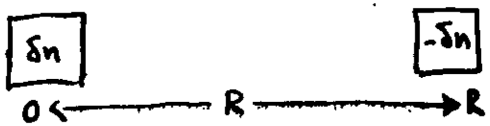
$$|BCS\rangle = \prod_k \left(|u_k| + |v_k| e^{i\theta} c_k^\dagger c_{-k}^\dagger\right)|0\rangle$$

$$= \sum_{n=0,1,\dots} e^{i2n\theta} |a_n|\, |2n\rangle$$

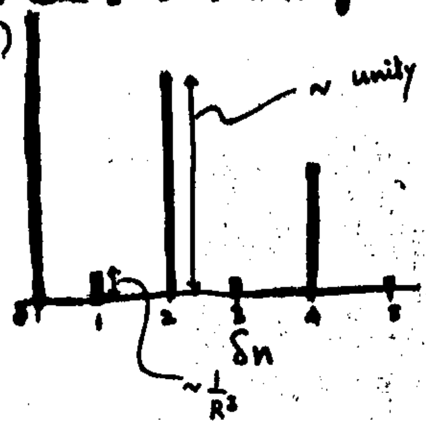


PHASE COHERENT GRAND CANONICAL ENSEMBLE

CORRELATION IN THE LOCAL NUMBER FLUCTUATION



Probability amplitude for the above to occur in the ground state is $P_R(\delta n)$



in a current carrying ground state

$$P_R(\delta n) \rightarrow e^{i(\theta(0) - \theta(R))} P_R(\delta n)$$

Thus in the bulk of a superconductor coherent quantum mechanical number fluctuations occur.

Supercurrent flow indicates the tendency of the system to maintain this coherant fluctuation.

The phase discussed untill now is of pure quantum mechanical origin.* It becomes ill defined and does not exist when $\hbar \to 0$.

This is not true of any other order parameter in condensed matter physics.

Phase of the spin density wave

" " " Charge density wave

⋮

* it has no reference to any spatial pattern.

- Quantum mechanics on a macroscopic scale
- Macroscopic quantum tunnelling
- Nature of dissipation
- Quantum theory of measurement.

TIME EVOLUTION OF A BCS STATE

Eigen states: $e^{iE_n t}|n\rangle$

BCS: $e^{iHt}|BCS\rangle = \sum e^{i2n\theta}|a_n|\, e^{iHt}|n\rangle$

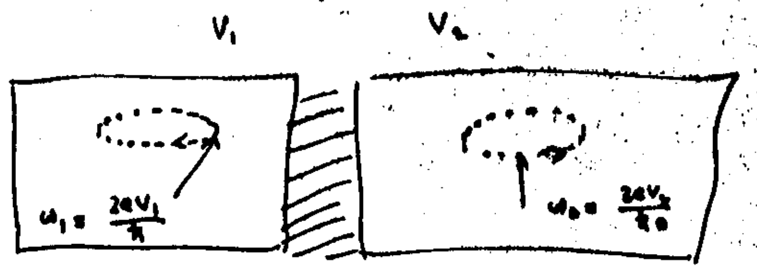
$$= \sum e^{i\left(2n\theta + \frac{2\mu + 2eV}{\hbar}\right)t}|n\rangle$$

$$\Downarrow$$

$$\prod_k \left(|u_k| + |v_k|\, e^{\frac{i}{\hbar}(2\mu + 2eV)t}\, e^{i\theta}\, c^\dagger_k c^\dagger_{-k}\right)|0\rangle$$

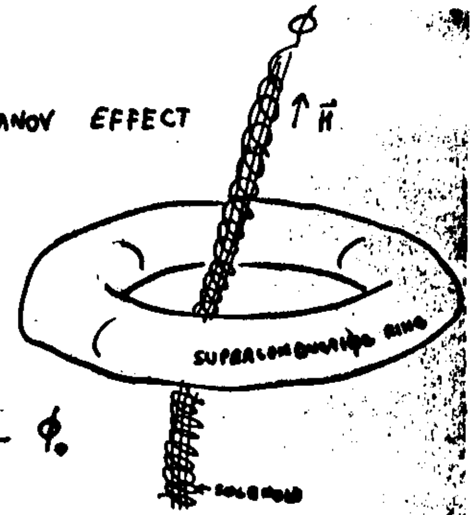
thus $\Delta \longrightarrow |\Delta|\, e^{i\theta}\, e^{\frac{2i}{\hbar}(\mu + eV)t}$

This is the origin of a.c. Josephson effect.



Josephson frequency: $\omega_1 - \omega_2 = \dfrac{2e(V_1 - V_2)}{\hbar}$

FLUX QUANTISATION – BOHM-AHARANOV EFFECT



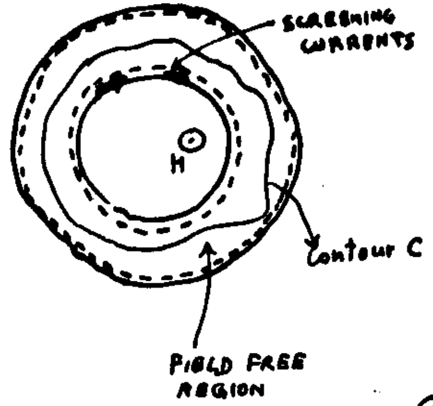
IN EQUILIBRIUM (SUPER CURRENT INSIDE THE BULK OF THE RING IS ZERO) THE FLUX ENCLOSED IN THE RING $\phi = n\left(\frac{hc}{2e}\right)$ — ϕ_0

WHERE n IS AN INTEGER

$$\psi(r_1 \ldots r_N) = \psi_0(r_1 \ldots r_N)\, e^{i[S(\vec r_1) + \ldots S(\vec r_N)]}$$

$$\vec j(\vec r) = \frac{ne}{m}\left(\hbar \vec\nabla S - \frac{e\vec A}{c}\right)$$

Deep inside the superconductor $\vec j(\vec r) = 0 \Rightarrow \vec A = \frac{c\hbar}{e}\vec\nabla S$

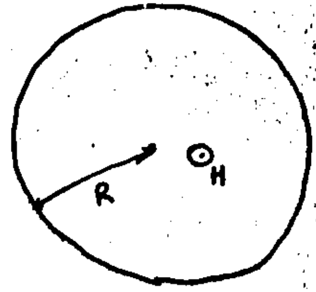


$$\oint \vec A \cdot d\vec l = \frac{c\hbar}{e}\oint \vec\nabla S \cdot d\vec l = \frac{c\hbar}{e}\not{S}$$

$\not{S}$ is the increase in S when one goes around one complete circuit along the curve C.

ANOTHER EXAMPLE

ELECTRON CONFINED TO MOVE ON A RING. THE RING ENCLOSES A SOLENOID



$$H = \frac{p^2}{2m} = \frac{-\hbar^2}{2mR^2}\frac{d^2}{d\theta^2} \quad \text{for zero field}$$

$$H = \frac{-\hbar^2}{2mR^2}\left(\frac{d}{d\theta} + i\,\phi\alpha\right)^2$$

$$H\psi_m = E\psi_m \qquad \psi_m = e^{im\theta}$$

$$E_m = \frac{-\hbar^2}{2mR^2}(m+\alpha)^2$$