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EXPERIMENTAL WORKSHOP ON
HIGH TEMPERATURE SUPERCONDUCTORS
(11 - 22 April 1988)

JOSEPHSON JUNCTIONS, WEAK-LINK RINGS AND SQUIDS

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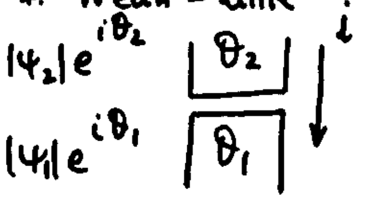
Lecture 2 — JOSEPHSON JUNCTIONS, WEAK-LINK RINGS + SQUIDS

[van Duzer, Barone]

* Weak-link



$$i = i_c \sin(\theta_2 - \theta_1) = i_c \sin\theta$$

Josephson relationships:

dc $i = i_c \sin\theta$

ac $V = \dfrac{\phi_0}{2\pi}\dfrac{d\theta}{dt}$ — note - to remember $[V] = [\frac{d\phi}{dt}]$

* ENERGY STORED IN J.J

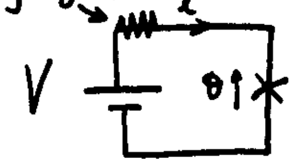
$$\mathcal{E}_J = -\frac{i_c \phi_0}{2\pi}\cos\theta$$

Origin - increase θ to increase i but change in $\theta \rightarrow$ voltage (ac JE) ∴ work done

$$dW = i\,V\,dt = i_c \sin\theta \, \frac{\phi_0}{2\pi}\frac{d\theta}{dt}dt$$

$$W = -\frac{i_c\phi_0}{2\pi}\cos\theta = \epsilon_J$$

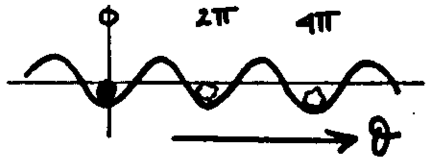
* GIBBS free energy (external ~~voltage~~ current source)



$$G = \epsilon_J - i\,\frac{\phi_0}{2\pi}\theta$$

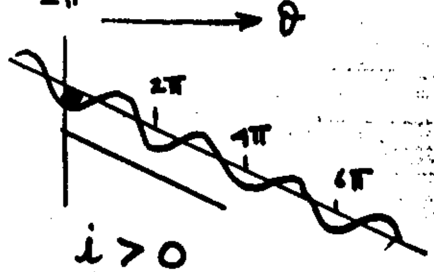
Show that $\frac{\partial G}{\partial \theta} = 0 \rightarrow i = i_c \sin\theta$

"WASHBOARD POTENTIAL" $V = \frac{\phi_0}{2\pi}(-i_c\cos\theta - i\theta)$



$i = 0$

Many equiv. minima — absolute value of θ has no meaning.

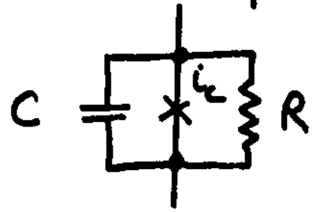


$i > 0$

$i > i_c \rightarrow \frac{d\theta}{dt} \equiv V$

Metastable

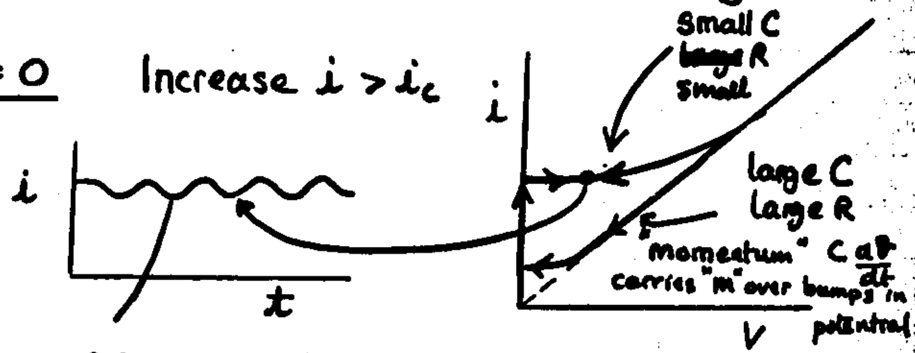
RSJ model for junction



C ≡ mass

R ≡ damping

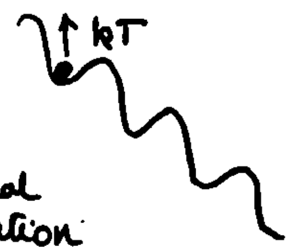
T = 0 Increase $i > i_c$



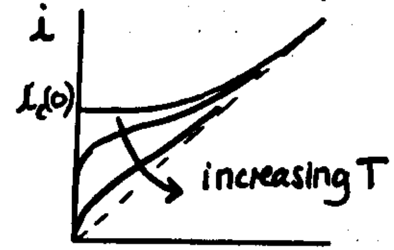
ac component as m rides over bumps in potential

ac voltage

T > 0

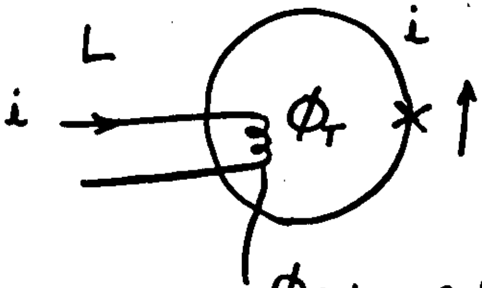


Thermal activation



"i_c" measured < I_c intrinsic — depends on meas. sensitivity

WEAK-LINK RINGS – SQUIDs or granular s/c



single valued wave function

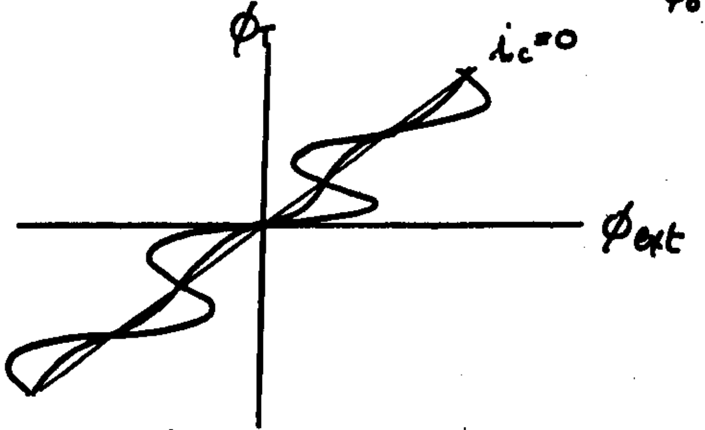
$$\theta = 2\pi \frac{\phi_T}{\phi_0} + 2n\pi$$

ϕ_{ext} externally applied flux (induces a current)

$$\phi_T = \phi_{ext} - Li$$

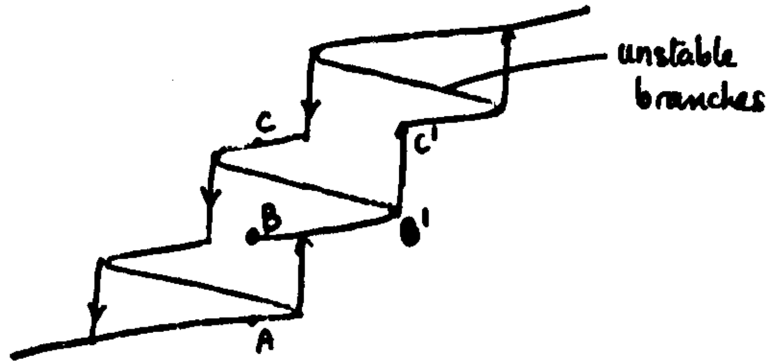
$$= \phi_{ext} - L\, i_c \sin \frac{2\pi \phi_T}{\phi_0}$$

$$\therefore \quad \phi_{ext} = \phi_T + L\, i_c \sin 2\pi \frac{\phi_T}{\phi_0}$$



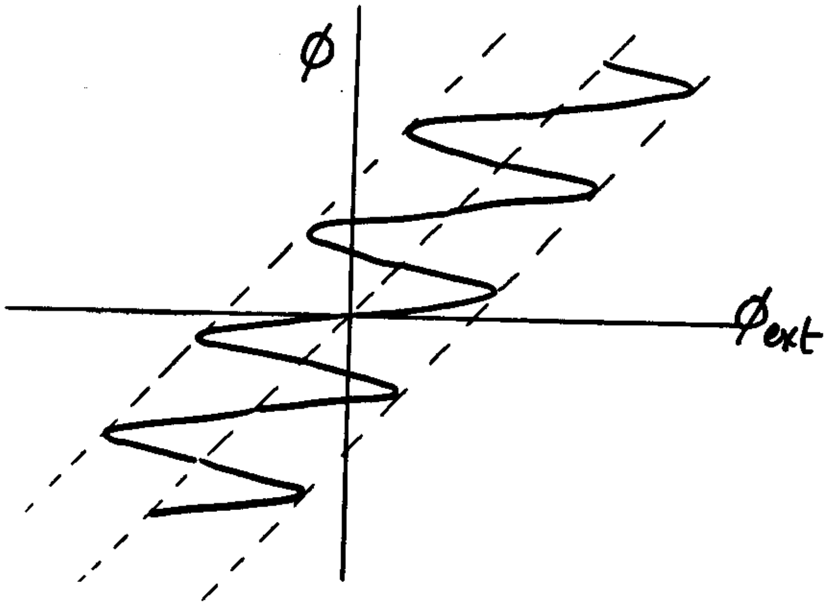
If $\quad L i_c / \phi_0/2\pi < 1 \quad$ REVERSIBLE

$\quad \frac{2\pi L i_c}{\phi_0} > 1 \quad$ IRREVERSIBLE



hysteretic magnetic properties: granular s/c

But properties remain periodic in ϕ

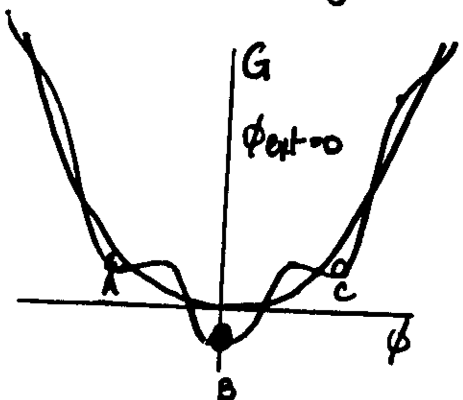


Note ϕ is no longer exactly quantised

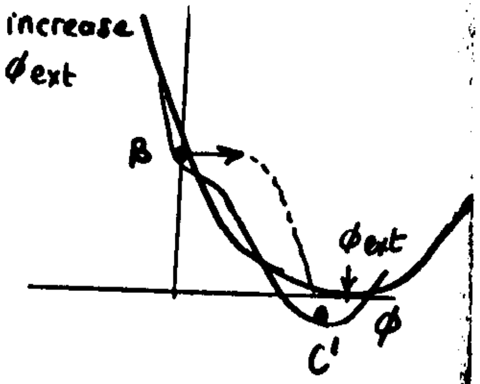
Energy may be written as

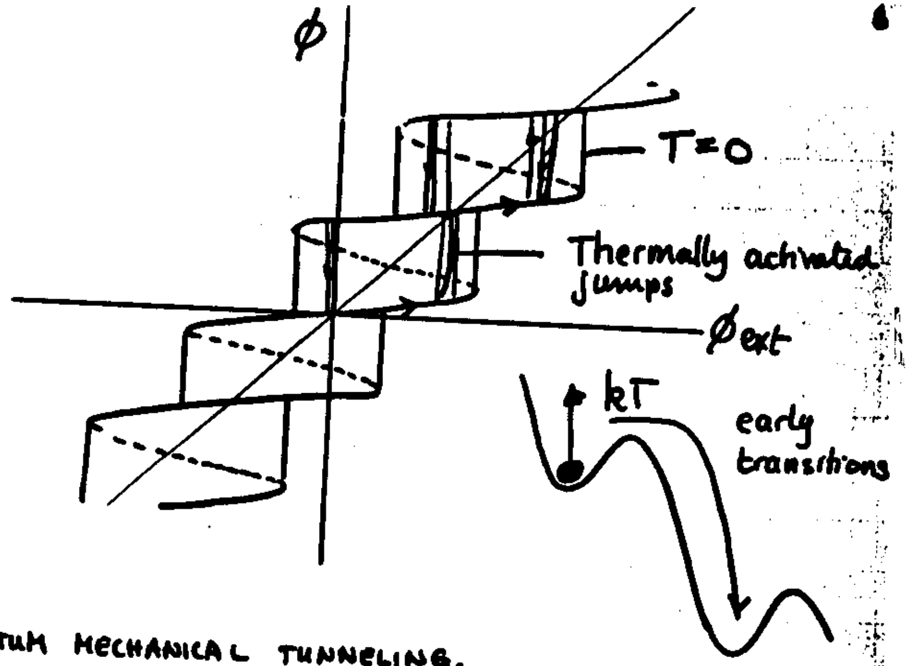
$$G = \frac{(\phi - \phi_{ext})^2}{2L} - \frac{i_c \phi_0}{2\pi} \cos \frac{2\pi\phi}{\phi_0}$$

Prove that this gives the correct ϕ ϕ_{ext} relationship



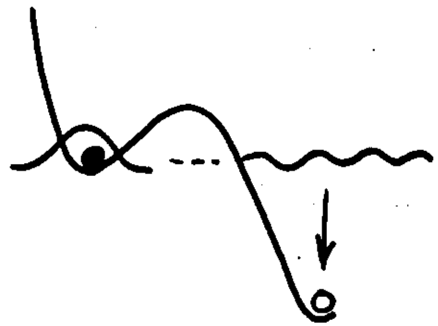
∃ metastable equilibrium





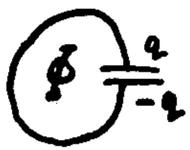
ALSO

QUANTUM MECHANICAL TUNNELING.
(major topic of s/c research before high-T_c)



Flux Φ in ring described by a wave function

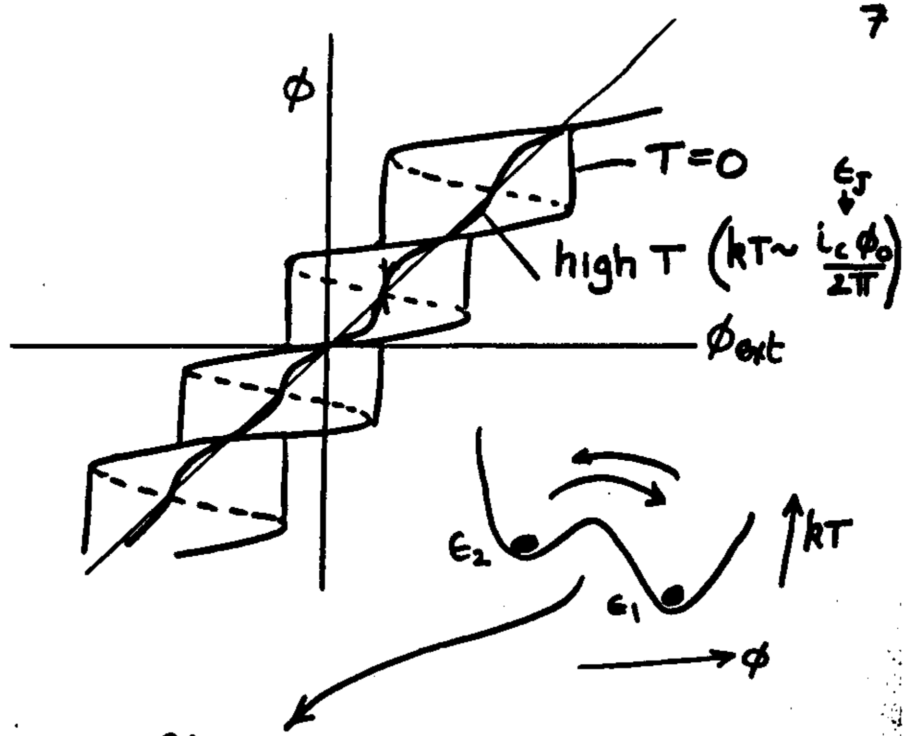
q charge on junction conjugate variable



$$[\Phi, q] = i\hbar$$

$$\Delta\Phi \, \Delta q \sim h$$

MACROSCOPIC QUANTUM EFFECTS
ONLY OBSERVABLE < 1K



$\tilde{\phi}$ determined by average time spent in the 2 states. Determined by Boltzmann statistics

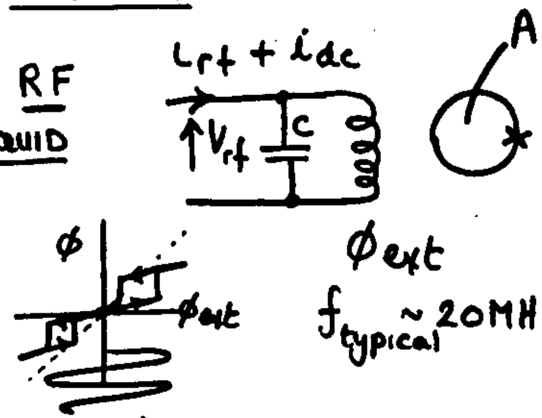
$$\tilde{\phi} \approx \frac{\phi_1 e^{-\epsilon_1/kT} + \phi_2 e^{-\epsilon_2/kT}}{e^{-\epsilon_1/kT} + e^{-\epsilon_2/kT}}$$

→ A reversible function of T, B ϕ_{ext}.

Note magnetisation of ceramic s/c become reversible as T → T_c

SQUIDs

RF SQUID



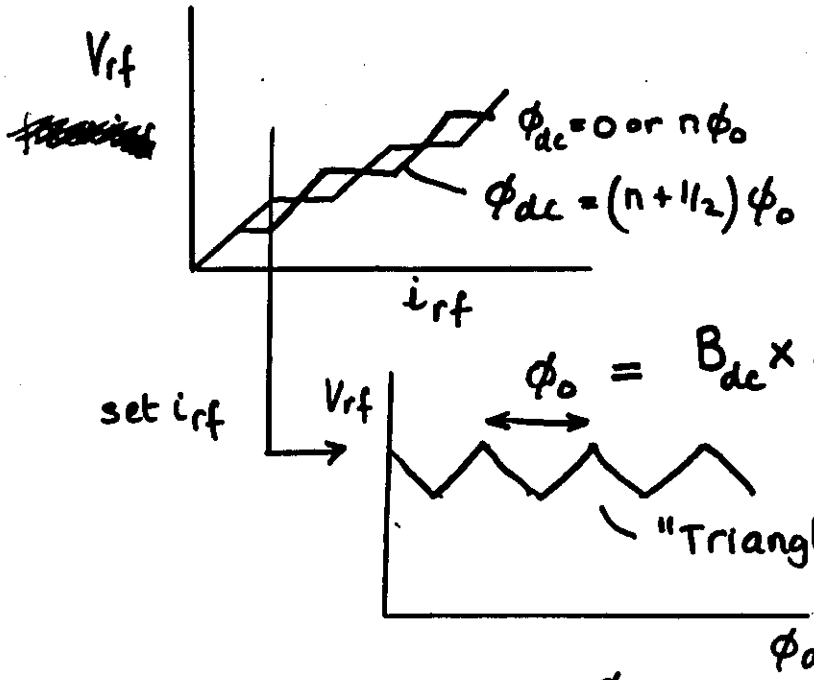
Hysteretic Magnetisation loops at 20 MHz → loss

ϕ_{ext}

$f_{typical} \sim 20$ MHz

Loss in r.f. coil is periodic in $\phi_{DC\,ext}$

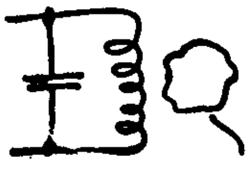
See * van Duzer
* Barone for full treatment
also Gifford, Webb & Wheatley paper.



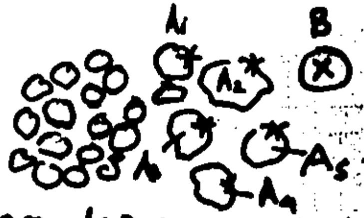
$\phi_0 = B_{dc} \times$ Area of loop

Periodicity in $B = \frac{\phi_0}{A}$

Birmingham bulk SQUID [Nature 328, 47('87)]
Colclough et al

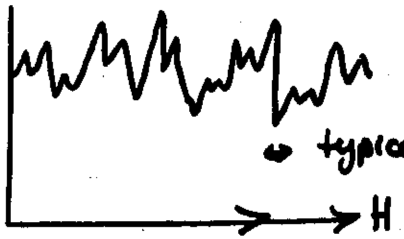


Green phase
Poor s/c



granular superconductor
An assembly of weakly-coupled rings

→ Superposition of "triangle patterns" with periodicity ϕ_0/A_i
with $A \sim$ grain loops $\sim 10\mu$ in size



→ typical scale = $\frac{\phi_0}{\tilde{A}}$

$\Delta H \sim \frac{\phi_0}{10^{-10}} \sim 2\times10^{-5}$ T ~ 0.2 gauss

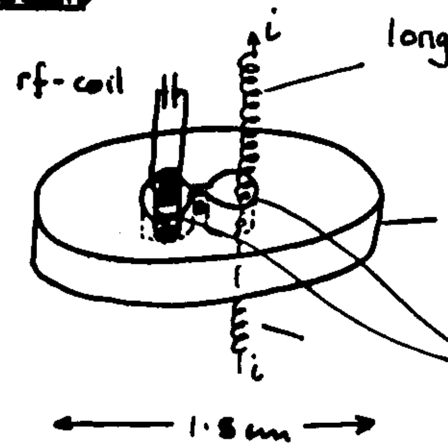
Uses

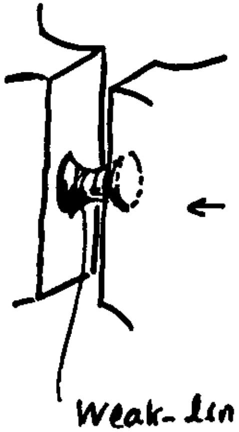
* Simple demonstration of a genuine quantum effect at LN_2 temperatures
* Detection of minority s/c phase in multi-phase sample $< (1\text{ mm})^3$ with suitable electronics
* encouragement that real SQUID operation in single larger loops might be possible.

Quantum sensitivity - large area - RF SQUIDs

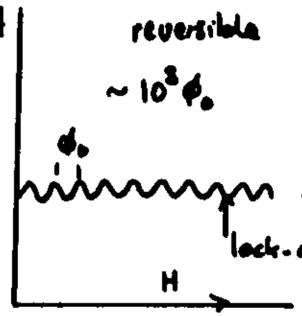
Zimmermann — NBS — crack junction
Zavaritskii + Zavaritski
~~Vasiliev~~ + ~~Vasiliev~~ — Moscow
Harrop, Colclough, Muirhead, Gough - Birmingham } INTERLAKEN

Birmingham SQUID





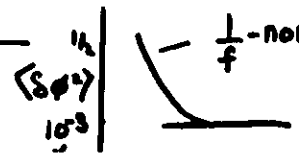
← Superconductor between holes filed away to leave a small bridge section $\sim (1/10\text{ mm})^2$ area



$\phi_0 \sim \frac{hc}{2e}$ to ~2% accuracy

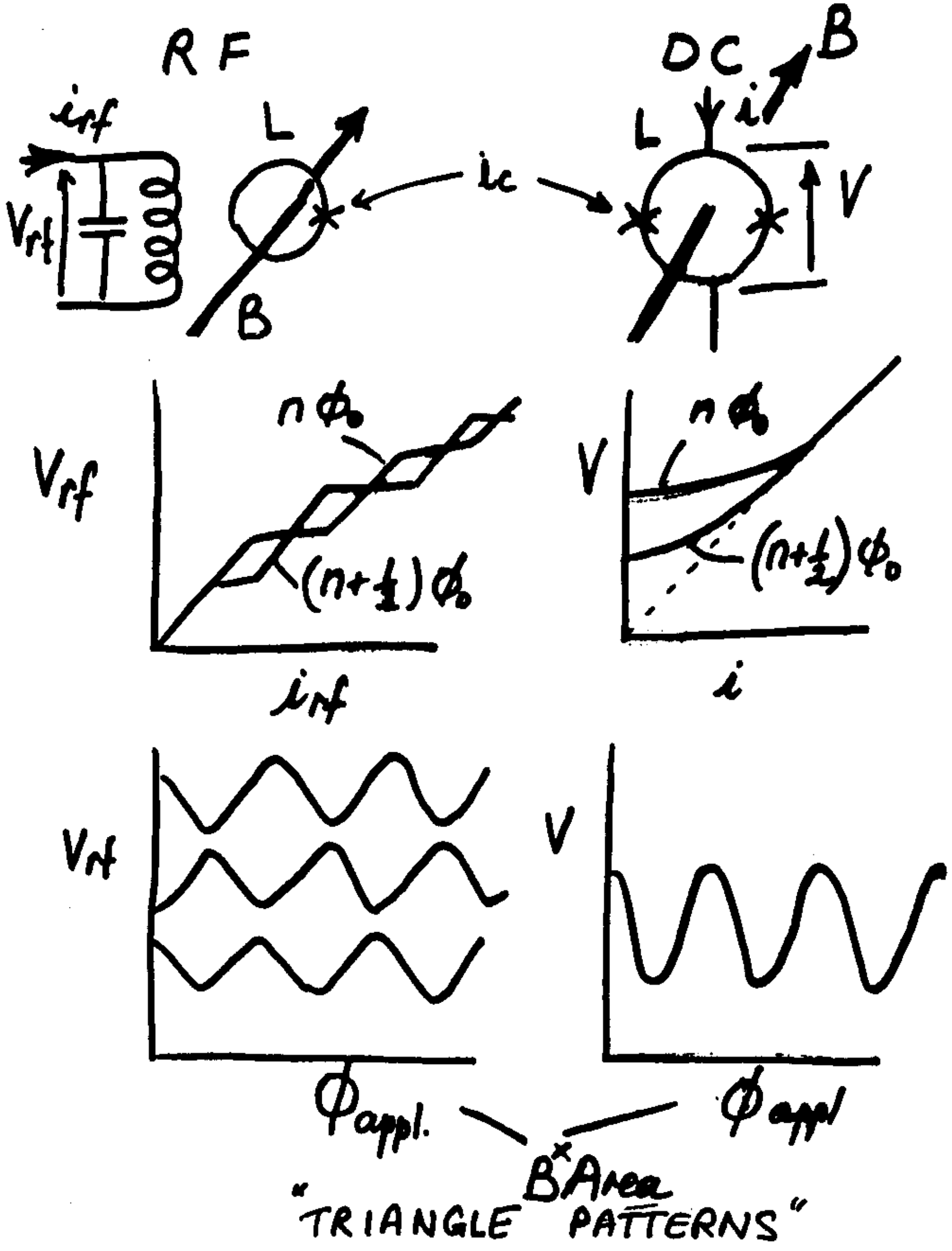
$\langle S\phi^2\rangle^{1/2} \sim 10^{-3}\,\phi_0/\sqrt{Hz}$

$\langle SH^2\rangle^{1/2} \sim 10^{-12}\,T/\sqrt{Hz}$



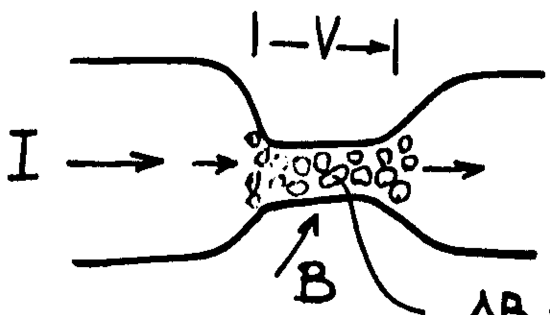
$\frac{1}{f}$-noise at low f

SQUIDS (SCHEMATIC ONLY)



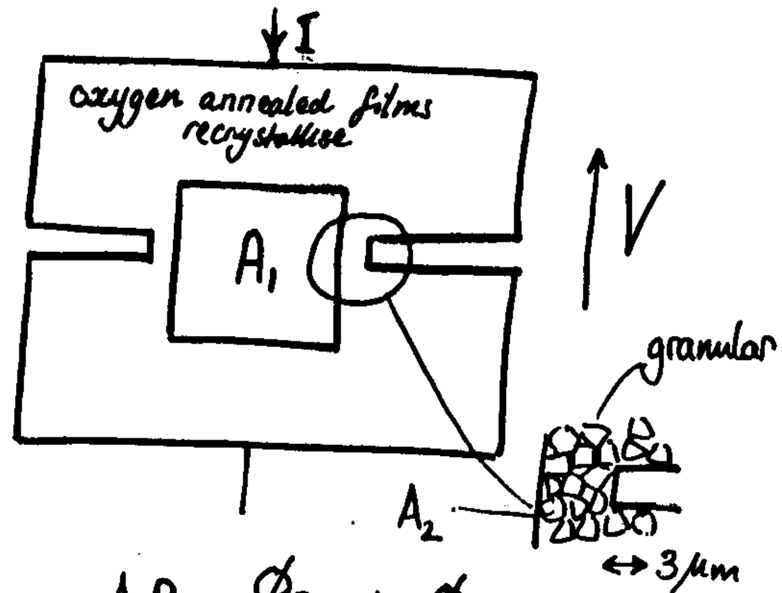
DC-SQUID CONFIGURATIONS

1. CERAMIC CONSTRICTION JUNCTIONS



$$\Delta B = \frac{\phi_0}{A\,(\mu m^2)}$$

2. THIN FILM PLANAR DC SQUIDS



$$\Delta B \sim \frac{\phi_0}{A_1} + \frac{\phi_0}{A_2}$$

IBM DC-SQUID (May '87)

THIN FILM

ALSO HITACHI + NEC

68K

28K

4.2K

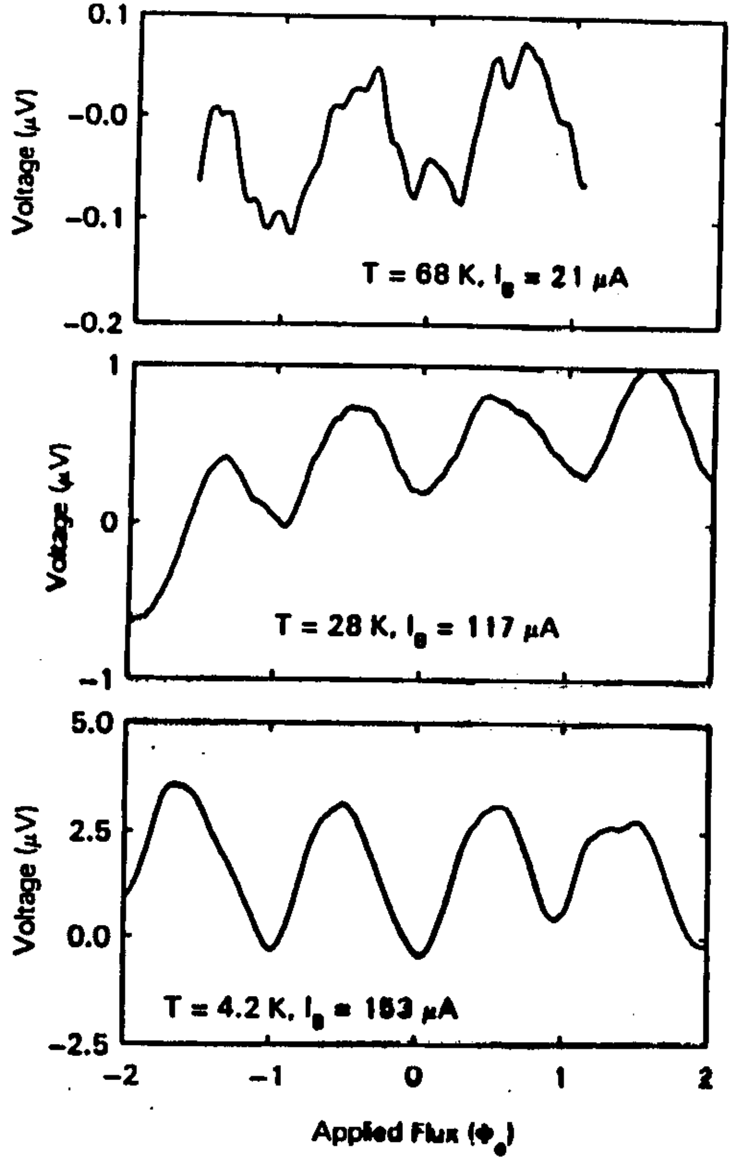
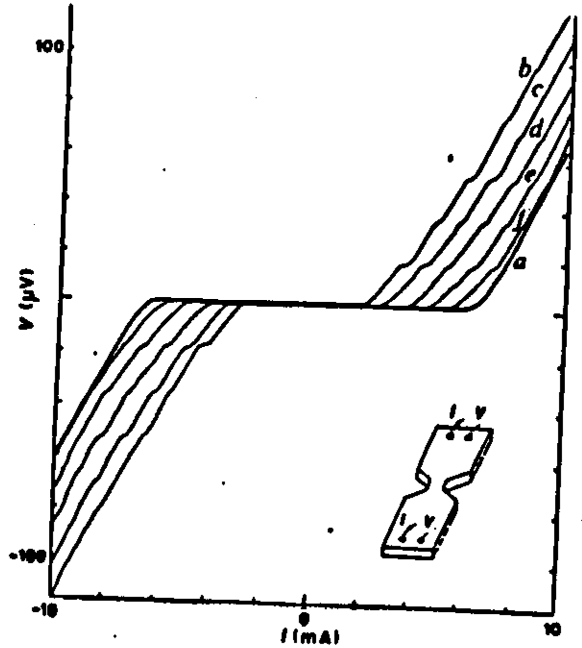


FIG. 4. Voltage across the SQUID, *V*, vs the applied flux for three temperatures 4.2, 28, and 68 K. The zero of the voltage scale is arbitrary.



Robbes et al – CAEN ~100μm

Constriction junction SQUID

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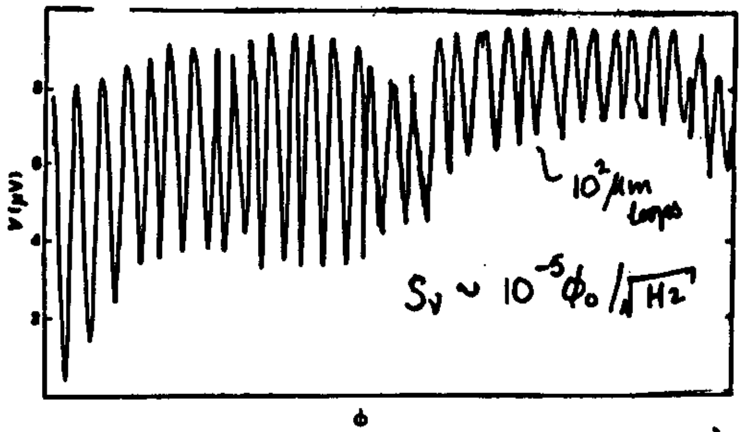

Fig. 3 Plot of the voltage *V* across the SQUID versus the applied flux ϕ at 77 K. The voltage reference is arbitrary. Bias current I = 320 μA; bandwidth, 5 Hz.