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EXPERIMENTAL WORKSHOP ON
HIGH TEMPERATURE SUPERCONDUCTORS
(11 - 22 April 1988)

JOSEPHSON AND RELATED MACROSCOPIC QUANTUM PHENOMENON - II

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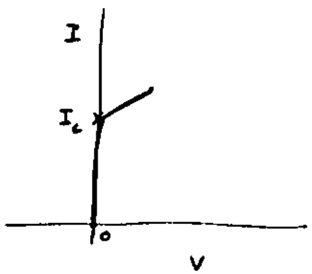
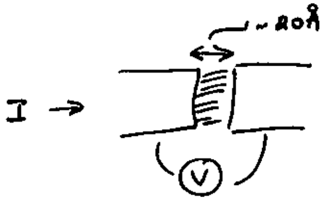
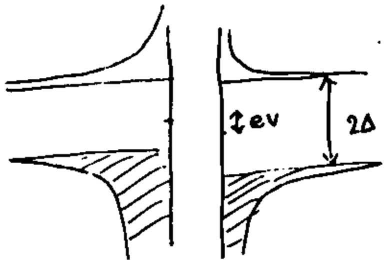
D.C. JOSEPHSON EFFECT

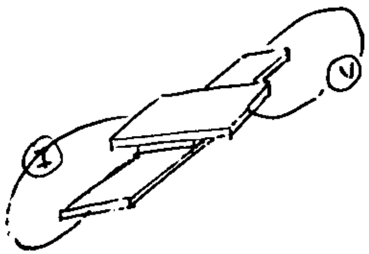




$I \sim 10^{-2} A/cm^2$

QUASI PARTICLE TUNNELLING





Current density of a superconductor

$$\vec{J} = \frac{e^*}{m^*}\Big(\frac{i\hbar}{2}\underbrace{(\psi\nabla\psi^* - \psi^*\nabla\psi)}_{-i\vec{\nabla}\phi} - \frac{e^*}{c}\vec{A}|\psi|^2\Big)$$

The Gauge transformation

$$\vec{A} \to \vec{A} + \vec{\nabla}\chi$$

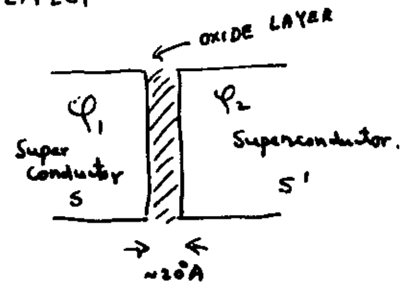
$$\phi \to \phi + \frac{2e}{\hbar c}\chi$$

Keeps the current density invariant.

WEAK LINKS AND JOSEPHSON EFFECT

1) QUASIPARTICLE TUNNELLING (we won't discuss it)

2) CONDENSATE TUNNELLING



$$H = H_S + H_{S'} + H_T$$

$$H_T = \sum_{kl} T_{kl} c^\dagger_{kS} c_{lS'} + h.c.$$

In Equilibrium chemical potentials are identical.

The Eigen states in the absence of tunnelling are:

$$\psi_\nu = \phi^{(S')}_{2(N-\nu)}\ \phi^{(S)}_{2\nu} \qquad (H_S + H_{S'})\psi_\nu = E_\nu \psi_\nu$$

$$t_0 = \sum_{\substack{k,l\\k',l'}} \langle \nu+1 |\, T_{kl}\, c^\dagger_{kS}\, c_{lS'} \,| I \rangle \frac{1}{E - E_I} \times$$

$$\times \langle I |\, T_{k'l'}\, c^\dagger_{k'S}\, c_{lS'} \,|\nu\rangle$$

$$= -4 \sum_{kl} |T_{kl}|^2 \frac{u_k v_k u_l v_l}{\epsilon_k + \epsilon_l}$$

$$H\psi_\nu = E\psi_\nu + t_0 (\psi_{\nu+1} + \psi_{\nu-1})$$

$$\cancel{\psi_\phi}\ \mp E(\phi) = E + t_0 \cos(\phi)$$

Current $J(\phi) = \frac{\partial(\Delta E(\phi))}{\partial\phi}$

ENERGY OF COUPLING

$$\Delta E(\phi) = E_0 \cos(\phi)$$

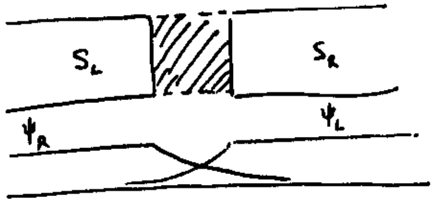
Eigen states in the presence of pair tunnelling: $\psi(\phi) = \sum_\nu e^{i\nu\phi}\psi_\nu$ phase coherence among various Cooper pair numbers

Simple derivation of DC & AC Josephson Effects.

All the cooper pairs are in the same "extended" state.

This leads to the macroscopic wave function.

Hence enough if we consider one-cooper pair problem.



$$|\psi\rangle = \psi_R|R\rangle + \psi_L|L\rangle$$

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle \quad ; \quad H = H_L + H_R + H_T$$

$E_L = 2\mu_R$

$E_R = 2\mu_R$

$E_L - E_R = 2eV$

$$H_L = E_L|L\rangle\langle L| \qquad H_T = t_0\left(|L\rangle\langle R| + |R\rangle\langle L|\right)$$

$$H_R = E_R|R\rangle\langle R|$$

$$i\hbar\frac{\partial\psi_R}{\partial t} = E_R\psi_R + t_0\psi_L \qquad \psi_L = \rho_L^{\frac{1}{2}}e^{i\phi_L}$$

$$i\hbar\frac{\partial\psi_L}{\partial t} = E_L\psi_L + t_0\psi_R \qquad \psi_R = \rho_R^{\frac{1}{2}}e^{i\phi_R}$$

$$\frac{\partial\rho_L}{\partial t} = \frac{2}{\hbar}t_0\sqrt{\rho_L\rho_R}\,\sin(\phi_L - \phi_R)$$

$$\frac{\partial\rho_R}{\partial t} = -\frac{2}{\hbar}t_0\sqrt{\rho_L\rho_R}\,\sin(\phi_L - \phi_R)$$

pair current $J \equiv \frac{\partial\rho_L}{\partial t} = -\frac{\partial\rho_R}{\partial t} = \underbrace{\frac{2t_0}{\hbar}\sqrt{\rho_L\rho_R}}_{\equiv J_1}\sin\phi$

$$\frac{\partial\phi_L}{\partial t} = \frac{t_0}{\hbar}\sqrt{\frac{\rho_L}{\rho_R}}\cos\phi + \frac{eV}{\hbar}$$

$$\frac{\partial\phi_R}{\partial t} = \frac{t_0}{\hbar}\sqrt{\frac{\rho_L}{\rho_R}}\cos\phi - \frac{eV}{\hbar}$$

$$\therefore \frac{\partial(\phi_L - \phi_R)}{\partial t} = \frac{2eV}{\hbar}$$

$$\therefore \phi_L - \phi_R = \frac{2eV}{\hbar}t + \text{Const.}$$

$$J(t) = J_1\sin\left(\varphi_L - \varphi_R + \frac{2eV}{\hbar}t\right)$$

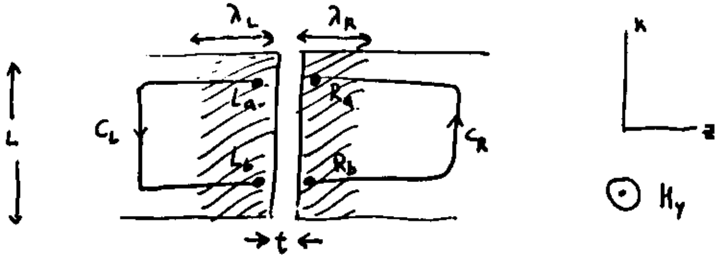
Energy of Coupling:

$$|\psi\rangle : \psi_L e^{i\varphi_L}|L\rangle + \psi_R e^{i\varphi_R}e^{i\frac{2eV}{\hbar}t}|R\rangle$$

$$\Delta E = \langle\psi|H_T|\psi\rangle = |\psi_L||\psi_R|\cos\left(\varphi_L - \varphi_R - \frac{2eV}{\hbar}t\right)$$

$$\therefore J(t) \propto \frac{\partial\Delta E(\phi)}{\partial\phi} \qquad \phi \equiv \varphi_L - \varphi_R$$

— MAGNETIC FIELD EFFECTS —



MAGNETIC FIELDS PENETRATE THE WEAK LINKS RATHER EASILY.

THIS LEADS TO A DISTRIBUTION OF CURRENTS LEADING TO INTERESTING MAGNETIC FIELD DEPENDENCE.

$$\vec\nabla\phi_{L,R} = \frac{2e}{\hbar c}\left(\frac{mc}{2e^2\rho}\vec J_s + \vec A\right)$$

$$\varphi_{Ra}(x) - \varphi_{Rb}(x+dx) = \frac{2e}{\hbar c}\int_{C_R}\left(\vec A + \frac{mc}{2e^2\rho}\vec J_s\right)\cdot d\vec\ell$$

$$\varphi_{Lb}(x+dx) - \varphi_{La}(x) = \frac{2e}{\hbar c}\int_{C_L}\left(\vec A + \frac{mc}{2e^2\rho}\vec J_s\right)\cdot d\vec\ell$$

$$\varphi(x+dx) - \varphi(x) \equiv \left[\varphi_{Lb}(x+dx) - \varphi_{Rb}(x+dx)\right] - \left[\varphi_{La}(x) - \varphi_{Ra}(x)\right]$$

$$= \frac{2e}{\hbar c}\left[\int_{C_L}\vec A\cdot d\vec\ell + \int_{C_R}\vec A\cdot d\vec\ell\right]$$

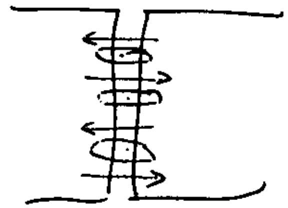
$$\varphi(x+dx) - \varphi(x) \approx \frac{2e}{\hbar c}\oint\vec A\cdot d\vec\ell = \frac{2e}{\hbar c}H_y\underbrace{(\lambda_L+\lambda_R+t)}_{d}$$

$$\therefore\ \boxed{\varphi = \frac{2e}{\hbar c}\,d\,H_y\,x + \varphi_0}$$

$$\boxed{\therefore\ J = J_1\sin\left(\frac{2e}{\hbar c}dH_y x + \varphi_0\right)}$$

(5)

$$\therefore\ J = J_1\sin\left(\frac{2e}{\hbar c}\,d\,H_y\,x + \varphi_0\right)$$



Electrodynamics of Josephson Junctions: (EXCITATIONS AT THE JOSEPHSON JUNCTION)

$$\frac{\partial\varphi}{\partial x} = \frac{2e}{\hbar c}H_y\,d$$

(when the magnetic field lies in the xy plane)

$$\frac{\partial\varphi}{\partial y} = -\frac{2e}{\hbar c}H_x\,d$$

Maxwell's eqns:

$$\vec\nabla\times\vec H = \frac{4\pi}{c}\vec J + \frac{1}{c}\frac{\partial\vec D}{\partial t}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \frac{4\pi}{c}J_z + \frac{1}{c}\frac{\partial D_z}{\partial t}$$

$$\frac{\hbar c^2}{8\pi e d}\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right) = J_1\sin\varphi + C\frac{dV}{dt}$$

Capacitance $C = \frac{\epsilon_r}{4\pi t}$

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} - \frac{1}{\bar c^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{1}{\lambda_J^2}\sin\varphi$$

$$\bar c = c\left(\frac{1}{4\pi C d}\right)^{\frac{1}{2}}\ ;\quad \lambda_J = \left(\frac{\hbar c^2}{8\pi e d J_1}\right)^{1/2}$$

(6)

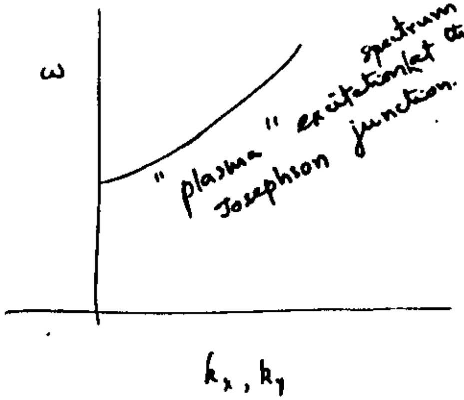
φ : Spatially uniform case

$$\frac{d^2\varphi}{dt^2} = -\omega_J^2 \sin\varphi$$

$$\omega_J \sim \frac{\bar{c}}{\lambda_J}$$

$$\nu_J = \frac{\omega_J}{2\pi} \sim 10^9 \sim 10^{11}\ \mathrm{Hz}$$

$$\bar{c} \sim \frac{c}{20}\ ; \quad \lambda_J \sim 100\,\mu m$$



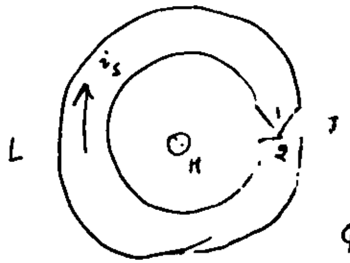
Anderson Plasma

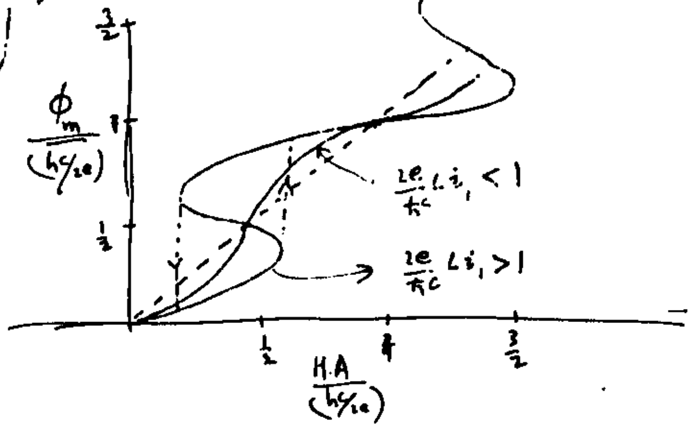
$$\omega^2 = \omega_J^2 + k^2 \bar{c}^2$$

Superconducting Link Shunted in parallel with a Superconductor.

(Related to SQUIDS)

(see Prof. Gough's lectures for more details)





$$\phi_2 - \phi_1 = \Delta\phi_J = \Delta\phi_L + 2\pi n = \frac{2e}{\hbar}\int_1^2 A\cdot d\vec{l} + 2\pi n$$

$$\Delta\phi_J^* = \Delta\phi_J - \frac{2e}{\hbar c}\int_{1,J}^{2} \vec{A}\cdot d\vec{l} = -\frac{2e}{\hbar c}\oint \vec{A}\cdot d\vec{l} + 2\pi n$$

$$= -\frac{2e}{\hbar c}\phi_m + 2\pi n$$

$$= -2\pi\frac{\phi_m}{\phi_0} + 2\pi n$$

Total magnetic flux thro' the hole is

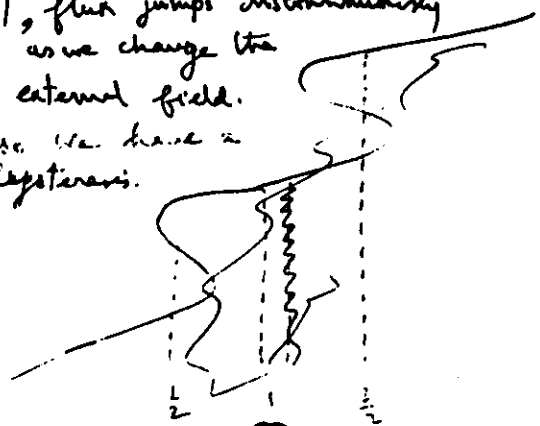
$$\phi_m = HA + L i_{cir.} = H.A + L i_1 \sin(\Delta\phi_J^*),$$

$$\phi_m = HA - L i_1 \sin\left(\frac{2\pi\phi_m}{\phi_0}\right)$$

If $\frac{Li_1}{\phi_0} < 1$, the total flux inside the ring changes continuously

If $\frac{Li_1}{\phi_0} > 1$, flux jumps discontinuously as we change the external field.

Also we have a hysteresis.



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