INTERNATIONAL ATOMIC ENERGY AGENCY
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SMR.378/21

WORKSHOP ON THEORETICAL FLUID MECHANICS AND APPLICATIONS

(9 - 27 January 1989)

CONVECTIVE HEAT TRANSPORT (II)

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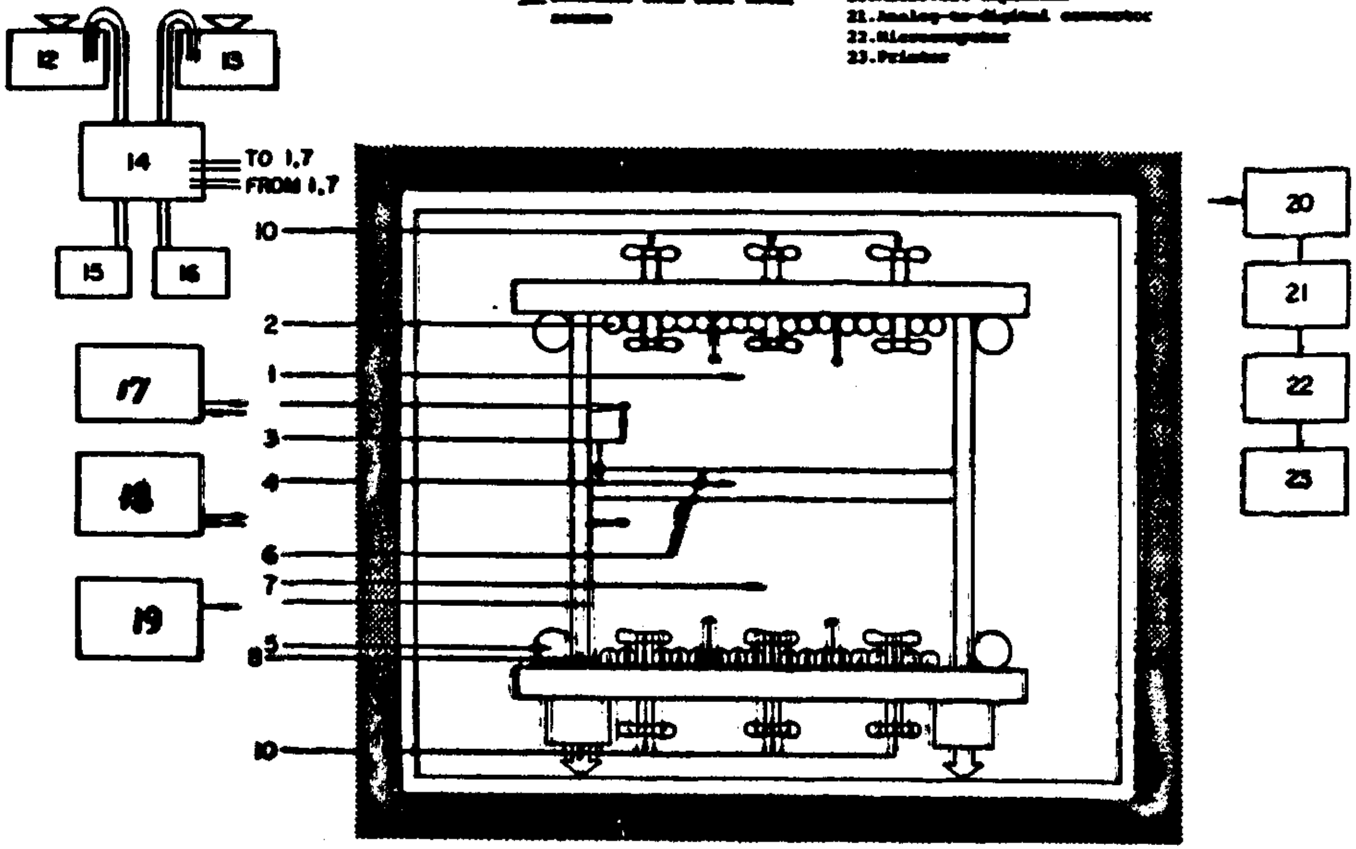
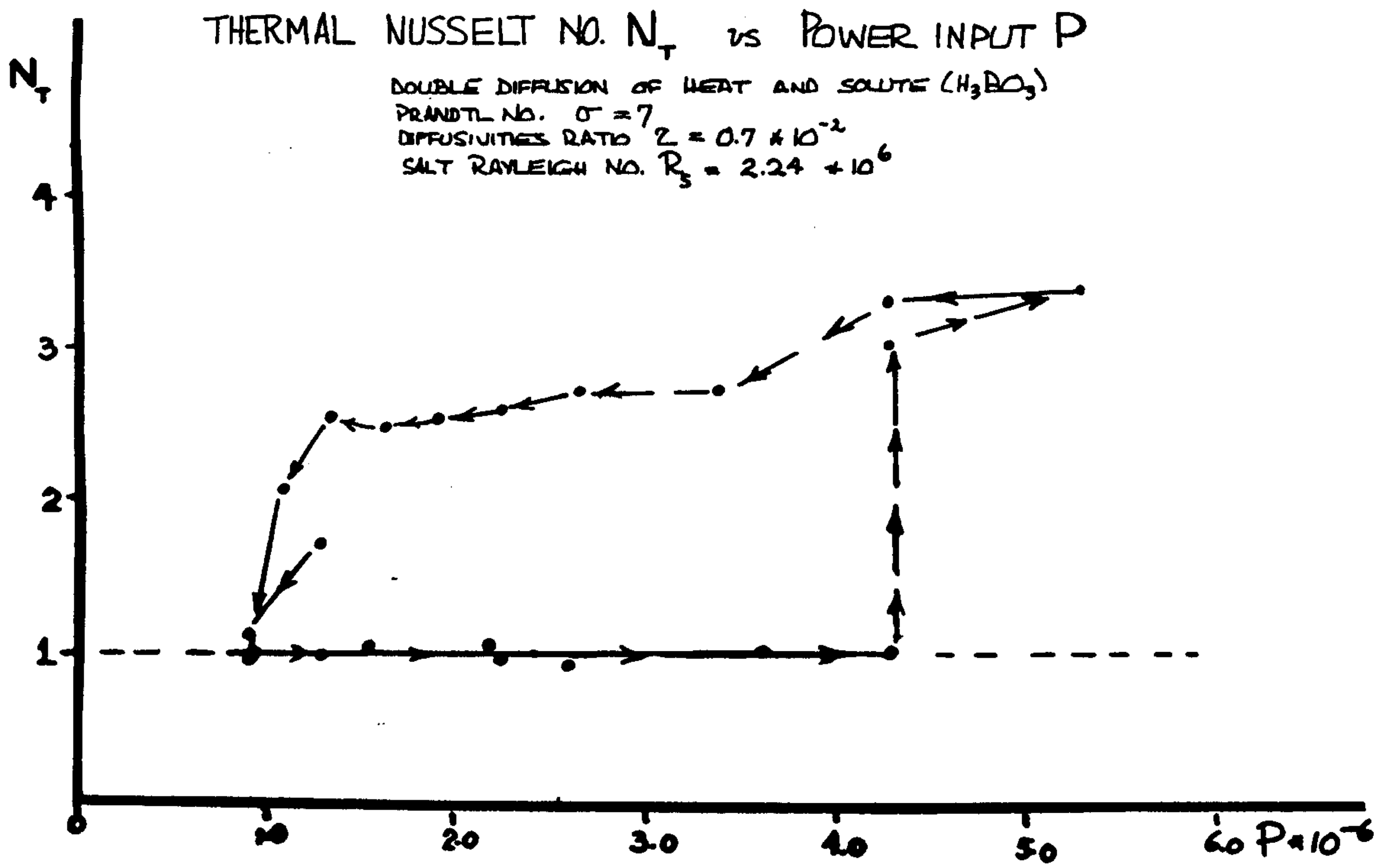
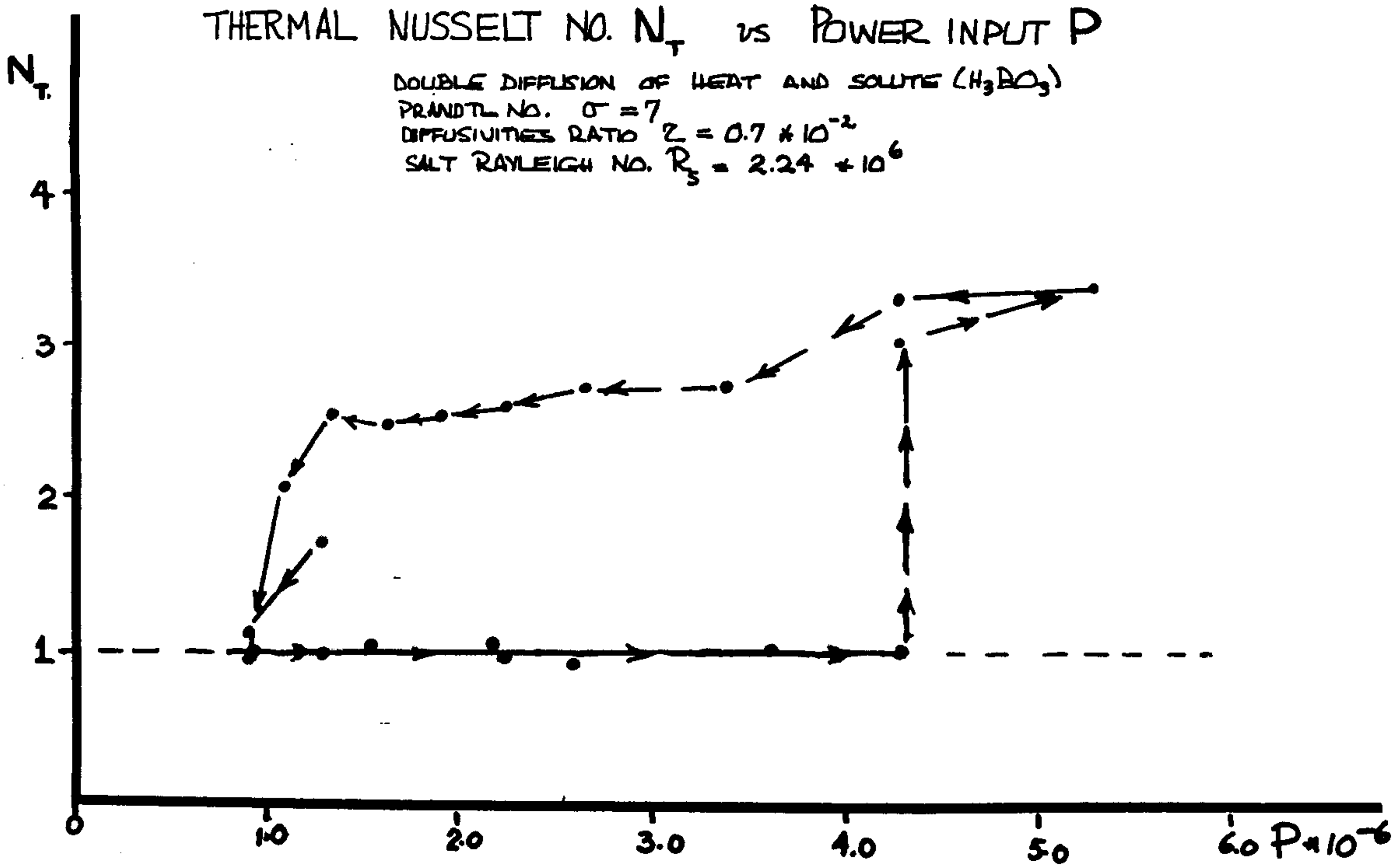
Figure 2. Schematic diagram of the apparatus.

1. Upper reservoir
2. Cooling coils
3. Thermocouples
4. Working fluid layer
5. Guard bath warm water source
6. Porous membrane
7. Lower reservoir
8. Heater
9. Magnetic stirring bar
10. Driving magnet
11. Guard bath water return
12. Constant head cold water source
13. Constant head fresh water source
14. Bank of solenoid valves
15. Collector of efflux from bottom reservoir
16. Collector of efflux from top reservoir
17. Constant temperature cold water circulator
18. Constant temperature warm water circulator
19. Constant voltage power supply
20. Microvolt amplifier
21. Analog-to-digital converter
22. Microcomputer
23. Printer





THERMAL NUSSELT NO. N_T vs POWER INPUT P
DOUBLE DIFFUSION OF HEAT AND SOLUTE (H3BO3)
PRANDTL NO. σ = 7
DIFFUSIVITIES RATIO τ = 0.7 × 10^-2
SALT RAYLEIGH NO. R_S = 2.24 × 10^6
N_T
4
3
2
1
0
1.0
2.0
3.0
4.0
5.0
6.0
P × 10^-6



THERMAL NUSSELT NO. N_T vs POWER INPUT P
DOUBLE DIFFUSION OF HEAT AND SOLUTE (H_3BO_3)
PRANDTL NO. $\sigma = 7$
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SALT RAYLEIGH NO. $R_S = 2.24 \times 10^6$
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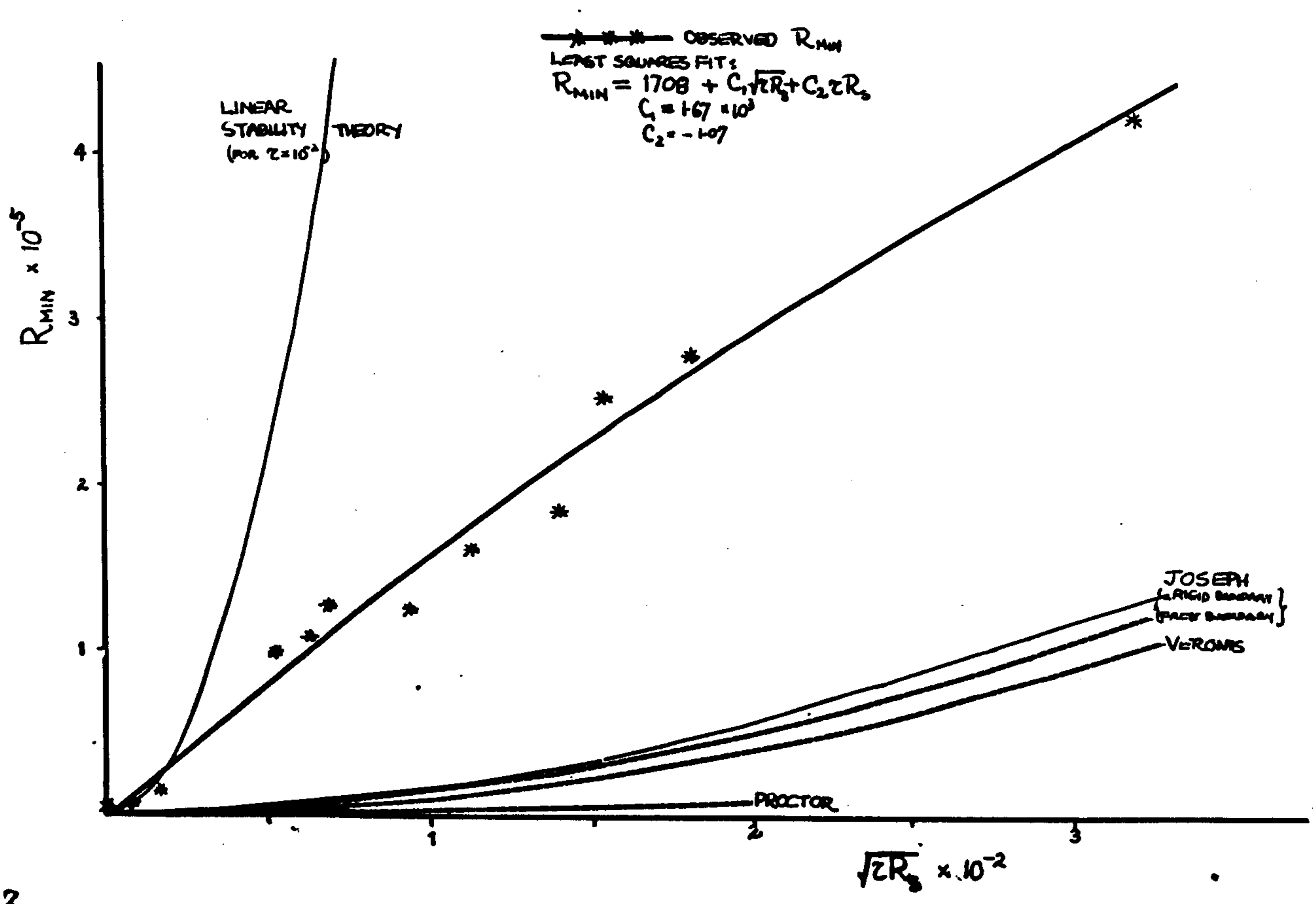
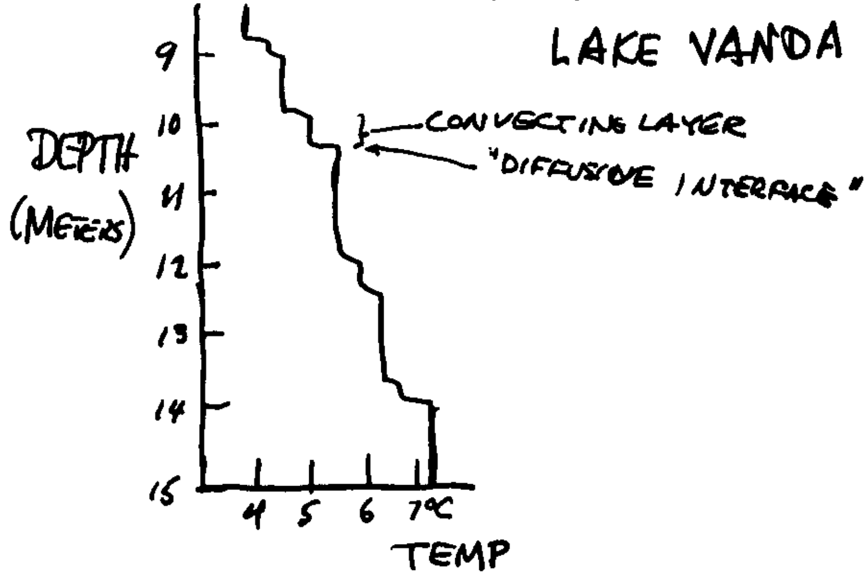
3
2
1
0
1.0
2.0
3.0
4.0
5.0
6.0
$P \times 10^{-6}$



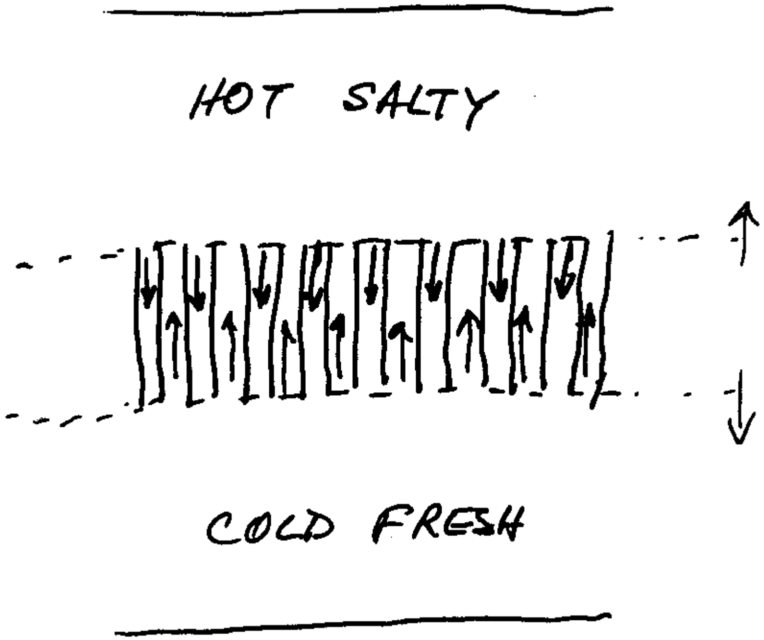
OBSERVED R_{MIN}
LEAST SQUARES FIT:
$R_{MIN} = 1708 + C_1\sqrt{\tau R_s} + C_2 \tau R_s$
$C_1 = 1.67 \times 10^3$
$C_2 = -1.07$
LINEAR STABILITY THEORY (FOR $\tau = 10^{-2}$)
JOSEPH
RIGID BOUNDARY
FREE BOUNDARY
VERONIS
PROCTOR
$R_{MIN} \times 10^{-5}$
$\sqrt{\tau R_s} \times 10^{-2}$
1
2
3
4

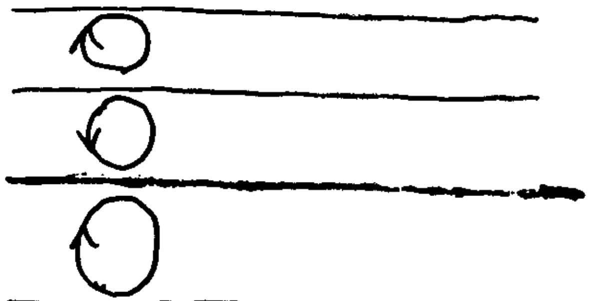
3

LAYERS IN LAKE VANDA

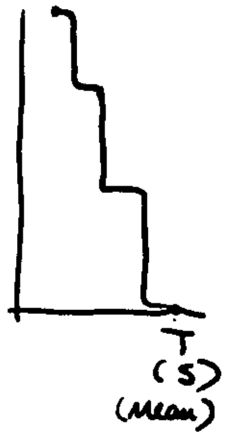


SALT FINGERS.





(Really probably turbulent convection in each layer)



IDEALIZED MODELS FOR SALT FINGERS.

L. STERN:

1) INFINITELY LONG
2) 'BACKGROUND' GRADIENTS OF T AND S, OTHERWISE INDEP. of z.
3) PERIODIC IN HORIZONTAL

$$w_t(x) + p_z - g(\alpha T - \beta S) = \nu w_{xx}$$
$$T_t + w\bar{T}_z = \kappa_T T_{xx}$$
$$S_t + w\bar{S}_z = \kappa_S S_{xx}$$

$p = 0$, $w, T, S \propto \underline{\sin kx\, e^{\lambda t}}$

LEADS TO A FAMILY OF EXACT SOLUTIONS OF THE NAVIER-STOKES (BOUSSINESQ) EQUATIONS (THERE IS A RELATION BETWEEN λ AND k.) (FOR THE 'RIGHT' k, $\lambda = 0$)

2. LNH + G. VERONIS $\tau \to 0$

INFINITELY LONG (AT ZERO ORDER)
BACKGROUND GRADIENT OF TEMP
SALINITY DIFFERENCE

[SALINITY BOUNDARY LAYER
(CARLEMAN INTEGRAL EQ.)

$$\int_0^1 |x-y|^{-\alpha} f(y)\,dy = g(x)$$
]

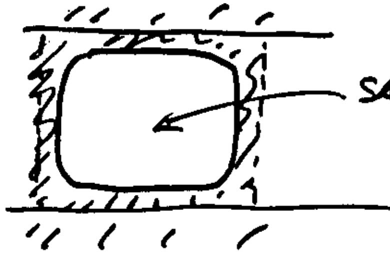
IMPORTANT LENGTH SCALE (HORIZONTAL)

$$L = \left(\frac{4\nu\kappa_T}{g\alpha\bar{T}_z}\right)^{1/4}$$

"BOUYANCY LAYER"

IDEALIZED MODEL FOR SOLAR POND CASE (PROCTOR)

SMALL τ
BUT NOT TOO BIG R_S!



SALINITY ESSENTIALLY UNIFORM, EXCEPT IN BOUNDARY LAYERS.

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