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Notes on Radio Frequency Spectrum Management

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NOTES ON RADIO FREQUENCY SPECTRUM MANAGEMENT

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1. Introduction

When Jean-Baptiste Fourier (1768 - 1830) introduced his transforms of mathematical functions from time-domain into frequency-domain and back, this was considered nothing more as a curious method of solving differential equations. His approach was strongly criticized and generated lots of doubts. Only after resolving the doubts by Dirichlet and Riemann, the Fourier Transforms and Fourier Spectrum became basic tools in many branches of science. The term "radio-frequency spectrum" appeared when it has been found that the concept of Fourier spectrum allows for defining conditions of interference-free operation of radio stations.

Today, the radio frequency spectrum is considered as a physical, measurable quantity, and not only as an abstract mathematical concept. The ability to carry energy and messages at a distance, at the speed of light, and at no cost, made the spectrum of radio waves recognized as a valuable resource of the mankind. Free access to it, from any place and at any time, added to its attractiveness and made it a common welfare from which everyone can profit. Its role is considered so important that its use is regulated by an international treaty - the International Telecommunication Convention. One of the articles of the treaty reads:

"... radio frequencies and geostationary satellite orbit are limited natural resources [...]; they must be used efficiently and economically [...] so that countries [...] may have equitable access to both..." (I.T.C., Art. 33-154)

2. Resource sharing

Pasture model

Common resources have one disadvantage. The disadvantage has been explained best by Hardin taking as an example a pasture open to all. We will follow his example. Because the pasture is free of charge, it is to be expected that each herdsman try to maintain as many cattle on it as possible. Such an arrangement works well until the number of beast reaches the carrying capacity of the

land. At this point, the inherent logic of the commons proceeds as follows:

"As a rational being, each herdsman seeks to maximize his gain. Explicitly or implicitly, more or less consciously, he asks 'What is the utility to me of adding one more animal to my herd?' This utility has one negative and one positive component:

1) The positive component is a function of the increment of one animal. Since the herdsman receives all the proceeds from the sale of the additional animal, the positive utility is nearly +1.

2) The negative component is a function of the additional overgrazing created by one more animal. Since, however, the effects of overgrazing are shared by all herdsmen, the negative utility for any particular decision-making herdsman is only a fraction of -1.

Adding together the component partial utilities, the rational herdsman concludes that the only sensible course for him to pursue is to add another animal to his herd. And another; and another... But this is the conclusion reached by each and every rational herdsman sharing a commons.

Therein is a tragedy. Each man is locked into a system that compels him to increase his herd without limit - in a world that is limited. Ruin is the destination toward which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons." [Hardin, 1968]

Is this applicable to the radio frequency spectrum?
The pasture and the spectrum are quite different objects, but there are some analogies. Indeed, the spectrum cannot be fenced. It goes un-priced at international conferences. It has limited carrying capacity. A portion of spectrum space occupied by one radio system is denied to other ones - like as in the case of cattle and pasture. If, therefore, one replaces the word "animal" in Hardin's text by "radio station" and the word "herdsman" by "administration", one obtains a simple "pasture" model of unrestricted use of the radio spectrum.

Sharing problems

One conclusion from the free pasture model is that the complete freedom in use of a limited common resource may work satisfactorily only under conditions of low-population density. Low population density means that competitive interactions between the users of the resource are negligible. As the population grows, competitive interactions develop, the resource becomes scarce, and the concept of its free use has to be abandoned.

Our past confirms that the approach to common resources changes, as does our understanding of their social role and value. Firstly, the commons in food gathering were abandoned. Farmland has been enclosed, and there is no free farmland now. Later, open pastures and free hunting and fishing areas have been restricted. Finally, using the commons as a place for waste disposal has been abandoned and restrictions on the disposal of sewage are now widely accepted throughout the world. Not so long ago, the radio spectrum and geostationary satellite orbit were added to the list of critical resources that must be shared among nations around the world. Finally, a concept of environment and environmental protection was developed, and restrictions were also imposed on the pollution of land, water, air, and electromagnetic environment. Suddenly, we have discovered that some of our resources, considered long time as being inexhaustible, have become scarce, even at critical levels for further development of mankind. The issue of their rational use, conservation, and protection has become an essential element, both on national and world-wide scales.

When discussing the future of resource sharing one can indicate several general approaches:

> "We might sell them off as private property. We might keep them as public property, but allocate the right to enter them. The allocation might be on the basis of wealth, by the use of an auction system. It might be on the basis of merit, as defined by some agreed-upon standards. It might be by lottery. Or it might be on a first-come, first-served basis... " [Hardin, op.cit.]

Consultations, negotiations, and coordination among all interested parties are necessary elements here. Any of the options listed above implies that an organizational framework exists to allow for such coordination activities, and to monitor if all agreed restrictions imposed on the resource use are observed by all parties involved.

Preferences

Which of the approaches above is the best one?
Each of them can be questionable and objectionable, dependent on the criteria applied. The final answer results from human preferences of goals and hierarchies of values. Although discussion of non-technical issues is beyond the scope of this contribution, one must realize that, in practice, it is often impossible to separate technical aspects of resource sharing from their economical, social and political contexts, and from the interests affected by them. The sharing problem cannot be solved by technical means only, without involving systems of human values and ideas. Firey points out here a significant role of the social tradition and experience:

> "Every adoptable set of resource process will be one which is valued by some population in terms of that population's own system of activities. ... Where a resource process involves beliefs and techniques that are incongruous with a people's system of activities, it will not be adopted by that people, however superior it may be by other criteria." [Firey, 1977]

However, in a pluralistic society the goals and hierarchies of values are often

> "...inconsistent and conflicting. The hierarchies of values and preferences of each individual are inconsistent among themselves, and different individuals and [...] groups have different hierarchies of preferences which are partially in conflict with those of other groups and individuals. Furthermore, the capacities of different groups to implement their preferences [...] are widely different." [Brooks, 1972]

"Inconsistency" here means that progress toward realization of one value or goal is destructive of another value or goal held by the same individual or group, and the lack of consistency may be not obvious to the individual or group concerned. "Conflicting" goals or values mean such goals or values of two groups that cannot be served by the same policies: what enhances one will degrade the other. These remarks are of special significance in an international context of the world-wide issues of radio spectrum sharing.

3. Competition and cooperation

Early years

It was at the turn-out of the century that a national and then world-wide competition in radio communication has been initiated, and soon the population of radio equipment reached a threshold at which mutual interference begun to limit further developments. After a first period of anarchy and uncontrolled interference, major parties interested came to a conclusion about potential benefits resulting from mutual coordination of technical and operational issues in radio communication. Such an activity started on a local scale, the global nature of the problem required, however, an international cooperation. Indeed, only two years after the first transatlantic wireless communication had astonished the world, the first international conference was called to improve the use of the radio frequency spectrum. This was the preliminary conference held in Berlin, in 1903.

The first radio conference held three years later, also in Berlin, agreed upon the principles of radio spectrum use. A major step at the conference was, among others, the establishment of the International Bureau in Bern. The operating frequencies and other relevant details of radio stations, were to be sent to that bureau which became a center of information about the actual use of the radio frequency spectrum, worldwide. Today, that function is being continued by the International Frequency Registration Board (IFRB) of the International Telecommunication Union (ITU).

International treaties are all part of a world-wide game nations agree to play following certain rules. Agreement is an inevitable ingredient here, and there is nothing to force nations to abide by these rules. If competitive forces are stronger than cooperative ones, no progress could be made. In Berlin, due to competition and conflict of interests, it was not possible to reach consensus on issues related to inter-communications. Soon, a test of life showed in full light tragic consequences of that fact. It was the well-known disaster of Titanic in April 1912. Only three months later, and not without the influence of public opinion shocked by the disaster, the second radio conference was held in London. It finally settled the problem of inter-communication between ships on the sea.

The World War I interrupted international cooperation in the radio spectrum use for several years. Science and technology was harnessed to military applications. Radio got an enormous impetus. When the war finished, all the scientific progress, technical developments and operational experience gained during the war time could be used again in peaceful service for the mankind. In new circumstances, however, the old radio regulations were not appropriate, and the next international radio conference was called on in Washington in 1927. The problem of frequency demand exceeding the available spectrum resource appeared sharply there, and the unending battle of frequencies had started.

The spectrum shortage problem was settled in Washington by two actions. On the one hand more stringent technical standards, limiting the radiation out-of the band necessary for transmission of information were imposed. (The demand for a complete outlawing of spectrally inefficient spark-type transmitters was pressed strongly, but not passed because of competition and conflict interests.) On the other hand the resource limits defined in old radio regulations were moved to embrace additional portions of spectrum, not yet regulated. That approach was copied also at other conferences. The drawing up of the first Frequency Allocation Table, regulating the use of the spectrum within these limits, was considered as one of the most important results of the Washington conference. Another one was to set up the International Radio Consultative Committee, CCIR.

The first radio conferences established principles of: (a) the use of radio spectrum as a public resource, free of charge, (b) equitable access to it, (c) negotiations and consensus, (d) allocation and administrative regulation of its use, (e) electromagnetic compatibility. It is interesting to note that these principles have been maintained until the present time, with minor changes only.

International Telecommunication Union

The use of the radio frequency spectrum is now coordinated through the International Telecommunication Union (ITU). The ITU, a Specialized Agency of the United Nations since 1947, is the oldest of all existing intergovernmental agencies, with a line-age dating back to 1865. The purposes and structure of ITU are prescribed by the International Telecommunication Convention.

In order to not interfere with the principle of sovereignty, the degree of mutual collaboration is purposely limited. This is formulated as follows:

"While fully recognizing the sovereign rights of each country to regulate its telecommunications, [...] the plenipotentiaries of the Contracting Governments [...] have agreed to establish this Convention [...] with the object of facilitating peaceful relations, international cooperation, and economic and social development... " (I.T.C., Preamble)

The ITU Members, totalling of 162 countries, agreed that the Union shall, in particular:

"a) effect allocation of the radio frequency spectrum and registration of radio frequency assignments in order to avoid harmful interference between radio stations of different countries;

b) coordinate efforts to eliminate harmful interference between radio stations of different countries and to improve the use made of the radio spectrum;..." [I.T.C., Art.4-18, 19]

The agreed general rule of rational use of the resource is

"... to limit the number of frequencies and the spectrum space used to the minimum ... (and) ... to apply the latest technical advances as soon as possible." [I.T.C., Art.33]

In accordance with the Convention, the Members of the Union meet, at intervals of normally about five years, at a Plenipotentiary Conference. This is the supreme authority which lays down the general policy of the ITU, reviews the Union's work and revises the Convention itself, if it considers this necessary. It also establishes the calendar of all ITU conferences, and sets a limit on expenditure until the next plenipotentiary conference. The last such a conference was held in Nairobi 1982, and the next one is scheduled for 1989.

4. Spectrum management

Radio conferences

Further to the tradition originated in Berlin, London, and Washington, the use of the radio frequency spectrum is based on allocations of frequencies by geographical regions and service categories as agreed by all Members of ITU at Radio Conferences, called "Administrative". These may be world-wide or regional, general or specialized. They may establish and revise Radio Regulations, including frequency allocations, operational rules, standards, and procedures relevant to the spectrum sharing.

Since 1947, when ITU joined the United Nations System, there were two general world administrative radio conferences (WARC), one in 1959, and the other in 1979, and several specialized and/or regional conferences. The general WARCs were authorized to deal with virtually all aspects of spectrum use. The specialized WARCs dealt with particular services and/or particular portions of the spectrum. The regional conferences were held to solve specific spectrum use problems within particular geographic regions. As known, there are three ITU regions: Region 1 which consists of Europe, Africa and part of Asia (USSR and Mongolia), Region 2, containing both Americas, and Region 3, consisting of Australia, Oceania, and the remainder of Asia. Appendix 1 lists the Administrative Radio Conferences.

Dual approach

As mentioned, the use of the radio frequency spectrum has been based on frequency allocation principles, which differentiate between geographical regions and service categories. The basic rules agreed upon by the ITU Members are published in the Table of Frequency Allocations of the Radio Regulations. "Allocation" means the distribution of a frequency band to a service, "allotment" - to a country or area, and "assignment" - to an individual radio station.

Some allocations are world-wide, i.e. identical throughout the world. In other cases allocations are regional, i.e. uniform throughout a particular region. In still other cases there are allocations specific for a single country, or a group of countries, in addition to, or different from, allocations approved by a majority of ITU Members (so-called "footnotes" to the Table of Frequency Allocations). Frequencies allocated to a service are available for use by any country, subject only to the limitations contained in the Radio Regulations.

A country can make an assignment to an individual station, or an assignment plan to a whole network of its stations. If that use could cause interference outside the territory of the country, or if it is intended for international communication, or if the assigning country seeks an international recognition for its assignment, it has to notify the IFRB and seek registration in the international list of frequency assignments and geostationary orbit positions. Each frequency and orbit assignment notified to IFRB is examined for its conformity with the radio regulations (including the Table of Frequency Allocations) and for compatibility with other registered frequency assignments and plans. If the result of examination is favourable, the assignment is registered, if not - it is returned for modification as appropriate. An advice concerning selection of proper frequency is given by IFRB, if needed.

This is so-called "ad hoc" frequency distribution method. There is one exception to that rule: there are certain services which are subject to so called "a priori" frequency plans. In "a priori" frequency plans specific bands allocated to specific services are parcelled among individual countries well in advance of their real use. Individual regions may have various allotment plans for specific services e.g. broadcasting, within their respective areas. "A priori" plans make a one-time distribution of the spectrum resource on the basis of present needs and predicted future requirements of all parties interested.

For services subject to "a priori" planning, a frequency assignment in accordance with the plan receives protection from any other assignment. In the case of "ad-hoc" managed services, the protection is given in accordance with priority of registration dates - a system frequently described as "first-come, first-served". In addition to that, one differentiates between three service categories: primary, permitted, and secondary. Primary and permitted services have equal rights, except when planning, stations of a primary service have priority in choice of frequencies. Stations of a secondary service should neither cause interference to, nor claim protection against interference from, stations of primary and permitted services.

Advocates of the "a priori" approach indicate that the "ad hoc" method is not fair because it transfer all the burden of coordination to the latecomers which must accommodate their requirements to the existing uses of the resource. It also excludes application of some optimization techniques that require full information about the spectrum uses. Replacement of "ad hoc" methods by systematic plans allows for fair sharing the spectrum resource and for its rational use, they say. Opponents, on the other hand, point out that "a priori" planning approach freezes technological progress. Moreover, is impossible to predict future requirements with a needed degree of accuracy, and

plans based on un-realistic data are of no practical value. In addition, as the radio spectrum is available at no cost, there is no mechanism to limit the requirements. Although the International Telecommunication Convention calls for minimizing the use of the spectrum/orbit resource,

" ...each country has an incentive to overstate its requirements, and there are few accepted or objective criteria for evaluating each country's stated need. In fact, the individual country itself may have only the dimmest perception of its needs over the time period for which the plan is to be constructed ... Under these circumstances, it is easy to make a case that allotment plans are not only difficult to construct, but when constructed will lead to a waste of resources as frequencies and orbit positions are 'warehoused' to meet future, indeterminate needs..."[Robinson, 1980]

One of the main advantages of the "ad-hoc" spectrum distribution is that it eliminates the "warehousing" problem, according to its advocates. Although that problem could be - at least in theory - alleviated also in the "a priori" planning approach, either by allowing frequency allotments to be transferred to another country or by imposing a condition of use within a specific period of time, or by imposing a tax or price system, nothing has been done till now.

5. Technical difficulties

As mentioned, a "used" or "occupied" portion of the spectrum/orbit resource means "denied to others". In this connection we have to distinguish between the physical and administrative denials [Berry, 1977]. The space physically denied depends on numerous processes and parameters which can be only roughly estimated, and is difficult to define with a precision. Thus, in order to simplify spectrum management, the administrative denial is usually used in organizational processing as a practical approximation to physical denial. Consequently, the space is administratively denied by spectrum management rules even if it is really not physically denied. One of the means to improve the use of the spectrum resource consist, therefore, in making (1) more precise estimations for physical denials, and (2) better approximations of administrative denials to them. To do that, however, further progress in radio sciences seems to be needed.

There are millions and millions of transmitters and receivers around the world, mutually interacting by means of radio waves. They would dissolve into chaos without a proper place assigned to each of them. Spectrum managers have to determine their operating frequencies and times, powers, antenna locations, heights, and patterns, as well as signal structures and signal processing methods. At the same time they have to fit in with budget, technology and other constraints. One can found here some similarity to the problems encountered in other fields. Indeed, radio spectrum management problem is a special case of a more general problem, which can be formulated as follows: given a collection of consumers who place demands upon a set of resources, find an assignment of consumers to resources that satisfies various constraints and that minimizes (or in some cases maximizes) a given objective function [Hale, 1980].

The usual question in optimization procedures is (in addition to the problem of the selection of proper objective function and identifying relevant constraints), how much time or how much work is required to find the solution. Sometimes, the exhaustive inspection of all possible solutions, their inter-comparison, and selection the best one, is the only exact method available. In our case, unfortunately, due to the complex interactions and an enormous amount of required data, any attempt to apply rigorous formal methods to real-life situations leads to the "intrinsically difficult" or "intractable" mathematical problems which cannot be solved in practice [Stockmeyer and Chandra, 1979].

As no exact solution method realizable is known, various informal, approximate, and heuristic methods have to be applied instead [Hale, 1980, Struzak, 1982]. Despite all efforts made, frequency management is still not yet based on stringent scientific and technical criteria [Sviridenko, 1977]. One may hope that some techniques and approaches developed in other areas, such as operation research, mathematical programming, graph theory, system theory, game theory, and computer simulation techniques could find applications in spectrum management problems, still waiting for efficient solutions.

6. Concluding remarks

The disproportion between the demand for radio spectrum and geostationary orbit positions on one hand, and the available resource capacity on the other hand, calls for reconsideration of the existing order in the spectrum/orbit use. As mentioned, the current practice is based on the administrative allotment concept and "service separation" philosophy, elaborated at the times of the first radio conferences. In the meantime, the population of radio stations increased enormously, new radio technologies have been introduced, and the political situation over the world has changed dramatically. Digital signal processing, microelectronics, and computer technology opened new horizons for integration of services and techniques, and spread-spectrum techniques revolutionized the concept of frequency channel use. To follow these changes, proposals are published from time to time that the existing administrative regulation system should be replaced by a competitive market economy mechanism. Such a deregulation is urged as a means of allowing market forces to distribute spectrum/orbit equitably to the market sectors where demand is greatest. Advocates of this idea indicate that it match the demand to the available resource capacity. Moreover, relying upon administrative decision-making is inferior to relying on market forces because decisions are arbitrary and often mistaken in determining what is the best interest of users [Webbing, 1977, Deregulation, 1987].

One expected that WARC 79, the largest general conference in the history of the ITU, would be a major vehicle for debating a new order of the spectrum/orbit use. In reality WARC'79 did not established any new principles, and

> "...it was rather to adjust existing regulations governing spectrum allocations and use to accommodate new and future requirements. The debate on general principles was left essentially untouched by the WARC..."[Robinson 1980].

How radio communication activities can be developed to the advantage of all nations, raises global implications. Although the necessity of international coordination in the spectrum use is generally understood, it is not always clear whether coordination is desired in order to resolve conflicts where parties adhere to their own individual objectives, or in order to achieve certain common, collectively agreed objectives. Spectrum/orbit sharing, world-wide, requires every nation to study the best ways of the use of these resources, as well as to study the nations' needs and elaborate an adequate position and effective interface for negotiations in the international framework. The technical elements of that process remain very important and deserving the most competent attention . It seems that many problems encountered in the present use of the radio spectrum and geostationary orbit are symptoms of inadequacies in our knowledge, and that further scientific efforts may offer practical solutions. The specific role of science has been best described by Gvishiani:

> "...By its very nature, science is well equipped for internationally coordinated efforts directed to the solution of common problems. Science is universal, independent of nationality, ideological convictions or political orientation, which makes joint efforts much easier than in any other field..." [Gvishiani, 1982]

As the time between scientific discovery and its application becomes shorter and shorter, it is more and more difficult to separate the pure and applied aspects of disciplines, and the need for closer collaboration between scientist and engineers becomes essential more then ever.

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The views expressed herein are those of the author and do not necessarily reflect those of CCIR or ITU.

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ANNEX 1

POWER OPTIMIZATION EXAMPLE

1. Introduction

The following describes how the optimization techniques can be used to solve a spectrum management problem. Transmitter power is a fundamental parameter determining the strength of both the wanted and unwanted signal. The rational assignment of power to a transmitter is one of the methods of enhancing electromagnetic compatibility. The power assignment problem can be formulated as a linear programming problem for which efficient computer programs already exist [Bock and Ebstein, 1964].

2. Problem statement

There are n transmitters and m receivers distributed over a given territory. The power, in watts, to be assigned to the jth transmitter is denoted by x_j, $j = 1, \dots n$. The sum of the powers assigned to all the transmitters is then:

$$F = x_1 + x_2 \dots x_n$$

and this is the objective function that should be minimized. The constraints follow the requirement of satisfactory reception at each receiver.

The receivers are indexed by $i = 1, 2, \dots m$. For each receiver there is a minimum acceptable signal to noise-plus-interference power ratio R_i.

The requirement concerning the satisfactory reception of the signal is therefore:

$$S_i > R_i (N_i + I_i)$$

and is called the receiver No. i constraint.

The background noise N_i at receiver i, in watts, is defined as the total effective power due to all unwanted and uncontrolled sources of energy including outside transmitters to which power cannot be assigned, internal circuit noise, man-made radio noise, etc.

S_i is the signal power from the wanted transmitter at receiver No. i and I_i is the total (interference) power from unwanted transmitters to which powers are being assigned. In order to discriminate between wanted and unwanted transmissions, a desirability factor D_{ij} is introduced.

D_{ij} = 1 if the signal from transmitter j is desired at receiver i under given circumstances, and

D_{ij} = 0 in the opposite case.

For each transmitter-receiver pair of the system, there is also a transmission factor t_{ij}, which, when multiplied by the transmitter power x_j, gives the effective power at the receiver input. The coefficients t_{ij} account for the transmitter output and receiver response characteristics, antenna gains and their relative position, distance between antennas, terrain and atmospheric effects, etc. Values of N_i and t_{ij} may derive either from measurement or from the solution of signal prediction equations.

Therefore:

$$S_i = \sum_{j=1}^{n} D_{ij} t_{ij} x_j$$

and

$$I_i = \sum_{j=1}^{n} (1 - D_{ij}) t_{ij} x_j$$

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3. Numerical example

Consider a simple two-dimensional case with two co-channel transmitters and two receivers, $n = m = 2$. The case is described by the following numerical data:

Acceptable $S/(N+I)$ ratios:	$R_1 = R_2 = 1$
Background noise powers:	$N_1 = N_2 = 10^{-6}$ W
Desirability factors:	$D_{11} = D_{22} = 1$; $D_{12} = D_{21} = 0$
Transmission factors:	$t_{11} = t_{22} = 10^{-7}$; $t_{12} = 0.5 \times 10^{-7}$; $t_{21} = 0.25 \times 10^{-7}$
Upper power limits:	$x_1 \leq 70$ W $x_2 \leq 60$ W

The following expressions can be deduced from these data:

Objective function to be minimized:	$F = x_1 + x_2$
the constraints:	$2x_1 - x_2 \geq 20$ (receiver No. 1)
	$-x_1 + 4x_2 \geq 40$ (receiver No. 2)
	$70 \geq x_1 \geq 0$ (transmitter No. 1)
	$60 \geq x_2 \geq 0$ (transmitter No. 2)

These are the boundaries of the feasible region.

4. Geometric interpretation

A geometric representation of the essential structure of the problem is given in Fig. A III-1, which corresponds to the numerical example above.

The power of transmitter No. 1 is plotted on the abscissa and that of transmitter No. 2 on the ordinate. Thus, any point in the plane corresponds to the pair of powers for the two transmitters considered. The regional constraints are represented by straight lines, which limit the feasible region of the plane. Note that the solution points must lie within, or on the boundary of, the feasible region. The parallel broken lines are lines of constraint value of the sum F, the objective function to be minimized. What is sought is a point of the feasible region for which F is as small as possible; this is obviously the point lying at the intersection of the two receiver constraints.

The optimum solution obtained is $x_1 = 17.1$ W, $x_2 = 14.3$ W. The sum $F = 31.4$ W is the smallest total power that can be assigned under the constraints stated above.

The mutual interference is manifest in the increased power required from each of the transmitters as compared with the 10 W required to overcome the background noise. This value results from the first constraint for $x_2 = 0$ and the second one for $x_1 = 0$.

An optimization problem of this kind involving k variables must be considered in a k-dimensional space. The region of allowable points then becomes a hypersurface in that space; however, the interpretation is similar to that given above. Since the illustration above was two-dimensional, the search for an optimum was possible simply by a visual inspection. In a k-dimensional space, the visual inspection of allowable points is not possible and it must be replaced by an appropriate mathematical optimization procedure which can be executed automatically by computers.

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Thus, the receiver constraint can be rewritten in equivalent form (for each receiver separately):

$$\sum_{j=1}^{n} \left[D_{ij} - R_i(1 - D_{ij})\right] \cdot t_{ij} \cdot x_{ij} - R_i N_i \geq 0$$

Additionally, transmitter powers should be non-negative values:

$$x_j \geq 0$$

Moreover, they have an upper limit:

$$x_j \leq x_{j\,max}$$

Note that all those expressions are linear in respect to variables x. Therefore, the problem can be formulated; minimize the linear objective function with linear regional constraints. This problem can be solved using a simplex algorithm from linear programming, as mentioned in § 3.5. A simple geometric interpretation of the problem is given in Fig. A III-1 with the illustrative example that follows.

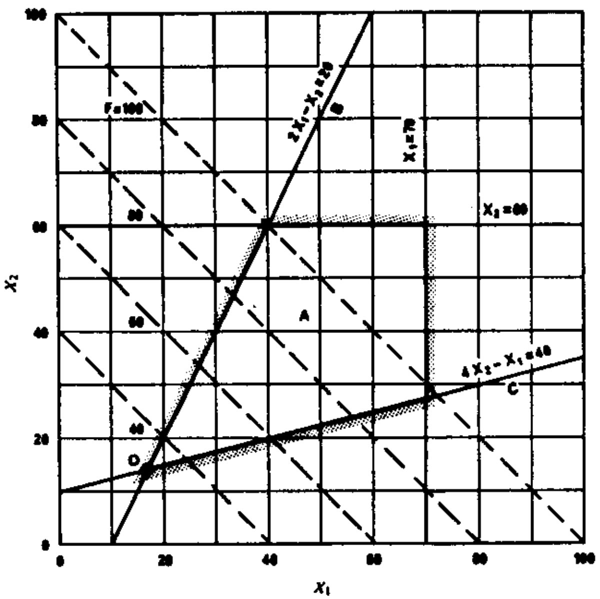

FIGURE A III-1 – *Optimum assignment of transmitter powers*

O: optimum point
A: feasible region
B: receiver No. 1 constraint
C: receiver No. 2 constraint