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Detection of Coded Signal - Equalization

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DETECTION OF CODED SIGNAL - EQUALIZATION

by

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Up to now, we have been considering the transmission of digital information over the AWGN channel with the assumptions that the channel has unrestricted bandwidth, that it introduces an attenuation factor as well as a phase shift.

Now, we are going to consider the case where the channel is band limited while maintaing the other assumptions. In this case we expect that some intersymbol interference will result since the transmission rate is high. However, in practical digital communications systems the frequency response of the channel is both not known precisely and time varying such that to design a proper demodulator for combatting interferences we have to employ adaptive techniques for equalization.

On the other hand, if $C(f)$ is the frequency response of a band limited channel to W Hz, then within this band

$$C(f) = |C(f)|.\exp(\theta(f))$$

A channel is said to be nondistorting if $|C(f)|$ is a constant and $\theta(f)$ is linear with f, otherwise it is said to be a distorting channel.

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There are several types of distortion that could be described. However, we mention only the most usual ones which are: 1) amplitude distortion, which occurs when $|C(f)| \neq$ constant; 2) phase or delay distortion, which occurs when $\theta(f)$ is not linear with f; 3) nonlinear distortion, which occurs when any nonlinear element is present in the system; 4) phase jitter, which occurs when a low index frequency modulation of the transmitted signal with the low frequency of the power line; 5) impulse noise, which occurs from switching equipment.

For the sake of mathematical tractability, we are going to consider the case when the channel behaves like a linear filter and introduces amplitude and delay distortion besides the AWGN noise.

Before doing the performance analysis for the most general case of a periodically time varying channel, we are going to consider the problem of signal design for band limited channels.

It can be shown that different types of digital modulation techniques can be represented by

$$\sum_{n=0}^{\infty} u_n.p(t-nT)$$

where $\{u_n\}$ is the set of discrete information symbols and $p(t)$ is a pulse with $P(f) = 0$ for $|f| > W$. This signal is transmitted over a band limited channel $C(f)$, with $C(f) = 0$ for $|f| > W$.

The output of the channel is

$$r(t) = \sum_{n=0}^{\infty} u_n.h(t-nT) + n(t)$$

where

$$h(t) = \int_{-\infty}^{\infty} p(a).c(t-a)da$$

and n(t) is the AWGN noise.

It has been shown that from the point of view of signal detection, the optimum receiver filter is the filter matched to the received pulse h(b-t).

The output of the receiving filter is

$$y(t) = \sum_{n=0}^{\infty} un.x(t-nT) + w(t)$$

when x(t) is the pulse representing the response of the receiving filter to h(t) and w(t) is the response of the receiving filter to n(t).

Now, y(t) is sampled at times t = kT + t', and so

$$y(t+kT+t') = \sum_{n=0}^{\infty} un.x(t+kT+t'-nT) + w(t+kT+t')$$

$$y(k) = \sum_{n=0}^{\infty} u(n).x(k-n) + w(k)$$

$$= u(k).x(o) + \sum_{n=0}^{\infty}{}' u(n).x(k-n) + w(k)$$

where $\sum_{n}'$ does not contain the term n = k . Letting x(o) = 1, we have

$$y(k) = u(k) + \sum_{n}' u(n).x(k-n) + w(k)$$

Therefore, at the k-th sampling instant u(k) is the desired information, $\sum_{n}'$ is the intersymbol interference and w(k) is the AWGN noise random variables.

Now, in order to simplify the signal design problem, we consider that C(f) = 1 for |f| ≤ W. Under this condition X(f) = $|P(f)|^2$ and consequently

$$x(t) = \int_{-W}^{W} X(f).\exp\{jwt\}df$$

We are interested in determining the spectral properties of x(t) and consequently the transmitted pulse p(t) for no intersymbol interference. Since

$$y(k) = u(k) + \sum_{n}' u(n).x(k-n) + w(k)$$

then for no intersymbol interference x(k) = 1 for k = 0 and 0 for k ≠ 0.

On the other hand, since x(t) is a band limited signal it can be represented by

$$x(t) = x(n/2w).\{ \sin(2.\pi.W(t-n/2W))/2.\pi.W(t-n/2W) \} \quad (1)$$

where

$$x(n/2W) = \int_{-W}^{W} X(f).\exp\{j.w.n/2W\}df$$

Now, suppose that the transmission rate for { u(k) } is 2W symbols/sec (Nyquist rate). Letting T = 1/2W in (1), we have

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT).\{ \sin(\pi(t-nT)/T)/(\pi(t-nT)/T) \}$$

No intersymbol interference means that

$$x(t) = \sin(\pi t/T)/(\pi t/T)$$

such that

$$X(f) = \begin{cases} T, & |f| \le 1/2T \\ 0, & |f| > 1/2T \end{cases}$$

This signal is not physically realizable, and so it has only an academic interest. However, if we lessen the assumption that T

= 1/2W to T < 1/2W, then following the same reasoning we end up with the condition for no intersymbol interference as

$$Xeq(f) = \begin{cases} T, & |f| \le 1/2T \\ 0, & |f| > 1/2T \end{cases}$$

where $Xeq(f) = \sum_{n=-N}^{N} X(f + n/T)$.

For practical purposes if W < 1/T < 2W, then a variety of pulses present no intersymbol interference and a good spectral characteristic. One of these is the one having the raised cosine spectrum. Note that these pulses represent the impulse response of the modulator-demodulator pair since the channel was assumed an ideal one.

Next, we are going to consider the case where the transmission rate is 2W symbols/sec and that some controlled intersymbol interference will be allowed. The pulses which satisfy these assumptions is called partial response signals. Time invariant as well as time varying partial response signals will be presented.

The easiest of all digital modulation to be considered is the pulse amplitude modulation (PAM). For the sake of simplicity, let us remove the AWGN noise of consideration. Then, the output of the receiving filter is

$$y(t) = \sum_{n=0}^{\infty} u(n).x(t-n/2W)$$

When x(t) is the sinc type of pulse, we have

$$x(n/2W) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

and { u(n) } is recovered by sampling the received signal at a rate of 1/T.

Now, suppose that it is allowed to have at least two nonzero values of x(n/2W). As a consequence, we encounter intersymbol interference at the sampling times. However, due to the fact that it is known apriori it is deterministic, and so the receiver can take it into consideration for further processing.

This controlled intersymbol interference technique have two special cases: 1) the duobinary signal; and 2) the modified duobinary signal.

The duobinary signal is represented by:

$$x(n/2W) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$X(f) = \begin{cases} (1/W).[\ \exp(-j\pi f/2W)\ ]\cos(\pi f/2W), & |f| \le W \\ 0, & \text{otherwise} \end{cases}$$

The modified duobinary signal is represented by

$$x(n/2W) = \begin{cases} 1, & n = -1 \\ -1, & n = +1 \\ 0, & \text{otherwise} \end{cases}$$

$$X(f) = \begin{cases} (j/W).\sin(\pi f/W), & |f| \leq W \\ 0 & , \text{otherwise} \end{cases}$$

The main difference between the partial response signals is that the modified duobinary signal can avoid error propagation by pre-coding the data at the transmitter instead of eliminating intersymbol interference (by subtraction) at the receiver.

To give some insight, let us consider the duobinary signal with binary PAM. The received signal at the sampling times is

$$A(n) = u(n) + u(n-1), \quad n = 1, 2, \ldots$$

where { u(n) } is the amplitude of the transmitted pulses and $u(n) \in \{ +1, -1 \}$. Thus, $A(n) \in \{ +2, 0, -2 \}$ with $A(n) = 2$ for $(u(n), u(n-1)) = (+1, +1)$, $A(n) = -2$ for $(-1, -1)$, and $A(n) = 0$ for $(-1, +1)$ or $(+1, -1)$. Error propagation occurs if $u(n-1)$ is decoded erroneously.

It can be shown [1] that the probability of error for symbol-by-symbol demodulation when using digital PAM signaling with duobinary and, modified duobinary pulses on a AWGN channel is given by

$$Pm \leq (1 - 1/M^2).\mathrm{erfc}[\{(3/M^2-1).(\pi^2/4) .T.Pav/No\}^{1/2}] \qquad (2)$$

If one desires to compare (2) with the case of ideal sinc type of pulse or the raised cosine one, it can be shown [1] that the probability of error is given by

$$Pm \leq (1 - 1/M).\mathrm{erfc}[\{(3/M^2-1).T.Pav/No\}^{1/2}]$$

Therefore, the partial response signal case is 2.1 dB worse than the ideal or raised cosine type of signals.

Performance Analysis of Periodically Time Varying Partial Response Signalling with Maximum Likelihood and Integrate & Dump Receivers

1. Introduction

Although high speed digital communication in band limited channels has been employed as an alternative solution to the placed demand, it presents intersymbol interference (ISI) as one of the major degrading factor affecting the over all performance of the communication system. In order to diminish the degradation impose upon the system by the ISI, band-limited signals with controlled intersymbol interference (partial response signals) have been introduced. This class of physically realizable signals have a signalling rate of 2W symbols per second with a known intersymbol interference at the sampling instants. Detection of partial response signals become a simple matter of sequentially subtracting, at the n-th sampling instant, the (n - 1)-th ISI value(s). Since this scheme allows error propagation, a "pre-coding" can be utilized to mitigate this effect.

In general, time invariant controlled as well as uncontrolled ISI, which for the sake of brevity we call time invariant ISI, has been the form of interference assumed in the analysis [1]-[6]. In [7]-[11], a maximum likelihood (ML) receiver for digital signals with time invariant ISI was associated with a finite state structure for uncoded as well as coded systems. Since finite state structure can be described by a trellis, transfer function technique (TFT) is a real asset in the performance evaluation of this type of communication systems.

Motivated by these previous works, we present performance analysis of a PAM communication system with periodically time varying controlled ISI

by use of the dynamic transfer function technique (DTFT) developed in [12]. Basically, this DTFT technique is a systematic way of counting error events that start in some node at time k in the trellis as well as all the previous error events prior to time k. This is the same as fixing an information digit in the time interval $[k, k+1]$ and count all those paths in the trellis that have a decoded information digit disagreeing with the fixed information digit.

We associate with the periodically time varying trellis structure a periodic sequence of channel impulse responses, $h^{\tilde{p}}(t)$ with $\tilde{p}$ an integer, $0 \leq \tilde{p} \leq M-1$. For each $h^{\tilde{p}}(t)$, we define the ISI coefficients by $\underline{h}^{\tilde{p}} = \{h_o^{\tilde{p}}, \ldots, h_{L-1}^{\tilde{p}}\}$, and we assume that $\underline{h}^{\tilde{p}}$ can take values on a set $H = \{\underline{h}^{\tilde{p}}, 0 \leq \tilde{p} \leq M-1\}$ where M is the number of channel impulse responses and L-1 is defined as the memory length of the intersymbol interference coefficients.

It is readily noticed that if for each $\tilde{p}$ the ISI coefficients are $\underline{h}^{\tilde{p}} = \{h_o, h_1, \ldots, h_{L-1}\}$ then time invariant is the resulting form of interference. On the other hand, if we assume that for each $\tilde{p}$, $\underline{h}^{\tilde{p}}$ can be any element of the set H then time varying ISI (dynamic ISI) is being considered. Multi path radio channels and, in general, time dispersive channels are examples where dynamic ISI takes place, [13].

In this paper, we derive upper bounds on the total as well as on the individual bit error probabilities by use of the DTFT for an uncoded PAM communication system with an optimum and suboptimum receiver when the channel varies periodically.

2. System Model

The channel considered in this analysis is in the class of continuous output discrete time channels with finite input alphabet $u_n \varepsilon U$, where without loss of any generality, $U = \{0, 1, 2, \ldots, W-1\}$. Each symbol is independent and identically distributed for uncoded communication systems or dependent according to some redundant coding rule.

Consider the following communication system as shown in Fig. 1. Let $\underline{u}_m = (u_{m,1}, u_{m,2}, \ldots, u_{m,N})$ be the m-th source symbol sequence of length N, where each $u_{m,j}$ has block length $\tilde{b}$, $1 \le m \le W^{\tilde{b}.N}$, $N >> L-1$ and $N >> \tilde{b}$. Let $\tilde{b}$ source symbols be input to the M channel impulse responses, $h^{\tilde{p}}(t)$, at a time and that only one of them is "selected" during T' seconds (T' equals the duration of $\tilde{b}$ source symbols). For each subsequent T' seconds a new selection is made. Let us define $A(i_1, i_2, \ldots, i_M) = \{h_1^i(t), h_2^i(t), \ldots, h_M^i(t)\}$ as the ensemble of all possible periodic combinations of the M channel impulse responses with $0 \le i_j \le M-1$. Without loss of generality, let us assume that the periodic sequence of the channel impulse responses identified by $A(0, 1, \ldots, M-1) = \{h^0(t), h^1(t), \ldots, h^{M-1}(t)\}$ is selected. That is, the first $\tilde{b}$ block goes through h(t), the second $\tilde{b}$ block goes through $h^1(t)$ and the M-th $\tilde{b}$ block goes through $h^{M-1}(t)$, and that subsequent blocks follow this pattern. We also assume perfect synchronization between transmitter and receiver.

Since the channel has the $A(0, 1, 2, \ldots, M-1)$ structure, we have from Fig. 1 that v(t) is given by

$$v(t) = \sum_n u_n \, \delta(t - nT)$$

and x(t) by

$$x(t) = \sum_n u_n(t - nT) * h^{\tilde{p}}(t) = \sum_n h^{\tilde{p}}(t - nT) \qquad (1)$$

where T is the symbol duration, $n(= \tilde{b} . \rho + r - 1)$ is the n-th source symbol

from a specified origin, ρ is an integer, $1 \le r \le \tilde{b}$, $\tilde{p} = n \bmod (\tilde{b} . M)$, and * represents convolutional operator.

Let $h^{\tilde{p}}(t) = 0$, for $|t| \ge LT$ and $0 \le \tilde{p} \le M-1$. Since $h_k^{\tilde{p}} = h_{-k}^{\tilde{p}}$ for $1 \le k \le L-1$, we define the memory length of each $h^{\tilde{p}}(t)$ as L-1. Therefore, optimum receivers no longer can be based upon observation at single time interval independent of the previous ones, which implies that we have to use receivers with memory. Among nonlinear receivers, the maximum likelihood receiver is the one which minimizes the probability of error for the entire data sequence [7]. Thus, it will be considered in the subsequent analysis. The condition of equal memory length for each $h^{\tilde{p}}(t)$ comes from the fact that the DTFT assumes that elements in the periodic combination have the same number of states. Since we are considering W-ary source during each T seconds, all impulse responses must have the same memory length in order to have the same number of states. Clearly, this implies that the same number of bits is being transmitted during each T seconds. It is possible, by use of the DTFT, to analyze cases where different number of bits are being transmitted during each T seconds. However, we require that the resulting number of states be the same during each one of the T seconds.

Let the noise at the front end of the receiver be one of the sample functions of the white Gaussian process with zero mean and two sided spectral density $N_0/2$. At the receiver, we have

$$y(t) = x(t) + n(t) \qquad (2)$$

3. Maximum Likelihood Receivers

From (1), we have that the corresponding channel signal $x_m(t)$ is given by

$$x_m(t) = \sum_n u_{mn} h^{\tilde{p}}(t-nT) \tag{3}$$

where we have assume for simplicity that $\tilde{b} = 1$.

Let us assumed that $x_m(t)$ was transmitted. Hence, the ML procedure will choose $x_m(t)$ if

$$p[y(t)/x_m(t)] \geq p[y(t)/x_{m'}(t)], \qquad \text{for all} \quad m' \neq m \tag{4}$$

Due to the finite energy of the signals, and by appropriated choices of orthonormal basis (Gram-Schmidt procedure) we can represent $y(t)$ and $x_m(t)$ by its coefficients in this new basis. Let $\underline{y}$ and $\underline{x}_m$ be the coefficients. Since the noise is white Gaussian

$$p[\underline{y}/\underline{x}_m] = (1/N_o)^{N/2} \exp\{(-1/N_o) \ \|\underline{y} - \underline{x}_m\|^2\} \tag{5}$$

Thus, $\underline{x}_m$ is chosen if and only if

$$\begin{aligned} x_m(t) &= \max_{\underline{x}}{}^{-1} \{\ln p(\underline{y}/\underline{x}_m)\} = \min_{\underline{x}}{}^{-1} \{\|\underline{y} - \underline{x}_m\|^2\} \\ &= \max_{\underline{x}}{}^{-1} \{2 \int_{-\infty}^{\infty} x_m(t)y(t)dt - \|x_m(t)\|^2 \ dt\} \\ &= \max_{\underline{x}}{}^{-1} \{\phi_m\} \end{aligned} \tag{6}$$

with ϕ_m being a metric. ϕ_m is explicitly given by

$$\phi_m = 2 \sum_n \int_{-\infty}^{\infty} u_{mn} h^{\tilde{p}}(t-nT)y(t)dt \quad \sum_n \sum_j \int_{-\infty}^{\infty} u_{mn} u_{mj} h^{\tilde{p}}(t-nT)h^{\tilde{q}}(t-jT)dt \tag{7a}$$

$$= 2 \sum_n u_{mn}\, y_n - \sum_n \sum_j u_{mn}\, u_{mj}\, h^{\tilde{p},\tilde{q}}_{n-j} \tag{7b}$$

$$= \sum_n \{2u_{mn}\, y_n - \sum_j u_{mn}\, u_{mj}\, h^{\tilde{p},\tilde{q}}_{n-j}\} \tag{7c}$$

$$= \sum_n \{\phi_{m,n}\} \tag{7d}$$

with

$$y_n = \int_{-\infty}^{\infty} h^{\tilde{p}}(t-nT)y(t)dt \tag{8}$$

and

$$\int_{-\infty}^{\infty} h^{\tilde{p}}(t-nT)h^{\tilde{q}}(t-kT)dt = h^{\tilde{p},\tilde{q}}_{n-k} \tag{9}$$

For $\tilde{p} = \tilde{q}$, we have that

$$h^{\tilde{p},\tilde{q}}_{n-k} = h^{\tilde{q},\tilde{p}}_{k-n} = h^{\tilde{p}}_{n-k} = h^{\tilde{p}}_{k-n} \tag{10}$$

Let us assume $\underline{h}^{\tilde{p}}$ is different for every $\tilde{p} \in \{0, 1, 2, \ldots, M-1\}$. Note that $\tilde{p}$ specifies each one of the M impulse response functions of the communication system under consideration. Substituting (10) in the second term of (7b), we have

$$\sum_n \sum_j u_n\, u_j\, h^{\tilde{p}}_{n-j} = \sum_n [u_n^2\, h^{\tilde{p}}_o - 2 \sum_{i=1}^{L-1} u_n\, u_{n-i}\, h^{\tilde{p}}_i] \tag{11}$$

From (8), we see that for each $\tilde{b}$ source symbol (T' seconds), the optimum receiver will have a filter matched to the impulse response of the channel, and (9) implies the knowledge of the characteristic of the channel by the receiver. Substituting (11) in (7c), we have that the metric ϕ_m depends on y_n, u_n, and $L-1$ past data input. Substituting (7c) in (6), one realizes that the resulting equation describes the Viterbi algorithm in a trellis structure with W^{L-1} states. Note that branch values in this trellis structure are characterized by the respective channel impulse response $\tilde{p}$ for each T' seconds.

Multiplying (2) by $h^{\tilde{q}}(t-kT)$, making use of (1) and integrating, we have

$$y_k = \int_{-\infty}^{\infty} y(t).h^{\tilde{q}}(t-kT).dt = \int_{-\infty}^{\infty} x_m(t).h^{\tilde{q}}(t-kT).dt + \int_{-\infty}^{\infty} n(t).h^{\tilde{q}}(t-kT).dt$$

$$= x_{mk} + n_k$$

where

$$x_{mk} = \sum_n \int_{-\infty}^{\infty} u_{mn}\, h^{\tilde{p}}(t-nT)h^{\tilde{q}}(t-kT)dt$$

$$= \sum_n u_{mn} \int_{-\infty}^{\infty} h^{\tilde{p}}(t-nT)h^{\tilde{q}}(t-kT)dt$$

$$= \sum_n u_{mn}\, h^{\tilde{p},\tilde{q}}_{n-k} = \sum_{i=-L+1}^{L-1} u_{m,k-i}\, h^{\tilde{p}}_i \tag{12}$$

and

$$n_k = \int_{-\infty}^{\infty} n(t)h^{\tilde{q}}(t-kT)dt \tag{13}$$

Thus, the channel output y_k can be written as

$$y_k = \sum_{i=-L+1}^{L-1} u_{m,k-i}\, h^{\tilde{p}}_i + n_k \tag{14}$$

We also have that $E\{n_k\} = 0$, and $E\{n_k\, n_j\} = (N_o/2)h^{\tilde{p}}_{k-j}$. Note that we can think of (14) as a finite state representation of the continuous case where n_k are correlated gaussian random variables. Figure 2, shows this equivalence. As shown in [10], there is no need to use a whitenning filter since the y_k's are sufficient statistics and that the metric Φ_m is additive and depends on y_n, u_n, and the L-1 past data input.

From the above conditions, we propose to analyze two cases: CASE I - the optimum receiver (matched filter); CASE II - the suboptimum receiver (the integrate and dump filter).

CASE I - OPTIMUM RECEIVER

Given that $x_m(t)$ was transmitted, an error will occur if during a span of N branches, a path in the trellis structure has accumulated metric $\Phi_{m'}$ greater than the accumulated metric Φ_m of the correct path. Hence, the pairwise error probability is given by

$$Pr[y_m \rightarrow y_{m'}] = Pr[\Phi_{m'} > \Phi_m / \underline{x}_m]$$

From equation (7c), one recognizes that $\Phi_{m'}$, is a Gaussian random variable, and so is $\{\Phi_m - \Phi_{m'}\}$.

Let K be the mean of $\{\Phi_m - \Phi_{m'}\}$. Then

$$K = E\{\Phi_m - \Phi_{m'}\} = \sum_n \Big[2(u_{m'n} - u_{mn})\hat{y}_n - (u^2_{m'n} - u^2_{mn})h^{\tilde{p}}_o - \sum_{i=1}^{L-1} 2(u_{m'n}\, u_{m',n-i} - u_{mn}\, u_{m,n-i})h^{\tilde{p}}_i\Big] \tag{15}$$

with the average value $\hat{y}_n$ of y_n given by

$$\hat{y}_n = \sum_{i=-L+1}^{L-1} u_{m,n-i}\, h^{\tilde{p}}_i \tag{16}$$

Let W' be the variance of $\{\Phi_m - \Phi_{m'}\}$. Then, a normalized variance W is

$$W = (1/4\sigma^2)Var\{\Phi_m - \Phi_{m'}\} = (1/4\sigma^2)E\{\sum_n 2(u_{m'n} - u_{mn})n_n\}^2 \tag{17}$$

where σ^2 is the variance of the gaussian random variables. Clearly, the pairwise error probability is

$$Pr[\Phi_{m'} - \Phi_m > 0 / \underline{x}_m] = Q(K/2\sigma\sqrt{W})$$

Since we have matched filter, one can show after some algebraic manipulations that K = W. Thus,

$$\Pr[\phi_{m'} - \phi_m > 0 / \underline{x}_m] = Q(\sqrt{W}/2\sigma) \quad (18)$$

Assume that a transformation from binary source symbols $u_n \in \{0, 1\}$ into $\tilde{u}_n \in \{-1, 1\}$ is provided. Doing some manipulations on equation (17), one can easily show that

$$\sqrt{W}/2\sigma = \left[(2/N_o) \sum_n \sum_j e_n\, e_j\, h^{\bar{p}}_{n-j}\right]^{1/2} \quad (19)$$

where the signal error, e_n, is given by

$$e_n = (1/2)(u_{m'n} - u_{mn}) = \begin{cases} 0, & u_{m'n} = u_{mn} \\ \pm 1, & u_{m'n} \neq u_{mn} \end{cases} \quad (20)$$

Upper bounding (18) by

$$Q(x) \leq \exp(-x^2/2)$$

we finally have

$$\Pr[\phi_{m'} - \phi_m > 0 / \underline{x}_m] \leq \exp((-1/N_o)(h^{\bar{p}}_o\, e_n^2 + 2 \sum_{i=1}^{L-1} e_n\, e_{n-i}\, h^{\bar{p}}_i) \quad (21)$$

Since (21) characterizes the conditional signal error probability for a given sequence $\underline{e}$, the unconditional signal error probability is

$$\Pr[\text{signal error}] < \sum_{\underline{e}} \Pr[\text{signal error}/\underline{e}]\ \Pr[\underline{e}] \quad (22)$$

It can be shown that the unconditional signal error probability is upper bounded by

$$\Pr[\text{signal error}] \leq \sum_{\underline{e}} \{\prod_n (z/2)^{e_n^2} \exp[(-1/N_o)(h^{\bar{p}}_o\, e_n^2 + 2 \sum_{i=1}^{L-1} e_n\, e_{n-i}\, h^{\bar{p}}_i]\} \quad (23)$$

where z accounts for the case $u_{m'n} \neq u_{mn}$. The expression in branckets characterizes paths in an error state diagram with $(2W-L)^{L-1}$ states.

We now introduce the dynamic and output equations to evaluate the upper bound on the bit error probability, P_b, for the periodically time varying controlled ISI. Let Y_i, E_i, S_i, and A_i be the state, input, output and transition matrices of the i-th error state diagram, where the matrices elements are of the form $\tilde{s} \cdot D \cdot z^c$ with $\tilde{s}$ a constant, D a pairwise error probability, z a decoding error, and c_i a distortion function for each bit corresponding to the i-th channel impulse response for $0 \leq i \leq (M-1)$. The dynamic state equations and respectives transfer functions can be obtained once the periodic structure of the channel is known. Since we have assumed the periodic structure $A(0, 1, \ldots, M-1) = \{h^o(t), h^1(t), \ldots, h^{M-1}(t)\}$ where during $\bar{b}$ source symbols the channel impulse response is $h^{\bar{p}}(t)$, the dynamic state and output equations can be shown (see [12]) to be given by

$$\begin{array}{ll} Y_o = E_o + A_o \cdot Y_{M-1} & T_o = S_o \cdot Y_{M-1} \\ Y_1 = E_1 + A_1 \cdot Y_o & T_1 = S_1 \cdot Y_o \\ \cdot \quad \cdot \quad \cdot \quad \cdot & \cdot \quad \cdot \quad \cdot \\ Y_{M-1} = E_{M-1} + A_{M-1} \cdot Y_{M-2} & T_{M-1} = S_{M-1} \cdot Y_{M-2} \end{array} \quad (24)$$

and the total transfer function by

$$T(z) = (1/M) \sum_{i=0}^{M-1} T_i(z) \tag{25}$$

Since $T(z)$ is a short representation for $T(z, c_1, c_2, \ldots, c_M, D)$, taking derivative of (25) with respect to z, results in the bit error probability which we represent by

$$P_b(c_1, c_2, \ldots, c_M) \leq (1/2) \left[d/dz\right] T(z) \Big|_{z=1,\ (c_1, c_2, \ldots, c_M)} \tag{26}$$

Since we are assuming binary source symbols, the distortion function $c_i = 0$ if the transmitted bit is equal to the decoded bit, and $c_i = 1$ if they are different, for each channel impulse response. The (individual) bit error probability for the i-th channel impulse response, P_{bi}, is obtained by letting $c_i = 1$ and $c_k = 0$ for all $k \neq i$. Finally, the total bit error probability is the sum of all individual bit error probabilities.

To show the procedure so far, let us take an example where, without loss of generality, $\tilde{b} = 1$, $L = 2$, $M = 2$. This is a typical example of duobinary signals. The discrete equivalent model for $L = 2$ is shown in Fig. 2. From (14), we have that

$$Y_k = \sum_{i=-1}^{1} u_{k-i}\, h_i^{\tilde{p}} + n_k \tag{27}$$

with

$$\tilde{p}(\tilde{q}) = \begin{cases} 0, \text{ for } jN \leq \tilde{p} \leq (j+1)N - 1 \text{ , } j \text{ even} \\ 1, \text{ for } jN \leq \tilde{p} \leq (j+1)N - 1 \text{ , } j \text{ odd} \end{cases} \tag{28}$$

The error state diagrams having $(2W-1)^{L-1} = 3$ states $\{-1, 0, +1\}$ with 1, and -1 equally likely are shown in Fig. 3 with the corresponding

branch values given by

$$a_1 = \exp\{-h_o^o/N_o\} \qquad a_2 = \exp\{-(h_o^o + 2h_1^o)/N_o\} \tag{29a}$$

$$a_3 = \exp\{-(h_o^o - 2h_1^o)/N_o\} \qquad a_4 = 1 \tag{29b}$$

$$b_1 = \exp\{-h_o^1/N_o\} \qquad b_2 = \exp\{-(h_o^1 + 2h_1^1)/N_o\} \tag{29c}$$

$$b_3 = \exp\{-(h_o^1 - 2h_1^1)/N_o\} \qquad b_4 = 1 \tag{29d}$$

The state and output equations for the optimum receiver are shown in (35).

CASE II – SUBOPTIMUM RECEIVER

Instead of a matched filter, let us assume that the "integrate and dump" is being used. We have that the observables are

$$Y_k = \int_{-\infty}^{\infty} y(t)\,p(t - kT)\,dt \tag{30}$$

where

$$p(t) = \begin{cases} (1/T)^{1/2} \ , & 0 \leq t \leq T \\ 0 \ , & \text{otherwise} \end{cases}$$

Let $h^{\tilde{p}}(t) = 0$ for $t \leq 0$ and $t \geq LT$. Multiplying (2) by $p(t - kT)$ and integrating, we have

$$x_{mk} = \int_{-\infty}^{\infty} x_m(t)\,p(t - kT)\,dt = \sum_n u_{mn} \int_{-\infty}^{\infty} h^{\tilde{p}}(t - nT)\,p(t - kT)\,dt =$$

$$= \sum_n u_{mn}\, h_{n-k}^{\tilde{p}} = \sum_{i=0}^{L-1} u_{m,k-i}\, h_i^{\tilde{p}} \tag{31}$$

With the above assumptions the "integrate and dump" filter output is

$$Y_k = \sum_{i=0}^{L-1} u_{m,k-i}\, h_i^{\bar{p}} + n_k \qquad (32)$$

Equation (32) is the discrete equivalent representation of the continuous case, which is shown in Fig. 6.

Without loss of generality, assume $\bar{b}=1$, $L=2$, $M=2$. Following the same procedure as in CASE I, we will end up with the signal error probability given by

$$\Pr[\text{signal error}] \leq \sum_{\underline{e}} \{\prod_n (z/2)^{e_n^2} \exp[(-1/N_o)\,(\sum_{i=0}^{L-1} e_{n-i}\, h_i^{\bar{p}})^2]\} \qquad (33)$$

Note that (33) describes the path's evolution in the error state diagram for each $\bar{p}$, which in this case are shown in Figure 5 with the corresponding branch values given by

$$a_1 = \exp\{-(h_o^o)^2/N_o\} \qquad a_2 = \exp\{-(h_o^o + h_1^o)^2/N_o\} \qquad (34a)$$

$$a_3 = \exp\{-(h_o^o - h_1^o)^2/N_o\} \qquad a_4 = \exp\{-(h_o^o)^2/N_o\} \qquad (34b)$$

$$b_1 = \exp\{-(h_o^1)^2/N_o\} \qquad b_2 = \exp\{-(h_o^1 + h_1^1)^2/N_o\} \qquad (34c)$$

$$b_3 = \exp\{-(h_o^1 - h_1^1)^2/N_o\} \qquad b_4 = \exp\{-(h_1^1)^2/N_o\} \qquad (34d)$$

From (24), the dynamic state equations and transfer functions of the reduced error state diagrams are given by

$$Y_1 = a_1\, z^{c_1}/2 + Y_2\, z^{c_1}(a_2 + a_3)/2 \qquad T_1 = 2a_4 \cdot Y_2 \qquad (35a)$$

$$Y_2 = b_1\, z^{c_2}/2 + Y_1\, z^{c_2}(b_2 + b_3)/2 \qquad T_2 = 2b_4 \cdot Y_1 \qquad (35b)$$

and the total transfer function by

$$T(z) = (1/2)\,[T_1(z) + T_2(z)] \qquad (35c)$$

Similarly to (26), the individual bit error probabilities are given by

$$P_{b1} = P_b(c_1=1,\ c_2=0) \leq (1/2)\,[d/dz]T(z) \qquad z=1,\ c_1=1,\ c_2=0 \qquad (35d)$$

$$P_{b2} = P_b(c_1=0,\ c_2=1) \leq (1/2)\,[d/dz]T(z) \qquad z=1,\ c_1=0,\ c_2=1 \qquad (35e)$$

and the total bit error probability by

$$P_b = P_{b1} + P_{b2} \qquad (35f)$$

From equations (35a) and (35b), we have that Y_1 and Y_2 are respectively given by:

$$Y_1 = \{a_1\, z^{c_1} + (b_1/2)\cdot(a_2 + a_3)z^{c_1+c_2}\}/\{1 - [(a_2 + a_3)(b_2 + b_3)/4]z^{c_1+c_2}\}$$

$$Y_2 = \{b_1\, z^{c_2} + (a_1/2)\cdot(b_2 + b_3)z^{c_1+c_2}\}/\{1 - [(a_2 + a_3)(b_2 + b_3)/4]z^{c_1+c_2}\}$$

Substituting Y_1 and Y_2 above in (35d) and (35e), the total bit error probability, (35f), can be shown after some algebraic manipulations to be given by

$$P_b = (1/4)\cdot\{(a_1 \cdot b_4 + a_4 \cdot b_1) + (1/4)\cdot(a_4 \cdot b_1 + a_1 \cdot b_4)\cdot(a_2+a_3)\cdot(b_2+b_3) + a_1 \cdot a_4 \cdot (b_2+b_3) + b_1 \cdot b_4 \cdot (a_2+a_3)\} / \{1 - [(a_2+a_3)\cdot(b_2+b_3)\cdot(1/4)]\}^2 \qquad (36)$$

Evaluation of the bit error probabilities for CASE I and CASE II is only a matter of substituting the corresponding values of a_i's and b_i's given by equations (29) and (34) respectively. We consider channel impulse responses given by

$$h_1(t) = \begin{cases} [(a/G) \cdot (E/T)]^{1/2} & \text{for } 0 \leq t \leq T \\ [(b/G) \cdot (E/T)]^{1/2} & \text{for } T \leq t \leq 2T \\ 0 & \text{otherwise} \end{cases} \quad (37)$$

$$h_2(t) = \begin{cases} [(c/G) \cdot (E/T)]^{1/2} & \text{for } 0 \leq t \leq T \\ [(d/G) \cdot (E/T)]^{1/2} & \text{for } T \leq t \leq 2T \\ 0 & \text{otherwise} \end{cases} \quad (38)$$

where a, b, c, d are integers such that $a+b = c+d = G$.

4. Results

In the analysis that follows, we have assumed the "channel impulse responses" are given by equations (37) and (38) with (a, b, c, d) being any integer satisfying $a+b = c+d = 18$.

We have evaluated the performance of PAM communication system with time invariant channel impulse response by using the upper bound on P_b as given by equation (36) under: 1) no ISI with parameters (a, b, c, d) = (18, 0, 18, 0); 2) controlled ISI with optimum receiver and parameters (a, b, c, d) = (9, 9, 9, 9); 3) controlled ISI with suboptimum receiver and parameters (a, b, c, d) = (9, 9, 9, 9). This is shown in Fig. 7. Note that the energy content of the channel impulse responses are equal to E.

By concentrating most of the total energy of $h_1(t)$ and/or $h_2(t)$ to

any one of the equally spaced time intervals, that is, by increasing the values of the parameters a and c or b and d, we have that less signal-to-noise ratio is required to achieve a prescribed bit error rate as can be seen in Fig. 8 for the cases (15, 3, 15, 3), (12, 6, 12, 6) when compared with (9, 9, 9, 9) which is the duobinary signal case. Note that (18, 0, 18, 0) is the case with no ISI.

In Table 1, we compare periodically time varying controlled ISI with period two with time invariant controlled ISI as well as with the case with no ISI. It is seen that periodically time varying attains better performance than the time invariant case with coding gains up to 1.27 dB, depending on the chosen periodic combination. It is also seen that a small degradation up to .44 dB, depending upon the periodic combination being selected, when comparing no ISI with periodically time varying. On the other hand, degradation up to 1.28 dB is seen when we compare no ISI with time invariant controlled ISI. Therefore, a savings of .83 dB is achieved by using periodically time varying instead of time invariant controlled ISI for the worst case.

It is also noticed that most of the gain was already achieved by using periodic combination with period two. Hence, only a small fraction is gained by increasing the periodicity in the combination of time invariant channel impulse responses with controlled ISI.

In Table 2, it is shown the degradation of an uncoded PAM communication system when there is a loss of synchronization. It is clear that the best periodic combination with perfect synchronization becomes the worstest when the synchronization is lost. This is due to the fact that the ISI coefficients assume the lowest possible values, therefore, increasing the total bit error probability. Consequently, in order to have the lowest upper bound on P_b as given by (36) and to have the lowest possible degradation when the receiver is out of synchronization, the energy content of the channel impulse responses in each equally spaced time interval should be approximately equal. This fact is easily shown when we compare the periodically time varying controlled ISI with parameters (9, 9, 10, 8) with and without synchronization from Table 3, with the time invariant controlled ISI with parameters (9, 9, 9, 9) from Tables 1 and 2. As can be seen, we have only a gain of roughly .03 dB when in synchronism whereas in out of synch a degradation of roughly .02 dB.

5. Conclusions

We have presented an analysis of an uncoded PAM communication system with periodically time varying controlled intersymbol interference by use of the DTFT technique. This analysis was based upon the assumption of an optimum and sub optimum receiver. It was shown by examples that periodically time varying controlled ISI with period two requires less signal-to- noise ratio to achieve the same performance as that of the time invariant with ISI when most of the energy content of the channel impulse response is confined to any one of the equally spaced time interval. We have also shown that time invariant ISI is a particular case of the periodically time varying ISI by simply taking $a_i = b_i$ for $1 \leq i \leq 4$, either for the optimum or suboptimum receiver. Therefore, the DTFT technique is a natural extension of the Viterbi's TFT technique used in the analysis of linear time invariant ISI. Generalization for any $\bar{b}$, any number of finite state structure, memory length $L-1$, and alphabet size W is straightforward. However, the computational aspect regarding this generalization becomes enormous for large values of L and W due to the exponential growth of the parameters involved.

Table 1

(Optimum Receiver)

	P_b at	10^{-2}	10^{-3}	10^{-4}		10^{-2}	10^{-3}	10^{-4}
	(a,b,c,d)		SNR(dB)		(a,b,c,d)		SNR(dB)	
	9,9,9,9	7.24	8.8	9.96	9,9,17,1	6.4	8.18	9.51
	10,8,10,8	7.19	8.75	9.91	10,8,17,1	6.4	8.17	9.5
	11,7,11,7	7.0	8.6	9.79	11,7,17,1	6.35	8.16	9.48
	12,6,12,6	6.8	8.45	9.65	12,6,17,1	6.3	8.15	9.44
	13,5,13,5	6.6	8.3	9.52	13,5,17,1	6.25	8.1	9.4
	14,4,14,4	6.4	8.15	9.43	14,4,17,1	6.2	8.05	9.36
	15,3,15,3	6.25	8.05	9.36	15,3,17,1	6.1	8.0	9.33
	16,2,16,2	6.1	8.0	9.32	16,2,17,1	6.05	7.95	9.32
	17,1,17,1	6.0	7.95	9.31	17,1,17,1	6.0	7.95	9.31
No ISI	18,0,18,0	5.96	7.94	9.3	18,0,17,1	5.97	7.95	9.305

Time Invariant with Controlled ISI — Periodically Time Varying with Controlled ISI

Table 2

(Integrated and Dump Receiver)

P_b at		10^{-2}	10^{-3}	10^{-4}		10^{-2}	10^{-3}	10^{-4}
	(a,b,c,d)	SNR(dB)			(a,b,c,d)	SNR(dB)		
	9,9,9,9	8.95	10.95	12.31	9,9,17,1	11.5	13.82	15.33
	10,8,10,8	8.98	10.97	12.34	10,8,17,1	11.51	13.82	15.33
	11,7,11,7	9.06	11.05	12.42	11,7,17,1	11.51	13.82	15.33
	12,6,12,6	9.21	11.2	12.57	12,6,17,1	11.52	13.82	15.33
	13,5,13,5	9.43	11.42	12.79	13,5,17,1	11.54	13.82	15.33
	14,4,14,4	9.75	11.74	13.11	14,4,17,1	11.57	13.83	15.33
	15,3,15,3	10.23	12.22	13.59	15,3,17,1	11.64	13.84	15.34
	16,2,16,2	10.97	12.96	14.33	16,2,17,1	11.81	13.91	15.36
	17,1,17,1	12.34	14.33	15.7	17,1,17,1	12.34	14.33	15.7
No ISI	18,0,18,0	5.96	7.93	9.3	18,0,17,1	5.97	7.95	9.305

Time Invariant with Controlled ISI — Periodically Time Varying with Controlled ISI

Table 3

(Optimum Receiver)

P_b	10^{-2}	10^{-3}	10^{-4}		10^{-2}	10^{-3}	10^{-4}
(a,b,c,d)	SNR(dB)			(a,b,c,d)	SNR(dB)		
9,9,10,8	7.21	8.78	9.93	9,9,10,8	8.97	10.96	12.32

Perfect Synch. — Out of Synch.

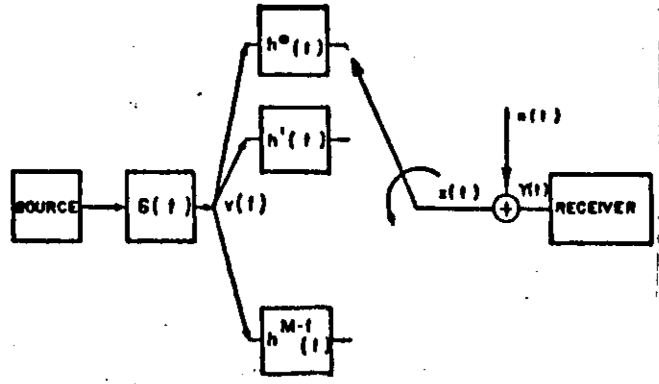

Figure 1 – Communication system.

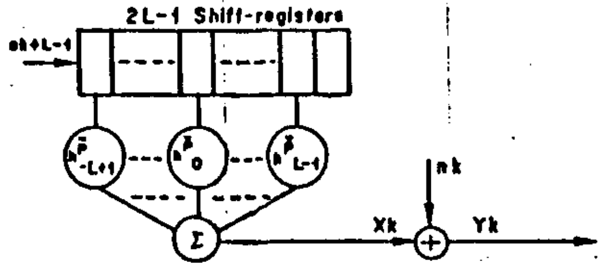

Figure 2 – Discrete time equivalence.

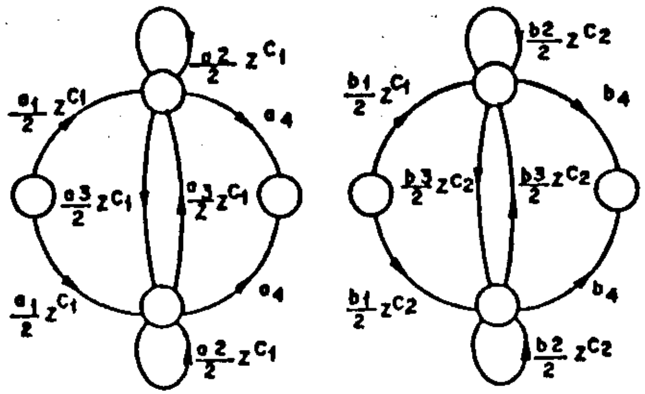

Figure 3 – Error state diagram for L=2 and M=2

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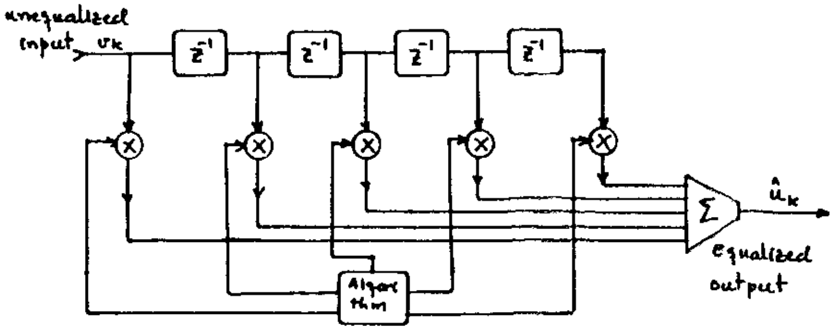
So far, we have considered the case where the channel was the ideal one with and without controlled intersymbol interference. The resulting pulse shapes characterized the modulator-demodulator having time invariant and time varying impulse responses.

Next, we are going to coment on the case where an equalizer will be used to compensate for the nonideal frequency response of the channel.

The most often used linear filter for equalization is the transversal filter shown below



where

$$v(k) = \sum_{n=0}^{L} f(n).u(k-n) + n(k)$$

and

$$Q(k) = \sum_{j=-K}^{K} c(j).v(k-j)$$

The peak distortion criterion and the zero-forcing equalizer can be described as follows: let { f(n) } and { c(n) } be the impulse responses of a discrete time linear filter model and of an equalizer, respectively. The cascade of them can be represented by an equivalent filter with impulse response given by

$$q(n) = \sum_{j=-\infty}^{\infty} c(j).f(n-j)$$

Assume that the equalizer has infinite taps, then the equalizer output is given by

$$Q(k) = q(o).u(k) + \sum_{n}{}' u(n).q(k-n) + \sum_{j=-\infty}^{\infty} c(j).n(k-j)$$

Let q(o) = 1, and the second term in the right hand side be denoted by D, it is possible to select tap weights q(n) such that D = 0 with q(n) = 1 for n = 0 and q(n) = 0 for n ≠ 0. Therefore, intersymbol interference is eliminated. But

$$q(n) = \sum_{j=-\infty}^{\infty} c(j).f(n-j) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases} \qquad (3)$$

Taking the Z transform in (3), we have Q(z) = C(z). F(z) = 1 which implies that C(z) = 1/F(z).

Therefore, to eliminate intersymbol interference the equalizer has to be the inverse filter to F(z). Such a filter is called zero-forcing filter. By cascading a noise whitening filter with transfer function $1/F^*(z)$ and the zero-forcing equalizer results in an equivalent zero-forcing equalizer given by

$$C'(z) = 1/F(z).F^*(z^{-1}) = 1/X(z)$$

The mean square-error (MSE) criterion aims at finding the tap weights { c(j) } to minimize the mean square value of the error

$$e(k) = u(k) - Q(k)$$

where u(k) is the symbol transmitted and Q(k) is the estimated value of u(k), that is,

$$J = E\{ |e(k)|^2 \} = E\{ |u(k) - \hat{u}(k)|^2 \}$$

where

$$\hat{u}(k) = \sum_{j=-\infty}^{\infty} c(j).v(k-j)$$

By invoking the Orthogonality Principle e(k) is orthogonal to $\{ v^*(k-j) \}$, we have that

$$\sum_{j=-\infty}^{\infty} c(j).E\{ v(k-j).v^*(k-i) \} = E\{ u(k).v^*(k-i) \}, \text{ for all } i$$

Since $v(k) = \sum_{n=0}^{L} f(n).u(k-n) + n(k)$, then

$$E\{ v(k-j).v^*(k-i) \} = \sum_{n=0}^{L} f^*(i).f(n+i-j) + No.d(i,j)$$

$$= \begin{cases} x(i-j) + No.d(i,j), & |i-j| \le L \\ 0, & \text{otherwise} \end{cases}$$

and

$$E\{ u(k).v^*(k-i) \} = \begin{cases} f^*(-i), & -L \le i \le 0 \\ 0, & \text{otherwise} \end{cases}$$

Taking the Z transform in the above equation, we have

$$C(z).[F(z).F^*(z^{-1}) + No] = F^*(z^{-1})$$

or

$$C(z) = F^*(z^{-1})/[F(z).F^*(z^{-1}) + No]$$

When the noise whitening filter is included, then

$$C'(z) = 1/[F(z).F^*(z^{-1})) + No] = 1/(X(z) + No)$$

Therefore, when No tends to zero, the MSE and the Peak distortion criteria give the same solution.