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COURSE ON BASIC TELECOMMUNICATIONS SCIENCE

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Noise

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N O I S E *

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SUMMARY

The physical sources of noise are discussed, and illustrated with an example of noise calculation in a transistor amplifier. The concepts of noise figure and noise temperature are introduced and the effect of cascading is analysed.

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1. Introduction

Noise is an inevitable phenomena in all areas of human activity, particularly in communication. Any corruption to the desired signal is termed noise. Noise in a communication system may arise inside the system, or externally. Examples of the latter are atmospheric and man made noise, while internal noise arises due to random motion of charge carriers within the devices used in the system. In this short discussion, we shall be concerned with the analysis of such internal noise sources.

2. Thermal Noise

At any temperature above absolute zero, the charge carriers in a device move randomly and give rise to thermal noise. Obviously, noise being a random phenomenon, it does not make sense to talk about its variation with time. Instead we characterize noise by some average values. One of them is the mean square value. Nyquist's theorem for thermal noise states that the mean square thermal voltage across a resistor of R ohms at $T^{o}K$ in the frequency band B Hz is given by

$$v_{rms}^2 = 4kT\ B \quad (1)$$

where k=Boltzmann constant = 1.38×10^{-23} J/K.

A noisy resistor can therefore be represented by either of the equivalent circuits shown in Fig.1, where the second circuit is obtained by taking the Norton equivalent of the first one.

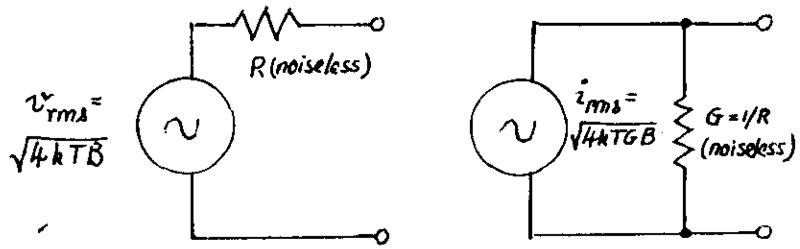


Fig.1 - Equivalent circuits of a noisy resistor

What about the mean square noise voltage across a one port resistive network containing many nontrivially interconnected resistors? Nyquist's formula is useful in this context and applies to any RLC one-port. It states that the mean square noise voltage across such a one-port is given by

$$v_{rms}^2 = 2kT \int_{-\infty}^{\infty} R(f)\, df \tag{2}$$

where R(f) = real part of the driving point impedance at frequency f. For a purely resistive network, (2) gives

$$v_{rms}^2 = 4kT\, R_{eq}\, B \tag{3}$$

where R_{eq} is the equivalent resistance of the network.

3. Shot Noise

Shot noise arises due to discrete nature of current flow in a device. Consider a saturated thermionic diode; the current in this is due to electrons emitted from the cathode, which arrive randomly at the anode. The total current is then an average value I_o plus a randomly fluctuating component, whose mean square value is given by Schottky's theorem as

$$i_{rms}^2 = 2\, e\, I_o\, B \tag{4}$$

where e is the electronic charge.

4. An example

At this point, it is instructive to calculate the noise in a common emitter transistor amplifier, whose circuit is shown in Fig.2(a). Assuming the effects of the base biasing resistors R_1 and R_2 to be negligible, and C_1, C_2 and C_3 to act as short circuits at the frequency of opration, the equivalent circuit becomes that shown in Fig. 2(b), where the effects of C_π, C_μ and r_μ have been neglected. We wish to determine the mean square noise voltage at the output; to this end, the equivalent noise circuit is drawn in Fig. 2(c), where the following transistor noise sources may be identified: (i) v_x^2 due to thermal noise in the base spreading resistance r_x, i.e. $v_x^2 = 4kTr_xB$. (ii) i_b^2 due to shot noise in the base; and (iii) i_c^2 due to shot noise in the collector. The other noise sources are due to thermal noise in R_s and R_L, i e.

$$v_s^2 = 4kTR_s \, B \text{ and } v_L^2 = 4kT\, R_L \, B \tag{5}$$

Now,

$$i_b^2 = 2e\, I_b\, B = 2kT \, \frac{I_b}{kT/e} \, B = \frac{2kTB}{r_\pi} \tag{6}$$

where I_b is the quiescent base current. Also

$$i_c^2 = 2e\, I_c \quad B \tag{7}$$

where I_c is the quiescent collector current. Assuming noise

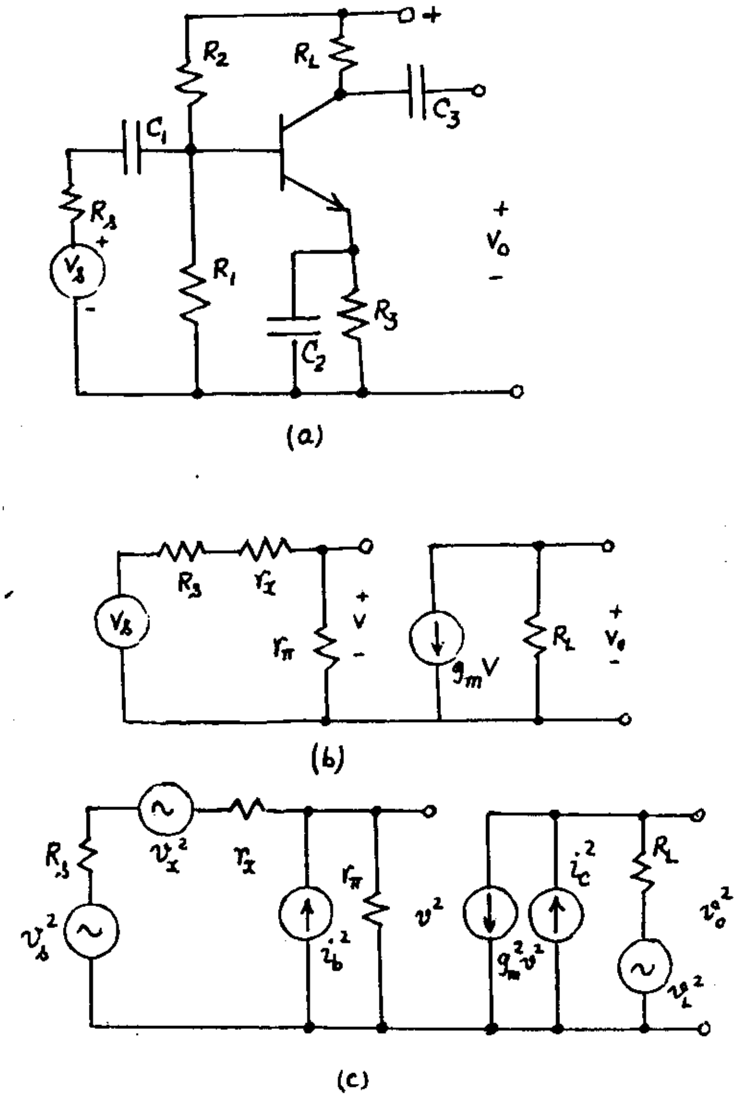


Fig. 2 – (a) CE transistor amplifier
(b) Equivalent circuit of (a)
(c) Equivalent noise model of (a)

sources to be uncorrelated, and using superposition, we obtain

$$v^2 = \left(\frac{r_\pi}{r_\pi + r_x + R_s}\right)^2 (v_s^2 + v_x^2) + \left(\frac{r_x + R_s}{r_x + R_s + r_\pi} i_b r_\pi\right)^2 \tag{8}$$

Similarly,

$$v_0^2 = g_m^2 \, v^2 \, R_L^2 + i_c^2 \, R_L^2 + v_L^2 \tag{9}$$

$$= g_m^2 R_L^2 \left(\frac{r_\pi}{r_\pi + r_x + R_s}\right)^2 \left[4kT \;\; R_s \;\; B + 4kT \;\; r_\pi \;\; B + \frac{(r_x + R_s)^2 \;\; 2kTB}{r_\pi}\right] + 2e \; I_c \; B \; R_L^2 + 4kTR_L B \tag{10}$$

To get an idea of the magnitudes, assume R_s=1K, r_x=100 Ω, R_L=10K, I_c=1mA, β=100 Then, at room temperature,

$$g_m = I_c/(kT/e) = 0.04 \text{ mho}$$

$$r_\pi = \frac{kT/e}{I_b} = \frac{\beta}{g_m} = 2500\,\Omega \tag{11}$$

Substituting these values in (10) gives

$$\frac{v_o^2}{B} = 1.69\times10^{-12} \quad v^2/\text{Hz} \tag{12}$$

so that for a bandwidth of B=10 KHz, the rms noise voltage is

$$v_o = 0.13 \text{ mV} \tag{13}$$

5. Available Power

The available power from a source is the power it can deliver to a matched load. Hence the available power from a noisy resistance R is (see Fig.3)

$$P_a = \frac{(v_{rms}/2)^2}{2} = kTB \tag{14}$$

Note that this result is independent of the value of the resistor. The unit of P_a/B is watts/Hz. The quantity $10\log_{10}$ (P_a/B) is expressed in dBW while $10\log_{10}[(P_a/B)/10^{-3}]$ is expressed in dBm. Notice that P_a/B is precisely the power spectral density.

6. Noise figure of a System

The noise figure F of a system is defined as

$$F = \frac{\text{Signal to noise ratio at input}}{\text{Signal to noise ratio at output}} \tag{15}$$

For a noiseless system F=1; for actual phsyical systems, F>1. the noise figure is usually expressed in dB, where $F_{dB}=10 \log_{10}F$.

The signal to noise ratio at any point of a system is independent of the load because both signal power and noise power appear across the same load. Hence one can work in terms

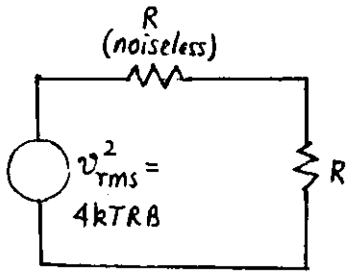


Fig.3 - Calculation of available power from a noisy R

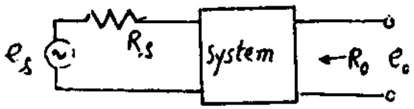
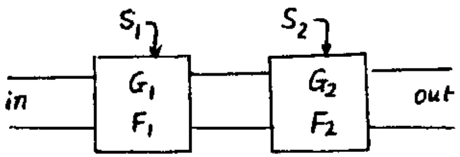


Fig.4 - Noise figure calculation



G = available power gain
F = noise figure

Fig.5 - Cascade of two systems

of any convenient load; matched load is, of course, the appropriate choice, because then one can work in terms of the available signal and noise powers. Consider the system of Fig.4; in this, the available signal power at input is

$$P_{ai} = e_s^2/(4R_s) \tag{16}$$

Also assume that only thermal noise is present at input, so that the available noise power for a source temperature of T_s is

$$P_{nai} = kT_s \; B \tag{17}$$

Thus input signal to noise ratio is;

$$(S/N)_i = \frac{e_s^2}{4k\; T_s\; R_s \; B} \tag{18}$$

The available signal power at the output is

$$P_{ao} = e_o^2/(4\; R_o) = G \; P_{ai} \tag{19}$$

where G is the available power gain. Thus assuming P_{nao} to be the available noise power at the output, the noise figure of the system is

$$F = \frac{P_{ai}/P_{nai}}{P_{ao}/P_{nao}} = \frac{1}{G} \quad \frac{P_{nao}}{P_{nai}} \qquad (10)$$

Equation (20) shows that the noise figure is the ratio of the actual output noise power to the noise power which would have appeared at the output had the system been noiseless Also, if P_{int} be the available noise power at output due to internal noise sources of the system, then

$$P_{nao} = G\ P_{nai} + P_{int} \qquad (21)$$

Thus, from (20),

$$F = 1 + \frac{P_{int}}{G\ kT_s\ B} \qquad (22)$$

It is usual practice to use $T_s = T_o = 290^\circ K$ so as to standardize the noise figure. Thus

$$F = 1 + \frac{P_{int}}{G\ k\ T_o\ B} \qquad (23)$$

Note that if $G >> 1$, then $F \approx 1$ i.e. the effect of internally generated noise becomes negligible.

7. Noise Temperature

We have seen that the available noise power of a resistor R is kTB watts. This fact is used to define the equivalent noise temperature of a noise source as

$$T_n = P_{na}/(kB) \qquad (24)$$

where P_{na} is the available noise power of the source in a bandwidth of B Hz.

For example, if two resistors R_1 and R_2 at temperature T_1 and T_2 are connected in series, then the mean square voltage generated by the combination is

$$v^2_{rms} = 4\ kT_1\ BR_1 + 4\ kT_2\ B\ R_2 \qquad (25)$$

The equivalent resistance is R_1+R_2; thus

$$P_{na} = v^2_{rms}/\left[4(R_1+R_2)\right] \qquad (26)$$

The equivalent noise temperature is, therefore,

$$T_n = \frac{P_{na}}{k\ B} = \frac{R_1T_1 + R_2\ T_2}{R_1+R_2} \qquad (27)$$

8. Effective Noise Temperature

Equation (23) can be written as

$$F = 1 + \frac{T_e}{T_o} \qquad (28)$$

where $T_e = P_{int}/(GkB)$ is called the effective noise temperature of the system. Recall that $P_{nao} = G\ P_{nai} + P_{int}$ and that $P_{nai} = kT_s\ B$. Thus

$$P_{nao} = G\,k\,T_s\,B + G_a\,k\,T_e\,B$$

$$= Gk(T_s + T_e)B \quad (29)$$

9. Cascaded Systems

Consider a cascade connection of two systems S_1 and S_2 (Fig.5) having available power gains G_1 and G_2. Then noise at the output consists of the following components: (i) Amplified source noise: $G_1G_2\,k\,T_s\,B$, (ii) Noise generated in S_1 and amplified by S_2 : $G_2\,P_{int_1} = G_2(G_1\,k\,T_{e1}B)$, and (iii) noise generated in S_2: $P_{int_2} = G_2\,k\,T_{e2}\,B$.

Thus the total available noise power at the output is

$$P_{na2} = G_1\,G_2\,k\left[T_s + T_{e1} + (T_{e2}/G_1)\right]B \quad (30)$$

Since the available power gain for the cascade is $G_1\,G_2$, a comparison of (29) and (30) shows that the effective noise temperature and noise figure of the cascade are:

$$T_e = T_{e1} + (T_{e2}/G_1) \quad (31)$$

$$F = 1+(T_e/T_o)=1+(T_{e1}/T_o)+T_{e2}/(G_1T_o)$$

$$= F_1 + \frac{F_2-1}{G_1} \quad (32)$$

These results can be generalized to a cascade of any number of stages, as given below:

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1\,G_2} + \ldots\ldots \quad (33)$$

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1\,G_2} + \ldots\ldots \quad (34)$$

Clearly, the succeeding stages in the cascade have decreasing effects on the overall noise performance

Acknowledgements

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