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COURSE ON BASIC TELECOMMUNICATIONS SCIENCE

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# "Electromagnetism : A refresher course" Part III

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**ELECTROMAGNETISM : A REFRESHER COURSE**

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**ANTENNAS**

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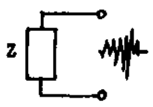
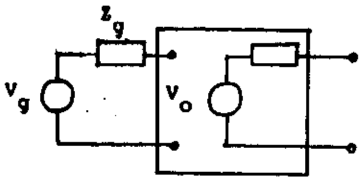
## 9. The radar equation

### Noise output




(a)




(b)

Fig. 9.1.

The thermal noise at the terminals of an impedance Z is given by (Fig. 9.1a)

$$\overline{dv^2} = 4kTR(\omega)df \qquad (9.1)$$

The quadratic values add up. Let Q be the quadratic gain of the system (Fig. 9.1b)

$$Q(\omega) = \frac{|V_o|^2}{|V_g|^2} \qquad (9.2)$$

If no additional noise were created the quadratic fluctuation would be

$$\overline{dv_o^2} = 4kTR_g(\omega)Q(\omega)df \qquad (9.3)$$

Because of additional noise sources, a noise factor $F > 1$ is introduced, hence

$$\overline{dv_o^2} = 4\;k\;T\;R_g(\omega)Q(\omega)F(\omega)df \qquad (9.4)$$

If the input system has $n_g$ times as much noise as $R_g$ (where $n_g$, the noise source factor, is > 1) :

$$\overline{dv_o^2} = 4\;k\;T\;R_g(\omega)Q(\omega)\underbrace{\left[F(\omega)+n_g(\omega)-1\right]}_{F_{eff}}df \qquad (9.5)$$

This gives (Fig. 9.2)

$$(\overline{v_o^2})_{noise} = 4\;k\;T\int_{\omega_1}^{\omega_2} R_g\;Q\;F_{eff}\;df$$

$$= 4\;k\;T\;R_g\;Q_{ref}\;(F_{eff})_{av}\;B \qquad (9.6)$$



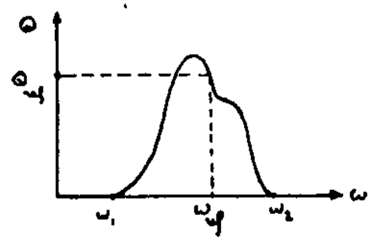



Fig. 9.2.

where

$$B = \int_{f_1}^{f_2} \frac{Q}{Q_{ref}}\,df$$

$$F_{av} = \frac{1}{B}\int_{f_1}^{f_2} F\,\frac{Q}{Q_{ref}}\,df \qquad (9.7)$$

### Minimum detectable signal

$$\frac{(\overline{v_o^2})_{signal}}{(\overline{v_o^2})_{noise}} = \frac{Q(\overline{v_g^2})}{(\overline{v_o^2})_{noise}} \geqslant \min.\left(\frac{S}{N}\right)_{output} = N \qquad (9.8)$$

The minimum signal to noise ratio at the output depends on the detection method, the equipment etc ...

### Radar signal

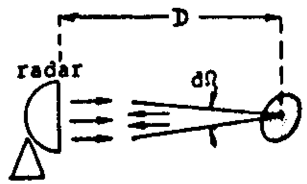



Fig. 9.3

From (2.12) the power density incident on the target is

$$W_i = \frac{|E_i|^2}{2R_{co}} = \frac{P_{tr}}{4\pi D^2}G \qquad (9.9)$$

where G is the gain (DΩ) of the antenna. The power scattered in a solid angle dΩ is (Fig. 9.3)

$$P = W_i\;\sigma_{rad}\;\frac{d\Omega}{4\pi} \qquad (9.10)$$

The solid angle of concern is, from (7.8),

$$d\Omega = \frac{S_{eff}}{D^2} = \frac{1}{4\pi D^2}G\;\lambda^2\;M\;P \qquad (9.11)$$

Therefore

$$\mathcal{P}_{rec} = \frac{G^2\lambda^2}{64\pi^3 D^4}\,\sigma_{rad}\,M\,P\,\mathcal{P}_{tr} \quad , \tag{9.12}$$

$$= \frac{\left|\overline{v_g^2}\right|}{4R_g}$$

Radar equation

Averaged over all frequencies :

$$\boxed{\mathcal{P}_{rec} = \frac{G^2\lambda^2}{64\pi^3 D^4}\,\sigma_{rad}\,M\,P\,\mathcal{P}_{tr} \geq k\,T\,B\,F_{eff}\,N} \tag{9.13}$$



Bibliography

The literature on antennas is extensive. On an elementary level :

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On a more professional level :

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## List of symbols

$\bar{a}$ = magnetic potential (T m)

$\bar{b}$ = magnetic induction (T)

c = $(\epsilon_o \mu_o)^{-0.5} = 3.10^8$ = velocity of light in vacuum (m $s^{-1}$)

$\bar{d}$ = electric induction (C $m^{-2}$)

$\bar{e}$ = electric field (V $m^{-1}$)

$\bar{e}_a$ = impressed electric field (V $m^{-1}$)

$\bar{e}_i, \bar{h}_i$ = incident fields

$\bar{h}$ = magnetic field (A $m^{-1}$)

$\bar{j}$ = volume current density (A $m^{-2}$)

$\bar{j}_a$ = applied volume current density (A $m^{-2}$)

$\bar{j}_s$ = surface current density (A $m^{-1}$)

$k_o = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ = wave number in vacuum ($m^{-1}$)

$\bar{u}_a$ = unit vector in direction $a$

D = directivity (dimensionless)

$\bar{F}$ = radiation vector (V)

G = gain of an antenna (dimensionless)

I = current (A)

M = mismatch factor (dimensionless)

P = polarization factor (dimensionless)

$\bar{P}_e$ = electric dipole moment (C m)

$\bar{P}_m$ = magnetic dipole moment (A $m^2$)

R = distance to the origin (m)

$R_{co} = (\mu_o/\epsilon_o)^{0.5} = 120\pi$ = characteristic resistance of vacuum ($\Omega$)

$S_{eff}$ = effective cross-section of an antenna ($m^2$)

W = electromagnetic energy density (J $m^{-3}$)

$Z_a = R_a + jX_a$ = antenna impedance ($\Omega$)

$Z_L$ = a load impedance ($\Omega$)

$\mathcal{E}$ = electromagnetic energy (J)

$\mathcal{P}$ = a power (W)

$\mathcal{R}$ = array factor (dimensionless)

$\epsilon_o = \frac{1}{36\pi} 10^{-9}$ F $m^{-1}$

$\lambda$ = wavelength in vacuum (m)

$\mu_o = 4\pi\ 10^{-7}$ H $m^{-1}$

$\sigma$ = conductivity (S $m^{-1}$)

$\rho$ = volume charge density (C $m^{-3}$)

$\phi$ = electric potential (V)

$\Omega$ = a solid angle (sr)