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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) · P.O. B. 506 · MIRAMARE · STRADA COSTIERA 11 · TELEPHONE: 2940-1
CABLE: CENTRATOM · TELEX 400892-1

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COURSE ON BASIC TELECOMMUNICATIONS SCIENCE
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"Electromagnetism : A refresher course" Part III

J. Van Bladel
Rijksuniversiteit Gent

These notes are intended for internal distribution only.

ELECTROMAGNETISM : A REFRESHER COURSE

by J. Van Bladel
Rijksuniversiteit Gent

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ANTENNAS

by J. Van Bladel
Rijksuniversiteit Gent

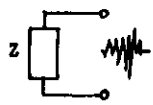
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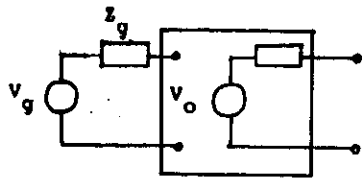
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9. The radar equation

Noise output



(a)



(b)

Fig. 9.1.

The thermal noise at the terminals of an impedance Z is given by (Fig. 9.1a)

$$\overline{dv^2} = 4kTR(\omega)d\omega \quad (9.1)$$

The quadratic values add up. Let Q be the quadratic gain of the system (Fig. 9.1b)

$$Q(\omega) = \frac{|v_o|^2}{|v_g|^2} \quad (9.2)$$

If no additional noise were created the quadratic fluctuation would be

$$\overline{dv_o^2} = 4kTR_g(\omega)Q(\omega)d\omega \quad (9.3)$$

Because of additional noise sources, a noise factor $F > 1$ is introduced, hence

$$\overline{dv_o^2} = 4kTR_g(\omega)Q(\omega)F(\omega)d\omega \quad (9.4)$$

If the input system has n_g times as much noise as S_g (where n_g , the noise source factor, is > 1):

$$\overline{dv_o^2} = 4kTR_g(\omega)Q(\omega) \underbrace{[F(\omega) + n_g(\omega) - 1]}_{F_{eff}} d\omega \quad (9.5)$$

This gives (Fig. 9.2)

$$\begin{aligned} (\overline{v_o^2})_{\text{noise}} &= 4kT \int_{\omega_1}^{\omega_2} R_g Q F_{eff} d\omega \\ &= 4kTR_g Q_{ref} (F_{eff})_{av} B \end{aligned} \quad (9.6)$$

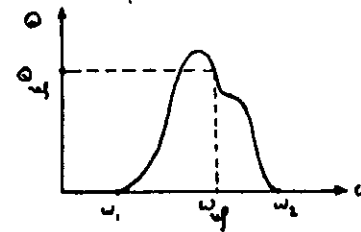


Fig. 9.2.

where

$$B = \int_{f_1}^{f_2} \frac{Q}{Q_{ref}} df \quad (9.7)$$

$$F_{av} = \frac{1}{B} \int_{f_1}^{f_2} F \frac{Q}{Q_{ref}} df$$

Minimum detectable signal

$$\frac{(\overline{v_o^2})_{\text{signal}}}{(\overline{v_o^2})_{\text{noise}}} = \frac{Q(\overline{v_g^2})}{(\overline{v_o^2})_{\text{noise}}} \gg \min. \left(\frac{S}{N} \right)_{\text{output}} = N \quad (9.8)$$

The minimum signal to noise ratio at the output depends on the detection method, the equipment etc ...

Radar signal

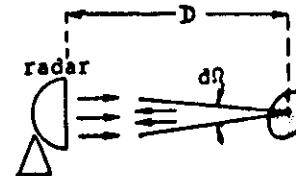


Fig. 9.3

From (2.12) the power density incident on the target is

$$W_1 = \frac{|E_1|^2}{2R_{co}} = \frac{P_{tr}}{4\pi D^2} G \quad (9.9)$$

where G is the gain ($D\Omega$) of the antenna. The power scattered in a solid angle $d\Omega$ is (Fig. 9.3)

$$P = W_1 \sigma_{rad} \frac{d\Omega}{4\pi} \quad (9.10)$$

The solid angle of concern is, from (7.8),

$$d\Omega = \frac{S_{eff}}{D^2} = \frac{1}{4\pi D^2} G \lambda^2 M P \quad (9.11)$$

Therefore

$$\begin{aligned} \rho_{\text{rec}} &= \frac{G^2 \lambda^2}{64 \pi^3 D^4} \sigma_{\text{rad}} M P \rho_{\text{tr}} \\ &= \frac{\left| \frac{v^2}{g} \right|}{4 R_g} \end{aligned} \quad (9.12)$$

Radar equation

Averaged over all frequencies :

$$\rho_{\text{rec}} = \frac{G^2 \lambda^2}{64 \pi^3 D^4} \sigma_{\text{rad}} M P \rho_{\text{tr}} \gg k T B F_{\text{eff}} N \quad (9.13)$$

Bibliography

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List of symbols

$\bar{a}$ = magnetic potential (T m)
 $\bar{b}$ = magnetic induction (T)
 $c = (c_0 \mu_0)^{-0.5} = 3 \cdot 10^8$ = velocity of light in vacuum ($m \cdot s^{-1}$)
 $\bar{d}$ = electric induction ($C \cdot m^{-2}$)
 $\bar{e}$ = electric field ($V \cdot m^{-1}$)
 $\bar{e}_a$ = impressed electric field ($V \cdot m^{-1}$)
 $\bar{e}_1, \bar{h}_1$ = incident fields
 $\bar{h}$ = magnetic field ($A \cdot m^{-1}$)
 $\bar{j}$ = volume current density ($A \cdot m^{-2}$)
 $\bar{j}_a$ = applied volume current density ($A \cdot m^{-2}$)
 $\bar{j}_s$ = surface current density ($A \cdot m^{-1}$)
 $k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ = wave number in vacuum (m^{-1})
 $\bar{u}_a$ = unit vector in direction $\underline{a}$

 D = directivity (dimensionless)
 $\bar{F}$ = radiation vector (V)
 G = gain of an antenna (dimensionless)
 I = current (A)
 M = mismatch factor (dimensionless)
 P = polarization factor (dimensionless)

 $\bar{P}_e$ = electric dipole moment (C m)
 $\bar{P}_m$ = magnetic dipole moment ($A \cdot m^2$)
 R = distance to the origin (m)
 $R_{co} = (\mu_0 / c_0)^{0.5} = 120\pi$ = characteristic resistance of vacuum (Ω)
 S_{eff} = effective cross-section of an antenna (m^2)
 W = electromagnetic energy density ($J \cdot m^{-3}$)
 $Z_a = R_a + jX_a$ = antenna impedance (Ω)
 Z_L = a load impedance (Ω)
 $\mathcal{E}$ = electromagnetic energy (J)

P = a power (W)
 $\mathcal{R}$ = array factor (dimensionless)
 $\epsilon_0 = \frac{1}{36\pi} 10^{-9} F \cdot m^{-1}$
 λ = wavelength in vacuum (m)
 $\mu_0 = 4\pi 10^{-7} H \cdot m^{-1}$
 σ = conductivity ($S \cdot m^{-1}$)
 ρ = volume charge density ($C \cdot m^{-3}$)
 ϕ = electric potential (V)
 Ω = a solid angle (sr)

