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SMR.379/6

COURSE ON BASIC TELECOMMUNICATIONS SCIENCE

9 January - 3 February 1989

Optimum Frequency Planning: a New Concept

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by
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SUMMARY

The problem of optimum frequency and power assignment in a transmitter network is formulated in this paper, and an appropriate solution method is outlined.

The paper describes an heuristic technique capable of solving a class of frequency planning problems involving: co-channel, adjacent-channel, spurious, and intermodulation interferences; fixed pre-existing frequency assignments and frequency-separation requirements; non-repetitive zone structures and frequency resource lists that contain gaps and vary from zone to zone; natural or political boundaries; specific terrain topography and irregular transmitter locations; signal and noise environment, transmitter powers, and directive antenna patterns that vary from transmitter to transmitter. The method is based on normalization, decomposition, and reduction techniques and on graph-colouring procedures. It assures that all relevant constraints and requirements are fulfilled and the total power radiated by all transmitters is minimum.

The paper is addressed to persons involved in preparation, revision, or approval of frequency plans: to spectrum planners, spectrum managers, frequency assigners, electromagnetic compatibility analysts and all those responsible for efficient radio spectrum utilization.

1. Introduction

General

As evidenced by the recent World and Regional Administrative Radio Conferences, there have been growing demands for spectrum sharing by many users (see, e.g. Kirby and Rutkowski [9]). Unfortunately, radio transmission systems may not employ the same area without interfering with one another. Thus a fundamental problem in sharing the spectrum is the elimination of harmful interferences that limit the effectiveness of radio services. Hence, use of the spectrum resource involves careful co-ordination of several factors by which one radio communication may be distinguished from another, and interference between them avoided. In addition to precluding interferences, the rational use of the spectrum should assure accommodation for as many users as possible with the limited frequency resource available. The most efficient way to achieve these goals consists in a preventive activity: that is, in a minimization of possible conflicts and spectrum wastage before their occurrence.

Both national and international wireless communications would dissolve into chaos without a system for allocating proper places in the radio spectrum among the claimant transmitters. As the number of transmitters in use grows, more requirements concerning electromagnetic compatibility must be given due consideration, and the problem becomes more and more complex. Since the earliest days of radio, this problem has been of great international interest. Initially, the rule "the higher power radiated the better" was the only guide in planning transmitter networks, and it was not so long ago that it became evident that this approach leads to pitfalls: to raising of interference background level; to power racing; to natural environment degradation; to capital investment losses.

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Relatively soon it became clear that the effective use of the radio spectrum depends on both the operational characteristics of systems and frequency planning techniques. This has been confirmed recently by the CCIR which endorses special studies on optimum network planning and frequency assignment techniques that could be recommended for use on world, regional, or national, scales (CCIR [1]).

A frequency plan is a function that assigns appropriate operational characteristics to each of the claimant transmitters. Generally, this may be one, a few, or all, of the factors by which radiocommunications can be distinguished from one another: operating frequency (frequency discrimination); power radiated (power level discrimination); antenna location, height, and radiation pattern (direction discrimination); polarization (polarization discrimination); time-structure of the signal (signal structure discrimination); operating time. Initially, only the frequency was assigned, and this explains the expression "frequency planning".

Although there are numerous publications on frequency planning (see e.g. Hale's bibliography [6] and special issues of the Institute of Electrical and Electronics Engineers (IEEE) and European Broadcasting Union (EBU) journals [5, 7, 8]) the general problems of optimum frequency planning with all the above-mentioned factors taken into account have not yet been solved, and are among the main topics of CCIR activity [2]. The reason is that mathematical models accounting for numerous factors and complex interactions, as well as a great amount of required input data, create severe problems in attempts to apply rigorous formal methods to real-life situations.

Paper organization

The chief aim of this paper is to introduce a new formulation of the problem of optimum frequency and power assignment in a transmitter network and to outline its possible solution.

The remaining part of the paper is organized as follows:

In section 2 we formulate the problem. We formulate it as an optimization issue. It is worth noting here that, generally, optimization is the determination of the value of design parameters subject to specific constraints that yield the best result. The following elements of any optimization problem are implicit:

- there are variables subject to control;
- the result to be optimized is described in mathematical terms of controllable parameters;
- a measure, or criterion, of system performance which should be optimized is selected;
- all requirements and assumptions, jointly named constraints, are stated explicitly.

We base our approach on mathematical models in which variables describing relevant components, states and performances of a system are described, and relationships between them represented through mathematical expressions. Many factors of a physical, technical, economic and even social and political nature, must be given due consideration when planning transmitter networks. There is no realistic way to include all of them into a mathematical model explicitly, and their effect is evaluated by handling them as constraints.

In section 3 we outline the proposed solution method, and main difficulties encountered. The usual question in any optimization problem is how much work or how much time is required to find the solution. In our approach we apply the decomposition principle. The original problem is decomposed into several subproblems of a lower dimensionality, and each of them is solved separately. Their order is determined so that results of preceding sub-problems are used in subsequent ones. Normalization, reduction, and graph-colouring procedure are the key elements of the solution method proposed.

Section 4 introduces important definitions of two graphs: the signal graph and the bond graph, on which our solution method is based. Two algorithms for consruction of these graphs are also offered here.

Sections 5 and 6 present two key subproblems concerning the transmitter ordering and frequency channel ordering. Appropriate algorithms are also proposed.

In section 7, guided by our efforts in sections 4, 5, and 6, we present an outline of the proposed solution method to the problem, along with a relevant algorithm. In this paper, a frequency plan is a function which assigns to each of the claimant transmitters a pair: an operating frequency (from a set of available frequencies, called the frequency catalogue), and an operating power (from a set of available powers, called the power catalogue). This contrasts with the traditional methods, which are still widely used, and which account for only one of them: frequency or power (see, e.g. Strużak [12]). There are three main reasons for proposing the new concept. Firstly, the frequency and power are both the crucial factors that control the interference and coverage zones in any transmitter network. Secondly, they are operational parameters that can be modified easily during the planning process. Finally, it is natural to expect that an optimum sought over the plane frequency-power should be closer to the "ideal" than an optimum sought with the single variable: power or frequency only.

In section 8 we conclude with a summary of our findings. Main concepts of this paper are developed from the ideas presented at the Seminar on optimization methods, Institute of Computer Science, Mathematics Department, University of Wrocław, and the lectures given by the author in January 1980, and partially published (Strużak [11, 12]).

2. Problem formulation

This paper deals with the following problem: given a collection of test points and a collection of radio transmitters to be assigned frequencies and powers, find an optimum assignment that satisfies specific constraints and that minimizes the total power radiated by all transmitters. The constraints are of three types:

1) catalogue constraints (powers and frequencies must be selected from the specified catalogues);

2) environmental constraints (EM compatibility with a specified environment signal must be assured at the test points);

3) quality constraints (reception quality must be guaranteed at the test points).

It is assumed that complete information is available.

It is assumed also that a specified test receiver with a test antenna is situated at each test point.

Locations, heights, directive radiation patterns, orientations, polarizations and other relevant parameters of both transmitters and test receivers are optional, however, their choice is beyond the scope of this paper. They should be established before the frequency and power assignment.

Any number of transmitters, test points, and constraints is acceptable; the total number of transmitters, test points, freɪencies, powers, environmental signals ɪd other constraints that determine the ·oblem's dimension, is, however, finite.

is assumed moreover that a list of en- ronment signals is attributed to each st point. The list contains data on sig- ıl power at the receiver input (or ɔwer-flux density, polarization and azi- uth of signal arrival).

:veral real-life situations can be reduced the form presented above. The test ɔints, for example, can represent the ıits of area. Then, with a large number ' them and their regular distribution, ɔntinuous coverage area problems can : approximated. The test points can so represent cities or other concentra- ɔns of people. In that case, population ɔverage problems are manageable. Sin- ılar transmitter test point locations can mulate point-to-point radiocommuni- ıtions. Very specific distributions may : required to fit the coverage area to na- ıral or political boundaries or to fill in ıps in the coverage by other networks.

Solution outline

·ometric interpretation

ɪgure 1 shows a general interpretation of ır problem. Three abstract spaces are stinguished: the "transmitter", the ɪignal" and the "receiver performance" ɪaces. Each of the transmitters consi- :red is represented by a point in the ransmitter" space, and each of the test ceiver input signals is represented by a ɔint in the "signal" space: the same is ue of the test "receiver performance" ɪace. Relations between these spaces are so shown: each point from the "trans- ıitter" space is mapped in the "signal" ɪace, and points from this space are ıapped in the "receiver performance" ɪace. These relations are dependent on ıany factors, some of them uncontroll- ɪle (noise, propagation effects). This in- oduces some uncertainty in the map- ıng process. Generally, the problem ɔnstraints cut out allowable regions in ıch of the spaces separately. The prob- m consists, firstly, of transforming the lowable regions from the "receiver per- ɪrmance" and "signal" spaces into the ransmitter" space and, secondly, of nding the optimum allocation of points ithin the resultant allowable region ere.

regularity

ue to irregular structures allowed, the oblem is insoluble by the classic lattice techniques (Arnaud in [8]). As is known, these techniques are based on the following assumptions:

a) all transmitters are identical; their powers and antenna heights are the same;

b) all antennae are omnidirectional;

c) terrain effects are ignored; the Earth is supposed to be flat and propagation is isotropic;

d) no natural or political boundaries; each transmitter is located at a node of a boundless, regular lattice (no other locations allowed);

e) all lattice nodes are occupied by transmitters (no free nodes or coverage gaps);

f) interference environment is the same for each transmitter;

g) one set of channels is regularly re-used throughout the whole boundless plane (other frequency allocations ignored).

These assumptions make it impossible to take into account optional locations and antenna patterns, non-continuous coverage area, effects of other radio services sharing the same area, etc.

Discontinuity

The discontinuous nature of the problem excludes differential calculus, Lagrangian multipliers and, generally, all continuous-type techniques, including linear programming. Integer and non-linear programming general techniques are not directly applicable either.

Problem dimension

One of the distinct characteristics of our problem is the large number of possible cases that must be taken into account. Even with a moderate number of frequencies, powers, transmitters and test points, the number of computations required for exhaustive inspection of all possible combinations is enormous. If the catalogues contain N_F frequencies and N_p powers are available, then there are $N_F \times N_p$ pairs of frequency power. With N_T claimant transmitters, this makes N different frequency plans possible

$$N = (N_F \times N_p)^{N_T} \qquad (1)$$

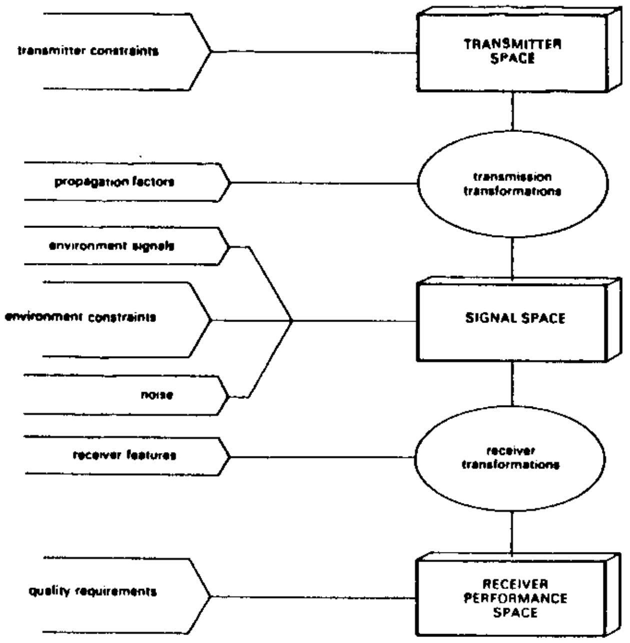


Figure 1—"Transmitter", "signal", and "receiver" performance, spaces and relations between them. The problem constraints and influencing factors are also shown.

(all possible permutations with repetitions allowed). **

If there are N_c test points, then each of these plans should be checked N_c times for compatibility and quality constraints. For $N_F = 20$ frequencies, $N_p = 5$ powers, $N_T = 10$ transmitters and $N_c = 100$ test points, the total number of tests is $100 \times 100^{10} = 100^{11} = 10^{22}$. Even if each test takes only 1 millisecond of computer time, the job would not be finished in 3×10^{11} years! Note, for comparison, that the estimated age of the Earth is 5×10^9 years or so.

For this reason no solution method based on such an exhaustive inspection is acceptable. The author conducted a search of the literature but found no other formal solution to the problem.

Proposed solution method

It is well known from practice that the signal environment usually varies from one test point to another and that the degree of difficulty in finding a proper frequency also varies from transmitter to transmitter. We make use of these inequalities by applying the "worst-case" rule. Thus, we identify the "most difficult" transmitter and the "most critical" test point and we do an assignment that optimally "fits in" this particular transmitter to its environment at this particular test point. After assigning a frequency and power to the first transmitter, all signals of this transmitter are considered as new environmental signals, and the next "most difficult" transmitter is fitted in to them at its "worst" test point. The first transmitter is matched to the original environment, whereas the last is fitted in to the environment modified by all previous assignments. This method equalizes chances of the claimant transmitters by offering more freedom to the transmitters which are initially in a more difficult situation. It reduces the number of different frequency plans and the number of verifications required to check whether all constraints are fulfilled or not. The number of possible plans is:

$$N = N_F \times N_p \times N_T \qquad (2)$$

With, as previously, $N_F = 20$ frequencies, $N_p = 5$ powers, $N_T = 10$ claimant transmitters, $N = 1000$.

Decomposition

In order to apply this method, the original problem is decomposed into sub-problems of lower complexity. These sub-problems are ordered and solved separately, in a determined sequence, so that results of preceding stages are used in the subsequent ones. Specific constraints are imposed on each sub-problem, so that their sum can be equivalent to the original problem.

In the first sub-problem, a transmitter priority list is arranged. All transmitters are intercompared, and the transmitter with maximum interference potential is selected and placed at the top of the list. The assignment begins with this transmitter. In the second sub-problem, a channel priority list is prepared for a given transmitter. Here all relevant environmental signals are intercompared channel by channel, and the channel that assures minimal radiated power in the worst case is placed at the top of the list, and is the first assigned. The last sub-problem consists of tests and an assignment of optimum frequency and power. Here the solutions of the previous sub-problems are used, and specific test procedures are applied. This is a systematic transmitter-by-transmitter and channel-by-channel procedure that stops when an optimum combination is found. More "prominent" transmitters have more freedom here and others are automatically adjusted to them, because each new assignment takes into account all assignments made previously. After a limited number of computation, ordering, and testing steps, the final solution is found or any inconsistency in the problem conditions is stated.

** *Note:* Formula *(1)* results from the following considerations:
A frequency plan is a list of transmitters, each with a specific frequency and power assigned. When two such plans are intercompared, they are considered identical if they contain the same transmitters and each transmitter has the same frequency and power in both plans. With a single transmitter, a single power and N_F different frequencies, N_F different plans are possible, because each of the frequencies can be assigned to the transmitter. If N_p different values of the power are available instead of a single value, the number of different plans is $(N_F \times N_p)$, because for each of the N_p powers each of the frequencies can be assigned to the transmitter. Let $N(i)$ be the number of different frequency plans for i transmitters, with N_F frequencies and N_p powers available, with an extra transmitter added $(N_F \times N_p)$ different assignments are possible for this transmitter, and each of them can be combined with each of $N(i)$ former plans. Therefore the resultant number of different plans with this extra transmitter added is $(N_F \times N_p) \times N(i)$. For $i = 1, 2, 3, \ldots N_T$, there is $N = (N_F \times F_p)^2, (N_F \times F_p)^3, \ldots (N_F \times N_p)^{N_T}$.

4. Graphs

In our attempt some graph-theory concepts are applied. A graph is a set of vertices partially or completely interconnected by lines (see Christofides [4]). We introduce here graphs of two different kinds: signal graph G, and bond graph H.

Signal graph G

Imagine a map with all claimant transmitters and test points marked. Mark also sources of all environment signals at convenient places on the map and all the vertices of graph G will be displayed. In this graph the vertices represent signal sources (transmitters, environment) and signal sinks (test points) whereas the lines represent the power flow. If the signal from a transmitter reaches a test point with a significant power, there is an oriented line (arrow) from the transmitter to the test point. Lines outgoing from test points or ingoing to signal sources are forbidden in graph G, so that there are no lines interconnecting the sources or interconnecting the sinks. There are no isolated (unconnected) sources or sinks. An example is shown in figure 2.

The graph may also be memorized within a computer, without delineating it on paper. All elements of such a graph are labelled so that full information concerning the transmitting stations and their signal environment is contained in the graph (see table 1). At the beginning, the information about the operating frequencies and powers is not available, and graph G is constructed according to the following algorithm:

Algorithm: signal graph G construction

Step 0

Start with the list of test points (TEP), list of claimant transmitters (TRA) and list of environment signals (ESI), assuming tentatively that all transmitters radiate the same unit power and all transmitters and test receivers use the same frequency. Introduce all transmitters and test points as the vertices of the graph.

Step 1

Stop if there are no points on list TEP, otherwise select a test point from the top of this list.

Step 2

Go to step 7 if there are no transmitters on list TRA, otherwise select a transmitter from the top of this list and determine its distance d from the test point. If the transmitter is desired at the test point and $d > A$ go to step 5, if it is undesired and $d > B$ go to step 6.

Step 3

Determine the signal power p at the input of the test receiver accounting for propagation losses and directive antenna patterns. If the transmitter is desired and $p < C$ go to step 5, if it is undesired and $p < D$ go to step 6.

Step 4

Insert a line from the transmitter to the test point, note the signal at the receiver input, remove the transmitter from list TRA and return to step 2.

Step 5

Stop: problem inconsistency (quality constraints not fulfilled). Revise the problem and return to step 0.

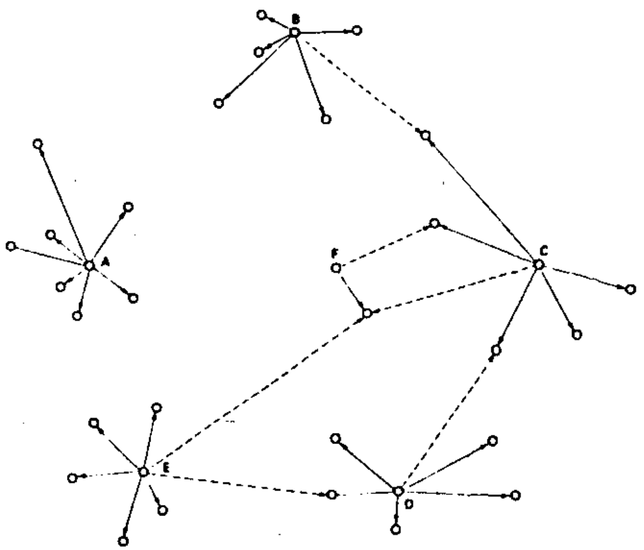


Figure 2—Signal graph G. It represents 5 transmitting stations (A-E), 1 environment signal (F), 28 test points (not labelled), and 34 signal paths: 28 desired (continuous arrows) and 6 undesired (dotted arrows)

Table I

Information contained in signal graph G

No	graph element	information carried
1	claimant transmitter vertex	— geographical location, — antenna* — frequency and power of each signal radiated
2	test point vertex	— geographical location, — antenna* — test receiver nominal and spurious response (or signal-to-interference ratios) — environment signal lists
3	environment signal vertex	— no specific information
4	oriented line	— frequency, power, and desirability index of each signal at the test receiver input

* Height, polarization, orientation, radiation pattern.

Step 6

Signal path irrelevant: remove the transmitter from list TRA and return to step 2.

Step 7

Select and note all relevant environment signals, insert corresponding lines in the graph, remove the test point from list TEP, and return to step 1.

The limiting values A-D are determined according to the problem constraints and nominal usable field strength.

Transmitters from the outside of the circle with radius A are unable to produce any signal that could be usable, and transmitters from the outside of the circle with radius B are unable to produce any signal that could be usable or interfering at the test point considered, even in the most favourable conditions (maximum allowable power radiated, minimum propagation losses, maximum antenna directive gains).

There is no restriction concerning the propagation model used in signal power density computations (see CCIR [1]), but models that are based on detailed terrain data analyses are the most appropriate. An example for UHF/VHF can be found in Sęga *et al.* [10] A discussion of this question is, however, beyond the scope of this paper.

The number of distance determinations N_d, required in step 2, equals

$$N_d = N_T \times N_t \qquad (3)$$

The number of necessary power calculations N_p (step 3), is much lower: its lower bound is

$$N_p \geqslant \max [N_T, N_t] \qquad (4)$$

and the upper bound is given by *(3)*.

Graph G consists, therefore, of $(N_s + N_t)$ vertices and a number of oriented lines for which the lower and upper bounds are *(3)* and $\max [N_s, N_t]$. N_s is the number of the signal sources. Note that graph G is a sparse one and contains only a small percentage of the lines of the complete graph with the same vertices, which is

$$\frac{1}{2}\,(N_s + N_t) \times (N_s + N_t - 1) \qquad (5)$$

because in the complete graph all nodes are mutually interconnected.

For example, if—as previously—$N_s = N_T = 10$ transmitters, and $N_t = 100$ test points, graph G consists of 110 vertices and less than 1000 lines, which is only 17% of the total 5995 lines of the complete graph. The algorithm requires 1000 calculations of the distance and from 100 to 1000 power evaluations. With 1 ms of distance calculation time and 100 ms of power calculation, the job would take 10 to 100 seconds of computer time.

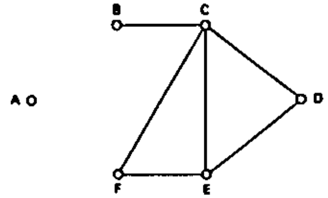


Figure 3—Bond graph H corresponding to figure 2. Its 6 vertices (A-F) represent signal sources and its 6 lines represent potential transmitter influences

The information contained in graph G is modified during the planning process. At the beginning, all transmitters have the same frequency.

After the assignment of an actual frequency-power pair to the *i*-th transmitter, this power and frequency along with all signals generated by the transmitter, modify the existing data in graph G.

Finally, all its vertices and lines are labelled by proper data, as listed in table I.

We define the transmitter zone as the set of all test points at which its signal is relevant. In graph G this zone is the set of all test-point vertices incident to the vertex representing this transmitter (signal source).

Bond graph H

Bond graph H is generated from the signal graph G according to the following algorithm:

Algorithm: bond graph H construction

Step 0

Start with graph G, test point list TEP, transmitter list TRA and signal list ESI.

Step 1

Go to step 3 if there are no points on list TEP; otherwise select a test point from the top of the list, identify all source vertices in graph G that are connected with the test point and insert them as vertices in graph H (without duplication).

Step 2

If there is only one transmitter or signal source connected, remove the test point from the list TEP and return to step 1. In the opposite case, insert lines non-oriented in graph H between each pair of transmitters or signal sources connected, remove the test point from list TEP and return to step 1.

Step 3

If there are parallel lines in graph H, replace them by single lines and stop.

As it follows from this algorithm, graph H is of a lower dimension than graph G because it contains only signal sources as vertices. A line exists between two vertices in graph H if these vertices are connected with a common test point in graph G. The number of vertices in graph H is therefore N_s and the lower and upper bounds for the number of lines are zero and $\frac{1}{2}N_s \times (N_s - 1)$ (the latter is valid if the graph is complete). For N_s = 10 signal sources, graph H consists of 10 vertices and less than 45 lines. An example is shown in figure 3.

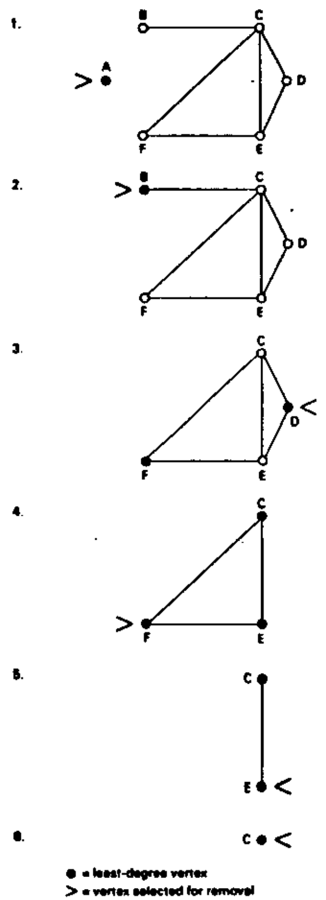
As shown, isolated vertices are allowed in graph H. The structure of graph H indicates how many frequencies are required. If all its vertices are isolated, that is, there are no influences between the signal sources, all transmitters can use the same frequency, and a single frequency channel is sufficient for the whole network.

In the opposite case, when graph H is complete and each one of its vertices is connected with its $(N_s - 1)$ neighbours, all transmitters are mutually related, and N_s frequency channels are required.

5. Transmitter ordering

This sub-problem consists in ordering of all transmitters waiting for the frequency and power assignment in such a way that the "most difficult" or the "most prominent" one is in the top position. The criterion is the number of potential conflicts, i.e. the number of frequency channels that must be co-ordinated with the neighbouring transmitters and environment signals. We call this number the "interference potential".

The interference potential of a transmitter can be evaluated using bond graph H, and we apply the known colouring procedure in order to establish proper transmitter order. In colouring the graph, interconnected vertices may not receive the same colour. The objective of the classical colouring problem is to find the minimum number of colours required to colour all vertices. Here we are interested in ordering the vertices to be coloured. We base our approach on the known heuristic procedure described elsewhere (e.g. Christofides [4], Zoellner [7]), according to the following reduction algorithm. The degree of a vertex equals the number of lines incident to it. The procedure consists of the consecutive exclusion of the least degree vertex from the graph.



Priority list of signal sources

No.	signal source
1	transmitting station C
2	transmitting station E
3	environment signal F
4	transmitting station D
5	transmitting station B
6	transmitting station A

Figure 4—Example of transmitter ordering by graph reduction. Graph H shown in upper left part of the figure corresponds to figures 2 and 3

Algorithm: transmitter ordering

Step 0

Start with graph H.

Step 1

Remove a signal-source vertex of the least degree from the graph along with lines connected to it, and place it at the last position on the signal-source list.

Step 2

Repeat step 1 with the reduced graph, place the removed vertex at the last as yet unoccupied position on the list, until the last vertex is removed from the graph.

Step 3

Remove vertices representing environment signals from the list and stop.

An example is shown in figure 4. As can be seen, this sub-problem is of lower complexity than the original problem because all test points and original constraints are not involved directly. The number of steps in this algorithm equals the number of the signal sources.

6. Frequency channel ordering

Frequency channel ordering concerns the selection of the best operating frequencies for a given transmitter. As follows from the original problem formulation, there are several test points, each with a specific signal environment. With signal sources active, all these signals can be observed on the screen of a spectrum analyser connected to the test antenna. The quantitative characterization of these signals may derive from direct measurement or from solution of prediction equations. The signal environment, or spectrum occupancy, may be different at each of the test points. Due to their geographical dispersion it is difficult to intercompare the reception conditions over the transmitter zone.

In order to omit this difficulty we normalize all environment signals. The normalization consists in compensation for transmission losses which each signal suffers on its path from the transmitter to the test point. In other words, the normalized environment signal power is the power that should be radiated by the transmitter in order to equalize the environment signal at a given test point. The idea is illustrated by figure 5. With all signals normalized, the intercomparison of signal reception over the whole transmitter zone is easy, independently on direction and distance from the transmitter. Thus it now becomes possible to select the critical test point at each frequency channel and find that channel which contains the minimum of signal power at the least favourable test point. We use the following algorithm:

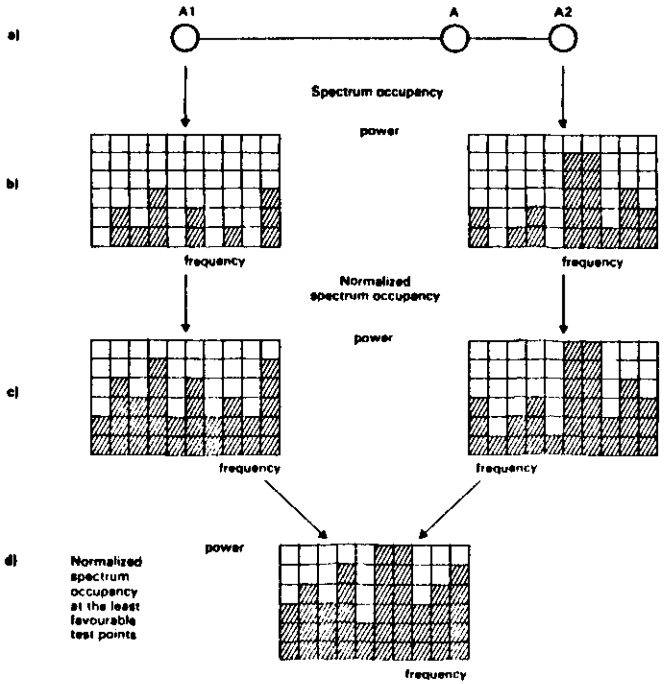


Figure 5—Idea of environment signals normalization and selection of the best frequency channel

a) *geographical location of transmitter A and its zone consisting of test points A1 and A2*

b) *spectrum occupancy at A1 and A2*

c) *normalized spectrum occupancy at A1 and A2*

d) *envelope of normalized spectrum occupancy at A1 and A2 = spectrum occupancy over the transmitter zone. See text for explanation*

Algorithm: frequency channel ordering

Step 0

Given a transmitter, start with its zone in graph G.

Step 1

For each test point and each environment signal (ES) divide the ES power by the test receiver input power due to the unit power transmitter, accounting for frequency difference—if necessary.

Step 2

For each frequency channel select the test point at which the normalized signal power found in step 1 is the highest (critical test points selection).

Step 3

Order the channels according to non-decreasing normalized signal power at critical test points.

Note that all required data are already contained in graph G. This sub-problem is generated sequentially, once per transmitter. As a result, a channel priority list is obtained in which the frequency channel that assures a minimum transmitter power in the worst case is in the top position.

The dimension of this sub-problem is lower than the original problem because a single transmitter and relevant signals are considered simultaneously. The number of comparisons equals the number of test points multiplied by the number of relevant environment signals.

7. Optimum frequency and power assignment

Here, the results of earlier sub-problems are used, according to the following algorithm:

Algorithm: optimum frequency and power assignment

Step 0

Start with the transmitter priority list (A) and frequency priority list (B).

Step 1

Stop if there are no transmitters on list A; otherwise select a transmitter from the top of the list.

Step 2

Go to step 6 if there are no channels on list B; otherwise select a channel from the top of the list and assign it tentatively to the transmitter.

Step 3

Determine the power required for reception quality at the least favourable test point and select the nearest higher power from the catalogue. If there is no such allowable power go to step 6.

Step 4

Check if constraints are fulfilled in the critical test points. If they are not, go to step 7.

Step 5

Assign power and frequency channel tested, include all signals generated by this assignment to the environment, remove the transmitter from list A, and return to step 1.

Step 6

The constraints cannot be fulfilled (problem inconsistency). Verify the problem data and return to step 0.

Step 7

Remove the channel from list B and return to step 2.

This is a systematic reduction method which stops after a limited number of steps: when the job is done, i.e. when the transmitter list is exhausted, or when a problem inconsistency is discovered. This sub-problem is of lower dimensionality than the original problem because only a single transmitter is considered at each step. The possible number of tests are N_T for the most favourable case (final solution found at first attempt) and $N_T N_F$ for the least favourable one, where N_T and N_F are the numbers of transmitters and frequency channels, respectively.

If an assignment is made, then the transmitter is best fitted into its environment, all constraints are fulfilled, and transmitter power is minimum. If problem data are inconsistent it is automatically indicated where and why the assignment cannot be made.

The methods of verification where the compatibility and quality requirements are fulfilled or not are described elsewhere (see e.g. White [13]) and are not repeated here.

8. Conclusions

In this paper we have generalized the classical frequency planning problem. Instead of the traditional assignment of an operating frequency only, we have considered the assignment of a pair: an operating frequency and power to the transmitter. We have formulated this issue as a constrained optimization problem. As shown, several real-life situations can be reduced to the form presented. An heuristic method for solving this problem has been proposed, based on decomposition, reduction, and normalization principles.

New concepts have been introduced, the most signifiant being the signal and bond graphs; the critical points of the transmitter zone; the transmitter and frequency channel priority lists. We have proposed several algorithms for constructing these graphs and lists. In one of them we use the known graph-colouring procedure. The systematic solution method proposed offers a solution that fulfills all problem constraints and ensures the best fit-in of each transmitter to its local environment in the sense of minimum power radiated. If there is any inconsistency in the input data, such that all constraints cannot be fulfilled everywhere, the method offers indications of where, why, and which of them cannot be fulfilled. This makes it possible to modify the data and repeat the solution with corrected data.

Owing to the breaking up of the original problem into sub-problems of lower complexity and solving them separately one after the other, economy in memory size and number of computations required can be achieved in comparison with the exhaustive search methods. Similar to other heuristic procedures, our method does not guarantee any unique, ideally optimum solution but offers a sub-optimum or near-to-optimum one. We believe that our approach and ideas presented here can serve as an efficient tool to cope with new tasks facing radiocommunications, and open new vistas in theory and practice of frequency planning.

Acknowledgement

Valuable comments were received from Dr. Jerzy Kucharczyk of the Institute of Computer Science, Mathematics Department, University of Wrocław, and from Mr. Jerzy Rutkowski, Senior Counsellor of the CCIR. It is the pleasant duty of the author to express his gratitude to them.

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Frequency planning methods for sound and television broadcasting

by H. EDEN

SUMMARY

Because of the growing need for the assignment of frequency channels in sound and television broadcasting, efforts have been made in the course of the last 40 years to improve spectrum utilization efficiency. Among other things, these efforts have produced three important methods of frequency planning which are described and discussed below in sufficient detail to enable their practical application.

Under the first method, interference is limited by providing for a minimum separation distance between interfering transmitters. In the second place the lattice planning method, developed on the basis of uniform propagation conditions and characteristic transmitter data, permits the effective limitation of interference by means of optimized channel arrangements. The third method, which makes use of elements of graph theory, is appropriate for the optimization of spectrum utilization in realistic conditions.

1. Introduction

The problem of distributing channels taken from a limited frequency band among a given number of transmitters situated within an area of limited size, in order to permit the establishment of an effective broadcasting service, is as old as broadcasting itself. However, as long as the number of available frequency channels was not substantially smaller than the number of transmitters, this problem could be solved fairly easily and did not require elaborate theoretical studies. This was approximately the situation that prevailed during the first quarter-century after the coming into operation of the first broadcasting stations.

It was mainly in conjunction with the implementation of nation-wide television services that the need arose to base frequency planning on at least some simple but efficient rules. This need arose from the relatively large bandwidth required for a television channel and the consequential use of "very high" carrier frequencies. As VHF transmitters can, in principle, provide a useful signal only to within-horizon distances and as, beyond the horizon, the signal strength falls off rather rapidly, frequency reuse is possible. On the other hand, frequency reuse is in fact absolutely necessary because of the limited number of channels available in the frequency bands allocated to the broadcasting service; otherwise reasonable coverage could not be provided to a large area.

2. The concept of interference limitation

The need for a simple but efficient rule to harmonize the use of the available frequency channels first became apparent in the United States during the 1950s, i.e. at a time when the development of television in Europe had slowed down drastically because of the war. In the United States the problem was solved by the establishment of strict rules for the limitation of interference. [1] According to these rules, the minimum separation distance between two transmitters T_1 and T_2 likely to cause interference to one another is determined in such a way that the nuisance field of the interfering transmitter, i.e. its field strength increased by the relevant RF protection ratio, does not exceed, at any point of the coverage contour of the wanted transmitter, a predetermined permissible interference level (which depends on the type of service envisaged), e.g. the minimum usable field strength. [2] Hence, the minimum distance consists of two distance elements: the coverage range d_c of the wanted transmitter, and the interference range d_i of the interfering transmitter, and is in fact the larger of the two sums $d_{c1} + d_{i2}$ and $d_{c2} + d_{i1}$ (figure 1). The necessary field strength predictions are based on statistical propagation curves [3] and take account of such diverse parameters as effective transmitting antenna height, transmitter power, transmitting antenna directivity, polarization, etc. The relevant RF protection ratio takes account of the type of interference, e.g. co-channel interference (including the influence of frequency offset between carriers), adjacent channel interference, etc. [4, 5]

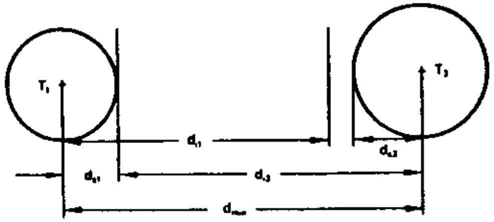

Figure 1—Minimum distance separation d_{min} *between two transmitters* T_1 *and* T_2

As the minimum distance between transmitters of different power is in general determined by the higher-power transmitter, improvement in spectrum utilization efficiency is achieved by two means:

- transmitters of certain power classes are grouped together in specific sub-bands;
- the permissible interference level is determined as a function of the power class: the higher the power level, the lower the permissible interference level.

It is obvious that this concept of interference limitation can equally well be applied, subject of course to proper adjustment of the parameters according to the problem, to sound broadcasting in all available bands. It is worth mentioning that this concept is still used in numerous countries, particularly in Region 2, and in fact served as a basis for planning during the Regional Administrative MF Broadcasting Conference (Region 2) (Rio de Janeiro, 1981).

The concept outlined above is particularly valuable for solving the problem of fitting additional transmitters into an existing transmitter network; it is less useful for the establishment of an initial plan. The optimization procedure required in the latter case means that, apart from respecting the interference limitation rules in each case, either frequency spectrum consumption is to be minimized or coverage by those transmitters to which frequencies have been assigned is to be maximized.

It should also be noted that, due to the multiple effects of interference, the field strength available during (100 − T) per-cent of the time will in general exceed the minimum usable field strength, although interference from each individual source is limited to the latter value except for a small percentage (T%) of the time. However, since interference from each individual source is thus limited throughout the coverage area, high-power transmitters are less affected by multiple interference effects than transmitters having a lower power.

3. Regular lattice planning methods

3.1 General

In Europe the need for vigorous planning method arose only a few years later than in the United States or more precisely when, following the European VHF Broadcasting Conference (Stockholm, 1952), a further conference was contemplated to establish the frequency assignment plans required for television in the UHF broadcasting bands.

However, the problem solved in the United States by means of interference limitation was different from that existing in Europe, which was to provide complete coverage to a large area:

- either with a maximum number of programmes, using a given number of channels;
- or with one programme, using a minimum number of channels.

These are in fact two different ways of describing one and the same optimization problem, the solution of which requires that the following two conditions be met simultaneously:

- a minimum density of transmitters, to permit noise-free reception throughout the area;
- a minimum number of channels, to allow the necessary minimum separation distance between transmitters liable to interfere with one another, thereby ensuring reception free from unacceptable interference.

It is obvious that use was made in Europe of the experience gained in the United States. A set of minimum separation distances was established to meet the second requirement, taking due account of the differences between the television systems in the two areas. [7]

Additionally, for determination of the minimum number of channels necessary to provide full coverage to an extended area with one programme, certain simplifying assumptions were made as regards propagation conditions and network configuration, and channels were arranged so as to minimize potential mutual interference. The planning method resulting from this theoretical approach [8, 9] encompassed two separate elements:

- geometrically regular lattices,
- linear channel distribution schemes,

and it was capable of meeting both the aforementioned requirements.

3.2 Geometrically regular lattices

Because of the many parameters and effects that may have an impact on frequency planning, e.g. varying propagation conditions, transmitter power, transmitting-antenna height and directivity, and terrain irregularities, the problem was first simplified by assuming that all transmitters had the same power, antenna height and directivity as well as the same polarization, that they were situated on an infinitely extended plane area forming a geometrically regular lattice and that propagation conditions did not show variations throughout the area considered.

As regards geometry, the transmitters were supposed to be situated at the points of intersection of two sets of parallels, each of them equidistant and covering the whole area. Thus the distance between transmitters along the parallels of either set are equal but may differ at intersecting lines (figure 2, unbroken lines).

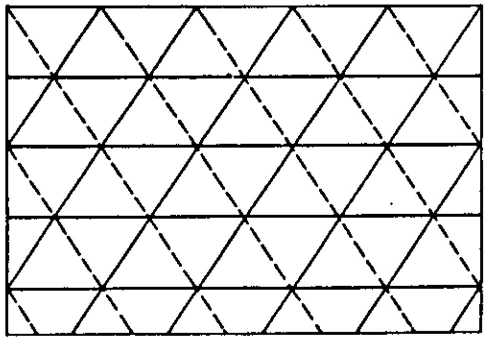

Figure 2—Geometrically regular lattice obtained from sets of parallels intersecting each other

The two sets of parallels subdivide the infinite area into parallelograms, the number of which, although infinite in principle, is equal to the number of intersection points, i.e. of transmitter sites. If each transmitter could provide coverage to an area equal in size to that of a parallelogram, then complete coverage of the infinite area could be obtained. This implies that, as long as there is no interference, distances between adjacent transmitters and effective radiated powers are suitably adapted to one another and include, if necessary, transmitting-antenna directivity. For the purpose of coverage considerations it is convenient at this stage to introduce a third set of parallels joining the existing points of intersection along the shorter of the two diagonals of the parallelograms (figure 2, broken lines). In the elementary triangles of the resulting lattice none of the angles exceeds 90°.

In the absence of interference, complete area coverage can be obtained only if the coverage range d_c of each transmitter is equal to the larger of the three distances between the corners and the centre of gravity of an elementary triangle. Clearly, complete coverage implies a certain amount of overlapping which, in principle, should be minimized.

Overlap minimization under the conditions described calls for a lattice of equilateral elementary triangles in which the parallelograms become rhombi and the distance of the centre of gravity from each corner of the triangle is $d/\sqrt{3}$. To determine the minimum unavoidable overlap it is appropriate to relate the coverage area of any one transmitter, $S_c = \pi d_c^2 = \pi d^2/3$, to its share in the whole area, $S = \sqrt{3} d^2/2$. The quotient which is known as the coverage factor c is $c = 2\pi/\sqrt{3} = 1.209$. Thus, full area coverage can only be obtained with a minimum overlap of about 21%.

In a non-equilateral triangular lattice (e.g. figure 2) the unavoidable overlap is greater than in the equilateral case because, under the assumptions made, i.e. uniform transmitter data and propagation conditions, full coverage calls for a coverage range equal to the greatest of the three distances between the corners and the centre of gravity. It is therefore desirable for the elementary triangle to be equilateral. However, the increase in overlap resulting from the use of a non-equilateral triangular lattice will only be slight if the shape of the triangle deviates only slightly from the equilateral shape. Thus the user of a non-equilateral lattice may be advantageous in a number of circumstances, some of which will be discussed when the question of channel grouping is taken up.

Because of the high co-channel protection ratio values—30 to 40 dB—involved in FM sound and television broadcasting, the coverage areas of neighbouring co-channel transmitters can never overlap. However, it is clear from the above explanations that the maximum coverage factor c will be obtained if the co-channel transmitters form an equilateral triangular lattice with distances D between neighbouring co-channel transmitters: $c = 2\pi d_c^2/\sqrt{3}\ D^2$. Full area coverage will require the use of a sufficiently large number of channels C. Identical co-channel triangular lattices for these channels C, should then be superimposed so that the resulting lattice becomes quite geometrically regular again. As one can easily imagine, the number of possible solutions to this problem is rather small.

All the possible solutions can be found by subdividing the co-channel parallelogram (rhombus) by two sets of $C - 1$ equidistant lines parallel to either pair of sides of the parallelogram, as shown in figures 3*a)* and 4*a)* for an equilateral co-channel lattice and $C = 7$ or $C = 12$, respectively. Geometrical regularity of the resulting lattice of elementary triangles is obtained when each of the $C - 1$ parallels of one set carries just one transmitter. The $C - 1$ parallels of the other set also carry just one transmitter if C is a prime number (e.g. 7). If C is the product of two or more primes P_i (e.g. $12 = 2 \times 2 \times 3$) there are also solutions where, in the other set, only parallels which are multiples of P_i (or of their products $\prod_{i=1}^{i<n} p_i$) carry P_i (or $\prod_{i=1}^{i<n} p_i$) transmitters (figures 4*c)* to 4*e)*). For $C = 7$, all the possible solutions (except one which need not be considered since all the transmitters would lie along one of the sides of the triangle) are shown in figures 3*b)* to 3*f)*.

It is advantageous at this stage to introduce a system of non-rectangular co-ordinates (x, y) having its origin (0.0) in the lower left-hand corner, with the co-ordinates directed towards the lower right-hand corner (x) or the upper left-hand corner (y), respectively. The subdividing parallels are at unity distance from one another.

In this system of co-ordinates the solutions shown in figures 3*b)* to 3*f)* have one transmitter at $y = 1$, the position of which with respect to x varies between $x = 1$ and $x = 5$. The co-ordinates of all the other transmitters in each of the solutions are multiples of the first one, e.g. in figure 3*b)* the co-ordinates are (1.1), (2.2), (3.3), (4.4), (5.5) and (6.6.). Co-ordinates exceeding C can be normalized by reducing the real multiple by C or by a multiple of C, since the original and resulting values of the co-ordinates are congruent to each other modulo C. [10]

It can be seen from figures 3*b)* to 3*f)* that some of the solutions are "symmetrical" (triangular symmetry), e.g. those of figures 3*c* and 3*e)* or those of figures 3*b)*, 3*d)* and 3*f)*. This reduces the number of "genuine" solutions to two. Hence, it follows that, except for $C = 3$ with one "genuine" solution, they can all be found if the co-ordinates of one of the transmitters are assumed to be $x \leq C/2 - 1$ and $y = 1$ (or vice versa), i.e. to the left of the broken vertical lines in figures 3*a)* and 4*a)* and, more precisely, inside the parallelograms formed by broken lines. Thus in the example of figure 3 ($C = 7$) only the co-ordinates (1.1) and (2.1) need to be considered, whereas in the example of figure 4 ($C = 12$) only "genuine" solutions are presented.

The solutions in figures 3 and 4 are derived from an equilateral co-channel triangular lattice in which the square of the distance d between co-ordinates (x_1, y_1) and (x_2, y_2), with $(x_2 - x_1) = q_1$ and $(y_2 - y_1) = q_2$, is $d^2 = q_1^2 + q_1 q_2 + q_2^2$. In figures 3 and 4, the co-channel rhombus has a side length D which corresponds in either case to C units, and an area $S_c = \sqrt{3}\ D^2/2 = \sqrt{3}\ C^2/2$. If this area is subdivided into C congruent elementary parallelograms, each area will be $S_e = \sqrt{3}\, C/2$ and, if the elementary area is a rhombus, $S_e = \sqrt{3}\, d^2/2$, i.e. $d^2 = C$. Figures 3 and 4 show that the elementary triangles are generally not equilateral and have side lengths d_1, d_2 and d_3. Only in exceptional cases (figures 3*c)* and 3*e)*) is the elementary triangle equilateral. Solutions where both the co-channel and the elementary triangles are equilateral exist only when C is a rhombic number, i.e. when there are two integers, q_1 and q_2, which fulfil the equation $C = q_1^2 + q_1 q_2 + q_2^2$. [11] This is because all the transmitter co-ordinates must be integers. The area of equilateral and non-equilateral triangles are equal if $3d^4 = 4d_1^2 d_2^2 - (d_1^2 + d_2^2 - d_3^2)^2$. Example: in figure 4*d)* the side lengths are $d_1 = \sqrt{13}$, $d_2 = 3$, $d_3 = 4$; hence $3d^4 = 4 \times 13 \times 9 - (13 + 9 - 16)^2$, or $d^2 = \sqrt{13 \times 12 - 12} = 12 = C$. Table 1 gives a series of rhombic numbers together with the originating integers q_1 and q_2.

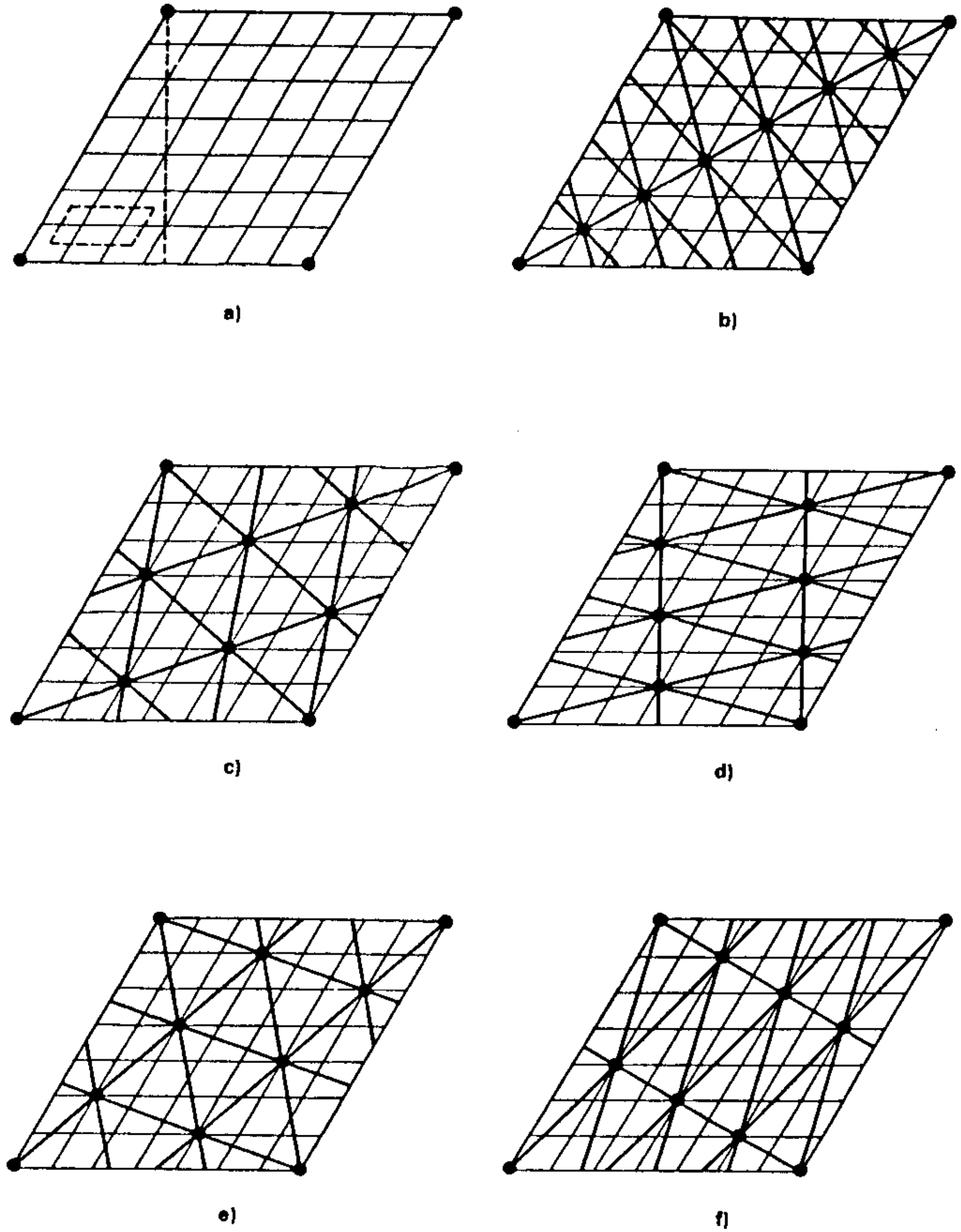

Figure 3—Superposition of C = 7 equilateral co-channel triangular lattices and the resulting elementary triangles: a) *principle;* b) *to* f) *possible solutions; co-ordinates of transmitters at parallel y = 1:* b) *1.1;* c) *2.1;* d) *3.1;* e) *4.1;* f) *5.1*

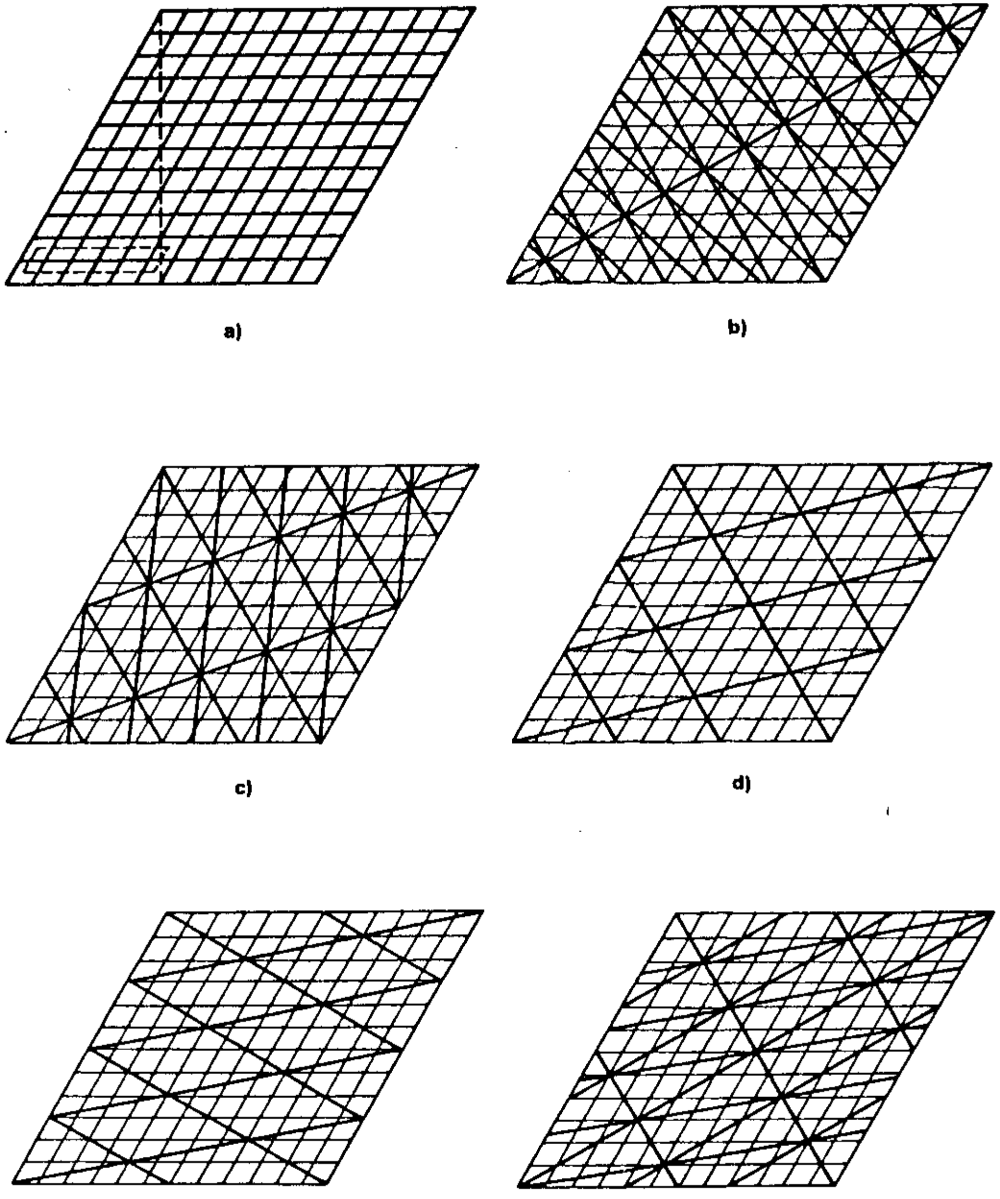

Figure 4—Superposition of C = 12 equilateral co-channel triangular lattices and the resulting elementary triangles: a) principle; b) to f) possible solutions; co-ordinates of transmitters at parallely = 1: b) 1.1; c) 2.1; d) 3.1; e) 4.1; f) 5.1

Table 1

Rhombic numbers C and their originating integers q_1 and q_2

C	q_1	q_2	C	q_1	q_2	C	q_1	q_2	C	q_1	q_2	C	q_1	q_2	C	q_1	q_2
1	0	1	9	0	3	19	2	3	28	2	4	43	1	6	52	2	6
3	1	1	12	2	2	21	1	4	31	1	5	48	4	4	57	1	7
4	0	2	13	1	3	25	0	5	36	0	6	49	3	5	61	4	5
7	1	2	16	0	4	27	3	3	37	3	4	49	0	7	63	3	6
									39	2	5						

The following conclusions may be drawn for geometrically regular lattices:

- Full area coverage can best be obtained with equilateral elementary triangles.

- Because of the high co-channel protection ratio values in FM sound and television broadcasting, more than one channel is required to obtain full area coverage.

- When co-channel interference is predominant, with the required RF protection ratio exceeding those for all other types of interference by 20 to 30 dB, the co-channel triangular lattice should be equilateral. However, if other types of interference are of similar magnitude, e.g. interference from adjacent channels at 100 kHz spacing in FM sound broadcasting, the use of equilateral co-channel triangles may not be appropriate.

- The co-channel and elementary triangles of one and the same lattice can only both be equilateral if C (the number of channels used) is a rhombic number.

- If full area coverage is to be obtained, some overlap is unavoidable. With equilateral elementary triangles this overlap is minimal (about 21%); otherwise it is larger.

- Thus a coverage factor $c \approx 1.2$ is required to obtain full area coverage, the actual value depending on the shape of the elementary triangle.

- If the coverage obtained with only one channel is c_1 and if there is only co-channel interference, then the number of channels required for full coverage is $C = 1.2/c_1$.

Some further conclusions may be drawn:

- If transmitters situated at the corners of the co-channel rhombus suffer interference from other transmitters situated elsewhere in that rhombus and using different channels, e.g. adjacent or second channels, then the distance of such transmitters from the corners should be maximized, i.e. they should be situated as closely as possible to the triangle's centre of gravity. Nevertheless, such interference will lead to a reduction in coverage and, to compensate for this loss, the required number of channels will have to be increased.

- The effect of interference depends mainly on the distance between the transmitters involved and the RF protection ratio required. Because of the smaller distance between the centre of gravity and the corners of the co-channel triangles it may happen, particularly in the case of adjacent channel interference at 100 kHz spacing in FM sound broadcasting, that co-channel interference is no longer predominant. In such circumstances equilateral elementary triangles may be preferable to equilateral co-channel triangles. However, regardless of the solution chosen, the number of channels required for full area coverage will increase.

- In cases where several types of interference are of similar magnitude, the solutions with non-equilateral elementary triangles obtained as a first result can easily be transformed into corresponding solutions with equilateral triangles, by retaining the largest of the three sides d_1, d_2 and d_3, and simultaneously rotating and extending the remaining two sides to make all three sides equally long (affine transformation).

3.3 Linear channel distribution schemes

The remaining part of the problem is to find an arrangement for the C channels, necessary for full coverage, in such a way that interference is minimized. At this stage it seems appropriate to recall that every co-channel rhombus which was, or will be, given as an example is part of an infinitely extended lattice consisting of C regularly superimposed co-channel lattices. Each result obtained by the lattice-planning method will therefore be characterized by a periodical repetition in all directions of the geometry and of the channel arrangement shown, by way of examples, on the area of a co-channel rhombus.

For the purpose of channel arrangement considerations, it seems appropriate to use channel number 0 as a reference and to assign it in any one example to the corners of the co-channel rhombus. Hence the numbers of the channels (1, 2, ..., $C - 1$) in the arrangement will automatically be equal to the difference in channel numbers between the transmitter under study and those at the corners of the co-channel rhombus. However, when considering, as an example, adjacent channel interference, it must be borne in mind that this type of interference does not exsit only between channels 0 and 1, but also between channels 1 and 2, 2 and 3, etc. For reasons of simplicity and the regularity of the resulting channel distribution scheme, it seems appropriate to assign channel 1 to a transmitter at co-ordinates (x_1, y_1) which, according to the previous explanations, should be fairly close to the centre of gravity, and channel 2 to the transmitter at co-ordinates ($2x_1$, $2y_1$), etc. This implies that channel numbers assigned to equidistant transmitters situated along a straight line will have the same difference. If this difference is greater than 1, channel numbers greater than C may result which should be normalized by subtraction of C or a multiple of C. The original and the normalized channel number are thus congruent to each other modulo C. [10] The resulting channel distribution schemes are called linear distributions. Of course, other solutions exist, but non-linear distributions [17], although not necessarily useless, are less manageable.

For the study of coverage efficiency, the consideration of linear channel distribution schemes is advantageous, since the interference situation in each of the C superimposed co-chan-

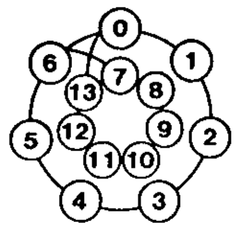

Figure 5—Cyclical system of C = 7 channels

nel lattices is identical, except for some irregularities linked to the lower and upper of the C channels. This exception can, however, be disregarded if the C channels are assumed to form a cyclical system of channels where channels 0 and $C-1$ are adjacent to each other. Such adjacency would incidentally result if an attempt were made to obtain coverage with n programmes by using a sequence of subsequent channels subdivided into n coherent groups of C channels each. A cyclical system of channels is shown in figure 5.

As the interference situation in all C channels as well as for each transmitter in any one of the C co-channel lattices is identical, the coverage areas of all transmitters are also identical, both in size and shape. Hence verification of the coverage obtained by all transmitters in the C channels does not require more than the determination of the coverage area of a single transmitter. The determination of the amount of overlap would in principle also be possible, but slightly more complex.

The use of linear channel distribution schemes is particularly advantageous when different types of interference are to be taken into account, such as co-channel interference, interference from adjacent channels or second channels or interference due to IF generation or radiation of the receiver's local oscillator. Table 2 presents two examples with different types of interference, together with the corresponding differences in channel numbers (interfering minus wanted channel).

Table 2

Types of interference to be taken into account and corresponding differences (interfering minus wanted) in channel numbers

example 1		example 2	
VHF FM sound broadcasting channel spacing: 100 kHz IF: 10.7 MHz		UHF television broadcasting channel spacing: 8 MHz IF range: 32-40 MHz	
type of interference	difference in channel number	type of interference	difference in channel number
co-channel	0	co-channel	0
adjacent channel	±1	adjacent channel	±1
2nd adjacent channel	±2	IF generation	±5
3rd adjacent channel	±3	oscillator radiation	-5
4th adjacent channel	±4	second channel	+9
IF generation	±107		
oscillator radiation	±107		

From the explanations given so far, it follows that all channels causing or subjected to interference require a position in the vicinity of the centre of gravity of one of the two triangles forming a co-channel rhombus. Their distance from the corners should in principle be the larger, the greater the RF protection ratio. In linear channel distribution schemes, for reasons of symmetry, channels with equal difference from that of the reference transmitter but opposite sign are at symmetrical positions relative to these two centres of gravity.

Remembering that channels z and $C-z$ are symmetrical to each other, there can obviously exist no more than $C/2$ different channel distributions, i.e. taking the example of figure 3*b)*, only three (see figure 6). The two centres of gravity are marked by the symbol G. The third solution would provide the largest distance from the corners to the adjacent channels. In the figure 3*c)* example, the distance of each transmitter from the nearest corner is identical; hence the possible three solutions are in this case equivalent, so that in reality there exists only one single "genuine" solution because of additional symmetry.

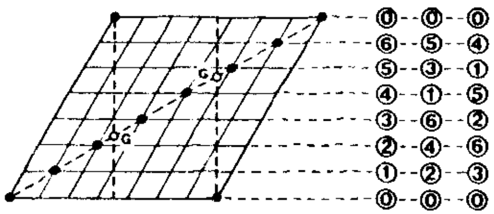

Figure 6—Possible channel distribution schemes applicable to the geometrical solution of figure 3 b) *($C=7$): G = centre of gravity*

In principle, the number of possible solutions increases with the number of channels necessary for full coverage. However, the difference in channel numbers, Δ, between neighbouring transmitters and the number of channels available C should have no factor (other than 1) in common, since in such cases only channel numbers that are multiples of this common factor would be used, whereas the remaining channel numbers would be left aside. It is obvious that in such circumstances no linear channel distribution scheme could ever be obtained. Hence the full number of $C/2$ solutions (the integral part of this fraction) will exist only if C is a prime number. If C is the product of two or more primes, the number of solutions is considerably lower. For $C=12$, for instance, there are only two solutions which are shown in figure 7 for the geometry of figure 4*f)*. Once again the two centres of gravity are marked by the letter G. None of the two solutions is satisfactory when used with this geometry.

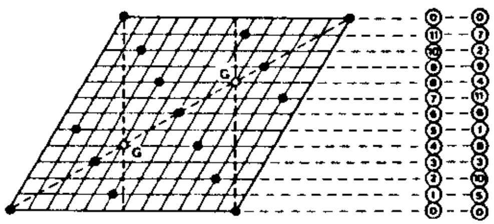

Figure 7—Possible channel distribution schemes applicable to the geometrical solution of figure 4 b) *($C=12$): G = centre of gravity*

Figure 8 shows an example for $C=19$; the elementary triangles of this solution are equilateral. The channel distribution scheme selected is fairly appropriate for UHF television broadcasting, as it makes optimum allowance for interference from channels ±1 and +9, the distance of these channels from the corners being √3 times or 2 times respectively the side length of the elementary triangle. The positions of channels ±5 are fairly unsatisfactory, however, as they are immediately adjacent to that of the reference channel. It can easily be verified that, with this geometry, no solution can be found for which the positions of channels ±1, ±5 and +9 are altogether equally satisfactory.

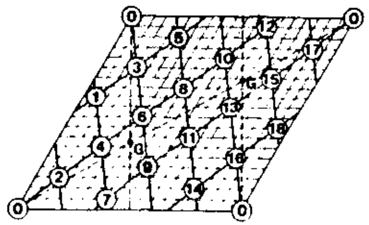

Figure 8—Example of a geometrically regular lattice and channel distribution scheme for $C=19$

If in figure 8 each channel is multiplied by 2 (and subsequently normalized, if necessary) the resulting solution would be fairly adequate for VHF FM sound broadcasting, since the positions of channels ±1, ±2 and ±3 are suitably remote from the corners, i.e. corresponding to the respective protection ratios. This can be seen from figure 8, if the correspondence between channel numbers in the two schemes is known: channels −1 (18), +2 and −3 (16) in the new scheme corresponds to channels +9, +1 and +8 in that of figure 8.

It can be concluded at this stage that the problem of achieving full area coverage can be solved, in a number of steps, by determining (figure 9):

- the coverage factor $c_{(1)}$ when only one channel is used and when, in the absence of noise, account is taken of co-channel interference alone: $C_{(1)} = 2\pi d_{c(1)}^2/\sqrt{3}\ D^2$
- the minimum number of channels C_{min} necessary to provide full coverage: $C_{min} \approx 1.2/c_{(1)}$;
- the most favourable geometry (equilateral or near-equilateral elementary triangles) for C_{min} or slightly larger values, if necessary or appropriate;
- the most appropriate channel distribution scheme for the geometrical solution of step 3;
- the coverage factor c_1 obtainable with any one of the C channels, taking account of all types of interference: $c_1 = 2\pi d_{c1}^2/\sqrt{3}\ D^2$;
- the coverage factor c obtainable with all channels: $c = c_1 \cdot C$;
- the power level necessary to permit noise-free reception at a distance d_{c1} from the transmitter.

It should be noted that, assuming complete uniformity of all transmitter and propagation data and complete lattice regularity, the coverage factor c will in the absence of noise not

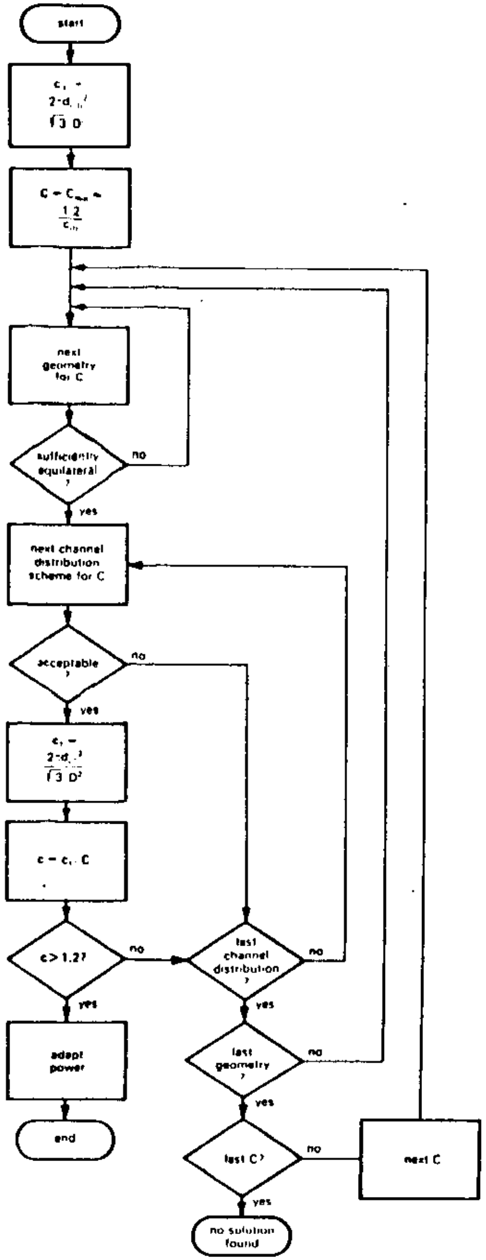

Figure 9—The optimization procedure of the lattice planning method

depend on the (uniform) level of transmitter power. Moreover, it should be pointed out that the distance D between co-channel transmitters is the sum of the coverage range d_r and the interference range d_i. Bearing this in mind, we shall have no difficulty in executing the aforementioned steps and making the appropriate corrections, as follows:

• To obtain the optimum result, steps 3 and 4 may require repetition in an iterative procedure.

• If in step 6 the resulting coverage factor c is not equal to (for equilateral elementary triangles), or sufficiently greater than (for non-equilateral elementary triangles) 1.2, the whole procedure has to be repeated, starting either with step 3 or, if necessary, from the beginning, but with an increased value of C. Affine transformation may be useful, if interference from other channels exceeds, or is comparable to, co-channel interference.

• If the power level determined in step 7 exceeds a reasonable or a predetermined value, either the co-channel distance D has to be reduced and the whole procedure has to be repeated, or the number of channels C has to be increased.

It is easy to imagine and important to know that the coverage factor C is not a monotonously increasing function of the number of available channels C. This is due to the fact that:

— in geometrically regular lattices the distance from the centre of gravity of the position which is closest to it varies with the geometrical solution selected and also with the number of channels available (compare, in this respect, the solutions of figures 3*b)* and 3*c)* or those of figure 4);

— the coverage factor c depends on the amount of interference from all sources and thus on the channel distribution scheme selected.

3.4 Multiple coverage and channel grouping

Having solved the optimization problem of obtaining full area coverage with a minimum number of channels, it is fairly easy to solve the problem of providing full area coverage with more than one programme too. It is obvious that the number of channels C_p necessary for coverage with P programmes is $C_p = C \cdot P$. There are two promising ways of solving this problem.

The first involves extending the cyclical system of C channels to comprise P cycles of C channels each; in figure 5 this solution is shown for $C = 7$ and $P = 2$. By selecting this solution, advantage is taken of the fact that in a cyclical system channels $C - 1$ and C are already adjacent to each other. Hence no further interference will in principle need to be considered, the only exception being those types of interference which were not previously involved because of the small number of channels used. As, for instance, in the figure 6 example, only seven channels are used, no second channel interference can arise from channel +9. This will no longer hold good when multiple coverage with seven channels per programme is envisaged. From figure 5 and the explanations given it is clear that the channels with numbers 9 and 2 are congruent modulo 7. Therefore, it would already have been possible in an earlier step to take account of this type of interference by considering channel 2 as a potential source of second channel interference.

The advantage of this first method is that the coverage obtained with the P programmes are identical, if the fact is overlooked that in reality the channels at the two extreme ends of the frequency range have only one adjacent channel and that the nine highest channels will not be subject to second channel interference. However, when more than two channels are grouped together at every transmitter site they have equidistant numbers (the difference being C); this may be considered as a drawback as it may give rise to third order intermodulation problems at the receiver input in areas of high field strengths. Such intermodulation problems can be eliminated by separating any two blocks of C channels by a varying number of extra channels. [13] However, the extra channels themselves must be used with particular care as their relationship to other channels will vary from one block to another.

Under the second method, all the C_p channels are treated as a single whole. As the number of possible solutions increases with the number of channels involved, this approach is more flexible than in the previous case. Of course the basic lattice resulting from this approach will provide one site per transmitter, which is inconsistent with actual practice. It would, however, be possible with such solutions to distort the basic lattice in such a way that all P transmitters and their channels are grouped together at one and the same site. No detrimental effects would result from this grouping provided that the distorted lattice is made completely regular again, that channels creating or subjected to interference are not brought into closer proximity than in the basic lattice, and that channels at the same site do not cause interference to each other. The P channels which are grouped together may have an arbitrary configuration in the basic lattice, but they should normally be in close proximity to one another. If more than two channels are taken from a straight line the difference in their channel numbers will be equal, as in the previous case, and here too intermodulation problems may arise when the transmitters are received in areas of high field strengths.

From the above explanation it is obvious that the elementary triangles of the distorted lattice rather than those of the basic lattice should be equilateral or near-equilateral. Because of this requirement, basic solutions with elementary triangles which are far from equilateral may be worthy of interest. Figure 10 illustrates this point for the basic solution of figure 3*b)*, which is repeated in figure 6 together with the second channel distribution. However, although the elementary triangle of the distorted lattice is much more appropriate than that of the basic solution, it must not be overlooked that, in this example, some of the resulting co-channel distances are severely reduced.

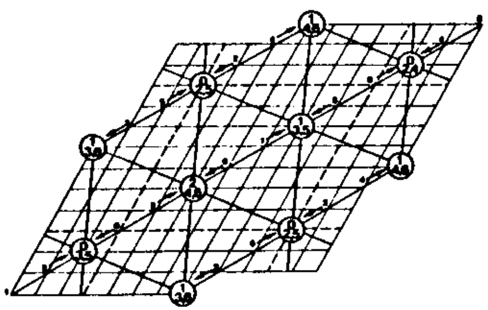

Figure 10—Channel grouping by lattice distortion

3.5 Practical application of the lattice planning method

The regular lattice planning method developed in theory can be considered as a useful tool for the practical solution of frequency assignment and related coverage problems in sound and television broadcasting. To apply this method in practice entails adapting the regular lattice to an existing or planned network of transmitters. Since in actual practice terrain data, population distribution, availability of electricity and feeder links, etc., are more important for the selection of transmitter sites than geometrical regularity, adaptation of the theoretical lattice to practical needs will inevitably lead to a more or less severe distortion of the theoretical lattice.

Once the regular lattice and the channel distribution scheme causing least interference are determined, practical application should start with the subdivision of the whole planning area into area elements by superimposing the regular lattice on a map showing the transmitter sites. During this operation, care must be taken to ensure that the number of transmitters (or transmitter sites) on an area element does not exceed the number of channels (or groups of channels) available on the same area element.

If the number of channels and transmitters on the area element are equal, each channel available in the superimposed regular lattice will have to be assigned to the nearest transmitter on the map; if there are fewer transmitters than channels, the distortion of the regular lattice in order to assign the channels should, in principle, be kept to the minimum.

In general, the density of the transmitters, which is related to the average distance between them, varies with the geographical area and hence from one area element to another. There are two possible ways of overcoming the difficulty resulting from this varying density for channel assignment:

• The planning area is subdivided into area elements similar in shape but different in size. This can be done by distorting the theoretical lattice to adapt the size of the area elements to the needs which arise from the varying density of the real transmitter network. This density adaptation [14, 15] will result in varying distances between interfering transmitters but retain (more or less) the original distance relations and, consequently, ensure that interference is kept to a relative minimum in all parts of the lattice.

• The planning area is subdivided into area elements of equal shape and size. [13] In this case, not all the channels available on the area element of the theoretical lattice can be assigned to transmitters in a region with reduced transmitter density. Special steps therefore have to be taken to ensure that on average all channels are used with equal frequency or, more precisely, that the resulting interference is kept to a relative minimum in every part of the planning area.

In addition to the measures taken to accommodate varying transmitter density, further measures are needed to take into account real network irregularities such as effective antenna heights, effective radiated powers and the like. In this respect it is important to note that less mutual interference, i.e. more efficient use of the spectrum, is preferable to less severe distortion of the original lattice.

The coverage provided in actual practice will usually be smaller than that obtainable in theory. This must be borne in mind not only when a decision is taken on the regular lattice and the corresponding channel distribution scheme, but also when the most appropriate transmitter sites are selected.

The lattice planning method has been successfully applied on several occasions, the most important of them being:

— the European VHF/UHF Broadcasting Conference—Stockholm, 1961 (for UHF television);

— the African VHF/UHF Broadcasting Conference—Geneva, 1963 (for television and FM sound);

— the Regional Administrative Conference for FM sound broadcasting in the VHF band (Region 1 and certain countries concerned in Region 3)—Geneva, 1982/84 (for FM sound).

The lattices and channel distribution schemes selected are reproduced in the Final Acts of those Conferences. At the two earlier Conferences, density adaptation was used before channels were assigned, whereas at the most recent of them neither of the two regular lattices was adapted to the transmitter density in the area where they were applied. All the frequency assignment plans established with the help of the lattice planning method at the three aforementioned Conferences supposedly meet expectations; at least this holds good for the UHF television plan for Europe, which has been in force for more than 20 years without any request for revision having been made.

3.6 Application to AM sound broadcasting

Before applying the lattice planning method together with linear channel distribution schemes to AM sound broadcasting, particular allowance must be made for the fact that propagation in the LF, MF and HF broadcasting bands takes place in more than one mode:

• Ground waves are propagating close to the Earth's surface and suffer attenuation which depends strongly on frequency.

• Sky waves are propagating through space above the Earth's surface and undergo attenuation and refraction in different regions of the ionosphere; attenuation and refraction depend a great deal, among other things, on the time of day and the frequency band considered.

The extensive variations of the coverage and interference ranges of the ground wave as a function of the frequency, seem to indicate that the use of this planning method for AM sound broadcasting might not be suitable. However, transmitter networks operating in the AM broadcasting bands are usually designed to take account of sky-wave interference. In such circumstances the interference range within each of the AM broadcasting bands becomes much less dependent on frequency and comparable in magnitude to the Earth's radius.

In the HF broadcasting bands, propagation conditions vary considerably from one band to another. Use of the ground wave is practically negligible because of severe attenuation. Hence coverage is provided almost exclusively by the sky wave, which propagates in various modes depending on the frequency band as well as on the time of day, season of the year and solar cycle. The lattice planning method does not enable these variables to be taken into account in addition to the usual parameters. Other methods were therefore developed and discussed during the First Session of the World Administrative Radio Conference for the planning of HF bands allocated to the broadcasting service (Geneva, 1984). [16] They are not considered in this article.

In the LF and MF bands, on the other hand, the required separation between co-channel transmitters when sky-wave propagation determines the interference range (i.e. during the hours of darkness) does not vary too much, either with frequency or with the season of the year or the solar cycle. It may therefore be assumed that the lattice planning method could also be applied to these two broadcasting bands. Studies carried out in this respect were confined to MF broadcasting. [6, 17] From the results obtained, however, it seems justifiable to apply the method to the LF band also.

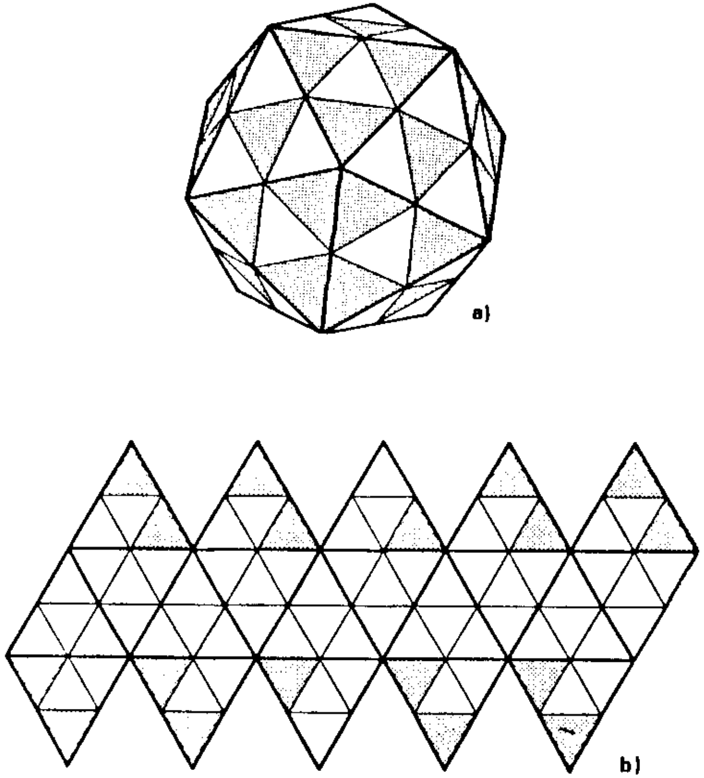

Figure 11—Icosahedron: a) *body;* b) *development (faces subdivided into four equilateral triangles of equal size)*

When applying the lattice planning method to LF and MF broadcasting, account must be taken of the Earth's spherical nature and finite surface. This can best be done by approximating the sphere to a polyhedron. Since the angles in a spherical equilateral triangle exceed 60°, no more than five such triangles can join at each apex of the polyhedron. Consequently, there are only three solutions to this approximation problem with 3 (tetrahedron), 4 (octahedron), or 5 (icosahedron) equilateral triangles joining at each apex. The most suitable of these solutions is the approximation to an icosahedron (figure 11*a)*), because of the dimensions of the resulting spherical equilateral triangles (side length: 7050 km). A subdivision into smaller equilateral (co-channel) triangles is impossible on the surface of the sphere but possible on the faces of the icosahedron. Figure 11*b)* shows, as an example, the development of the 20 faces of an icosahedron and their subdivision into 4 equilateral co-channel triangles. It should be noted that after reconstitution of the icosahedron, the triangles at the left- and right-hand sides of the figure join each other, as do the 2 times 5 single triangles above or below the central strip of the figure, respectively.

While the two ends of the central strip can join without creating undue additional interference, the same does not apply to the two groups of 5 triangles. The channel separations on any two of these triangles will shrink upon reconstitution of the icosahedron, and additional and unacceptable interference will arise if all channels are used. However, the resulting loss in channel assignment possibilities will be tolerable if care is taken to ensure that most of the losses occur in maritime areas. Triangles with channels that cannot be used are shown in figure 11*b)* as shadowed areas.

Taking the example of figure 11*b)*, the world is subdivided into 40 rhombi and therefore no channel can be used more than 40 times world-wide. However, these theoretical possibilities are considerably reduced because of the difficulties that arise, as has already been mentioned, upon reconstitution of the icosahedron and because a large number of the 40 rhombi are located, either entirely or in parts, in the seas which cover about 70% of the Earth's surface.

Because the Earth's surface is finite, unique interrelations exist between the co-channel distance D and:

— the number of co-channel transmitters N_0;

— the coverage factor c for a given protection-ratio value A;

— the transmitter power P for a given minimum usable field strength E_{min} necessary to ensure that interference rather than noise is the coverage-limiting factor.

Once a value for any one of these parameters D, $N_0(D)$, $c(D, A)$, $P(D, E_{min})$ is given, the values of the other parameters are determined.

In LF/MF broadcasting the lattice planning method, although applicable in principle, has hitherto been used only in planning exercises and never during conferences. This is due mainly to the fact that the Second Session of the Regional Administrative Conference (Regions 1 and 3) for drawing up frequency assignment plans for LF and MF broadcasting (Geneva, 1975) and the Second Session of the Regional Administrative MF Broadcasting Conference (Region 2) (Rio de Janeiro, 1981) were confined essentially to improving (and legalizing) the situation that actually existed. It must, however, be pointed out that because of the strong frequency dependency of the ground wave, application of the method might have resulted in a ground-wave coverage that was completely different from actual coverage for a vast number of transmitters and, consequently, in countless complaints by listeners.

3.7 Merits and demerits of the methods

From the explanations given, it is clear that the lattice planning method may be a valuable tool if the intention is to establish a frequency assignment plan permitting coverage of large areas with one or more programmes. As a first step in applying the method, a lattice grid with more or less rhombic meshes is spread over the whole planning area. Depending on the case, the size of the meshes may or may not be adapted to the varying transmitter density, i.e. to the number of transmitters per unit area. It is, however, important to recall in this respect that frequencies cannot be assigned to all transmitters if, within any one of the meshes, their number exceeds that of the available channels. If, as a consequence of density adaptation, the side lengths of the meshes in some parts of the planning area become smaller than the required minimum co-channel distance, the separation distance between transmitters causing or subjected to other types of interference will generally also become too small. However, it may be considered an advantage that the assignment of frequencies to all transmitters in such problem areas will still be possible. Moreover, since the relationship between the various separation distances remains unchanged, the detrimental interference effects of excessive transmitter density will be equally distributed among all transmitters in those particular parts of the planning area.

In a second step, individual frequencies can be assigned to all transmitters by adapting the linear channel distribution scheme to each mesh. In the environment of each transmitter site a limited number of frequencies promising minimum interference will be available and the most appropriate of these may be selected having due regard to further constraints, if any. This assignment procedure can be started simultaneously in various parts of the planning area, e.g. at the country level. Only slight adjustments will need to be made later to fit the resulting pieces of the frequency plan together along the seams. The establishment of a frequency assignment plan can thus be accomplished within a reasonable time and without computer aids.

Because of the unavoidable lattice distortion, the interference contributions from other transmitters and the resulting usable field strength will vary in practice from one transmitter to another. As long as all transmitters have the same characteristics the plan will be the better, the lower the mean value of the usable field strengths and the smaller their standard deviation. However, if in the entire network the effective radiated powers vary over a wide range it is preferable to determine both the mean value and the standard deviation according to power classes. Since high-power transmitters normally have a large coverage range, a small decrease in usable field strength will result in a noticeable increase in area coverage, whereas this increase would be much less pronounced if the same decrease in usable field strength were obtained for a lower-power transmitter. For this reason it is advisable to classify transmitters according to their (maximum) effective radiated powers and to adapt the linear channel distribution scheme to the real transmitter network so that, in principle, higher-power transmitters have a lower usable field strength than lower-power transmitters.

Apart from the advantage there are, of course, also a number of drawbacks. The most important of these is that terrain irregularities and varying transmitter characteristics cannot satisfactorily be taken into account. As the lattice planning method was developed assuming identical propagation conditions, antenna heights and directivities, and power levels throughout the planning area, actual deviations from these assumptions may lead either to inefficient use of the spectrum or to excessive interference, as the case may be. Excessive interference may perhaps be reduced or even eliminated subsequently on the basis of a careful analysis of the plan, e.g. by introducing directional antennas, reducing antenna heights, adapting powers, or the like. However, it remains questionable whether a plan modified in this way would not suffer some loss of spectrum utilization efficiency.

4. Planning methods using elements of graph theory

4.1 General

In view of the explanations given above it is obviously desirable to develop an algorithm according to which frequencies can be assigned to transmitters in such a way that either:

— a minimum number of contiguous channels is used to satisfy all requirements with the level of interference between transmitters being kept below a predetermined value; or

— the level of interference between transmitters is kept to a relative minimum when all requirements are satisfied with a predetermined number of contiguous channels.

In solving this optimization problem, real propagation conditions and the actual characteristic data of all the transmitters involved should be taken into account.

Normally, the first alternative is used in theory only, to determine the spectrum space necessary for full area coverage with one programme. The second alternative corresponds to frequency assignment practice. In either alternative it may be useful to assume that the permissible level of interference depends on the individual transmitter's effective radiated power. Association of the lower permissible interference levels with the higher effective radiated powers leads to improved coverage.

To reach this goal, it has been proposed that use be made of some elements of graph theory. [17] In the preparation of the Regional Administrative Conference for FM sound broadcasting in the VHF band (Region 1 and certain countries concerned in Region 3) (Geneva, 1982/84), this proposal was taken up and further developed [18, 19] for that particular purpose. However, it may equally well be applied to the planning of frequencies for television.

In the following description, the underlying problem of coverage maximization has been simplified by assuming that the usable field strength at the transmitter site is representative of the usable field strength at the coverage contour. The better the frequency assignment plan, the more justified this assumption.

4.2 Interference potential

Assuming that N transmitters belong to a network for which a frequency plan is to be established, it is possible to calculate on the basis of real propagation conditions and actual transmitter characteristics, all the $(N-1)$ field strengths to be expected at the site of each of the N transmitters.

Some of these field strengths will be below a threshold level which is low enough to ensure that even under conditions of strongest interference, i.e. co-channel interference in almost all cases, no noticeable contribution to the usable field strength of the affected transmitter is to be expected. This threshold level E_{th} may be assumed to be below the reference usable field strength E_{ref} by the co-channel protection-ratio value A_0, enlarged by a margin M which is determined according to experience and which takes account of the cumulative effect of multiple interference: $E_{th} = E_{ref} - A_0 - M$. It is convenient not to consider any field strength below the threshold and its respective potential source of interference. In a more refined procedure, the threshold level may be adapted to each wanted transmitter's effective radiated power P expressed in dB relative to a unit power, by applying a correction factor $f(P)$, e.g. $f(P) = 0.5\,(P_{max} - P)$.

The interference potential can be presented in the form of a field strength matrix M_E:

$$M_E = \begin{pmatrix} E_{1,1} & \cdots & E_{1,j} & \cdots & E_{1,N} \\ \vdots & & \vdots & & \vdots \\ E_{i,1} & \cdots & E_{i,j} & \cdots & E_{i,N} \\ \vdots & & \vdots & & \vdots \\ E_{N,1} & \cdots & E_{N,j} & \cdots & E_{N,N} \end{pmatrix}$$

The various elements in the matrix represent potential interference contributions, namely:

- those to transmitter i in the line i; $1 \leq i \leq N$;
- those from transmitter j in column j; $1 \leq j \leq N$; $j \neq i$.

As transmitters do not cause interference to themselves it is convenient to put "0" in those cases where $i = j$; this is also appropriate in cases where interference is below the threshold.

All elements that are different from 0 indicate the eventuality of interference when transmitters i and j are operated in the same channel. This result can be presented either as a graph (figure 12) or in the form of a simplified matrix, the so-called coupling matrix M_c which can be derived from matrix M_E. The elements of M_c are either 1 or 0, depending on whether the corresponding element in M_E exists or is 0.

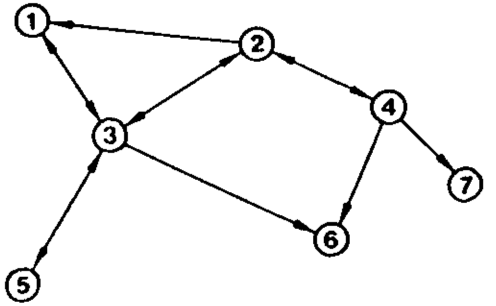

Figure 12—Transmitter network in graph presentation

Neither of the two matrices M_E and M_c is necessarily symmetrical, as the effective antenna heights and effective radiated powers of the transmitters involved are generally not identical.

In the graph, the N transmitters are represented by N vertices and their potential interference by connecting lines with arrows indicating the direction of the potential interference. In general, potential interference is bidirectional but where there are differences in the characteristic data of the transmitters it may seem to be unidirectional. Figure 12 shows an example with $N = 7$; the corresponding coupling matrix is:

$$M_c = \begin{pmatrix} 0&1&1&0&0&0&0 \\ 0&0&1&1&0&0&0 \\ 1&1&0&0&1&0&0 \\ 0&1&0&0&0&0&0 \\ 0&0&1&0&0&0&0 \\ 0&0&1&1&0&0&0 \\ 0&0&0&1&0&0&0 \end{pmatrix}$$

Coupled transmitters, i.e. transmitters involved in the same interference effect by either causing or suffering it, can apparently be identified much more easily from the graph than from the matrix, although there is no major difference in the two presentations. Coupled transmitters are connected by lines in the graph, and they have a "1" where line i and column j or, alternatively where line j and column i intersect.

4.3 Graph colouring

To facilitate the explanations, it is first assumed that co-channel interference alone is to be taken into account. In such simplified conditions the problem of assigning frequency channels is identical to the so-called graph colouring problem, i.e. the problem of colouring a graph so that coupled vertices will always have different colours, using the minimum number of colours. Without going into the details of graph theory, a heuristic way of solving the problem is to assign colour 1 (i.e. channel 1) to the vertex having the highest degree of coupling. Thereafter, the same colour (channel) may be assigned successively to other vertices also, following an order of decreasing degree of coupling, provided that none of these vertices is coupled with any one of those to which this colour (channel) has already been assigned.

The degree of coupling can be seen equally well from the graph or the corresponding coupling matrix. It may be helpful, however, to rearrange the coupling matrix according to a decreasing degree of coupling before assigning colours (channels).

In this simplified procedure the direction of interference is irrelevant since only bidirectional co-channel interference exists. Thus the arrows in the graph can be ignored and the corresponding coupling matrix can be symmetrized by replacing all elements $c_{i,j}$ and $c_{j,i}$ by the larger of the two values.

The symmetrized coupling matrix M_{cs} resulting from figure 12 or the corresponding coupling matrix M_c is

$$M_{cs} = \begin{pmatrix} 0&1&1&0&0&0&0 \\ 1&0&1&1&0&0&0 \\ 1&1&0&0&1&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&0&0&0 \\ 0&0&1&1&0&0&0 \\ 0&0&0&1&0&0&0 \end{pmatrix}$$

In this matrix, transmitter 3 (line 3) exhibits the highest degree of coupling; thus channel 1 is assigned to it. Transmitter 3 is not coupled to transmitters 4 and 7, of which transmitter 4 has the higher degree of coupling. Consequently, channel 1 can also be assigned to transmitter 4. Because of coupling between transmitters 4 and 7, no further use can be made of channel 1.

Before continuing the procedure and assigning channel 2, all reference to transmitters 3 and 4 should be eliminated from figure 12 and from M_{cs}, since these transmitters can never again become involved in interference as long as co-channel interference alone exists and channel 1 is not used any further. The elimination of this reference entails the deletion of vertices 3 and 4 and of all the lines connecting these vertices to others from figure 12, or the deletion of lines and columns 3 and 4 from M_{cs}, respectively. The result in the figure 12 example is that only the line connecting transmitters 1 and 2 is retained, or the two elements "1" in M_{cs} indicating coupling between these transmitters only. As the degree of coupling is equal for both these transmitters, channel 2 can be assigned at will to one of them, e.g. to transmitter 1. Subsequently, channel 2 can also be assigned to transmitters 5, 6, and 7, which are coupled neither to transmitter 1 nor to each other. Finally, channel 3 can be assigned to the only remaining transmitter, i.e. transmitter 2.

In this simplified example, appropriate assignment of three channels to the transmitters involved enabled the potential interference shown in the field strength matrix to be eliminated. It should be noted that the three channels are often used differently. It would, however, have been possible to equalize their utilization by substituting channel 3 for channel 2 at any one of transmitters 5, 6 or 7.

It is clear from this explanation that both presentations of the problem, the graph and the matrix, are equivalent and produce the same result. In cases involving a large number of transmitters, however, the graph method is preferable for manual treatment and the matrix method for computer-aided procedures.

4.4 Considerations of all types of interference

It is obvious that in practice all types of interference will have to be taken into account. Additionally, the number of available channels will in some circumstances be predetermined, so that it may be difficult to find a satisfactory solution. Thus the problem consists in eliminating, or at least minimizing, all potential interference by assigning "appropriate" frequency channels to the various transmitters. In this context a channel is "appropriate" when the differences between any two channels are such that either:

- no interference can arise at all; *or*
- the interference is below a corrected threshold. The correction is applied to the threshold in order to account for the real channel separation; its numerical value is the difference between the RF protection ratios required for co-channel conditions and for the channel separation actually envisaged: $E_{th\,corr} = E_{th} + (A_0 - A)$.

In VHF sound broadcasting with 100 kHz channel spacing, the most important types of interference are 1st, 2nd, 3rd, and 4th adjacent channel interference and IF generation. The latter will occur only in areas of high field strengths and is a problem of receiver immunity against strong signal effects rather than one involving the emissions. It is therefore not surprising that no RF protection ratios exist for this type of interference. Generally, IF generation is considered only when co-sited transmitters are involved. Interference caused by the receivers' local oscillators is not considered in the following explanations, as the pertinent channel separation is also involved in IF generation.

Generally speaking, the types of interference to be considered in relation to television are: adjacent channel interference, IF generation and second channel interference. Once again there are no RF protection ratios for IF generation, and consideration of this type of interference (as well as of oscillator radiation), is normally restricted to transmitters with identical or very similar coverage areas.

In both VHF sound broadcasting and television it may be necessary to respect some planning constraints, e.g. minimum channel separation between co-sited transmitters. Neither such constraints nor IF generation can be derived from the field strength matrix; according to requirements they will have to be introduced in the coupling matrix as an addition.

When determining the corrections to the threshold, it is important to take account of the fact that interference may be:

- equal in both directions, e.g. all interference in VHF sound broadcasting;
- different in each of the two directions, e.g. adjacent channel interference in most television systems;
- unidirectional, e.g. second channel interference in television.

When one of the latter two cases of interference is involved, the asymmetrical interference effect will influence the respective threshold values and, as a consequence, either reduce or amplify any asymmetry already existing in the interference potential.

The following explanations are based on the matrix form of presentation of the problem. Naturally, this does not mean that the graph form would be inadequate or useless. The matrix form is taken as it facilitates the use of computer aids.

Before starting the channel assignment procedure, it is advisable to replace the general coupling matrix by a more detailed channel separation matrix M_{ch} which can be derived from the field strength matrix by retaining all elements $E_{i,j} = 0$ and replacing all elements $E_{i,j} \neq 0$ by the exact channel separation required to eliminate potential interference, i.e. the separation for which the RF protection ratio has its greatest possible value.

As in the previous section, assignments will be made following an order of decreasing degree of coupling. This degree of coupling is equal to the larger of the two numbers of elements differing from 0 which appear in the line or column corresponding to the transmitter in question. To facilitate assignments, it may be helpful first to rearrange the channel separation matrix according to the degree of coupling.

If a completely new plan is to be established, it is advisable to start with the assignment of a channel situated at one of the two extremities of the frequency band to be planned. There are no strong grounds for preferring one or the other end. In the following examples planning will always begin with the lowest channel, i.e. channel 1. Care should be taken to ensure that equal use is made of the available channels C. Departures of more than 10 to 15% N/C assignments per channel should be avoided.

4.4.1 VHF sound broadcasting

In VHF sound broadcasting the RF protection ratio decreases monotonically with increasing channel separation, so that the channel separations (1, 2, 3 or 4, depending on the case) are minimum ones. It is convenient to enter a minimum channel separation "5" into the matrix when, even for the lowest RF protection ratio value, $E - E_{th\ corr} \geq 0$. Because of the symmetry in the interference effects, it is advantageous to symmetrize the channel separation matrix, i.e. to replace the elements $ch_{i,j}$ and $ch_{j,i}$ by the larger of the two values, before starting the assignment procedure.

The procedure itself starts with the successive assignment of channel 1, following an order of decreasing degree of coupling, to transmitters which are not coupled to any of the transmitters to which a channel has already been assigned. Once a channel has been assigned to a transmitter, the column corresponding to that transmitter may be eliminated from the matrix. The interference which might be caused by later assignments can still be recognized from the lines corresponding to that transmitter.

Again following an order of decreasing degree of coupling, channel 2 is then assigned so that no unacceptable interference is caused to the transmitter to which either channel 1 or channel 2 has previously been assigned. This can be seen from the columns of the channel separation matrix corresponding to the transmitter to which the channel is to be assigned. As the elements represent minimum channel separations, a "1" is required (a "0" would be even better) in lines corresponding to transmitters with channel 1 assignments, while a "0" is required in lines corresponding to transmitters with channel 2 assignments.

Further channels (3, 4, 5, etc.) are assigned successively, following the same rules as for channel 2. However, instead of checking only possible first adjacent channel interference, all other types of interference which may be involved will have to be checked as well. An assignment is always acceptable if the channel separation actually envisaged is equal to, or greater than, the minimum channel separation indicated by the respective element of the matrix.

The number of channels to be checked when assigning the first five channels will increase steadily and then remain constant until the last channel has been assigned. However, it should be borne in mind that additional checks may be required to take account of planning constraints or IF generations.

4.4.2 Television

In television, matters are more complicated because the interference effects are not symmetrical and do not necessarily decrease with increasing channel separation. Taking television system G as an example, typical RF protection ratio values are +3, −6 and −12 dB for second channel, lower adjacent channel and upper adjacent channel interference, respectively. The respective channel separations to be entered in the matrix are 9, −1 and +1.

Hence the elements of the matrix are "characteristic" rather than minimum separations. Where $E - E_{th\ corr} \geq 0$, another characteristic figure, e.g. "10", may be used to indicate the potential interference can only be eliminated when a channel which is not liable to cause any interference is selected. Hence in television the significance of the elements in the channel separation matrix is shown in table 3.

Table 3

channel separation	permitted type of interference from transmitter j to i
0	all types, including co-channel interference
9	all types except co-channel interference
−1	lower and upper adjacent channel interference only
+1	upper adjacent channel interference only
10	no interference at all

Because of the asymmetrical interference effects, it is not advisable to symmetrize the channel separation matrix before starting the assignment procedure, as this would endanger interference minimization. It is, however, appropriate to give further study to some of the consequences of the asymmetry in interference effects. If the channel assignment procedure is started with the lowest channel (as in this example), neither second channel nor upper adjacent channel interference can be caused *by*, but only *to*, transmitters to which a channel has already been assigned. On the other hand, lower adjacent channel interference can only be caused *by*, but not *to*, such transmitters. Thus, elements "9" and "+1" in columns pertaining to such transmitters are of no concern; in these particular circumstances, their significance is identical to "−1" or "10", respectively. Whether or not a second channel or an upper adjacent channel may be assigned to a particular transmitter can best be deduced from the elements in the relevant column of the matrix, where it intersects with lines representing transmitters to which a channel has already been assigned. These elements should be "9" or "0" in the case of a second channel assignment, or "+1", "−1", "9" or "0", i.e. any value except "10", in the case of an upper adjacent channel assignment, provided that in the latter case the symmetrical element in the corresponding column is "−1", "9" or "0". To facilitate checking of this condition it is expedient, immediately a channel has been assigned to a transmitter, to "symmetrize" the corresponding line and column by replacing "+1" by "10" in the line if the aforementioned condition is not met.

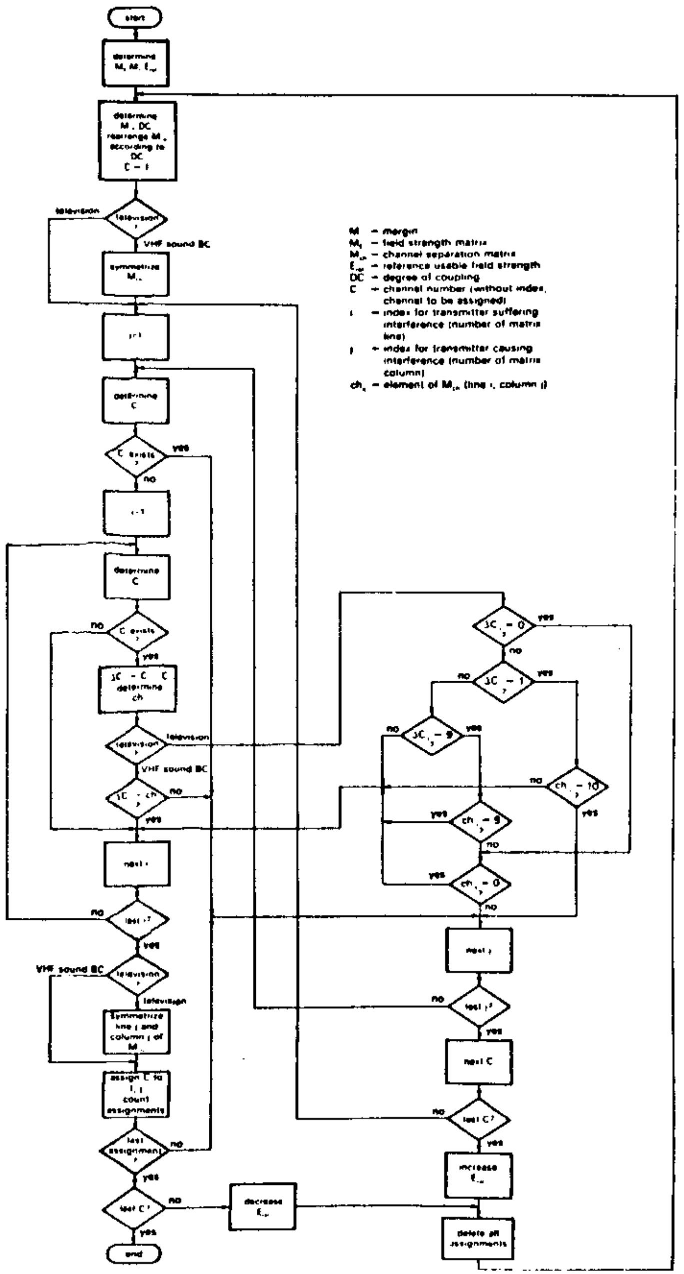

Figure 13—The optimization procedure according to the method using elements of graph theory

If this is done after each individual assignment, channel selection becomes fairly easy and columns corresponding to transmitters to which a channel has already been assigned may be deleted.

This assignment of channel 1 is now as easy as in the case of VHF sound broadcasting. Channel 2 may then be assigned to transmitters coupled, except by "10", with transmitters in the lower adjacent channel, and not coupled with transmitters to which the same channel has already been assigned. The permitted type of transmitter coupling can be identified from the element found at the point where the column of the transmitter to which a channel is to be assigned intersects with the line of the transmitter to which a channel has previously been assigned.

Channel 3 may be assigned to transmitters coupled, except by "10", or not coupled with transmitters in channel 2, coupled or not with transmitters in channel 1 and not coupled with transmitters to which the same channel has already been assigned. Assignment of channels 4 through 9 is carried out in the same way.

Channel 10 is liable to cause second channel interference to transmitters in channel 1. Therefore the possibility of second channel interference has to be checked in addition to the checks carried out for channels 2 through 9. The characteristic channel separation, i.e. the elements in the column with the new assignment and the lines of channel 1 transmitters, should be "9" or "0". Once the last assignment is made of channel 10 to a transmitter, those lines which refer to channel 1 transmitters may be deleted from the matrix, since no further cases of interference can occur. Assignments involving channels 11 and above will require checks corresponding to those for channel 10.

4.4.3 Supplementary information

The procedure described for VHF sound and television broadcasting has to be repeated, taking due account of the various types of interference involved, until a channel has been assigned to all transmitters. The objective which is to eliminate all potential interference, can however only be reached, if the number of available channels is large enough. Otherwise the interference has to be minimized, e.g. by gradually increasing, in an iterative process, the value of the reference usable field strength E_{ref}, which was used to determine the threshold. An increase in the reference usable field strength will lead to an increase in threshold and thus result in less potential interference and less coupling. As a consequence, more frequent use of individual channels will be possible and the number of channels required for eliminating the reduced amount of potential interference will be reduced. Figure 13 shows the whole channel assignment procedure in the form of a flow chart.

Because of the symmetry in the interference effects in VHF sound broadcasting, no additional steps need to be taken when the assignment procedure is started with the highest channel of the available spectrum. This is not the case when a plan for television is to be established. However, the detailed explanations given above will make it easy to develop the conditions to be met when the assignment procedure is started with the highest channel.

The above explanations are based on the tacit assumption that the co-channel protection ratio used to determine the interference potential corresponds to a frequency offset between carriers which is equal to (small) integral multiples of 1/3 the line frequency but which is not an integral multiple of the line frequency itself. In actual practice, these conditions can be met for three assignments of the same channel, whereas the fourth and any further assignment of the channel would involve non-offset operation. Fortunately, the distance separation between co-channel transmitters for which non-offset operation is unavoidable is usually considerably larger and, as a result, the interference potential is so much smaller than in cases where offset operation is practicable that the difference in RF protection ratio is compensated for or even exceeded. It seems, therefore, that no effort needs to be made to take non-offset interference into account from the outset. In desperate cases it may be sufficient to make full use of appropriate integer multiples of 1/12 the line frequency.

In some circumstances, the problem may be to find only appropriate channels for transmitters situated within a limited area, given that channel assignments for the remaining transmitters already exist and are to be preserved. The solution to this problem may be quite different from that described above. In such circumstances it is unlikely that channels can be assigned successively in their numerical order of sequence. However, it is still important that they should be assigned according to decreasing degree of coupling. Thus, following this order of priority, the "best" channel is selected to satisfy the "worst" requirement. [19] If more than one channel could be used, the best channel can be found by gradually reducing, in an iterative process, the reference usable field strength; if no channel can be found, it will be necessary gradually to increase the reference usable field strength of that transmitter. This procedure is to be followed for one transmitter after another until a channel has been assigned to each one.

5. Conclusions

Sections 2, 3 and 4 describe three methods for assigning appropriate frequency channels to transmitters for sound or television broadcasting. The channels are selected for assignment in such a way that interference will either not exceed a predetermined limit or be kept to the minimum practicable. In this way an attempt is made to maximize the use of the available frequency spectrum.

The minimum separation distance method developed in the United States provides a "go/no go" answer to the question whether a particular channel may be assigned to a transmitter. The method is therefore particularly useful when channels are to be assigned to transmitters added to an existing network. When establishing a completely new plan, use of the method would not necessarily lead to maximum spectrum utilization efficiency. Practical application of this method is reported from both the United States and the latest Regional Administrative MF Broadcasting Conference for Region2 (First Session, Buenos Aires 1980; Second Session, Rio de Janeiro 1981).

The lattice planning method developed in Europe is most suitable for the establishment of completely new plans providing coverage to large areas. Its usefulness is fairly limited, when channels are to be selected for assignment to transmitters supplementing existing networks. The method is particularly attractive because it provides, in the environment of the transmitter, a choice between several appropriate channels from which, depending on the circumstances, the best one can be selected. The method can be applied manually and it permits uninterrupted supervision of the planning process and the progress made. It also permits simultaneous planning in different parts of the planning area and requires very little mutual respect. Adjustment of the various pieces of the plan to one other will normally not be too difficult. The method has been used successfully at a number of regional conferences for the planning of sound and television broadcasting. Its major drawback is that it is based on complete regularity with respect to both propagation conditions and the characteristic data of the transmitters involved. Hence any major departure from this regularity may call in question the resulting spectrum utilization efficiency.

The method using elements of graph theory was developed fairly recently and has been supplemented by various authors. It is suitable for establishing completely new plans as well as for selecting the most appropriate channels for transmitters added to existing networks. It has the advantage of taking into account acutal propagation conditions and the real characteristic data of all the transmitters involved. It can be used far more efficiently with the help of computers. Its manual application may prove laborious, tedious and time-consuming. Simultaneous planning in different parts of the whole planning area is not possible and unsurmountable difficulties would arise in trying to fit together the various pieces of the plan. Since the method is fairly new, it has never been applied in practice for the establishment of completely new plans, but it was applied successfully at the Regional Administrative Conference for the planning of VHF sound broadcasting (Region 1 and parts of Region3) (Geneva, 1984) to improve channel assignments in a limited area involving about 100 transmitters.

It is therefore conceivable that the combined use of the lattice planning method and the method using elements of graph theory could be the best way of solving the problems of future conferences. Particularly when it is used simultaneously in different parts of an extended planning area, the lattice planning method might provide sound basic pieces of a plan which could subsequently be refined and improved and, finally, fitted together by applying the method using elements of graph theory.

(Original language: English)

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