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Detection of Coded Signals - Soft and Hard Decision

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DETECTION OF CODED SIGNAL - SOFT AND HARD DECISION

by

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Thus far, we have seen how to design a communication system that is capable of efficiently communicating one of M messages. The problem of implementing an efficient receiver was derived under the assumption that each member of the ensemble of communication systems has an optimum receiver.

Optimum receivers for M signals { si(t) } with N { gi(t) } as an orthonormal basis is implemented by a bank of matched filters to the { gi(t) } followed by circuits to compute the dot product of the received signal and the set of signals.

The main problem with this implementation lies on the fact that its complexity grows faster than linearly with N. If we accept some loss in performance by allowing a decoder to be included, then it will assume a role of central importance due to its flexibility in processing data. Basically, what it is being proposed is that each of the N components of the received vector be quantized to Q levels. Note that this operation is irreversible and so introduces degradation.

A measure of this degradation is the R_o' parameter, whereas the Ro measures the goodness of a communication system using parity check coders, transducers and optimum (unquantized) receivers.

Once knowing these facts, we are going to consider quantized channel models.

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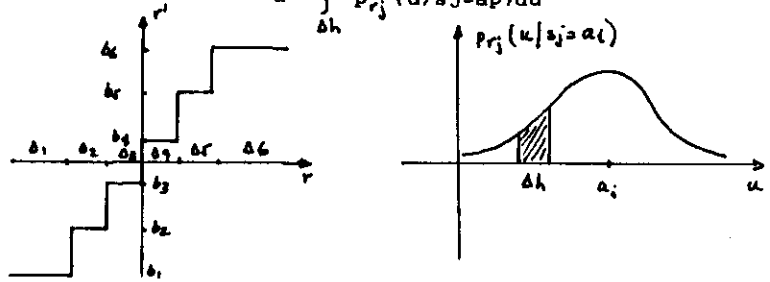
With AWGN channel each component sj of s = (s1, s2, ..., sN) is corrupted by addition of an i.i.d. Gaussian random variable. Thus, if { ap } is the transmitter alphabet, when sj = ap the j-th component of the received vector r has density function

$$p_{r_j}(u/sj=ap) = \sqrt{(1/\pi\, No)}\,.\exp\{-(u-ap)^2/No\}$$

Thus, the quantizer maps rj into an output component r'j that can not assume an arbitrary value but it is restricted to some quantizer output alphabet { bh } h = 1, 2, ..., Q. Given sj = ap, the probability that r'j is bh is

$$Qph = P[r'j = bh/sj = ap]$$

$$= \int_{\Delta h} p_{r_j}(u/sj=ap)du$$



Therefore, from the transmitter alphabet { ap }, 1 ≤ p ≤ A and quantizer output alphabet { bh }, 1 ≤ h ≤ Q, we have the transition probabilities { Qph } and so we have established a discrete memoryless channel with A-ary input and Q-ary output.

Once the discrete channel is established, the next step is related to evaluation of the measure of degradation. It can be shown [4], after some rather sophisticated argument, that

$$R_o' = -\log_2 \sum_{h=1}^{Q} \left[\sum_{i=1}^{A} p_i \sqrt{Q_{ih}} \right]^2$$

where $p_i = P[\, s_j = a_i \,]$.

It is also possible to show that as the quantization gets increasingly fine that R_o' tends to R_o, and it is given by

$$R_o = -\log_2 \int_{-\infty}^{\infty} \left[\sum_{i=1}^{A} p_i \sqrt{p_r(u/s=a_i)} \right]^2 du$$

which is the same as the unquantized measure.

Example: Let the transmitter alphabet be { +c, -c } and p(+c) = p(-c) = 1/2. For unquantized Gaussian noise

$$p_n(u) = \sqrt{(1/\pi N_o)}.\exp\{-u^2/N_o\}$$

then,

$$R_o = -\log_2 \int_{-\infty}^{\infty} [\ (1/2).[\sqrt{(1/\pi N_o)}.\exp\{-(u-c)^2/N_o\}]^{1/2} du\ +$$

$$+\quad (1/2).[\sqrt{(1/\pi N_o)}.\exp\{-(u+c)\ /N_o\}]^{1/2}]^2 du$$

After some algebraic manipulations, the cutoof rate Ro is given by

$$R_o = 1 - \log_2(\ 1 + \exp(-c/N_o)\ )$$

Now let us apply the measure Ro to some quantization schemes of interest.

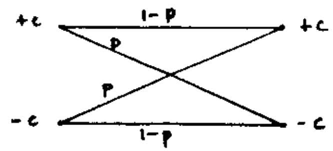
CASE I: A = 2 and Q = 2 (Hard decision)

This is the binary input binary output channel. If a1 = +c and a2 = -c, the matched filter output at the receiver is also quantized to two levels. Therefore,

$$Q_{12} = Q_{21} = p \qquad \text{and} \qquad Q_{11} = Q_{22} = 1-p$$

where $p = Q(\sqrt{(2.c/N_o)}\ )$.

This channel is called Binary Symmetric Channel (BSC) and it is represented by



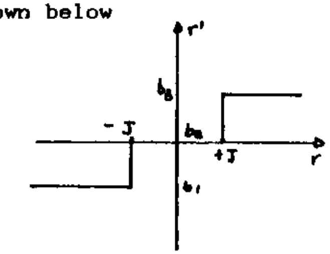
Due to symmetry, p1 = p2 = 1/2, and so

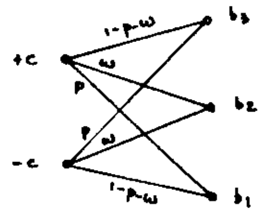
$$R_o' = -\log_2 \sum_{h=1}^{2} \left[\sum_{i=1}^{2} p_i \sqrt{Q_{ih}} \right]^2$$

$$= 1 - \log(\ 1 + \sqrt{4.p.(1-p)}\ \ )$$

CASE II: A = 2 and Q = 3 (soft decision)

A significant improvement in Ro resulting from binary quantization can be achieved by going to ternary quantization. The quantizer and the corresponding channel for A = 2 and Q = 3 are shown below





with

$$Q_{12} = Q_{21} = p, \quad Q_{13} = Q_{31} = w, \quad \text{and} \quad Q_{11} = Q_{22} = 1 - p - w$$

where

$$p = \int_{J}^{\infty} \sqrt{(1/\pi No)}.\exp\{-(u+c)^2/No\}du$$

$$w = \int_{-J}^{J} \sqrt{(1/\pi No)}.\exp\{-(u+c)^2/No\}du$$

such a channel is called Binary Symmetric Erasure Channel (BSEC). Again, p1 = p2 = 1/2, and so

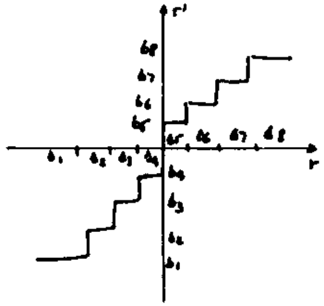
$$R'o = -\log_2 \sum_{h=1}^{3} [\sum_{i=1}^{2} pi \sqrt{Qih}\ ]^2$$

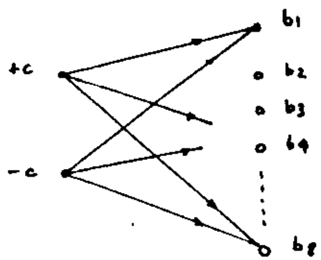
$$= 1 - \log_2(\ 1 + w + \sqrt{4.p.(1-p-w)}\ )$$

Note that $R'o$ can be optimized by choosing J conveniently.

CASE III: A = 2 and Q = 8 (3 bit quantizer)

This is the most frequently used quantizer in digital communication systems. Again a1 = +c and a2 = -c, and the quantizer output alphabet is { b1, b2, ..., b8 }. The quantizer and the corresponding channel for A = 2 and Q = 8 are shown below





with

$$p_{rj}(u/sj=ap) = \sqrt{(1/\pi\ No)}.\exp\{-(u-ap)^2/No\}$$

and

$$Qph = P[r'j = bh/sj = ap]$$

$$= \int_{\Delta h} p_{rj}(u/sj=ap)du$$

Up to now, we have been doing analysis for one dimensional quantizers. It is also possible to generalize these analysis to N-dimensional quantizers. The procedure is analogous to the one done so far.

As a breef comment, for two dimensional quantizers we can have hard and soft decision quantizers. We show this in the Figs. below

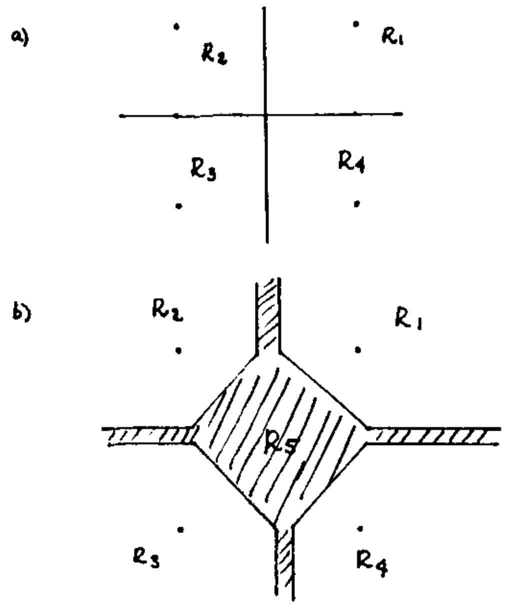


Fig. 1 - a) 2-dimensional hard decision quantizer

b) 2-dimensional soft decision quantizer