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REGRESIVE METHODS COMPLEX DEMODULATION-INVERSE PROBLEM

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Same additions to "Statistical Analysis of Ionospheric Parameters ..."

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Introduction

If ionosphere were an unchanged system a lot of problems can be solved easily because in nowadays there are mathematical models which describe the behavior of the ionosphere. But the ionosphere is like a living organism and it is different in every moment and a model which can adequately describe time variations just is not created. The ionosphere is a system of particles having a charge and its behavior is controled by the earth magnetic field. The axis of the magnetic dipole and the earth rotation axis are differ in about 11 degree that's why we have a daily disturbances in ionospheric parameters. Daily, monthly, seasonal, annual and more long period variations are present in ionospheric parameters. And of course the situation in ionosphere has a strong dependence upon the solar activity. The Earth is plunged into solar wind which have a non-regular structure. Due to "freezing in"-effect the solar wind carry solar magnetic field. At every time the solar wind parameters differ: magnetic field named IMF, velocity and density of particles and some other.

One limit aspect in regresive analysis

You can see what kind of complex situation should search the scientists. And of course all questions should be putted in the correct way. Mathematic is a powerfull instrument and we should use it carefully. Both for the mathematic and for the computers you use there are no differences in what sort of data you put into memory and process: the solar wind parameters or number of dots on every second page of some book. After processing of this data you obtain some answer but it can be misunderstud what it means from the physical point of view.

Let's take the dependence of the IMF and ground measured magnetic variations. This variations are created by currents in the ionosphere. On fig. 1 we present an example of the three components of the IMF from near Earth satellite and variations of three components of Earth's magnetic field measured on the ground in near magnetic pole region.

Is there any relationship between these fields or not? How to understand and how to start the search? We can start the search from extracting some periods of the variations. For this porpose we can use for example the low-pass frequency filter. On the matematic language the averaging of data sets i.e. summation and dividing on a common number of

elements

$$B = \frac{A1 + A2 + A3 + \ldots + An}{n}$$

is equivalent to filtering of data. But the averaging as follow

A1+A2+A3	A4+A5+A6	...	An-2+An-1+An
B1	B2		Bk

is not a filtering from the point of view you heared earlier. For filtering we sould use the moving average. For example the three points average should look like as follows

$$(A1+A2+A3)/3 = B1$$

$$(A2+A3+A4)/3 = B2$$

...

$$(An+An+1+An+2)/3 = Bn.$$

The parameters of such kind of filter (main frequency, frequency band, quality) depends on the number of points and weights used for averaging. We can modify the filter parameters using different weights, for example a 6-points average can look like this:

(1*A1+2*A2+3*A3+3*A4+2*A5+1*A6)/12 = B1

(1*A2+2*A3+3*A4+3*A5+2*A6+1*A7)/12 = B2

...

In the appendix B the Fortran program for low-pass filtering of data by means of 6-points averaging are given.

After we select the sort of data and extract some periods in data sets we can go to regression analyses. And in this moment is's very important to understand the real physical situation. For the above example the situation is as follows: the flow of the solar wind has its own magnetic field and due to the electromagnetic effect, an electric field vector E in coordinate system connected with Earth is created.

$$E \sim -[U \times B]$$

were U is the solar wind velocity and B - the interplanetary magnetic field. The electric field is created by that components of IMF which are across vector U, i.e. Bz and By in most of cases. If this electric field penetrate till to the ionosphere it can create currents in ionosphere which produce the magnetic field disturbances. If this assumption is correct we can expect a linear dependence between IMF and

ground magnetic field. On fig. 2 the dependence of ground magnetic field disturbances on Vostok station for summer season of 1979 from Bz component of IMF are presented.

Computer allows us to processe this data set and we can obtain a linear regression but the scatter of points is too large. Why? In this moment we are meeting the next underwater stone: to search the regression and correlation between some values we should make the data sets if possible of course, free from other dependences. After avoiding in the above data set the influence of By component IMF on the ground magnetic field data we receive the fig. 3.

After processing this data we obtain the correct regression line and small points scatter

$$H = (8.7 \pm 3.5)Bz \pm 2.1.$$

It means that variations of ground magnetic field in near magnetic pole regions have a linear dependece from the IMF.

One more example showing the importance of correct selection of data sets. On fig. 4 the comparison of two auroral indexes from north and south hemisphere is presented. The physical base of this comparison is the supposition that magnetic field disturbances in conjugate points should go synchronously. In this example the standart AE-index by use of 12 auroral station for the north hemisphere and data from all antarctic stations for the south hemisphere are used.

The computation gives regresion line factor 0.62 and correlation coefficient as 0.64. After separation Antarctic data set on two parts: data from auroral stations and data from polar cup stations we obtain a more better plot (see fig. 5).

The dotted line is the regression line with regression coefficiente 0.87 and correlation coefficient as 0.91. This result means that in the first data set both dependence and undependence value are presented and the second data set is a undependence value free.

Complex demodulation technique

Very often different parameters are registred simultaneosly. On fig. 6 you can see a synchronous registration of interplanetary and Earth magnetic field in diapason of pulsations.

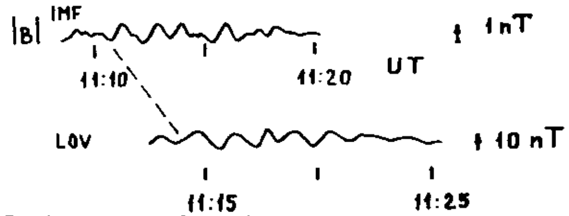


Fig.6 An example of simultanioely registrated magnetic pulsations in the near Earth space and on the Earth surface. The ground pulsation curve was shifted for near 4 min.

Nevertheless the simultaneously changing of some parameters does not mean that the one parameter changing is a cause of changing the other value. To prove it we should a) show for this case that the ground base pulsations is always present when there are IMF pulsation for the same periods and b) phases of variations change synchronously.

For the estimation of phases and amplitudes of signals in one moment you can use one more interesting technique based on FFT the so called complex demodulation. FFT provide a measure of power at different frequencies ranging from zero (linear trend) to the Nyquist frequency. The spectral estimates thus obtained are essentially an average parameter for they give the integrated effect of a frequency component during the whole span of the analysed event. The band-pass filters are the common means of obtaining the variation of amplitude with time. But sometimes we require the temporal changes in the amplitude and phase variations of the oscillation of interest during the period of the event.

The complex demodulation method consists of the following steps:

a) taking the spectrum through FFT of the original time-series;

b) chosing the frequency band around the peak of interest and filtering in frequency domain by using for example Hanning weighting:

$$W(w) = [1-\cos(2\pi*(w-w0)/dw)]$$

for $w0-dw/2 < w < w0+dw/2$, and $W(w)=0$ elsewere;

c) shifting the center of the band to zero and a new series forming by combining the positive and negative frequency components wich is subjected to inverse Fourier transform to obtain the demodulates.

The advantage of demodulates over raw data is that the former gives the uncontaminated amplitude and phase variations of a desired signal with time whereas the raw data are a combination of several signals superimposed on noise due to which the correct amplitude and phase of any desired signal can be estimated with great difficulties. On fig. 7 an example of using complex demodulation method is presented.

One example of inverse problem solving

The inverse problems are widely used in geophysics. They arise always when we try to find the parameters of the source by registrating of signals by him produced. The formulation of the inverse problem is as follows

$$y = \int K(x)dx$$

where y a signal we registrate, K(x) some function of source or the more often used formula for discret data

$$y = Ax$$

were A some m*n source depended matrix, y - n-vector of registrated data, x - m-vector source parameters. Knowing y we should find x that is obtain with:

$$x = By.$$

But the main difficulties in inverse problem are the following: for one y exist no one x and the solution is not stabl that is a small changings in y produce infinit large changing in x or gives an oscillating solution. It's due to a bad definition of A matrix.

For example let us take the magnetic variations on meridonal chain of magnetometers. It can have 8 - 12 stations which are measuring points. Let us suppose that magnetic time variations are created by currents in ionosphere on fixed altitude, in some latitude limits and the currents flow across the chain. It's more or less correct for the currents in auroral zone. The question is: what kind of fine structure of currents we can obtain? If we divide the ionosphere above the chain on current sheets then with the help of Biot-Savart law we calculate the magnetic field from each current sheet in each point of mesurement:

$$H = j/2\pi\,(\mathrm{arctg}\,[(b-x)/h]-\mathrm{arctg}[(b+x)/h])$$

$$Z = j/4\pi\,\ln[(2\,b\,h)/(h*h + (b-x)*(b-x)]$$

where H and Z are horizontal and vertical components of magnetic field on the Earth surface, j current density (constant for each sheet) in sheet, b 1/2 of wide sheet, h altitude, on wich current flow, x a distance point of measurements from current sheet in some coordinate system. The situation when having 10 stations is to try to find current density in 10 or less current sheets in latitude limits 20 degrees that is more than 2000 km will be correct but rather far from real conditions because the experimental data show more fast currents changing with distance. If the width of the current seets were smaller the assumption that on this distance current density can be modeled as constant were not far from reality. Therefore when we wish to determine m current densites in m sheets we have in each point of

mesurement

$$H_i = j/2\pi \sum_{k=1}^{m} \{\mathrm{arctg}[(b-x_k)/h]-\mathrm{arctg}[(b+x_k)/h]\}$$

$$Z_i = j/4\pi \sum_{k=1}^{m} \ln[\,2bh/h^2 + (b-x_k)^2\,]$$

or in vector form

$$H = A\,j$$

$$Z = B\,j$$

where A and B are m*n matrix. And if m > n we have an infinit number of solutions that it's a typical underdefined problem. Therefore there must be some additional constraints. To get a stable solution in this situation we should use the so called regularisation algoritm. It means that we find the solution by minimising of some functional. For our case for the H component it is

$$F(j) = [\,A\,j - H\,]^2 + a\,[j]^2 = \min$$

where a is a small parameter defined from the condition that [Aj-H] = min. So formulated, the problem have always and only one solution. The minimisation (solving to define j) can be made by different ways.

On fig. 8 the current density above the meridional chain in Antarctica is presented. Parameter a is equel to 10-7.

The next literature are used in this paper:

Agarval A.K., Singh B.P. and Nityananda N. (1980) Proc. Indian Acad.Sci. (Earth Planet.Sci.), v.89, no 1.

Banks R.J. (1975) Geophys. J.R. Astron. Soc. 43 87.

Tikhonov A.N. and Arsenin V.Ya. (1974) Methods for solving incorrect problems. Moscow (in russian).

Appendix A

How to write and run Fortran-program on Personal Computer.

You should make the next steps:

1)-to create (to write) the program with help of editor;

2)-to translate the program;

3)-to link (connect) all parts of the program and compute libraries;

4)-to run the program for execution.

There are some editors to write programs: ED, EDIV, EDLIN, NE, CW. You can chose which you like.

For translation (and compilation) FL-program are used, for linking - LINK-program. Both this program are executed by one command FTN.

To run the program type just it's name.

Example

If you wish to create the program named TEST with help of NE -editor you should make the next steps after you swich on computer and answer the question it asks:

1) NE TEST.FOR

After that type the text of your program on the screen. To save the text in the file on disk klink the keyes F3 and E.

2) FTN TEST

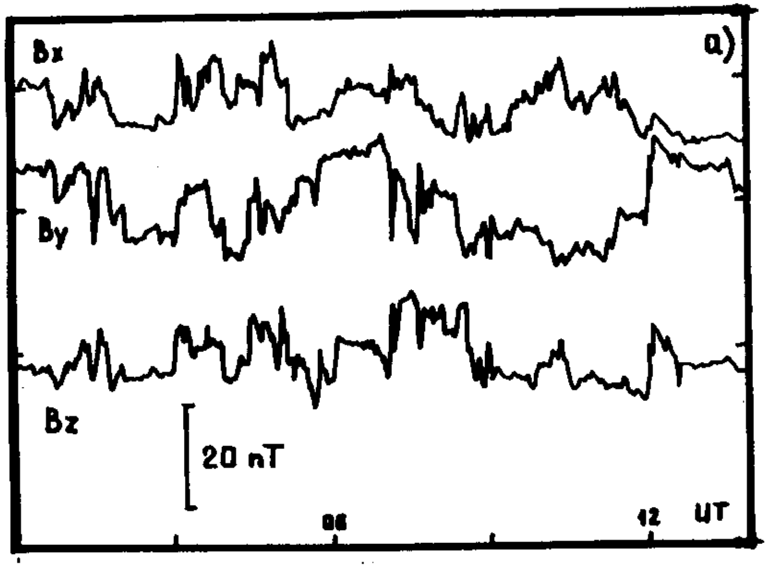
If there are any mistakes in program you should go to step 1 and correct text.

3) TEST

Appendix B

A simple Fortran-program for filtering of raw data.

```
      Program simpl
C Program for low-pass frequency filtering by use of 6-point
C        slipping averaging with difference weights.
C a - input data array, adata - name of file with input data
C b - output data array, bdata - name of file for output data
      dimension a(180),b(180)
C wei - array of weights
      dimension wei(6)
C nco - number of points when filtering
      nco=6
      nco=nco-1
      do 5 i=1,180
    5 b(i)=0.
      open(10,file='adata')
      open(11,file='bdata',status='new')
      write(*,*) 'put in the 6 weights'
      read(*,*) wei
      read(10,100) a
            do 20 ib=3,178
            ka=ib
            ic=1
            b(ib)=0.
               do 10 ia=ka,ka+nco
               b(ib)=b(ib)+a(ia)*wei(ic)
               ic=ic+1
10             continue
20          continue
      write(11,100) b
100   format(15i5)
      close(10)
      close(11)
      stop 'end of program'
      end
```



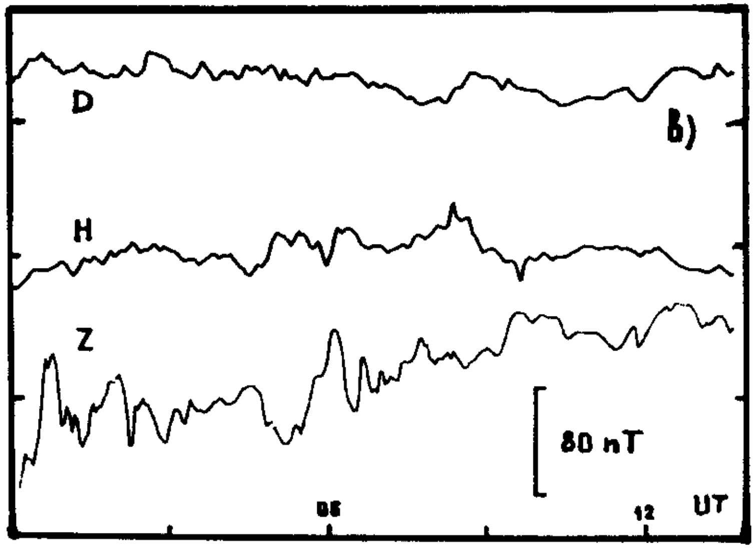


Fig.1 a) Three components of the IMF in the solar-magnetospheric coordinate system and b) three components of the ground magnetic variations on unmanned station in Antarctica with corrected geomagnetic coordinates φ =-83.7, λ = 110.7

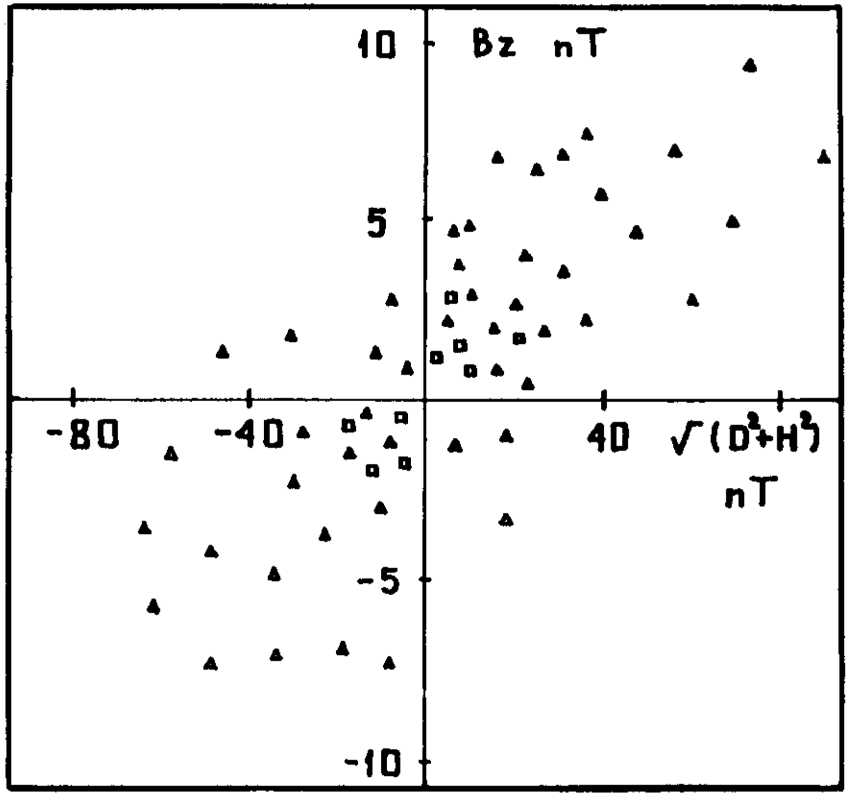


Fig.2 Amplitudes of the Bz-component of the IMF and magnetic variation on the Vostok station (corrected geomagnetic coordinates φ = -83.4, λ =43.8); every Δ is equal to 10 hour-, every □ is equel to 20 hour average measurements in period of summer season 1979.

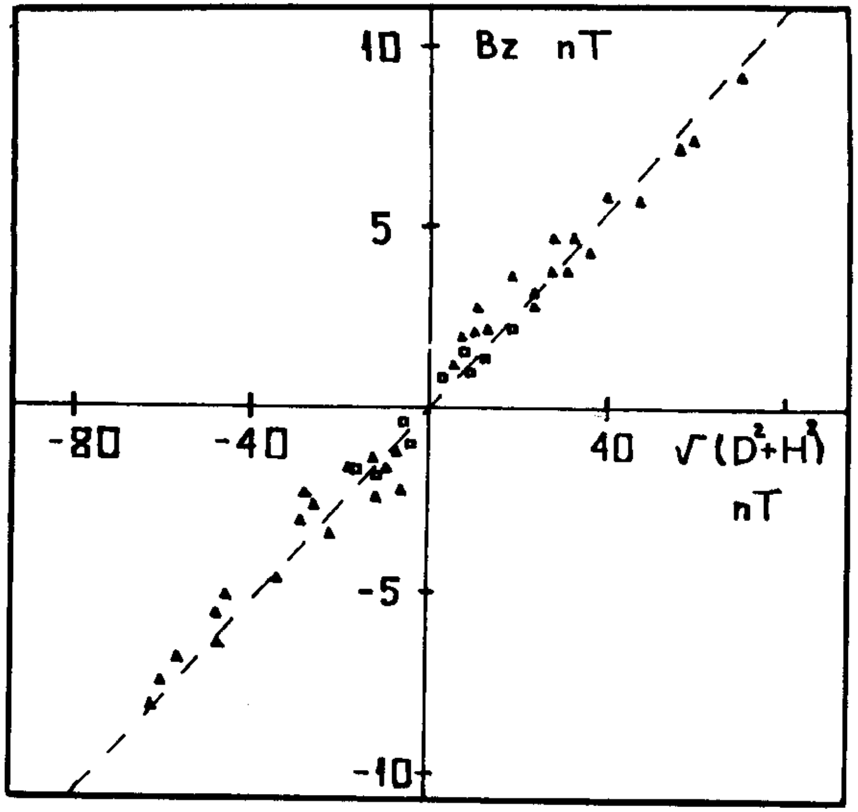


Fig.3 The same as fig.2 but the ground data are corrected for By dependence.

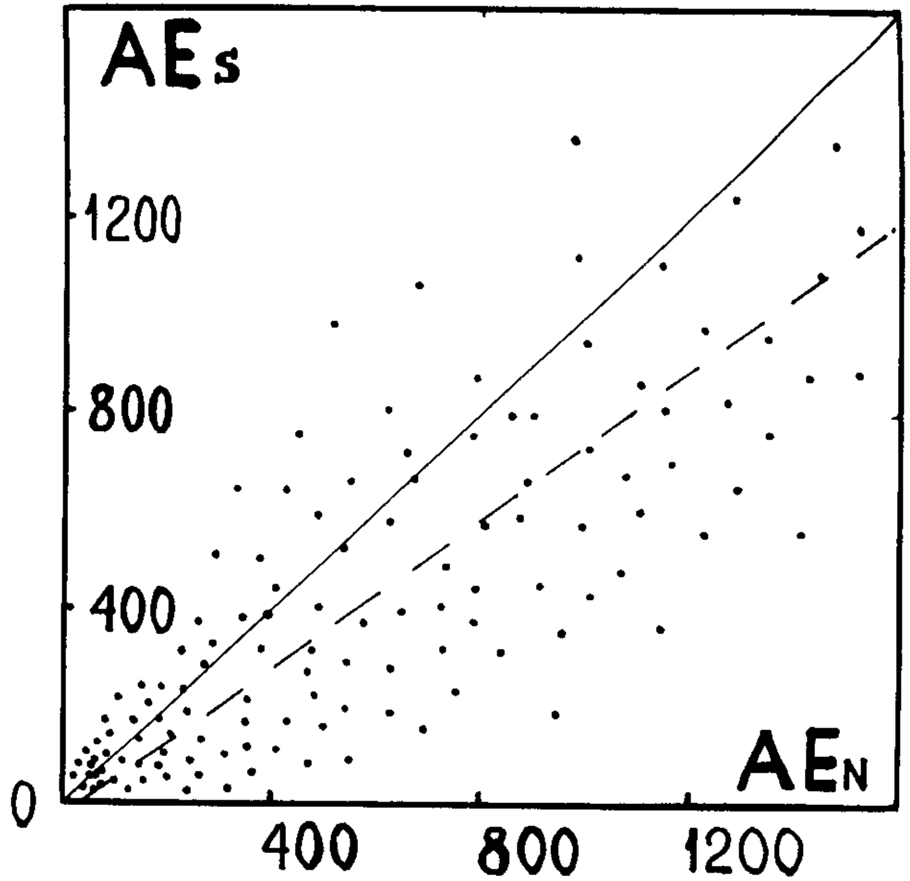


Fig.4 The comparison the Auroral indexes for the north (AEn) and south (AEs) hemispheres for period 10-13, 27-29 June 1982. Correlation coefficient R=0.65, the regresion line AEs=0.64 AEn - 92 nT and line AEs=AEn are presented. The hour average value are used.

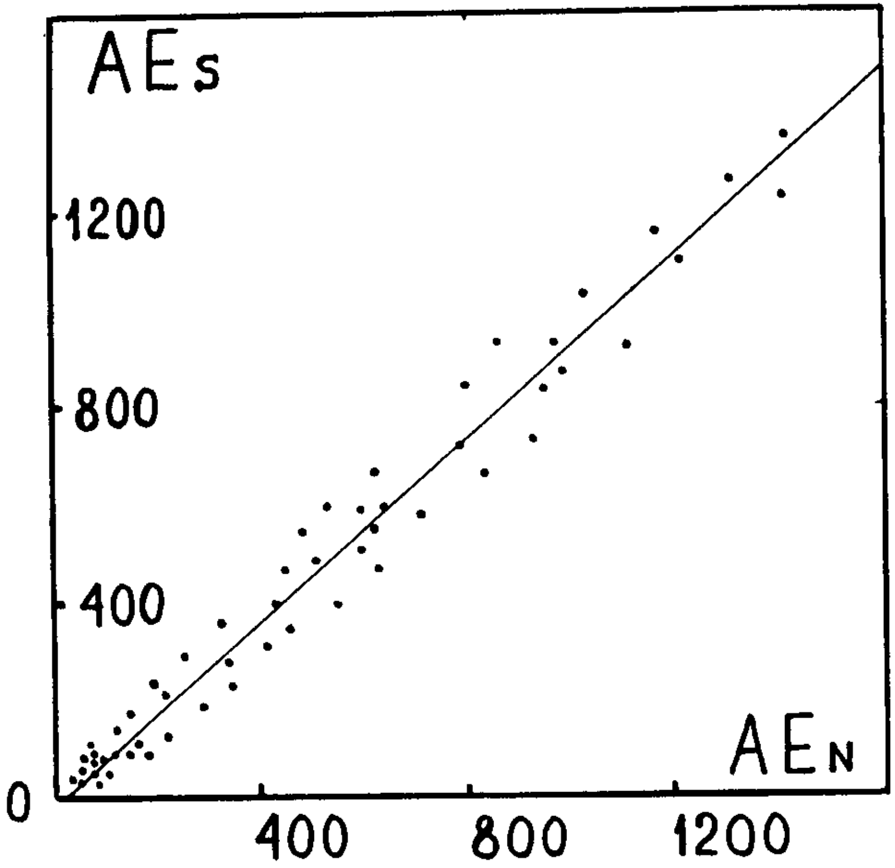


Fig.5 As fig.4 but for AEs-index only auroral stations data are used. Correlation coefficient R=0.91, the regresion line factors AEs=0.87 AEn - 25 nT.

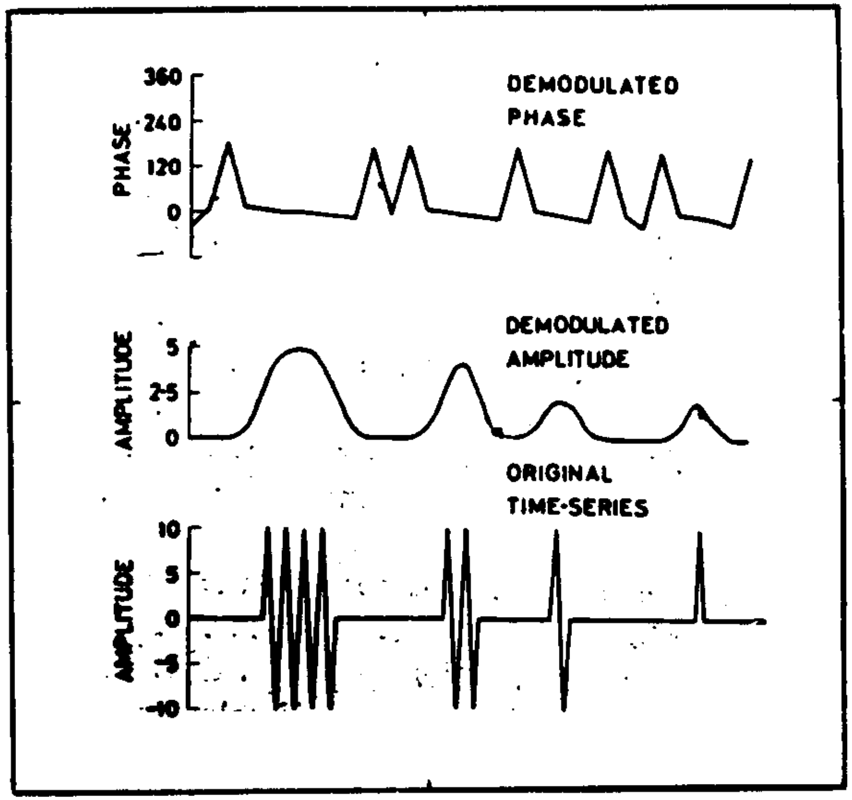


Fig.7 Amplitude and phase variations obtained through the use of complex demodulation together with the amplitude variations of the original time-series.

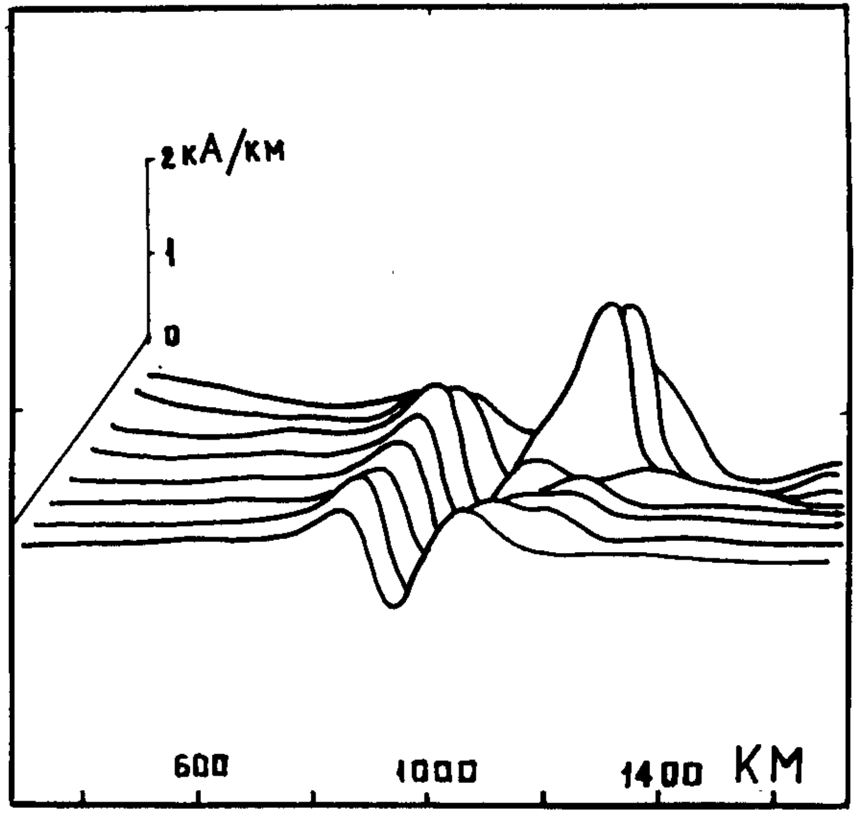


Fig.8 Currents in the near pole region obtained from meridinal ground magnetometer chain in Antarctica.