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UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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SMR.380/18

COLLEGE ON THEORETICAL AND EXPERIMENTAL RADIOPROPAGATION SCIENCE

6 - 24 February 1989

LECTURE 2: MAGNETOIONIC THEORY

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1 PROPAGATION IN AN IONISED MEDIUM

1.1 Introduction

The passage of a radio wave through free space has been discussed earlier. Here consideration is given to effects due to the presence of free electrons and ions in the ionosphere, and to the existence of the Earth's magnetic field.

The radiowaves set the electrically charged particles into oscillation and cause them to radiate tiny secondary wavelets in all directions. In the forward direction their path lengths are equal and the combined secondary wave is strong. This forward-going scattered wave travels in the same direction as the original wave and combines with it. It experiences a phase advance of $\pi/2$ so that the resultant also is slightly advanced with respect to the original wave. This means it appears to have travelled a little faster and arrived earlier. If the concentration of charged particles is greater, the re-radiated wave is more intense and the combined wave is further advanced in phase. The change of speed is greater for electrons, which can more easily be set into oscillation than for heavier ions.

For a wave obliquely incident on the ionosphere from below with an increase in the concentration of free charges with height, the speed of the wave increases upwards. Different parts of the wavefront find themselves in places where the charge concentration is different; the top travels more rapidly than the bottom, and so the resultant wave is refracted earthwards. Individual waves travel faster than the group as a whole. Reflection occurs when the group ceases to increase height.

During oscillations the charges often collide with the neutral air particles that surround them. After a collision they bounce off in random directions. The regular oscillations are interrupted and energy has to be fed in from the main wave. So the wave becomes weaker, or is absorbed as it travels.

When a charged particle, ion or electron, moves in a magnetic field, it travels in a spiral path, simultaneously moving along the field line and rotating around it. The speed of rotation depends on the charge and mass of the particle and on the strength of the field. In the Earth's magnetic field, ion and electron rotation rates are around 100 and 10^6 times per second respectively. Hence the refraction and the absorption of a wave is affected by this circular motion. The importance of the rotation depends on wave frequency. Greatest effect arises when the rotation rate matches the wave frequency.

The theory of wave propagation in an ionised medium in the presence of a magnetic field was first developed by Lorentz to explain light passage through crystals. However, when applied to radiowave propagation in the ionosphere, this failed to explain some observed features. The modified form developed by Appleton and Hartree is now known as the magnetoionic theory(1).

1.2 The Appleton-Hartree Equations

When a linearly polarised wave passes through an assembly of charged particles in the presence of a magnetic field it causes them to move round

rotates. The composite wave that results when these wavelets add to the original wave has its electric field rotating so that the polarisation is different from that of the original wave. If the original wave has its electric field rotating in a certain way it makes the charges rotate in the same way, they reradiate wavelets with the same kind of rotation, and when these are added to the original wave they produce a composite wave whose field also rotates in the original way. In this case the polarisation has not been changed, and the wave is a characteristic ordinary wave. A second characteristic extraordinary wave is possible in which the rotation is in the opposite sense.

The Appleton-Hartree theory applies for a medium which is electrically neutral with no resultant space charge and equal numbers of electrons and positive ions. A uniform magnetic field is assumed and the effect of positive ions on the wave is neglected. Steady state solutions for characteristic waves of plane polarisation are generated.

The complex refractive index n at angular frequency ω is given(1) as

$$n^2 = 1 - \frac{X}{1 - iZ - \dfrac{Y_T^2}{2(1-X-iZ)} \pm \left(\dfrac{Y_T^4}{4(1-X-iZ)^2} + Y_L^2\right)^{1/2}} \qquad (1)$$

where

$$X = \frac{Ne^2}{\varepsilon_0 m\omega^2} \qquad Y_L = \frac{e B_L}{m\omega} \qquad Y_T = \frac{e B_T}{m\omega} \qquad Z = \nu/\omega$$

N is the electron concentration, e and m are the electronic charge and mass and ε_0 is the permittivity of free space. ν is the electron collision frequency. The subscripts T and L refer to the transverse and longitudinal components respectively of the Earth's magnetic field B with reference to the direction of the wave normal. In particular, the refractive indices of the ordinary (upper sign) and extraordinary (lower sign) waves differ.

The corresponding wave polarizations R are

$$R = \frac{i}{2Y_L}\left[\frac{Y_T^2}{1-X-iZ} \mp \left(\frac{Y_T^4}{(1-X-iZ)^2} + 4Y_L^2\right)^{1/2}\right] \qquad (2)$$

1.2.1 No magnetic field, no collisions. This is the simplest case with Y and Z both zero, so that

$$n^2 = \mu^2 = 1 - X \qquad (3)$$

The square of the refractive index is real and lies between 0 and 1. For a given wave frequency it decreases with increasing electron concentration and for a given electron concentration it decreases with increasing wave frequency.

Curves showing the dependence of μ on X are called dispersion curves and the simple dispersion curve in the absence of a magnetic field and without collisions is depicted in Fig. 1. This indicates that below the ionosphere with N and X zero, μ is unity. As the wave penetrates into regions of higher electron concentration μ decreases. If N is sufficiently large then μ will become zero; otherwise the wave traverses the whole ionosphere and escapes.

According to Snell's law for a wave incident at an angle ϕ_o on a horizontally stratified ionosphere of refractive index μ_o the angle ϕ where the refractive index is μ is given by

$$\mu_o \sin\phi_o = \mu \sin\phi \qquad (4)$$

Outside the ionosphere $\mu_o=1$ so that at reflection with $\phi=90^0$ we have that $\mu=\sin\phi_o$. In the case of vertical propagation $\mu=o$ at reflection with X=1. Hence for wave frequency f at vertical incidence, reflection occurs at a height where the electron concentration N satisfies

$$f^2 = \frac{Ne^2}{4\pi\epsilon_o m} \qquad (5)$$

1.2.2 Magnetic field, no collisions. This condition implies Z=o but Y≠o. There are three values of X at reflection at vertical incidence (μ=o) of a wave of frequency f given from eq. (1) as

$$X = 1 \text{ (taking the positive sign)} \qquad (6)$$

and

$$X = 1-Y \text{ or } X = 1+Y \text{ (with the negative sign)} \qquad (7)$$

where $Y^2 = Y_L^{\,2} + Y_T^{\,2}$

Hence one magnetoionic component (the ordinary wave) is reflected as if the magnetic field where absent. The reflection level for the other component (the extraordinary wave) depends upon the magnitude of the field, but not its direction. For waves where $f>f_H$ the reflection level is given by X=1-Y and for $f<f_H$ by X=1+Y. This means that at HF reflection occurs at lower heights for the extraordinary wave.

Approximations to eq. (1) give the refractive index under quasi-transverse (QT) and quasi-longitudinal (QL) propagation, defined as follows:

$$\text{QT} \quad Y_T^{\,4} >> 4(1-X)^2$$

$$\text{QL} \quad Y_T^{\,4} << 4(1-X)^2 \qquad (8)$$

Note that these conditions depend both on θ and X. Hence QT propagation arises either for $\theta \sim 90^0$ or $X \sim 1$, which is the value of X near the level of reflection. The refractive indices are given from

Ordinary Wave

$$\text{QT} \quad \frac{X}{1-\mu^2} = 1 \quad \text{and} \quad \frac{X}{1-\mu^2} = 1 - (1-X)\cot^2\theta$$

$$\text{QL} \quad \frac{X}{1-\mu^2} = 1 + |Y_L| \qquad (9)$$

Extraordinary Wave

$$\text{QT} \quad \frac{X}{1-\mu^2} = 1 - \frac{Y^2}{1-X}$$

$$\text{QL} \quad \frac{X}{1-\mu^2} = 1 - |Y| \quad \text{and} \quad \frac{X}{1-\mu^2} = 1 - |Y_L|$$

Great care must be exercised in using these approximations since the range of validity can be very small. Dispersion curves for Y = and Y = 2 with $\theta = 45^0$ are given in Fig. 2.

The wave polarisations given from eq. (2) for the ordinary and extraordinary waves indicate the amplitude ratio and phase difference between the component electric vectors in the wavefront plane lying parallel to and normal to the projection of the magnetic field. In general wave polarisation is elliptical with the ordinary and extraordinary waves having equal axial ratios but opposite senses of vector rotation. In the case of no collisions (Z=o), $R_o.R_x = 1$ and the two waves have orthogonal major axes. With longitudinal propagation the two magnetoionic waves are circularly polarised. With transverse propagation the ordinary wave is linearly polarised with its electric vector parallel to the imposed magnetic field.

1.2.3 No magnetic field, with collisions. The effects of electron collisions are now examined and for simplicity we assume no magnetic field, i.e. $Y_T = Y_L = 0$. Eq. (1) now yields a complex refractive index:

$$n^2 = (\mu - i\chi)^2 = 1 - \frac{X}{1-iZ} = 1 - \frac{X}{1+Z^2} - \frac{iXZ}{1+Z^2} \qquad (10)$$

where μ and χ are the real and imaginary parts of n respectively.

When the refractive index is complex we have for a wave travelling in the z-direction

$$E = E_o \exp i\,(\omega t - \frac{\omega}{c} nz)$$

or

$$E = E_o \exp(-\chi \frac{\omega}{c} z) \exp i\,(\omega t - \frac{\omega}{c} \mu z) \qquad (11)$$

If χ is non zero this represents a wave whose amplitude decreases exponentially with distance. The quantity $(\omega\chi/c)$ is a measure of the decay of amplitude per unit distance and is called the absorption coefficient k

$$k = \frac{\omega\chi}{c} \qquad (12)$$

Dispersion curves for various values of χ are shown in Fig. 3. For a given value of χ the effect of a finite collision frequency is to diminish the reflecting properties of the medium. When Z ≠ 0, the real part of the refractive index never goes to zero and has a minimum value μ_m given by

$$\mu_m = 1 - \frac{1}{1+Z^2} \qquad (13)$$

Thus, in a lossy medium total reflection never really arises, although strong reflection still occurs near the levels where μ is small.

From eqs. (10) and (12) the absorption in nepers per metre (1 neper = 8.69 dB) is given as

$$k = \frac{\omega}{c} \cdot \frac{1}{2\mu} \frac{XZ}{1+Z^2} = \frac{e^2}{2\epsilon_0 mc} \frac{1}{\mu} \frac{N\nu}{\omega^2 + \nu^2} \qquad (14)$$

When N is small $\mu = 1$ and eq. (14) gives

$$k = \frac{e^2 N\nu}{2\,\epsilon_0 mc\omega^2} \qquad (15)$$

This is called 'non-deviative' absorption, since N is too small to produce considerable deviation of the ray. It arises primarily in the D-region. Near reflection when μ becomes small

$$k = \frac{\nu}{2c}\left(\frac{1}{\mu} - \mu\right) \qquad (16)$$

This is called (deviative) absorption, since it occurs in a region where considerable deviation of the ray takes place.

In the presence of collisions, the wave polarisations (eqs. 2) are complex. This means that the major axes of the polarisation ellipses of the ordinary and extraordinary waves are no longer orthogonal. The ellipses each rotate from the no-collision case by the same amount in opposite directions, such that each ellipse is the reflection of the other in the plane making an angle of 45° with the magnetic meridian.

1.2.4 Magnetic field, with collisions. The full Appleton-Hartree expressions must be used. Approximate solutions are not readily obtained and equations are solved numerically by computer.

1.3 Generalised Magnetoionic Theory

In the formulation of the Appleton-Hartree equations, it was assumed that all the electrons have the same average velocity and that the collision frequency ν is independent of the electron velocity. For waves reflected in the higher regions of the ionosphere the wave frequency $f \gg \nu$ and the exact value of ν is unimportant. In the D-region however, ν may be equal to or greater than the wave frequency and assumes considerable significance.

The assumption that ν is independent of electron velocity is one of the major limitations of the Appleton-Hartree theory and efforts have been made to modify the theory to allow for this effect. Phelps and Pack(2,3) established experimentally that ν is directly proportional to the electron energy for slow electrons in nitrogen. This result has been used by Sen and Wyller(4) to generalise the magnetoionic theory to include the energy dependence of the electrons. Their expression for the complex refractive index is

$$n^2 = (\mu - i\chi)^2 = 1 - \frac{X}{Z_m^2} C_{3/2}\left(\frac{1}{Z_m}\right) - i.\frac{5}{2}\frac{X}{Z_m} C_{5/2}\left(\frac{1}{Z_m}\right) \qquad (17)$$

$Z_m = \nu_m/\omega$ where ν_m is the monoenergetic collision frequency, $C_{3/2}(x)$ and $C_{5/2}(x)$ are the semi-conductor integrals, where

$$C_p(x) = \frac{1}{p!} \int_0^\infty \frac{\epsilon^p \exp(-\epsilon)}{\epsilon^2 + x^2} d\epsilon \qquad (18)$$

The use of the generalised expressions is particularly important in considering VLF and LF propagation, and in calculating the absorption of HF waves in the D-region.

1.4 Phase and Group Velocity

The phase velocity v is

$$v = \frac{c}{\mu} = c\left[1 - \frac{Ne^2}{m\epsilon_0\omega^2}\right]^{-1/2} \qquad (19)$$

for propagation with no collisions and no magnetic field. This indicates that the phase velocity in the medium is greater than the velocity of light and the wavelength in the medium is greater than in free space

$$\lambda = \lambda_0\left(\frac{v}{c}\right) \qquad (20)$$

If the phase velocity of a wave in a medium varies as a function of the wave frequency, it is said to be dispersive. Two waves with slightly different frequency will therefore travel with slightly different velocities. It is the interference pattern between two such waves that determines where, and with what velocity, the energy of the composite wave will travel. For a wave $\cos(kz - \omega t)$ the group velocity u is given by

$$u = \frac{\delta\omega}{\delta k} \qquad (21)$$

For a non-dispersive medium in which ω/k is constant, $u = v$.

The group refractive index μ' may be defined as

$$\mu' = \frac{c}{u} = c\frac{dk}{d\omega} = c\frac{d}{d\omega}\left(\frac{2\pi}{\lambda}\right) = \frac{d}{d\omega}(\mu\omega)$$

$$= \mu + \omega\frac{d\mu}{d\omega} = \mu + f\frac{d\mu}{df} \qquad (22)$$

For the no field situation where $\mu^2 = 1 - (f_N/f)^2$ we have that

$$\mu' = \frac{d}{df}(\mu f) = \frac{1}{\mu} \quad (23)$$

1.5 Propagation in an Anisotropic Medium

A medium is said to be isotropic if the phase velocity of a wave propagating within it is independent of direction. This is not the case within a magnetoionic medium where refractive index depends on direction of propagation relative to the field. In general the directions of the phase and ray paths then differ. It can be shown that the angle α between the wave normal and the ray direction is

$$\tan \alpha = -\frac{1}{v}\frac{dv}{d\theta} = +\frac{1}{\mu}\cdot\frac{d\mu}{d\theta} \quad (24)$$

where θ is the angle at which the wave normal direction cuts a reference axis. The phase path in an anisotropic medium is

$$P = \int_s \mu \cos\alpha \, ds \quad (25)$$

integrated over the raypath s. The corresponding group path is

$$P' = \int_s \mu' \cos\alpha \, ds \quad (26)$$

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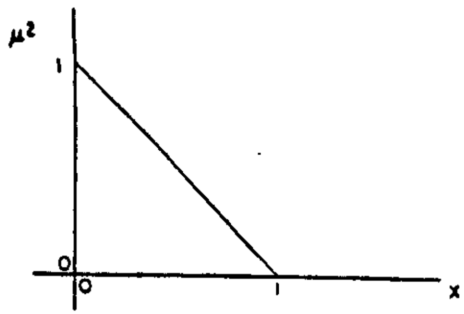

Fig. 1 Dispersion curve for no magnetic field and no collisions

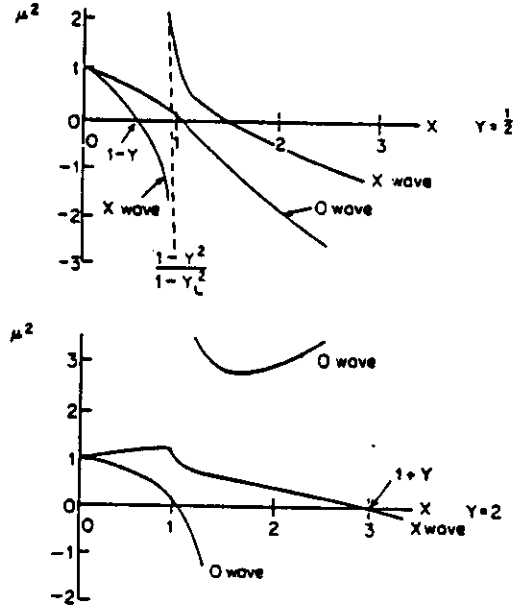

Fig. 2 Dispersion curves for Y=½ and Y =2 with θ=45°

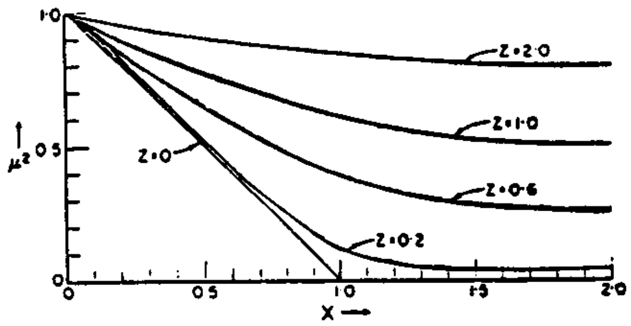

Fig. 3 Dispersion curves for no magnetic field, with collisions