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FARADAY ROTATION MEASUREMENTS AND TEC DERIVATION

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1. Radio receiver characteristics.

Receiving a radio signal requires to solve a great quantity of problems. Measuring the Faraday rotation of the polarization of a signal arriving from a geostationary satellite is a particular receiving situation. No other information than the ionospheric modification on the signal are to be taken into account and measured; no modulation signal must be demodulated; no tracking of the satellite must be done in real time ecc. This situation in general simplifies the design of a polarimeter. However some general elements of classic communication theory must be here recalled. In any case, for a deeper insight, an appendix and a bibliography can be found at the end of this writing.

1.1 Calculation of overall receiving system power budget

To have an idea of what kind of knowledge is necessary, it can be started with a very simplified evaluation of the overall power budget of a receiving system. The power budget is an accounting of the contribution of the many steps the signal undergoes from the transmitter to the final stage of the receiver.

This balance is needed to calculate the output signal to noise ratio, i.e. the validity of the system, or to detect the points were improvements should be made. An example: for a satellite down link transmitter, with an output power of 10 watts, it can be calculated, following the signal flow: (for the dB notation see Appendix)

- output power (10 watt)	40 dBm	
- transmitting antenna gain	3 dB	
effective radiated power		43 dBm
- transmission path loss	-150 dB	
received power		-107 dBm
- receiving antenna gain	12 dB	
- receiver noise figure	-3 dB	
- receiver gain	30 dB	
receiver useful signal level		-68 dBm
- noise power	-114 dBm	
Resultant System S/N	46 dB	

Tab.1

The more abstruse losses are not taken into account as well as the safety margins needed for important factors as low elevation angle, environment noise level, weather effect, equipment degradation, detuning etc.

The most realistic improvement of the receiver

system, to reach an adequate S/R (30-50 dB) are:
- use a higher gain antenna
- reduce the noise figure as much as possible
- reduce the bandwidth: remember that it can be reduced not more than the double of the signal band (or, in this case, of the phenomenon band).

To give more meaning to these sentences, further considerations must be made.

1.2 Noise

Noise is a phenomenon, inherent to any electrical circuit, and also introduced by external sources.

In general no absolute values of signal and noise are useful, but the relative ones: the S/N or S+N/N ratio (if signal is » than the noise, for example 100 times, the two ratios are equivalent. There is a theoretical limit by which the transmission of information is limited by noise (Shannon's law. See Abramsohn, 1963).

Many sources of noise contribute to the signal degradation and must be studied in designing a receiver:

Thermal noise: any electrical conductor is the source of thermal noise:

$$W = k \cdot T \cdot B$$

where:
W is in watt
k is the Boltzman's constant $1.38 \cdot 10^{-23}$ ($J \cdot K^{-1}$)
T is the absolute temperature, (°K)
B is the bandwidth in hertz
This noise is termed thermal or white noise, and it is caused by the free electron motion produced by thermal agitation. A voltage at the open ends of wires, conductors, resistors, is produced. The power is independent of the value of the resistors so that the voltage depends on the square of the resistors.

The temperature is an effective temperature T_e and is in general referred to a reference temperature T_0=290°K.

Receiver noise: One of the characteristics which give the merit of a receiver is its noise figure NF. It is the ratio between the actual noise power at its output and as compared to an ideal receiver at the same temperature and bandwidth. The ratio is always greater than one, and the larger is the noise figure, the poorer is the receiver. If the temperatures notation is used, it can be written:

$$NF = 10 \log(1+T_e/T_0) \quad dB$$

For example, a receiver with a noise temperature of 870°K has a noise figure of 6 dB.
Each component of a receiving system adds noise and arises the total temperature; this quantity depends on the temperature and the loss of the component: for example a transmission line with a temperature of 300 °K and a loss off the 1% adds about 3°K to the overall system temperature.

External noise: The major contribution of the overall temperature is produced by the sky and the Earth and received by the antenna, not produced by the antenna itself.

Manmade noise: the noise produced by other signals is generally denoted as interference and is very important at the most used frequencies for ionospheric measurement (30- 400 MHz).

1.3 Antennas at VHF

The most widely used antenna for Faraday rotation measurement are:
- the multielement Yagi array, mechanically rotating with a period of the order of some minutes
- the crossed Yagi, i.e. two dipole on the same boom (see below)
- an array of two helical antennas with opposite polarization
- the crossed Yagi electronically switched

For more details on the antennas see Blake L.W., 1966.

The Yagi array (the usual TV set antenna) is the most simple and the most used antenna for the polarimeter, due to its easy designing, building, and commercial availability. Its beamwidth is rather large (about 35° for a twelve element array) to make no critical the antenna pointing, although its gain cannot reach high values (10-12 dB).

1.4 Receiving system frequency stability, bandwidth

These considerations are limited only to the Polarimeter receiving system in the VHF band. For a most general treatment of this subject the bibliography must be consulted.
In satellite systems, the radio frequencies are usually generated and measured with an accuracy of 10^{-6} in the worse cases to 10^{-11} if a good stabilization is reached by appropriate electronics. For TEC measurement the incoming signal frequency must be precisely determined only if the phase differences are calculated through time intervals measurements. In fact if the time interval difference T between two components of the signal (for example the circular contra-clockwise components) is measured, its relation with the relative phase RP angle is: (see Fig. 1)

$$RP = 2\pi \cdot T \cdot f \quad (rad)$$

The receiving system frequency stability as well as the transmitter stability is not a very constrictive parameter in TEC measurement, due to the relatively broad bandwidth of the system (some KHz in the VHF band). Other causes of frequency changes are:

- the intrinsic band of the phenomenon - Faraday rotation can be considered as frequency change- (some cycles in a day)
- the slow motion of the satellite up and down -Doppler effect- (some Hz)
- the variability of the refraction index of the ionosphere, Doppler effect, (of the order of ten Hz in the VHF band).

2. Faraday rotation receiver.

A receiver suitable for polarization measurement is in general a two or more channels receiver. The phase coherence between the channels, necessary to measure phase differences, is obtained by using common local oscillators. Any phase error source must be minimized. Luckily, because of the little frequency change of the incoming frequency (100+200 Hz) and of the very narrow bandwidth of the "signal" introduced by the ionosphere (of the order of some Hertz) a precise tuning is allowed and a consequently good working point (at the middle of each bandwidth) of each device could be reached. A very schematic and simple but realistic receiver is presented in the following points. See Fig 2.

2.1 Crossed Yagi antenna

As said above, the most used antenna for Faraday rotation measurements is the crossed Yagi. A Yagi antenna consists (see fig. 3) of a L/2 folded dipole as driven element, a parasitic reflector and some director. It is designed for the frequency to be received, for the gain and the pattern requested. Two of this arrays are mounted on the same boom, tilted of 90 degrees. A balance unit (balun) adapts the antenna impedance with the cable impedance and provides a balanced antenna could be fed by an unbalanced signal. As the two antennas share the same boom, a phase difference between the signals will occur, therefore the transmission line must be different in length in order to retard the signal of the antenna closest to the satellite. The distance between the two antennas is of the order os some cm, the velocity of the EW in the cable is 60-70% of that in the air, so only a difference of some cm is needed. For example a distance of 5 cm in antennas mounting results in a difference of length of the lines of 3 cm if the velocity reduction is of 60%.

The antennas must be mounted 45 degrees with respect to the vertical in order to have the same ground interference.

The outputs of these antennas are the currents inducted by the components of the electric field of the incoming waves, parallel to the antennas themselves: two cable suitable for radio frequency signals connect antennas and the two channels of the receiver.

2.2 Preamplifier, hybrid

If antennas and receiver are more than 10+15 meter far, the cable power loss could reduce too much the S/N ratio and it must therefore be compensated by a first amplifier (preamplifier) situated as close as possible to the antenna. Acceptable characteristics of such preamplifier are:
gain: 20 dB, bandwidth: 4 MHz, noise figure: 3dB.

An 90° hybrid is mounted wether just before the preamplifier or directly after them. This device, by suitably combining the two inputs (i.e. the orthogonal components of the signal), supplies as outputs its two circular components of the signal. In the first case (mounting before the preamplifier) a worse noise figure is obtained, but any error (due to the possible different gains of the preamplifier) isn't introduced. In the second case on the contrary, the noise figure isn' t affected but some error can be introduced (see Fig.4).
Normal characteristics for such a device are:
gain: -0.2 dB, bandwidth 80-160 MHz, channels separation: 25 dB

2.3 Down conversion, intermediate frequency

The signal are down converted to 10.7 Mhz, amplified by an IF amplifier, and filtered to reduce the bandwidth and increase the S/N ratio. The mixers of each channel have the same local oscillator to ensure the coherence. Characteristics of the first mixer are: gain:35 dB, bandwidth: 2 MHz, noise figure 3 dB. The IF amplifier has the following characteristics: gain :45 dB, bandwidth: 8 KHz, noise figure less than 3 dB.

The signal frequency is finally converted to 10 KHz by two passive mixers.

2.4 Phase lock voltmeter

Two sinusoid with the same relative phase of the two circular components of the signal are now present at the input of two microvoltmeters (PLV). The S/N ratio is equal ti 3 dB in the actual case of this polarimeter. This device, using a phase lock loop network (Viterbi, 1966), supplies at its output a square wave of the same frequency and phase of its input signal, with an increment of the S/N ratio up to 20+25 dB. The amplitude information is available at these devices, after them this information is lost.

2.5 Phase meter

The phase difference between the two channels is converted by the phase meter to a continuous voltage linearly related to the phase difference itself. The linearity is assured up to a phase changing rate of ten cycles per second.

2.6 Chart recorder

A chart record galvanometer records the phase related voltage. An integrating time constant of the order of the second must be at this point expected. Furthermore a time mark is recorded on the chart strip every fifteenth minute.

3. Recording Faraday rotation data.

All the actions to be performed to make working a polarimeter are described in this chapter. These points must be intended as a trace for the experimental work in the laboratory.

3.1 Satellite data bulletins

Many publications (as, for example, the Spacewarn Bulletin published by the Goddard Space Flight Centre, Maryland) are available to know the characteristics of the usable satellites: satellite position, nominal frequency and transmitted power.

3.2 Geostationary satellite tracking

Some easy computer work supplies the antenna pointing (azimuth and elevation), given the satellite and station latitudes and longitudes.

3.3 Polarimeter placement and mounting

The nominal power transmitted by a geostationary satellite (36,000 km far from the ground) is generally of the order of the watt, therefore the signal to the receiver is weak due to the distance and the very small antenna gain. For example it can be calculated in the case of the Sirio:

$W_t \cdot G_t$ is 7.55 watt, G_r is 12 dB i.e.15.85 (power ratio) then the received power W_r results $2.8 \cdot 10^{-15}$ watt, corresponding to 37 microvolt on 50 ohm. The same calculation for the ATS 6 satellite gives a received signal of 0.2 microvolt. This situation gives some constrains in the choice of the polarimeter and the antenna location:

a) the polarimeter must be far from power lines, airports and aircraft ways, radio transmitters, broadcasting antennas, any source of RF signal and noise;

b) the apparatus must be in a controlled environment to avoid excess in temperature and humidity and its distance from the antenna must be as shorter as possible (not more than 20 - 30 m);
c) the horizon of the antenna must be free from East to West toward South in the northern emisphere and toward North in the southern emisphere.

d) in any case the antenna must be mounted preferably on the ground rather than on a building roof to avoid the ground reflection polarizations, which are minimized on the ground. This is more important for small elevations (less than 40 degrees).

3.4 Calibration of chart record

The chart record is calibrated to read the actual phase value by reading the pen trace position, by inserting subsequently two signals of the same phase, with π and 2π of phase difference. The calibration on the $0 \div 2\pi$ range is assured by the linearity of the phase meter and of the chart recorder.

3.5 Receiver tuning and beacon signal recognition

The received signal is in general a carrier at a frequency very close to the nominal frequency. Sometimes it could be modulated in frequency or amplitude. In the case of amplitude modulation or no modulation the received signal is a sinusoidal signal merged in the noise. At the IF level a sinusoidal signal is available and a beat note can be produced on a receiver tuned at very close frequency (about 1 KHz of difference), to makes audible the signal. So it is possible the accurate tuning of the receiver, i.e. of the first local oscillator.

3.6 Phase lock voltmer tuning

When the signal has been detected, the PLV must be tuned to work. An accurate tuning allows the PLV to follow possible frequency change of the signal (not exceeding the locking band).

4. Data reduction technique.

This chapter requires the knowledge of the theory of the radio propagation technique in the ionosphere in order that all the routines could be understood. In any case all the computer programmes as well as the theory supporting them can be studied a part from this lesson, accepting formulas and procedures.

4.1 Data transfer from chart record to computer files

The chart record contains:
- the phase layout
- the epoch information, i.e. the date and the fifteen minutes divisions
- the calibration, i.e. the zero and the 2π of the phase measurement, tacking into account the linearity of the scale. In case of not linearity, the entire range must be calibrated step by step.

To read the phase data and insert them into the computer, some work must be done:
- the time scale must be completed with five minutes divisions with the suitable mask;
- from the midnight (UTC: Universal Time Coordinated), every fifth minute, the actual phase must be read. As the phase is measured from 0 and 2π (corresponding to a polarization angle from 0 to π), it is displayed as a sawtooth, information therefore must be furnished every time an edge is reached.

4.2 Initial polarization angle, $n\pi$ ambiguity, M factor evaluation

As it has been outlined in other lessons, the relation between Total Electron Content (TEC) and Faraday rotation can be written:

$$\Omega = k/f^2 \cdot \int B \cos\theta \, N \, dl$$

where:

Ω is the amount of Faraday rotation in radian and it can be written: $\Omega=\Omega_m+\Omega_i+\Omega_0$, where Ω_m is the measured rotation, Ω_i is the initial polarization at the satellite, Ω_0 is the ambiguity.

k is a constant that includes the electron's charge and mass, the velocity of light, and other physical constants. Its value is, for our purposes, is $2.36 \cdot 10^{-5}$.

f is the frequency in Hertz

B is the magnetic field intensity in gammas (1 gamma=10^{-5} gauss).

θ is the angle between the direction of propagation and

θ is the angle between the direction of propagation and the magnetic field direction

the integral of Ndl is the TEC in electrons per square m.

Some geometrical calculation (see Klobuchar,1966) furnishes the initial polarization angle Ω_i at the satellite as saw by the receiving antenna. It depends on the angle of the antenna axis with respect to the spin axis of the satellite, the position of the spin axis with respect to the local vertical.

It is impossible to know how many 2π the polarization has rotated during the path i.e. Ω_0, however is possible to hope to solve this ambiguity if:
- long time measurement without interruption has been made.
- an ionosonde is placed near the subionospheric point (the intersection point between the ray path and the ionosphere at 400 km, projected to the Earth surface) and if it are able to furnish the critical frequency f_0 of the F_2 layer at the same time of Faraday measurement. In fact these two quantity are related and it is possible derive the correction to the polarization by some computer work.
- the satellite beacon transmits two coherent frequencies as, for example, 40 and 41 Mhz or 140 MHz.

For a deeper insight to these topics see (Spalla et alii, 1978 and 1983).

The last quantity to be taken into account is: $\int B \cdot \cos\theta \cdot N \cdot dl$

It can be written:

$$\int B \cdot \cos\theta \cdot N \cdot dl = \overline{M} \cdot \int N \, dl = TEC$$

where $\overline{M}$ is the weighted media of $B\cos\theta$ over the path with weight N. This parameter must be done by some computer calculation.

Appendix

- Frequency and wavelength

The frequency is the more usual unit in radio technology as the wavelength in Optics. The number of cycles per second is the frequency f, the angular frequency ω is radian/sec: $\omega = 2\pi f$.
The unit symbol Hz, for hertz, has been here adopted- accordingly to the International System of Units-.

- Radio bands

For communication and general transmission purposes the spectrum of radio waves from 3 KHz to 300 GHz has been broken into bands. Each band is devoted to a peculiar use as the so called Citizens Band around the 27 MHz; for each particular use of this resources it must be determined the frequency of the carrier in the devoted band and the bandwidth necessary (and possible for that frequency: remember the Shannon's theorem): in the case of a satellite telemetry in the VHF band, a frequency of 136-140 Mhz with a band of 50-100 KHz is usual.

For the complete breakdown of the frequencies see the tables 3 and 4 where the International and Alphabetical band designation are reported.

- Decibel notation

In studying the radio transmission very large numbers are often encountered. A shorthand is therefore necessary in expressing the magnitudes of power, gain, amplitude ecc.

Furthermore ratios between magnitudes rather than absolute values are necessary, so a very convenient way to write these ratios is a logarithm notation. For example the decibel notation for the power ratio is very convenient:

$$dB = 10 \log W_1/W_2$$

In the table below some power ratios and their dB equivalent are shown. This notation can give an absolute value of W_1 if W_2 assume unit values: more precisely if W_2 is assumed to be 1 Watt W_1 is expressed in dBw, if W_2 is equal 1 milliwatt, W1 is expressed in dBm. Thus:

	1 watt=	0 dBw=	30 dBm
1 kw=	10^3 watt=	30 dBW=	60 dBm
1 mw=	10^{-3} watt=	-30 dBw=	0 dBm

The decibel notation is useful also to express amplitude ratios. To be coherent with the power notation it must be written

$$dB = 20 \log A_1/A_2 \ .$$

- Antenna received power

The incident power flux at receiving antenna P_r can be written:

$$P_r = \frac{W_t \cdot G_t}{4\pi R^2}$$

where:
P_r = power flux at the receiving antenna in watt/square meter
W_t = total transmitted power
G_t = transmitter antenna gain
R = transmitter distance

The power received by the receiver antenna, W_r in watt is:

$$W_r = A_r \cdot P_r = \frac{W_t \cdot G_t \cdot A_r}{4\pi R^2}$$

replacing the effective area of the receiving antenna A_r by its equivalent in power gain G_r and the wavelength L:

$$W_r = \frac{W_t \cdot G_t}{4\pi R^2} \cdot \frac{G_r \cdot L^2}{4\pi}$$

and:

$$W_r = \frac{W_t \cdot G_t \cdot G_r \cdot L^2}{16 \pi^2 R^2}$$

It should be clear that this path loss is not loss in the sense of EM power converted into heat, rather it is caused by the spreading of the signal over a larger area as the distance increases. Neither atmospheric absorbtion nor scatter has been considered. This amount of power loss over a transmission path is, in principle, independent of frequency, depending only on the distance. An apparent violation of this rule has been introduced. But a great convenience can be reached, if in the calculation of path loss the wavelength (or the frequency f=c/L) is used as a tool.

- Antenna beamwidth, gain polarization

The antenna beamwidth is the angular separation between the two points, one on each side of the main beam, where the antenna response is one half the power of the maximum power on the axis.

Each kind of antenna has its own radiation diagram shape, maximum and beamwidth: for the details concerning the antennas of interest in polarization measurement, see the Reference Data in the Bibliography.

The gain G of an antenna, meaning power gain on an isotropic point source, is related to its beamwidth, because gain is realized by concentrating the most power as possible in a preferred direction. The power amount in the other directions can be considered as losses: the integral of these losses in a sphere around the antenna related to the total power gives the antenna gain. The theory of reciprocity assures that the same value can be accepted wether for transmitting or for receiving antenna.

Captions of the figures

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- OUTPUT POWER (10 WATT)	40 dBm	
- TRANSMITTING ANTENNA GAIN	3 dB	
EFFECTIVE RADIATED POWER		43 dBm
- TRANSMISSION PATH LOSS	-150 dB	
RECEIVED POWER		-107 dBm
- RECEIVING ANTENNA GAIN	12 dB	
- RECEIVER NOISE FIGURE	-3 dB	
- RECEIVER GAIN	30 dB	
RECEIVER USEFUL SIGNAL LEVEL		-68 dBm
- NOISE POWER	-114 dBm	
Resultant System S/N	46 dB	

Tab.1

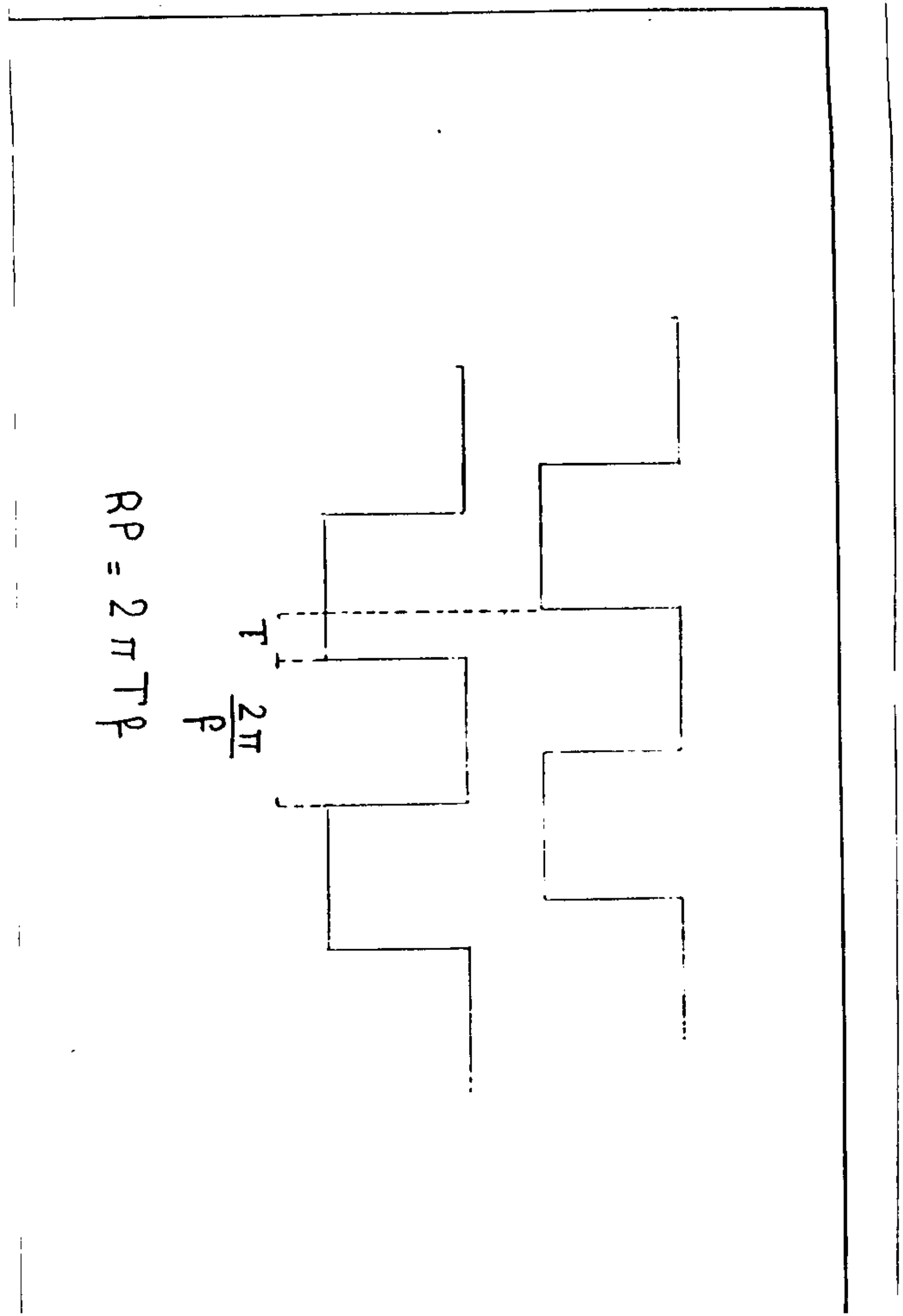


FIG. 1

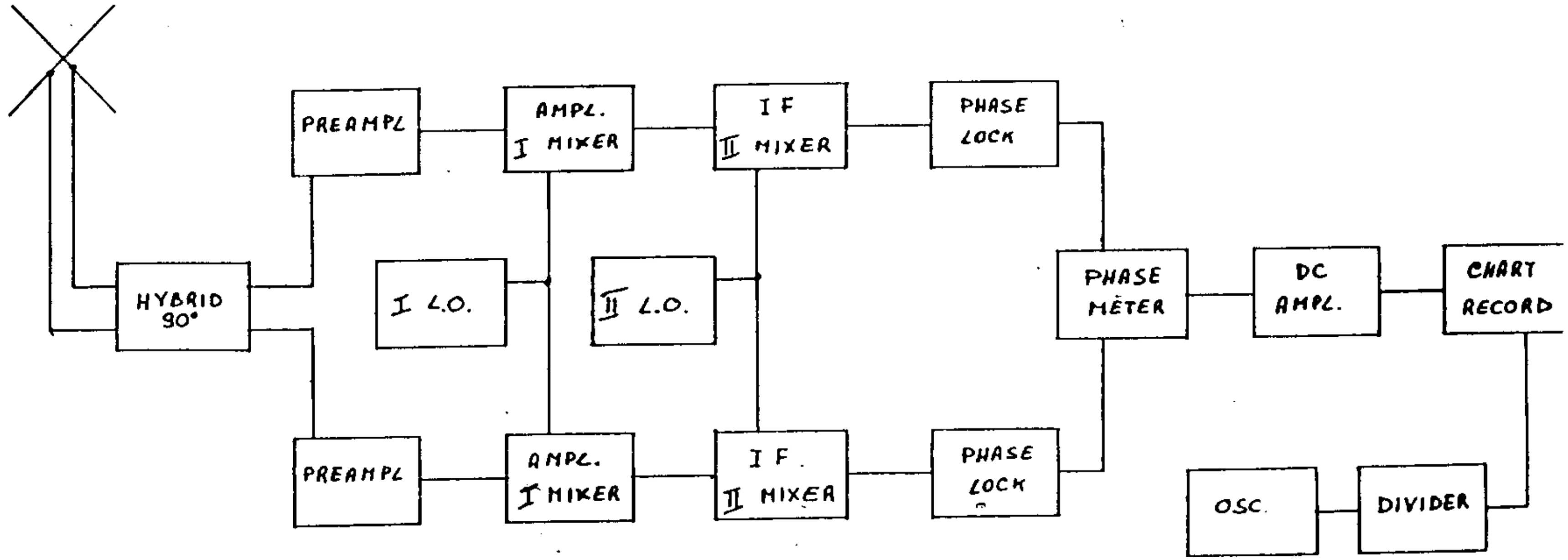

PREAMPL
AMPL. I MIXER
IF II MIXER
PHASE LOCK
HYBRID 90°
I L.O.
II L.O.
PHASE METER
DC AMPL.
CHART RECORD
PREAMPL
AMPL. I MIXER
I F. II MIXER
PHASE LOCK
OSC.
DIVIDER

FIG. 2

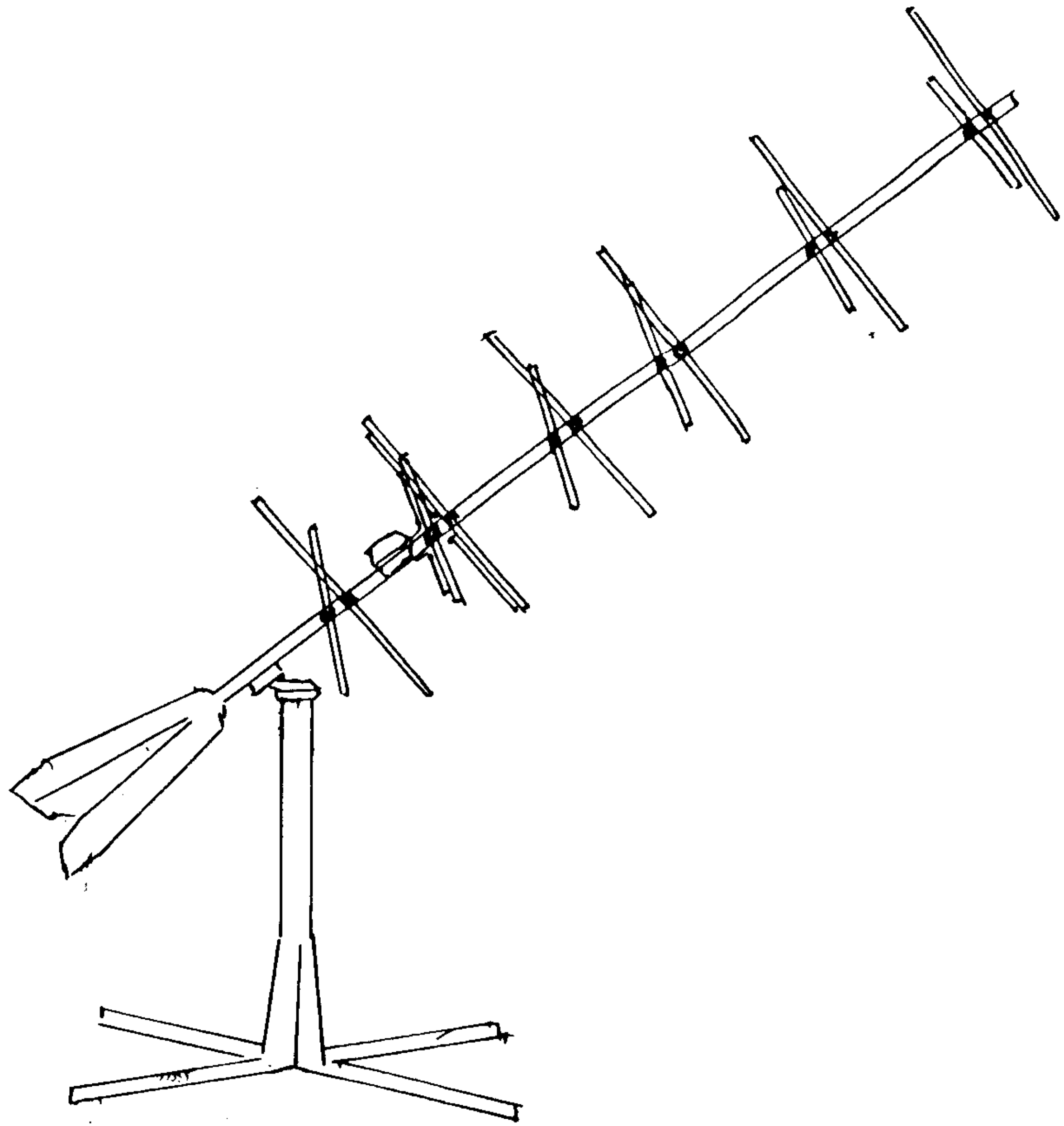
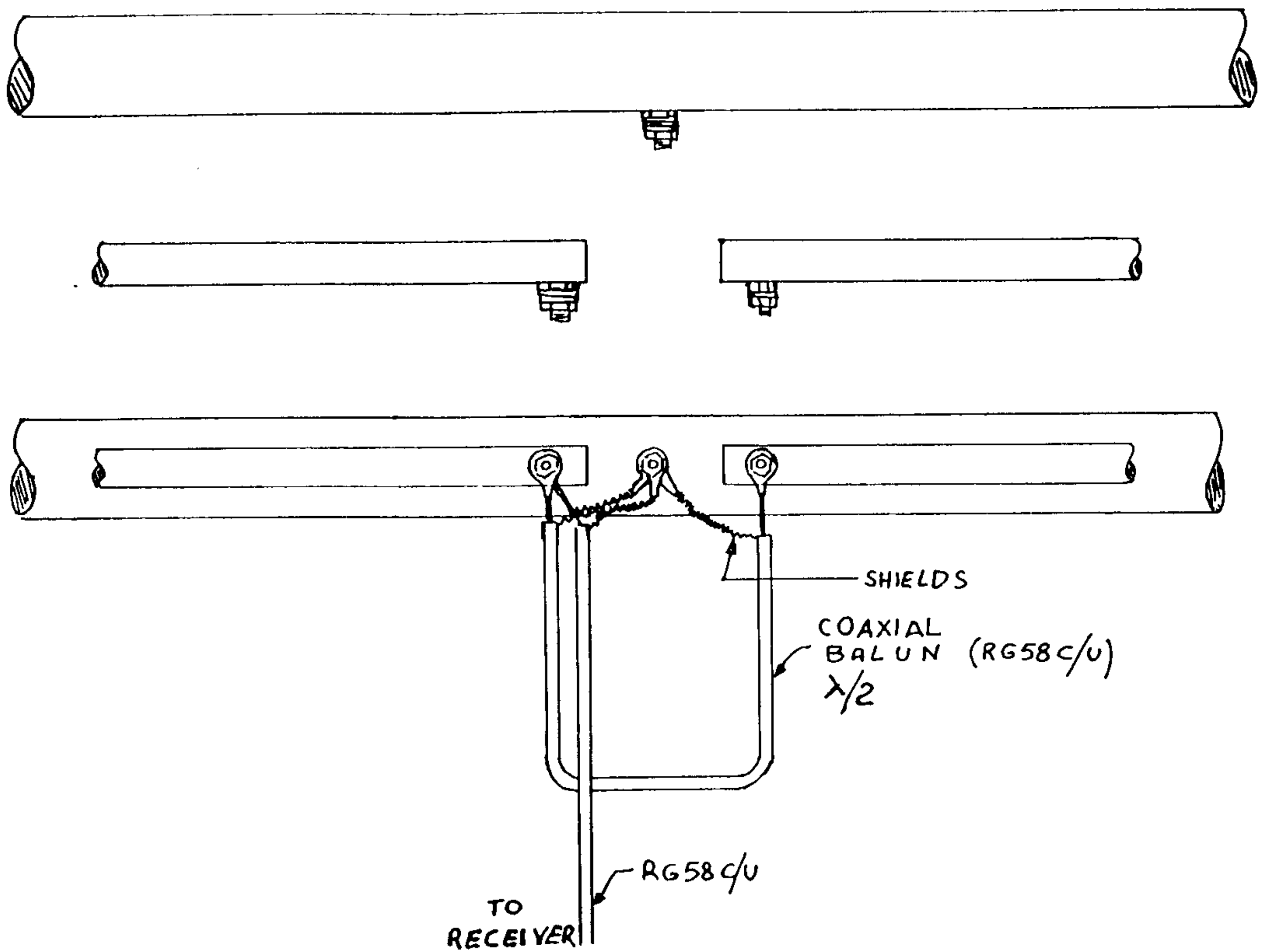


FIG 3a



FIG

	GAIN dB	BANDWIDTH MHz	NOISE FIG dB
ANTENNA	12	2	
HYBRID 90°	-.2	80 ÷ 160	CHANNEL SEPAR. 25
PREAMPLIFIER	20	4	3
FIRST CONV.	35	2	3
IF	45	8 KHz	

TAB 2

Frequency Band Designation of International Telecommunications Union

Abbrev.	Frequency, f	Wavelength, λ	Name
VLF	3 – 30 kHz	100 – 10 km	Very Low Frequency
LF	30 – 300 kHz	10 – 1 km	Low Frequency
MF	300 – 3,000 kHz	1,000 – 100 m	Medium Frequency
HF	3 – 30 MHz	100 – 10 m	High Frequency
VHF	30 – 300 MHz	10 – 1 m	Very-High Frequency
UHF	300 – 3,000 MHz	1,000 – 100 mm	Ultra-High Frequency
SHF	3 – 30 GHz	100 – 10 mm	Super-High Frequency
EHF	30 – 300 GHz	10 – 1 mm	Extremely High Frequency

TAB - 3

Frequency Band Designation by Alphabetic Format

Band	Frequency, f	Wavelength, λ
P	225 – 400 MHz	1.33 – 0.75 m
L	0.4 – 1.5 GHz	75 – 20.0 cm
S	1.5 – 5.0 GHz	20 – 6.0 cm
X	5.0 – 12 GHz	6 – 2.5 cm
K	12 – 36 GHz	2.5 – 0.83 cm
Q	36 – 46 GHz	8.3 – 6.5 mm
V	46 – 56 GHz	6.5 – 5.4 mm
W	56 – 100 GHz	5.4 – 3.0 mm
C	3.9 – 6.2 GHz	7.7 – 4.8 cm

TAB - 4