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COLLEGE ON ATOMIC AND MOLECULAR PHYSICS: PHOTON ASSISTED COLLISIONS IN ATOMS AND MOLECULES

(30 January - 24 February 1989)

ELECTRON-ATOM COLLISIONS IN LASER FIELDS

LECTURE II

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Further references: "Laser Physics" - M. Sargent, M. O. Scully, W. E. Lamb, Addison-Wesley Press 1974

"Density Matrix: Theory and Applications" K. Blum, Plenum Press 1981

scattering papers, e.g., "Correlations in electron-atom scattering" A Crowe pp 265-321 in Advances in Atomic & Molecular Physics Vol 24 1987, and other volumes ed. D.R. Bates & B. Bederson, Academic Press

"Alignment and orientation of the hyperfine levels of a laser-excited Na-beam" A. Fischer and I Hertel, Z. f. Phys. A 304, 103 (1982)

populations of $F=3$, $3^2P_{3/2}$ levels with: σ^+ light ---- σ^- light ---- polarization
with π light ---- allignment

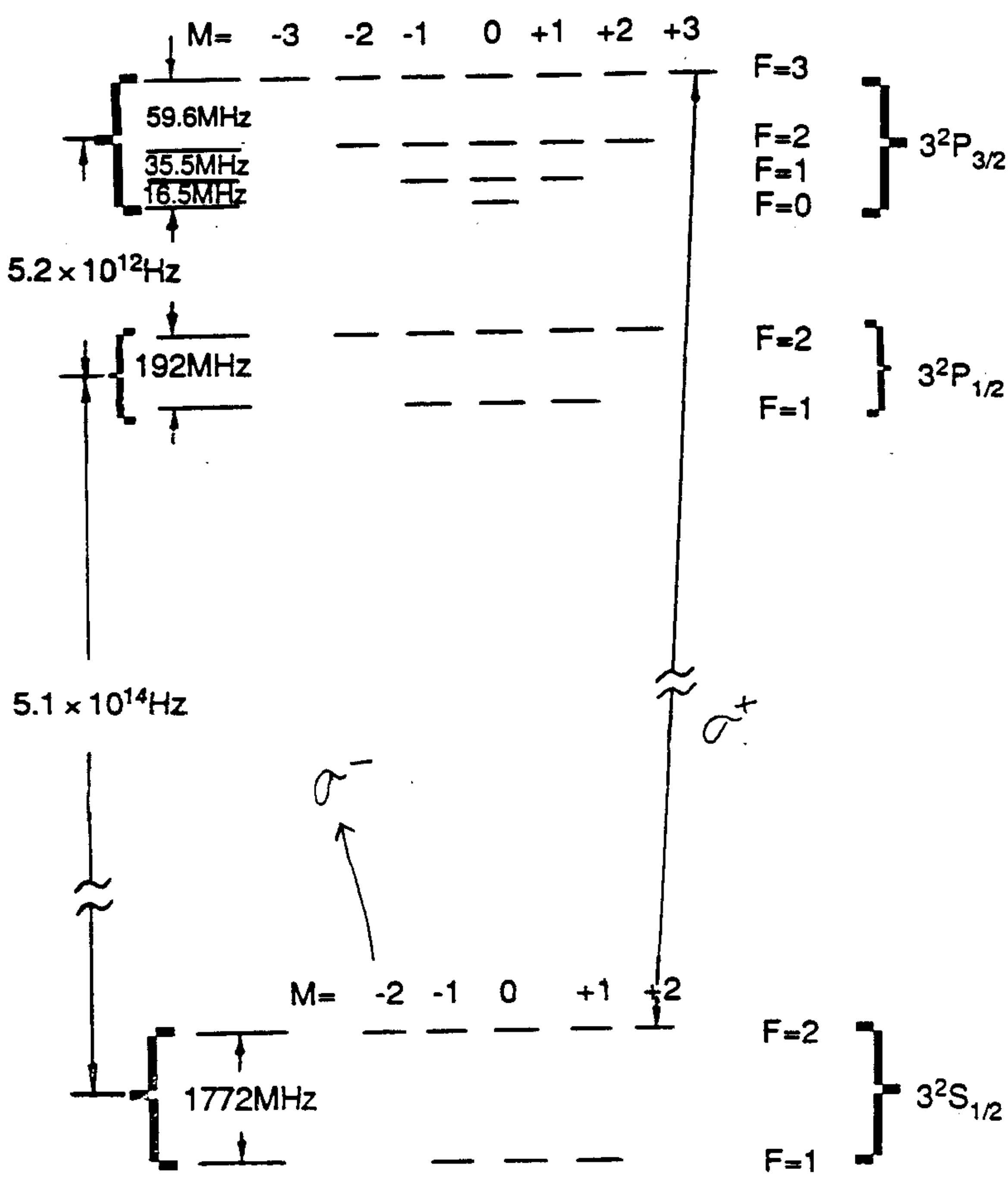


Figure 3. Hyperfine structure of ^{23}Na for the $3^2S_{1/2}$, $3^2P_{1/2}$, and $3^2P_{3/2}$ states (Not to scale).

Absorption cross section for resonant radiation

Einstein A, B coefficients

prob. of spontaneous decay per unit time

$$A(\omega) = \frac{1}{\tau} = \frac{2\omega^3}{3\hbar c^3} V_{AB}^2$$

where $V_{AB} = e\, z_{AB}$ ⟵ dipole matrix element

$P(\omega) = u(\omega) B(\omega)$ = prob. of induced transition per unit time [i.e., absorption probability]

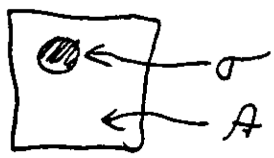
[$u(\omega)$ = radiation field energy density per unit volume per unit angular frequency]

and $$A = B \frac{\hbar \omega^3}{\pi^2 c^3}$$

[also $B_{AB} = B_{BA}$] i.e. $B = \frac{\pi^2 c^3}{\hbar \omega^3} \Delta f$ where $\Delta f = 1/\tau = A$

if one uses "cross section" concept, imagine a laser beam of total power I [watts] in area A, all of whose energy is within natural linewidth of transition.

then $B(\omega) u(\omega) = P(\omega) = \frac{I \sigma}{A \hbar \omega}$ · sec^{-1}



~~thus~~ But $I/A = J$ [power flux] $= J(\omega) \Delta\omega$

where $J(\omega)$ = power flux density per unit angular frequency

and $\Delta\omega$ = laser bandwidth, assumed to be narrower than atomic bandwidth.

now $\quad u(\omega) = \dfrac{J(\omega)}{c}$

thus

$$Bu = \frac{B\,J(\omega)}{c} = J(\omega)\,\Delta\omega\,\frac{\sigma}{\hbar\omega}$$

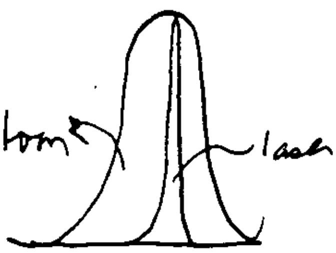
$$B = \frac{\pi^2 c^3}{\hbar\omega^3} A = \frac{\pi^2 c^3}{\hbar\omega^3 \tau}$$

and $\quad B = \dfrac{\Delta\omega\,\sigma c}{\hbar\omega}$

so

$$\sigma\,\frac{\Delta\omega\cdot\cancel{c}}{\cancel{\hbar\omega}} = \frac{\pi^2 c^{\cancel{3}\,2}}{\cancel{\hbar}\,\omega^{\cancel{3}\,2}\tau}$$

$$\sigma = \frac{\pi^2 c^2}{\Delta\omega\cdot\tau\ \omega^2} = \frac{\pi^2 \bar{\lambda}^2}{(\Delta\omega\cdot\tau)} = \frac{\pi^2\lambda^2}{8\pi^3 \Delta f\,\tau}$$

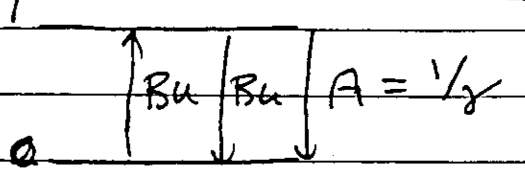


if laser linewidth < atom linewidth,

then use $\quad \Delta\omega = 2\pi\,\Delta f = 2\pi\frac{1}{\tau}$

and $\quad \boxed{\sigma = \dfrac{\pi^{\cancel{2}}\,\bar{\lambda}^2}{2\cancel{\pi}} = \dfrac{\lambda^2}{8\pi}\ \text{cm}^2}$

steadystate 2-level system rate equations



$$\cancel{Bu f_0 = Bu f_1 + A f_1}$$

or $f_0 = Bu f_1 + A f_1$

$$f_0 = \frac{Bu(\omega) + A}{2Bu(\omega)A}$$

$$f_0 = \frac{b/2 + 1}{b+1} \; ; \; f_1 = \frac{1}{2}\,\frac{1}{1 + 1/b}$$

where $b = \dfrac{2\pi^2 c^3 u(\omega)}{\hbar\omega^3} = \dfrac{2\pi^2 c^3 J}{\hbar\omega^3 \Delta\omega\, c}$

where J = laser power density (w/m^2)

time-dependent sol'n in terms of the density operator

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}, \quad T_r\, \rho = 1$$

the off-diagonal matrix elements are the coherence terms that connect 1, 2:

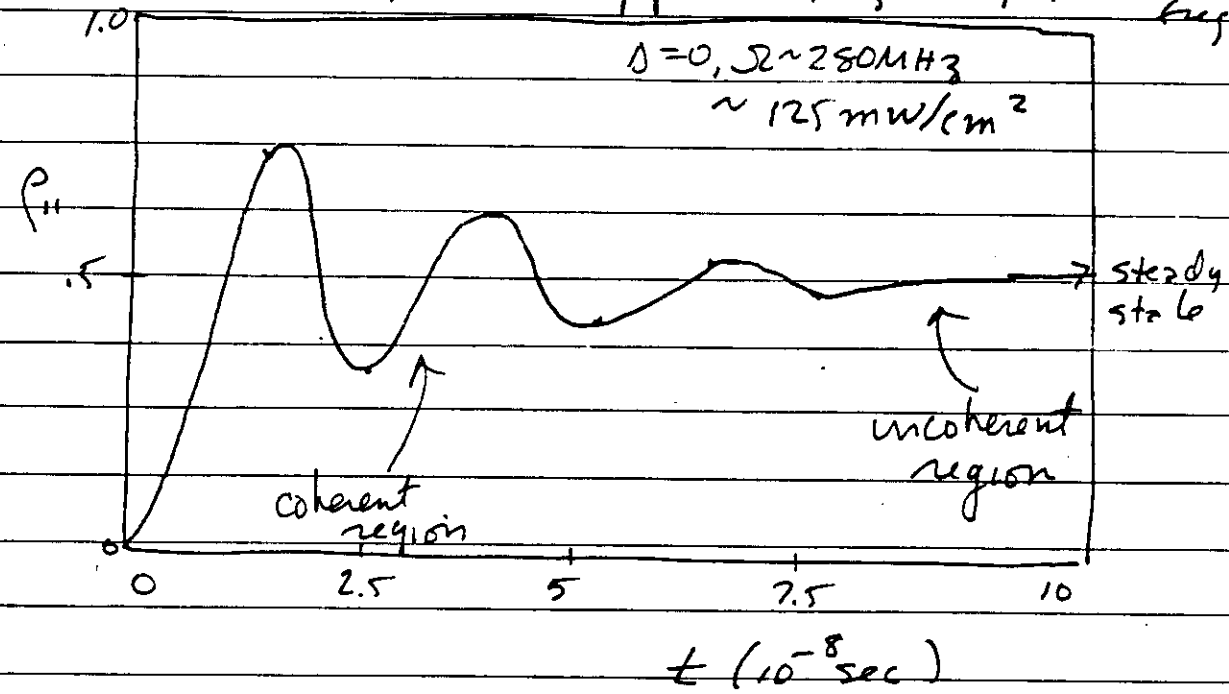
$$\dot\rho_{11} = -\gamma\rho_{11} + \frac{i\Omega}{2}\rho_{01}e^{i\Delta t} - \frac{i\Omega}{2}\rho_{01}^{*}e^{-i\Delta t}$$

$$\dot\rho_{10} = -\frac{i}{2}\Omega e^{i\Delta t}(\rho_{11} - \rho_{00}) - \frac{\gamma}{2}\rho_{10}$$

$$\rho_{01} = \rho_{10}^{*}, \quad \rho_{11} + \rho_{00} = 1 \; ; \; \gamma = \text{damping term} = 1/A$$

[see Sargent, Scully, Lamb p236 ff]

Ω = Rabi frequency. We show here the ρ_{11} solution
$\Delta = \omega - \nu$; ω = applied frequency, ν = transition frequency



this answers an important question about collisions in laser fields, when does the laser act as a state-preparer only, and does not alter the scattering process? Answer: when the steady state, incoherent region is reached, i.e., in about 10^{-7} sec, or, 1% of the interaction time for weak lasers. However, for strong lasers, coherence region might well dominate.

back to steady state: J. Bjorkholm et al Phys Rev A 23, 491 (1981)

$$b = \left(\frac{I}{I_s}\right) \frac{\Delta\nu_N^2/4}{\Delta^2 + \Delta\nu_N^2/4}$$

where $\Delta\nu_N = \gamma/2\pi$ (FWHM)

$\Delta = \omega - \nu$; laser bandwidth $\simeq 0$

I_s = "saturation parameter" $\equiv \frac{2\pi^2 \Delta\nu_N h \nu^3}{c^2}$

put in some numbers: $I_s = 19$ mw/cm^2

assume $I \simeq 190$ mw $\times 10$ [typical cw intensity, focussed

$\simeq 2000$ mw/cm^2

$$\Delta\nu_N = \frac{1}{2\pi \times 3\times10^{-8}} = \frac{10^8}{6.54} \ \mathrm{sec}^{-1}$$

at resonance $b = 100$

and $f_1 \approx 1/2$

more realistically, say $\Delta \sim 10\times10^6$ Hz, $b \simeq 4$

and $f_1 = .4$

this would correspond to $\therefore \frac{t_{atom}}{\tau} \cdot 4 \sim \frac{10^{-5}}{3\times10^{-8}}$

or about 130 photons per cm of interaction.

But there is a problem:

momentum transfer from laser to atom

momentum of a photon $= mc$

with $m = \frac{h\nu}{c^2}$

$\therefore$ momentum $= \frac{h\nu}{c}$

$$M\Delta V = \frac{h\nu}{c}; \quad \Delta V = \frac{h\nu}{Mc} \sim \frac{6\times10^{-27} \times 5\times10^{14}}{23\times1.6\times10^{-24}\times3\times10^{10}} \ \frac{\mathrm{cm}}{\mathrm{sec}}$$

$\Delta V \simeq 3$ cm/sec [for sodium]

the atom will rapidly be detuned! frequency change per photon

$$\Delta f = f \times \frac{\Delta v}{c} \simeq 5\times10^{14} \times \frac{3}{3\times10^{10}}\ Hz$$

$$= 50\ KHz/\text{photon}$$

or about 1 line-width per cm of interaction! this can be converted into an advantage, in fact, into an entire new field of atom-laser physics, namely atom cooling and trapping! I won't go into too much detail here, since this is not the primary subject of Winter College. However, it is important to note that (i) atom cooling (and velocity compression) will enable one to prepare very cold and "monoenergetic" atom beams. Atom traps will allow for collision experiments of types not previously possible.