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THEORY OF FAST ELECTRON H ATOM COLLISIONS IN THE PRESENCE OF A LASER FIELD

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THEORY OF FAST-ELECTRON H-ATOM COLLISIONS IN THE PRESENCE OF A LASER FIELD

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Abstract. We present some recent advances made in the theory of laser assisted collisions. We describe, in particular, analytical and numerical calculations of the cross sections of various processes occurring in the course of collisions of (relatively) fast electrons with hydrogen atoms, in the presence of a laser field. The topics covered in the discussion include free-free transitions, laser-assisted electron-impact excitation of atomic hydrogen and (e,2e) collisions in the presence of a laser field. Emphasis is put on the lowest order perturbative approach and on a treatment that allows us to include non-perturbative contributions. Both approaches are expounded and their results compared. We discuss also the influence of various laser parameters (polarization, frequency, and intensity) which strongly affect the dynamics of the collision. The results of our analysis show that the "dressing" of the target states by the laser field plays a determining role in the physics of such laser-assisted collisions.

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Summary

I. INTRODUCTION: DESCRIPTION OF THE (ELECTRON-ATOM-LASER) SYSTEM

In these lectures we will review some recent advances obtained in the theoretical description of a class of laser assisted electron-H atom collisions. The motivation for these studies was that, though hydrogen is the simplest atom, the system considered here displays the main ingredients essential to comprehend the dynamics of laser assisted collisions. It has, in addition, the advantage of lending itself to "exact" calculations. In this context "exact" means that, once an appropriate set of physical approximations has been chosen (typically: dipole approximation, single-mode laser, Born expansion for the scattering amplitude, ...), the calculation of the relevant transition amplitudes can be performed exactly, i.e. without further

approximation. This has the definite advantage of making clear the limits of validity of the theory.

The physics of the subsystems constituted of each pair of components -- electron-atom, electron-laser, atom-laser -- is now well understood, with, perhaps, the exception of the atom-laser system which still gives rise to numerous theoretical as well as experimental studies. Essentials of the theory of e-H atom scattering can be found in textbooks,[1] and the problem of the description of an electron in a plane wave field was solved more than fifty years ago by Volkov.[2] The state of the art concerning laser-atom interactions can be found in recent reviews[3,4] and also in the comprehensive book by Faisal.[5]

When those components are considered all together, i.e. when considering the dynamics of the (electron-atom-laser) system, one has to deal, in some sense, with an unsolvable "three-body" problem. This means that we must resort to more drastic approximations, in order to remain at a tractable theoretical (and computational!) level. See Ref. 5 Chapter 12, and also Ref. 6.

As an illustrative example of such computations we have chosen to describe, in some detail, recent calculations of transition probabilities and cross sections related to the collisions of relatively fast electrons with hydrogen atoms, in the presence of an external electromagnetic field. To this end we first write down the Hamiltonian operator for the system:

$$H = \sum_{i=0,1} \frac{1}{2} \left(\vec{p}_i + \frac{1}{c} \vec{A}\right)^2 - \frac{1}{r_1} - \frac{1}{r_0} + \frac{1}{r_{01}} + H_F \quad . \tag{1}$$

Here the indexes 0 and 1 label the projectile electron and the atomic electron respectively; $\vec{p}_i$ are the momentum operators; $c = 1/\alpha = 137$ a.u. is the velocity of light; $\vec{A}$ is the vector potential of the field and H_F the field Hamiltonian. (Note we have used atomic units: $m = 1$; $\hbar = 1$; $q/(4\pi\varepsilon_0) = e = 1$ or $\varepsilon_0 = 1/4\pi$).

For the sake of future discussions it is convenient to rewrite this Hamiltonian as follows:

$$H = H_{at} + H_e^{(0)} + H_e^{(1)} + H_I + H_F \tag{2}$$

where one has explicitly:

$$\text{i)} \quad H_{at} = \frac{1}{2} p_1^2 - \frac{1}{r_1} \quad , \tag{3}$$

which represents the H-atom Hamiltonian, such that

$$H_{at}|n\rangle = \varepsilon_n |n\rangle; \quad \varepsilon_n = -\frac{1}{2n^2} \quad ; \tag{4}$$

$$\text{ii)} \quad H_e^{(0)} = \frac{1}{2} p_0^2 \quad , \tag{5}$$

describing the projectile

$$H_e^{(0)}|\vec{k}\rangle = \frac{k^2}{2} |\vec{k}\rangle; \quad \langle \vec{r}|\vec{k}\rangle = e^{i\vec{k}\cdot\vec{r}} \quad ; \tag{6}$$

$$\text{iii)} \quad H_e^{(1)} = -\frac{1}{r_0} + \frac{1}{r_{01}} \quad , \tag{7}$$

which represents the interaction between the projectile and the atom;

$$\text{iv)} \quad H_I = \sum_{i=0,1} \left(\frac{1}{c} \vec{p}_i \cdot \vec{A} + \frac{1}{2c^2} A^2\right) \quad , \tag{8}$$

corresponding to the coupling between the field and both the projectile and the atomic electron. As regards the potential vector $\vec{A}$, we note that, since we will discuss radiative processes involving lasers operated in the IR, visible, or VUV ranges, we will rely on the dipole approximation. In addition, as we will not discuss effects related to the spatial dependence of the laser pulses,[7] it appears that the presence of the terms $A^2/2c^2$ in H_I, would shift the energy of each particle by the same quantity. Accordingly, we will use in the calculations, the following reduced form of H_I:

$$H_I = \frac{1}{c} \sum_{i=0,1} \vec{p}_i \cdot \vec{A} \quad . \tag{9}$$

This change results in a mere (non-observable) shift of the origin of the energy scale for the system.

Depending on the physical process considered and on the theoretical approach chosen, it is convenient to use either one of two distinct (though equivalent) representations of the field operators $\vec{A}$ and H_F.

1) If one considers spontaneous photon emission (as in bremsstrahlung) or lowest order contributions to a given radiative process, it is more convenient to use a quantized form of the vector potential $\vec{A}$, which reads:

$$\hat{A} = \frac{1}{2} \sum_i \alpha \left(\frac{8\pi}{\omega_i V}\right)^{1/2} \left(\hat{\varepsilon}_i^* a_i^+ e^{+i\omega_i t} + \hat{\varepsilon}_i a_i e^{-i\omega_i t}\right) \quad . \tag{10}$$

Note that, for the sake of comparison with the classical representation of the field, we have expressed $\hat{A}$ in the Heisenberg picture, in order to make explicit its time dependence. See also Ref. 5 for another derivation. Here the sum runs over all the modes and polarization states of the field; V is the quantization volume, and the factor 1/2 has been included for the sake of coherence with the classical description of the field.

Within this quantized framework, the operators a_i and a_i^+ act on occupation number states $|N_i\rangle$ with the following rules:

$$a_i|N_i\rangle = \sqrt{N_i}\,|N_i-1\rangle \quad , \tag{11a}$$

$$a_i^+|N_i\rangle = \sqrt{N_i+1}\,|N_i+1\rangle \quad , \tag{11b}$$

and the Hamiltonian operator reads explicitly:

$$H_F = \sum_i \omega_i \left(a_i^+ a_i + \frac{1}{2}\right) \quad . \tag{12}$$

If one considers a single-mode laser with frequency ω, polarization $\hat{\varepsilon}$ and occupation number $N \gg 1$, the observables of the field are readily obtained from the general expression:

$$\hat{E} = -\frac{1}{c}\frac{\partial \hat{A}}{\partial t} = -\frac{i}{2}\left(\frac{8\pi\omega}{V}\right)^{1/2}\left[a^+ \hat{\varepsilon}^* e^{i\omega t} - a\hat{\varepsilon} e^{-i\omega t}\right] \quad , \tag{13}$$

and the intensity of the laser is expressed in terms of the average value of $|\hat{E}|^2$:

$$I = \frac{c}{4\pi} \langle N|\, E^2 |N\rangle \quad . \tag{14}$$

As

$$aa^+|N\rangle = (N+1)|N\rangle \quad , \tag{15a}$$

$$a^+a|N\rangle = N|N\rangle \quad , \tag{15b}$$

and $\langle N|N\rangle = 1$, we obtain, after a simple algebraic manipulation,

$$I = \frac{1}{2}\frac{c\omega}{V}(2N+1) \quad .$$

Even for moderate laser intensities $N \gg 1$ (see Ref. 5, p. 125) and one gets eventually

$$I \approx \frac{Nc\omega}{V} \quad , \quad (\hbar=1) \quad , \tag{16}$$

which represents the quantity of laser radiation energy flowing through a unit surface per second.

2) If one considers intense single-mode lasers and, in addition, chooses to use a nonperturbative description of the process to be investigated, it may be more convenient to rely on a classical description of the field. In this classical limit, which correctly describes the field since we have assumed $N \gg 1$,[8] the vector potential $\hat{A}$ becomes

$$\hat{A} = A_0 \hat{\varepsilon} \cos \omega t \quad , \tag{17}$$

and the corresponding electric field is

$$\hat{E} = E_0 \hat{\varepsilon} \sin \omega t \quad , \tag{18}$$

with

$$E_0 = \frac{\omega}{c} A_0 \quad . \tag{19}$$

The corresponding instantaneous intensity I_t is

$$I_t = \frac{c}{4\pi} |\hat{E}|^2$$

and the time averaged (over one cycle of the laser field) intensity becomes

$$I = \frac{c}{8\pi} |E_0|^2 \quad . \tag{20}$$

We note that the correspondence with the quantum expression of I is ensured, provided we identify

$$E_0 = \left(\frac{8\pi N\omega}{V}\right)^{1/2} \quad . \tag{21}$$

As the laser intensity is one of the key parameters governing the physics of laser-assisted collisions, a remark concerning the so-called atomic unit of intensity I_0 may be in order here. I_0 is defined as the (time-averaged)

intensity of a laser field whose peak field strength E_0 is equal to the atomic unit of electric field strength intensity:

$$\mathcal{E}_0 = \frac{e}{a_0^2} = 5.142 \times 10^9 \text{ V cm}^{-1} \quad . \tag{22}$$

Since the atomic system of units uses the unrationalized Gaussian system, one must express $\mathcal{E}_0$ in statvolt cm^{-1} (see Ref. 9, Appendix p. 611), i.e.

$$\mathcal{E}_0 = 5.142 \times 10^9 \times \frac{10^8}{c} \text{ statvolt cm}^{-1} = 1.715 \times 10^7 \text{ statvolt cm}^{-1} \quad .$$

When substituting this value in the expression, Eq. (20), for the intensity one obtains the so-called atomic unit of intensity I_0:

$$I_0 = 3.509 \times 10^{23} \text{ erg s}^{-1} \text{ cm}^{-2} = 3.509 \times 10^{16} \text{ W cm}^{-2} \quad , \tag{24}$$

the latter value being often quoted in the literature.

Having defined the Hamiltonian of the global system we are (at least in principle!) able to treat any kind of laser assisted e-H-atom collision. Let us now turn to the description of the processes we will discuss in these lectures.

II. RADIATIVE ELECTRON-ATOM COLLISIONS

A. Bremsstrahlung

Let us first recall that, even in the absence of external sources, the coupling between the electron-atom system and the empty modes of the field (vacuum field), can lead to the spontaneous emission of one photon.[10] This is the well known bremsstrahlung process which can be symbolically represented, when specialized for H-atom, as follows:

$$e(\vec{k}_i) + H(1s) \rightarrow e(\vec{k}_A) + H(1s) + \gamma(\omega,\vec{\epsilon}) \quad . \tag{25}$$

This equation represents the collision of an incoming electron, with asymptotic momentum $\vec{k}_i$, with an H-atom, resulting in the (spontaneous) emission of one photon with frequency ω and polarization $\vec{\epsilon}$. The scattered electron has an asymptotic momentum $\vec{k}_A$ and the atom remains in the ground state. Conservation of energy requires that:

$$\frac{k_i^2}{2} = \frac{k_A^2}{2} + \omega \quad . \tag{26}$$

As spontaneous emission is involved here, the relevant approach is the lowest-order perturbative one (for higher-order corrections see Ref. 10). Within this framework, sophisticated theoretical analyses have been developed for the bremsstrahlung spectrum emitted in the course of collisions of fast electrons with atoms, including those for high Z elements.[11] The description of lower energy collisions is more involved and, again, the system e-H atom collisions is a good test case for the problem. It displays, in particular, the possibility of observing resonances corresponding to excitation frequencies of the atom, in the bremsstrahlung spectrum.[12,13] As we will show below, this is also the case for (stimulated) free-free transitions (FFT), which we will discuss next.

B. Free-free Transitions (FFT)

If the collision takes place in the presence of an external field, the e-atom system can exchange energy (i.e. photons) with the field. If the atom remains in its ground state when the exchange is completed, the corresponding process is usually referred to as FFT. It can be represented by the following equation:

$$e(\vec{k}_i) + H(1s) + N\gamma(\omega,\vec{\epsilon}) \rightarrow e(\vec{k}_A) + H(1s) + (N\mp\nu)\gamma(\omega,\vec{\epsilon}) \quad . \tag{27}$$

The symbols have the same meaning as in Eq. (26) above, except the term $N\gamma(\omega,\vec{\epsilon})$ in the lefthand side, which symbolizes the presence of a single-mode field with an occupation number $N \gg 1$, frequency ω, and polarization $\vec{\epsilon}$. In the final state the occupation number of the laser mode becomes $(N\mp\nu)$, ν being the number of exchanged photons between the field and the e-atom system; $-\nu$ corresponding to ν-photon absorption and $+\nu$ to (stimulated) emission. The conservation of energy relation is now:

$$\frac{k_i^2}{2} = \frac{k_A^2}{2} \pm \nu\omega \quad . \tag{28}$$

Studies on one-photon FFTs were first developed in an astrophysical context since they contribute to the opacity of stellar atmospheres.[14] We will address here the more specific case of FFTs induced by strong single-mode

sources, which can give rise to the exchange of several photons ($\nu>1$). Such processes have been observed in several instances, in studies of e-rare gas atom collisions in the presence of a far infrared (CO_2) laser.[15] Values of ν, up to 12, have been observed. Note also that absolute values of multiphoton FFT cross sections were recently measured for the same e-atom systems.[16] Comparison with theory is extremely difficult,[16,17] and detailed calculations have been reported mostly for e-H-atom collisions, see Sec. III, below.

C. Laser-assisted Electron-impact Atomic Excitation

The atom can be excited as a result of the joint action of the projectile impact and of the exchange of photons with the laser field. The corresponding equation reads:

$$e(\vec{k}_i) + H(1s) + N\gamma(\omega,\vec{\varepsilon}) \rightarrow e(\vec{k}_A) + H^*(n\ell) + (N\pm\nu)\gamma(\omega,\vec{\varepsilon}) \qquad (29)$$

and the energy conservation relation is now:

$$\frac{k_i^2}{2} + \varepsilon_{1s} = \frac{k_A^2}{2} + \varepsilon_n \pm \nu\omega \quad . \qquad (30)$$

Though predicted since the early days of quantum mechanics,[18] this process has been observed only very recently in helium (see Ref. 19 and Dr. Newell's lectures at this Winter College). Again, detailed calculations have been reported only for e-H atom collisions, see Sec. III below.

D. Laser-Assisted Electron-Impact Atomic Ionization: (e, 2e) Collisions

Electron-atom collisions resulting in the ionization of the target are often dubbed as (e,2e) collisions. Implicit to this designation is the fact that the two outcoming electrons are detected in coincidence. In spite of the considerable technical difficulties inherent into coincidence measurements, such processes (observed in the absence of laser!) have given rise to numerous experimental studies.[20] We note that such an interest was motivated by the fact that, under some well-defined conditions, the measurement of the angular distribution of the ejected electrons can furnish valuable information on the electronic momentum distribution in the target, thus leading to an actual (e,2e) spectroscopy. The physics of a large class of (e,2e) collisions is

well understood and in the special case of e-H atom collisions, the agreement between experiment and theory is now striking.[21]

In the presence of a laser, it is expected that the e-atom system could exchange one or several photons with the field. In the case of a single-mode laser, the process can be described via the following equation:

$$e(\vec{k}_i) + H(1s) + N\gamma(\omega,\vec{\varepsilon}) \rightarrow H^* + e(\vec{k}_A) + e(\vec{k}_B) + (N\pm\nu)\gamma(\omega,\vec{\varepsilon}) \quad , \qquad (31)$$

and the energy conservation relation is now:

$$\frac{k_i^2}{2} + \varepsilon_{1s} = \frac{k_A^2}{2} + \frac{k_B^2}{2} \pm \nu\omega \quad . \qquad (32)$$

Note that we have neglected here the possible recoil energy of the nucleus.

Up to the present, no experiments have been reported on such processes. However, as we will show in these lectures, very recent theoretical investigations[22,23] suggest that the (e,2e) cross sections, hence the angular distributions of the ejected electrons, are extremely sensitive to the dressing of the target by the laser field and could lead to envision an (e,2e) spectroscopy of dressed atoms. This point will be further elaborated below.

III. THEORETICAL APPROACHES

A. Coupling Parameters

All the above considered processes can be described in a unified way within the framework of the conventional, nonrelativistic, Quantum Electrodynamics theory (QED). However, depending on the physical conditions bearing upon the process considered, different strategies can be contemplated, which can be classified as either perturbative or nonperturbative. Indeed, two important parameters govern the physics of laser-assisted collisions: the kinetic energy of the projectile and the laser intensity. Their respective magnitudes will help to decide which approach is more appropriate to treat the problem considered.

Let us consider first the effect of the projectile kinetic energy on our choice of a theoretical method. It is indeed convenient to distinguish between low energy and high energy regimes. Here low energy can be defined as $E_c = k_i^2/2 \lesssim |\varepsilon_i|$, where $|\varepsilon_i|$ is the ionization energy of the target, and correspondingly high energy means $E_c > |\varepsilon_i|$. The magnitude of E_c affects our

choice of a theoretical description in the following way: In the low energy regime (typically a few eV) both the initial and final states of the electron-atom system can be conveniently considered as excited states of the H^- ion. We note that this picture is akin to a nonperturbative representation of the electron-atom system. Within this framework, any exchange of photons between this system and an external field may be considered as a radiative transition taking place between excited states (possibly doubly-excited states) of the H^- ion. The main difficulty, inherent to such an approach, lies in the difficulty in correctly representing the continuum states of the H^- ion. Many theoretical works have been nevertheless conducted along these lines; see, for example; Refs. 5 (Chapter 12), 6 and 14. One main outcome of these studies was the prediction, which has been indeed verified, that one could scan the resonances embedded into the e-atom continuum. As the main features of such low-energy radiative collisions have been thoroughly reviewed in the book by Faisal, Ref. 5, we will not dwell on this topic, and will turn now to the case of higher kinetic energies for the projectile electron.

Indeed, if the kinetic energy of the incoming electron is higher than the ionization potential of the target atom, one can rely on the usual tools of Collision Theory. Within this framework it is then natural to use the (perturbative) Born expansion, in terms of the electron-atom interaction potential $H_e^{(1)}$, to describe the dynamics of the projectile-target system. We note also that if the kinetic energy of the projectile is high enough (typically 250 eV) exchange effects can be neglected within a low order treatment of the collision. As we will show later on, such a treatment helps to visualize the contribution of the scattering event to the overall transition amplitude.

We will discuss now to what extent the magnitude of the laser intensity can affect our choice of a theoretical description of laser-assisted collisions. However, before we start the discussion a few remarks might be pertinent here. A careful analysis shows indeed that the relevant parameter, to discuss the influence of the laser on the collision, is not only its intensity, as often stated in the literature. It appears actually that the frequency also play an important role. This results from the fact that what is important here is the magnitude of the coupling between the field, on the one hand, and both the atom and the projectile, on the other hand.

In order to discuss the atom-field coupling a good yardstick is the atomic field strength intensity $\mathcal{E}_0 = 5.142 \times 10^9$ V cm^{-1}, which corresponds to the average field experienced by the atomic electron in its ground state. It is expected that lowest order perturbation theory predictions will no longer be reliable if the laser peak field strength intensity E_0 becomes comparable to $\mathcal{E}_0$.

The situation is not so clear when discussing laser-assisted collisions: one has also to take into account the effect of the field on the projectile. So long as one can neglect the influence of the target on its dynamic state, the projectile can be described by a Volkov wave[2,5,6]:

$$\langle \vec{r} | \chi_{\vec{k}} \rangle = \exp[i\vec{k}\cdot(\vec{r}-\vec{\alpha}_0 \sin\omega t) - iE_k t] \qquad (33)$$

where $E_k = k^2/2$, and

$$\vec{\alpha}_0 = \frac{A_0}{\omega}\hat{\epsilon} = \frac{E_0}{\omega^2}\hat{\epsilon} \quad , \qquad (34)$$

is a parameter (with the dimension of a length) which represents the amplitude of the (classical) oscillatory motion of the free electron into the laser plane-wave field. The magnitude of α_0 provides an indication of the strength of the coupling between the field and the projectile. It appears that the effect of the field on the collision process will depend on the relative magnitude of α_0 as compared to the spatial extension of the target, i.e. a few atomic units for a neutral atom in its ground state. As a consequence, a good criterion for the validity of a perturbative approach will be, in fact:

$$\alpha_0 = \frac{E_0}{\omega^2} \ll 1 \quad . \qquad (35)$$

This immediately shows that, even for moderate laser intensities, α_0 can become very large, provided $\omega \to 0$. This behavior, which is connected to the so-called infrared divergence of QED, indicates that the low- frequency limit presents some special features which lead to interesting developments.[5,6,24,25]

Another way to look at this point is to remark that the dimensionless quantity $\vec{k}\cdot\vec{\alpha}_0 \sin\omega t$ represents some (classical) coupling between the field and the electron. This coupling must remain "small" in order for a lowest order perturbative approach to be acceptable. For electron kinetic energies of the

order of a few a.u. this leads to a condition analogous to the relation Eq. (35). We note also the occurrence of the scalar product $\vec{k}\cdot\vec{\varepsilon}$, which can make the coupling vanish if $\vec{\varepsilon} \perp \vec{k}$, whatever the magnitude of ω_0 is. This point will be of interest later on, when discussing the kinematics of laser-assisted collisions.

Let us now summarize the results of this discussion concerning the expected range of validity of a lowest-order perturbative approach:

--For the (ground-state) atom-field interaction, the lowest-order perturbative description remains valid so long as:

$$E_0 \ll \varepsilon_0 = 5.142 \times 10^9 \text{ V cm}^{-1} \quad . \tag{36}$$

--For the field-projectile interaction, the lowest-order perturbative description will be adequate, so long as

$$\vec{\varepsilon}\cdot\vec{k}\,\frac{E_0}{\omega^2} \ll 1 \quad . \tag{37}$$

Since we will discuss here collisions involving relatively fast electrons: [$k^2/2 = 0$ (1 a.u.)], this condition becomes, discarding geometrical effects related to the orientation of the laser polarization:

$$E_0 \ll \omega^2 \times 5.14 \times 10^9 \text{ V cm}^{-1} \quad , \tag{38}$$

where ω^2 is expressed in atomic units.

As a consequence, to ensure the global validity of a lowest order perturbative approach one must choose the most conservative of the two criteria, Eqs. (36) or (38). As we will only consider here relatively low frequency laser sources, operated in the IR, visible or VUV ranges ($\omega \lesssim 1$), the condition Eq. (38) will have to be satisfied in order to safely develop a lowest-order perturbative scheme.

Before describing such an approach it might be worthwhile to mention that ω and E_0 are not the only laser parameters influencing the dynamics of the collision. As already mentioned, the kinematics of the collision, and in particular the orientation of the laser polarization $\vec{\varepsilon}$ with respect to the characteristic momenta of the system, will also play a determining role in the angular distribution of the outgoing electrons, see Sec. V below.

B. Lowest-order Perturbative Approach

As we shall consider collisions involving relatively fast electrons, we can use a doubly perturbative expansion of the transition amplitude, i.e. perturbative with respect to the e-atom interaction:

$$H_e^{(1)} = \frac{1}{r_{01}} - \frac{1}{r_0} \quad , \tag{39}$$

and to the field-charged particle interaction:

$$H_I = \frac{1}{c}\,(\vec{p}_0+\vec{p}_1)\cdot\vec{A} \quad . \tag{40}$$

The unperturbed states of the system are eigenstates of the uncoupled Hamiltonian:

$$H_0 = H_{at} + H_F + H_e^{(0)} \quad , \tag{41}$$

such as:

$$H_0|n,N,\vec{k}\rangle = (\varepsilon_n + N\omega + \frac{k^2}{2})|n,N,\vec{k}\rangle \quad . \tag{42}$$

Here the index n stands for the triad $n = (n,\ell,m)$ of atomic quantum numbers and $\varepsilon_n = -1/(2n^2)$; N is the occupation number of the considered single-mode field with frequency ω, and $H_e^{(0)}$ is the free electron Hamiltonian such that:

$$H_e^{(0)}|\vec{k}\rangle = \frac{k^2}{2}\,|\vec{k}\rangle \quad . \tag{43}$$

We have found it convenient to use the Schrödinger representation of the quantized version of the vector potential operator $\vec{A}$, which reads, in the dipole approximation:

$$\vec{A} = \frac{1}{2}\,c\left(\frac{8\pi}{\omega V}\right)^{1/2}(\vec{\varepsilon}a + \vec{\varepsilon}^{*}a^{+}) \quad . \tag{44}$$

Let us now illustrate the technique by considering an assisted collision resulting in the simultaneous absorption of one photon and excitation of the target to an unspecified state $|n\rangle$:

$$e(\vec{k}_I) + H(1s) + N\gamma(\vec{\varepsilon},\omega) \rightarrow e(\vec{k}_A) + H(n) + (N-1)\gamma(\omega,\vec{\varepsilon}) \quad , \tag{45}$$

the conservation of energy imposing:

$$\frac{k_1^2}{2} + \varepsilon_{1s} = \frac{k_A^2}{2} + \varepsilon_n - \omega \quad . \tag{46}$$

The corresponding lowest-order transition amplitude is second order (order one in $H_e^{(1)}$ and in H_I) and is accordingly of the general form:

$$T^{(2)} = \frac{1}{(2\pi)^3} \sum_{n'} \sum_{N'} \int d^3\vec{k}' \frac{\langle \vec{k}_A, N-1, n | H_I | n', N', \vec{k}' \rangle \langle k', N', n' | H_e^{(1)} | 1s, N, \vec{k}_i \rangle}{(\varepsilon_{1s} + N\omega + \frac{k_i^2}{2} - \varepsilon_{n'} - N'\omega - \frac{k'^2}{2})}$$

$$+ \text{sym.}(H_I - H_e^{(1)}) \quad . \tag{47}$$

The factor $(2\pi)^{-3}$ arises from the closure relation for the free electron basis set:

$$\langle \vec{r} | \vec{k}' \rangle = e^{i\vec{k}'\cdot\vec{r}}; \quad (2\pi)^{-3} \int d^3\vec{k}' |\vec{k}'\rangle\langle\vec{k}'| = 1 \quad ; \tag{48}$$

and the sum over the atomic states n' runs over the whole hydrogen spectum.

The corresponding transition probability can then be obtained from Fermi's Golden Rule:

$$W^{(2)} = 2\pi |T^{(2)}|^2 \rho(E) \quad , \tag{49}$$

the expression of the final density of states $\rho(E)$ depending on the particular process considered.

It should be clear at this point that such an approximation provides a description of the collisional step within the first Born approximation and the one-photon absorption to the lowest nonvanishing order of the theory. Such a transition amplitude can be conveniently represented with the help of the Feynman diagrams shown in Fig. 1.

Note that a more refined treatment of the same process could include a second Born description of the collisional stage, thus leading to the 6 Feynman diagrams displayed in Fig. 2.

One could also take into account the next higher order (actually third-order) radiative corrections to the one-photon absorption step, keeping the first Born treatment of the collision. This would lead to include 96 diagrams similar to those shown in Fig. 3. Although all of these diagrams are not

topologically distinct, this demonstrates, if needed, the complexity inherent to high order treatments of the radiative coupling with the e-target system.

It appears fortunately that a lowest order calculation in both the e-atom and field-(e-atom) couplings already contains most of the physical information and that further refinements would not alter significantly our analysis of the processes considered. More precisely, one can show that, in most cases, the low order laser-induced modifications will be much larger than those possibly produced by higher order corrections.

As an illustration of the perturbative approach, we will come back to the calculation of the transition matrix element in Eq. (48). Let us first explicit the first matrix element entering the expression of $T^{(2)}$:

$$T_A = \frac{1}{(2\pi)^3} \sum_{n'} \sum_{N'} \int d^3\vec{k}' \frac{\langle \vec{k}_A, N-1, n | \frac{\vec{p}\cdot\vec{A}}{c} | n', N', \vec{k}' \rangle \langle \vec{k}', N', n' | \frac{1}{r_{01}} - \frac{1}{r_0} | 1s, N, \vec{k}_i \rangle}{[\varepsilon_{1s} + (N-N')\omega - \varepsilon_{n'} + \frac{k_i^2}{2} - \frac{k'^2}{2}]} \quad . \tag{50}$$

The rightmost first order matrix element can be easily reduced:

i) the interaction Hamiltonian $H_e^{(1)}$ doesn't act on the photon state $|N\rangle$ which is accordingly left unchanged:

$$|N'\rangle = |N\rangle \quad \text{and} \quad \langle N|N\rangle = 1 \quad ; \tag{51}$$

ii) the spatial integration over the projectile position $\vec{r}_0$ can be readily performed, with the help of Bethe's integral; one has indeed[1]:

$$\int d^3\vec{r}_0 \, e^{i(\vec{k}_i - \vec{k}')\cdot\vec{r}_0} \left(\frac{1}{r_{01}} - \frac{1}{r_0}\right) = \frac{4\pi}{(\vec{k}_i - \vec{k}')^2} \left[e^{i(\vec{k}_i - \vec{k}')\cdot\vec{r}_1} - 1\right] \quad , \tag{52}$$

and this matrix element is eventually expressed in terms of an atomic matrix element of the following form:

$$\langle \vec{k}', N', n' | \frac{1}{r_{01}} - \frac{1}{r_0} | 1s, N, \vec{k}_i \rangle = \frac{4\pi}{(\vec{k}_i - \vec{k}')^2} \langle n' | [e^{i(\vec{k}_i - \vec{k}')\cdot\vec{r}} - 1] | 1s \rangle \delta_{N',N} \quad . \tag{53}$$

The second matrix element entering the expression Eq. (51) can also be reduced to a simpler atomic form but, as $H_I = 1/c\ \vec{p}\cdot\vec{A}$ acts on both the projectile and

atomic electron, it splits into two contributions:

$$H_I = \left(\frac{2\pi}{\omega V}\right)^{1/2} \vec{\varepsilon}\cdot(\vec{p}_0+\vec{p}_1)a \quad , \tag{54}$$

and the matrix element becomes (after the annihilation operator a has acted):

$$\langle \vec{k}_A, N-1, n|H_I|n', N, \vec{k}'\rangle = \left(\frac{2\pi N}{\omega V}\right)^{1/2} \vec{\varepsilon}\cdot[\vec{H}_0+\vec{H}_1] \quad , \tag{55}$$

where

$$\vec{H}_0 = \vec{k}'\langle \vec{k}_A|\vec{k}'\rangle\langle n|n'\rangle$$

or

$$\vec{H}_0 = \vec{k}'(2\pi)^3\delta(\vec{k}_A-\vec{k}')\delta_{n,n'} \quad . \tag{56}$$

(Note that we have used the fact that

$$\vec{p}|\vec{k}'\rangle = -i\vec{\nabla}\, e^{i\vec{k}'\cdot\vec{r}} = \vec{k}'|\vec{k}'\rangle).$$

Similarly $\vec{H}_1$ reads:

$$\vec{H}_1 = \langle \vec{k}_A|\vec{k}'\rangle\langle n|\vec{p}|n'\rangle \quad ,$$

or

$$\vec{H}_1 = (2\pi)^3\,\delta(\vec{k}_A-\vec{k}')\langle n|\vec{p}|n'\rangle \quad . \tag{57}$$

Eventually, the second-order matrix element, Eq. (51) can be rewritten[26,27]:

$$T_A = \frac{4\pi}{A^2}\left(\frac{2\pi N}{\omega V}\right)^{1/2}\left[\frac{\vec{k}_A\cdot\vec{\varepsilon}}{\omega}\langle n|(e^{i\vec{\Delta}\cdot\vec{r}}-1)|1s\rangle\right.$$

$$\left. + \sum_{n'} \frac{\langle n|\vec{p}\cdot\vec{\varepsilon}|n'\rangle\langle n'|(e^{i\vec{\Delta}\cdot\vec{r}}-1)|1s\rangle}{\varepsilon_{1s}+\frac{k_i^2}{2}-\frac{k_A^2}{2}-\varepsilon_{n'}}\right] \tag{58}$$

where we have introduced the momentum transfer $\vec{\Delta} = \vec{k}_i - \vec{k}_A$, a quantity which plays a determining role in the physics of the collision. We note also that the second term contains in fact the Coulomb Green's function:

$$G(\Omega) = \sum_{n'} \frac{|n'\rangle\langle n'|}{\Omega - \varepsilon_{n'}} \quad . \tag{59}$$

It is worth noting that the two terms contained in the expression of T_A, Eq. (58), can be associated with the contributions of the diagrams I and III, Fig. 1. The first term corresponds to a process in which the electron collides first with the atom, and then absorbs one photon (diagram I). Conversely the second term corresponds to a process in which the atom absorbs the photon after the collision (diagram III).

For computational purposes, we have transformed further the matrix elements, using the dipole-length form of the interaction operator instead of the momentum representation. It appears then, that when regrouping the contributions of the diagrams I - IV, Fig. 1, the overall transition amplitude can be rewritten as follows:

$$T^{(2)} = \frac{4\pi}{A^2}\left(\frac{2\omega N}{\omega V}\right)^{1/2}(i\omega[\langle n|\vec{\varepsilon}\cdot\vec{r}\; G(\varepsilon_n-\omega)e^{i\vec{\Delta}\cdot\vec{r}}|1s\rangle +$$

$$\langle n|e^{i\vec{\Delta}\cdot\vec{r}}\; G(\varepsilon_{1s}+\omega)\vec{r}\cdot\vec{\varepsilon}|1s\rangle] + \frac{\vec{\Delta}\cdot\vec{\varepsilon}}{\omega}\langle n|(e^{i\vec{\Delta}\cdot\vec{r}}-1)|1s\rangle) \quad . \tag{60}$$

We note that, when adding together the second order matrix elements we have been able to supress the (-1) terms contained in the original matrix elements, Eq. (58). Similarly, the change of gauge is equivalent to multiplying these matrix elements by an overall factor $i\omega$.[22,26]

The differential cross section for the fast electron being detected within $d\Omega(\vec{k}_A)$ and the atom being excited in the state $|n\rangle$, is obtained from Eq. (50), by replacing $\rho(E) = k_A/(2\pi)^3$ (a.u.) and dividing the transition rate by the flux of incoming electrons, one obtains eventually

$$\frac{d\sigma}{d\Omega(\vec{k}_A)} = 8\pi\alpha\,\frac{k_A}{k_i}\,\frac{I}{A^4}\,|T_{n,1s}|^2\,a_0^2 \quad , \tag{61}$$

where we have introduced the laser intensity $I = N\omega\omega/V$; α is the fine structure constants [$\alpha = 1/137 = c^{-1}$ (a.u.)] and the atomic matrix element $T_{n,1s}$ reads explicitly:

$$T_{n,1s} = i\{\langle n|\vec{\varepsilon}\cdot\vec{r}\; G(E_n-\omega)e^{i\vec{\Delta}\cdot\vec{r}}|1s\rangle + \langle n|e^{i\vec{\Delta}\cdot\vec{r}}\; G(E_{1s}+\omega)\vec{\varepsilon}\cdot\vec{r}|1s\rangle)$$

$$+ \frac{\vec{\Delta}\cdot\vec{\varepsilon}}{\omega}\langle n|(e^{i\vec{\Delta}\cdot\vec{r}}-1)|1s\rangle \quad . \tag{62}$$

We note that here $|n\rangle$ has not been specified and can still represent any accessible state of the hydrogen spectrum. More precisely, if:

- $|n\rangle = |1s\rangle$ the matrix element will correspond to bremsstrahlung or one-photon free-free transitions.[26-30]

- $|n\rangle = |nlm\rangle$ the matrix element will correspond to a joint electron-photon excitation of the atom, accompanied by the absorption of one photon.[26,28,31]

- $|n\rangle = |\phi^{(-)}_{\vec{k}_B}\rangle$, where $|\phi^{(-)}_{\vec{k}_B}\rangle$ represents an ingoing Coulomb wave with asymptotic momentum $\vec{k}_B$, the matrix element will correspond to a laser-assisted (e,2e) collision. Note, however, that the approximation developed here is only valid for some class of (e,2e) collisions, namely those occurring in the so-called "asymmetric geometry".[20-23] In this latter case there are two outgoing electrons in the final state and the definition of the cross section must be modified accordingly: The triple differential cross section (TDCS) for detecting a fast electron in $d\Omega(\hat{k}_A)$ with energy E_A and another (slow) electron within $d\Omega(\hat{k}_B)$, the system having absorbed one photon from the laser field is[22,23]:

$$\frac{d\sigma}{d\Omega(\hat{k}_A)d\Omega(\hat{k}_B)dE_A} = \frac{\omega}{\pi^2}\frac{k_A k_B}{k_i}\frac{I}{A}\,|T_{\vec{k}_B,1s}|^2 \quad , \qquad (63)$$

where $T_{\vec{k}_B,1s}$ has the same formal structure than $T_{n,1s}$, Eq. (62).

Several interesting features, characteristic of the physics of laser-assisted collisions, can be inferred from the general expressions, Eqs. (62), (63), of the cross sections, even before starting any computation:

--These expressions display the resonant structure of the transition amplitudes. Indeed, if the argument of one of the Green's functions matches (or nearly matches) the energy of an atomic bound state, the corresponding amplitude becomes very large and will dominate all other contributions to the total amplitude. We note that the unphysical infinities, which do appear in such a situation, arise from the fact that we neglected the shifts and widths of the atomic states. This is consistent with our lowest order treatment of the radiative process.

--One can identify the terms containing the Coulomb Green's function as arising from the lowest order radiative corrections to either the initial or final atomic states. This point will be made more explicit below, when discussing nonperturbative approaches.

--In the low-frequency limit, $\omega \to 0$, the last term of the amplitude can become dominant since its frequency dependence is in ω^{-2}. We note that this term stems from the contributions of the diagrams I and II, Fig. 1, associated with collisions events in which the projectile picks up the photon energy. This observation has given rise to numerous simplified treatments of the problem: We will discuss the limitations of such approaches in Sec. V. Let us only point out here that, for small scattering angles $\theta = (\hat{k}_i, \hat{k}_A) \to 0$, and in the low frequency limit $\omega \to 0$, this tendancy can be reversed. Indeed, if $\omega \to 0$ and $\theta \to 0$, then $A \to 0$ and $e^{i\vec{A}\cdot\hat{r}} \to 1$, which can make this contribution vanish even for small values of ω.

--In the limit of small momentum transfer ($A \to 0$), one can replace the exponential $e^{i\vec{A}\cdot\vec{r}} \to 1 + i\vec{A}\cdot\vec{r}$ (Bethe-Born approximation), which leads to a simplified version of the matrix element, Eq. (62).[27] It appears, however, that this approximation has a very limited range of validity.[30,31]

We will discuss now in what ways this lowest-order perturbative model can be modified when the conditions (36)-(38) are not verified, i.e. in the nonperturbative regime.

C. Nonperturbative Approaches

If one considers either the case of intense fields such that $E_0 \gg \omega^2 z_0$ or resonant situations such that the laser frequency matches an atomic excitation frequency, the perturbative approach is no longer valid (at least in its low order version discussed so far). In the latter case of resonant laser frequencies, a two level model of the atom is adequate: references to earlier works may be found in Ref. 32, see also Refs. 5 (Chapter 12), and 6 and references therein. On the other hand, in nonresonant situations, the role of the entire atomic spectrum cannot be neglected, which leads to a more difficult problem, and compells us to resort to more sophisticated nonperturbative approaches.

A nonperturbative description of the laser-projectile states is well known since it amounts a representation of the projectile as a Volkov wave, Eq. (33). No equivalent, ready-to-use prescription exists for the laser-atom interaction. Though different kinds of Floquet-like approaches have been proposed, mainly to deal with atomic multiphoton ionization, they lead to quite heavy computations and don't provide closed form expressions for the "dressed" atomic wave functions.[33-36]

One can nevertheless assume that, for a bound, negative energy, atomic state, the effect of the laser field will be less important than for the free electron. This observation leads to the idea of dressing the projectile to all orders, through the use of a Volkov wave, and representing the atom by a static potential. It has been shown indeed that such a simplified model is valid in the low frequency limit.[5,6,24,37,38]

A more refined model consists of treating perturbatively the laser-atom interaction. Earlier attempts have discussed the role of the perturbation within the relevant atomic multiplets,[39] while more general treatments include the contribution of the whole atomic spectrum.[40] We note that such an approach is certainly justified, in the low frequency regime, for laser field strength intensities E_0 obeying the following inequalities:

$$E_0 > E_0 > \omega^2 E_0, \text{ with } E_0 = 5.14 \times 10^9 \text{ V cm}^{-1} \quad . \tag{64}$$

We will now describe this latter approach which has given rise to several new interesting developments and presents the advantage of joining smoothly with the perturbative results, in the low intensity limit.

Within the framework of this model it is convenient to use the S-matrix formalism, which describes the laser-assisted process as transitions induced by the e-atom interaction $H_e^{(1)}$, Eq. (7), between dressed states of the atom-projectile system. For the sake of illustration of the method we will consider the lowest order term of such an S-matrix expansion:

$$S^{(1)} = -i \int_{-\infty}^{+\infty} dt \, \langle \chi_{\vec{k}_A}, \phi_n | H_e^{(1)} | \phi_{1s}, \chi_{\vec{k}_i} \rangle \quad . \tag{65}$$

This matrix element represents the first Born approximation for a collision between a dressed electron (Volkov wave $|\chi_{\vec{k}_i}\rangle$) and a dressed atom (wave function $|\phi_{1s}\rangle$) resulting in a dressed scattered electron $|\chi_{\vec{k}_A}\rangle$ and the atom ending in the (for the time being unspecified) dressed state $|\phi_n\rangle$.

The general form of Volkov waves entering this expression has already been given in Eq. (33). A lowest-order perturbative representation for the dressed atomic wave functions is:

$$|\phi_n\rangle = e^{i(\varepsilon_n t + \vec{A}\cdot\vec{r}/c)} \left(|n\rangle + \frac{i}{2} \left[e^{-i\omega t} \, G(\varepsilon_n + \omega) - e^{+i\omega t} \, G(\varepsilon_n - \omega) \right] \vec{E}_0 \cdot \vec{r} \, |n\rangle \right) , \tag{66}$$

where $|n\rangle$ is the unperturbed atomic eigenket, with eigenenergy ε_n, $G(\Omega)$ is the Coulomb Green's function and, for computational convenience we have used the dipole length form $H_I = -\vec{E}_0 \cdot \vec{r}$ of the interaction Hamiltonian. Accordingly, we have included the factor $e^{-i\vec{A}\cdot\vec{r}/c}$, to ensure gauge consistency with the Volkov waves, used to describe the projectile.

By using the generating function for Bessel functions:

$$\exp(i\vec{k}\cdot\vec{\alpha}_0 \sin\omega t) = \sum_{k=-\infty}^{+\infty} e^{ik\omega t} \, J_k(\vec{k}\cdot\vec{\alpha}_0) \quad , \tag{67}$$

the time integration in Eq. (65) is readily performed, and one obtains:

$$S^{(1)} = -\frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} \delta\left(\frac{k_A^2}{2} + \varepsilon_n - \frac{k_i^2}{2} - \varepsilon_{1s} - k\omega\right) f_{B1,k} \quad , \tag{68}$$

where $f_{B1,k}$ represents the first Born approximation for the scattering amplitude $|1s\rangle \to |n\rangle$, accompanied with the transfer of k photons. Within the approximation retained here, i.e. keeping only the lowest order contribution to the dressing of atomic states, one shows easily that $f_{B1,k}$ contains in fact three terms[23,40-42]

$$f_{B1,k} = f_I + f_{II} + f_{III} \quad , \tag{69}$$

which read explicitly:

$$f_I = -\frac{2}{A^2} J_k(\lambda) \, \langle n | (e^{i\vec{A}\cdot\vec{r}} - 1) | 1s \rangle \quad ; \tag{70a}$$

$$f_{II} = \frac{1}{A^2} \{ J_{k+1}(\lambda) \langle n | e^{i\vec{A}\cdot\vec{r}} \, G(\varepsilon_{1s} - \omega) \vec{E}_0 \cdot \vec{r} \, | 1s \rangle$$

$$- J_{k-1}(\lambda) \langle n | e^{i\vec{A}\cdot\vec{r}} \, G(\varepsilon_{1s} + \omega) \vec{E}_0 \cdot \vec{r} | 1s \rangle \} \quad ; \tag{70b}$$

$$f_{III} = \frac{i}{A^2} \{ J_{k+1}(\lambda) \langle n | \vec{E}_0 \cdot \vec{r} \, G(\varepsilon_n - \omega) e^{i\vec{A}\cdot\vec{r}} \, | 1s \rangle$$

$$- J_{k-1}(\lambda) \langle n | \vec{E}_0 \cdot \vec{r} \, G(\varepsilon_n + \omega) e^{i\vec{A}\cdot\vec{r}} \, | 1s \rangle \} \; ; \tag{70c}$$

where $\lambda = \vec{A}\cdot\vec{\alpha}_0$ and $\vec{A} = \vec{k}_i - \vec{k}_A$ is the momentum transfer. The corresponding cross section is then given by the usual scattering theory formula[1]:

$$\frac{d\sigma}{d\Omega(\hat{k}_A)} = \frac{k_A}{k_i} |f_{B1,\ell}|^2 \quad . \tag{71}$$

An interesting property of these matrix elements is that, for small values of the parameters $\lambda = \vec{\lambda}\cdot\hat{\alpha}_0$ (characteristic of the laser-projectile coupling) and E_0 (characteristic of the laser-atom coupling), one easily recovers the lowest-order perturbative expression Eq. (60) of the transition amplitude. This is achieved by merely retaining the leading terms in the Bessel function expansions and keeping the lowest order contribution in E_0.

Up to now, the expressions Eqs. (70) are general, since the final atomic state $|n\rangle$ is still unspecified. However, depending on the final state $|n\rangle$ considered, these amplitudes correspond to different physical processes. More precisely, if:

- $|n\rangle = |1s\rangle$, the transition amplitude $f_{B1,\ell}$ corresponds to multiphoton FPT, with exchange of ℓ photons.[40,41]

- $|n\rangle = |nlm\rangle \neq |1s\rangle$, where $|nlm\rangle$ represents any atomic bound state, he transition amplitude $f_{B1,\ell}$ corresponds to laser-assisted excitation, with exchange of ℓ photons.[42]

- $|n\rangle = |\phi^{(-)}_{\vec{k}_B}\rangle$, where $|\phi^{(-)}_{\vec{k}_B}\rangle$ represents an ingoing Coulomb wave with asymptotic momentum $\vec{k}_B$, the transition amplitude corresponds to laser-assisted (e,2e) collisions.[22,23]

This latter case, however, contains several particularities which deserve a few comments. Indeed, though this model is expected to provide sensible results in the so-called asymmetric geometry (see the preceding Section), one might question the validity of such a lowest-order treatment of the dressing of a positive energy Coulomb state. More precisely, the question arises of deriving a convenient form for a "dressed Coulomb wave", which could be valid for a low energy electron experiencing the effects of both a Coulomb potential and an intense laser field.

This difficult problem has not yet been solved and, for the time being, only various ansatz have been proposed. The first one, proposed by Jain and Tzoar,[43] consists on multiplying the Coulomb wave function $|\phi^{(-)}_{\vec{k}_B}\rangle$ by a Volkov-like factor $\exp(-i\vec{k}_B\cdot\hat{\alpha}_0 \sin\omega t)$, which leads to the following form of the dressed continuum wave function:

$$|\phi^{\vec{k}_B}\rangle = e^{-i(\vec{k}_B\cdot\hat{\alpha}_0 \sin\omega t)} |\phi^{(-)}_{\vec{k}_B}\rangle \quad . \tag{71}$$

Approximately valid in the low frequency regime, this ansatz has been used in earlier treatments of laser-assisted (e,2e) collisions.[44] When substituted into the expression Eq. (65) of the S-matrix element, and neglecting the dressing of the ground state, this approximation leads to the following simplified expression for the first Born amplitude:

$$f_{B1,\ell} \sim -\frac{2}{k^2} J_\ell[(\vec{\lambda}-\vec{k}_B)\cdot\hat{\alpha}_0] \langle \phi^{(-)}_{\vec{k}_B} | e^{i\vec{\lambda}\cdot\vec{r}} | 1s\rangle \quad . \tag{72}$$

It should be noted however, that, in the low intensity regime, this approximation does not permit recovery of the lowest order perturbative expression, Eq. (60), of the transition amplitude $T^{(2)}$. In fact, by retaining the leading term in E_0 in the expression Eq. (72) of $f_{B1,\ell}$ one would only recover the last term of the perturbative form for $T^{(2)}$.

Other, more refined, ansatz have been proposed to deal with this difficult problem.[45-48] Broadly speaking, they retain the general form of Jain and Tzoar's, Eq. (71), and allow, in addition, the momentum $\vec{k}_B$ contained in the Coulomb wave function to be shifted by a field-dependent term:

$$\vec{k}_B \rightarrow \vec{k}_B(\omega) = \vec{k}_B + \omega\hat{\alpha}_0 \cos\omega t \quad . \tag{73}$$

One has accordingly:

$$|\phi^{\vec{k}_B}\rangle \sim e^{-i(\frac{k_B^2}{2} t + \vec{k}_B\cdot\hat{\alpha}_0 \sin\omega t)} |\phi^{(-)}_{\vec{k}_B(\omega)}\rangle \quad . \tag{74}$$

It is still not clear to what extent such an approximation would permit the recovery of the perturbative expression for the transition amplitude, in the low intensity limit, see however, Refs. 49 and 50. We will not discuss further this approximation and will, instead, briefly describe an improvement to the Jain and Tzoar ansatz, which does not suffer from such a limitation. This improvement, first suggested by Banerji and Mittleman,[51] consists in looking for a dressed wave function of the general form:

$$|\tilde{v}_{\vec{k}}\rangle = e^{-i\left(\frac{k^2}{2}t + \vec{k}\cdot\vec{\alpha}_0 \sin\omega t\right)} |v_{\vec{k}}\rangle \quad . \tag{75}$$

Then, by substituting this ansatz form into the time-dependent Schrödinger equation:

$$\left(i\frac{\partial}{\partial t} - H_{at} - \frac{1}{c}\vec{A}\cdot\vec{p}\right)|\tilde{v}_{\vec{k}}\rangle = 0 \quad , \tag{76}$$

one gets the equation verified by the unknown function $|v_{\vec{k}}\rangle$:

$$\left(i\frac{\partial}{\partial t} - H_{at} - \omega\vec{\alpha}_0\cdot(\vec{p}-\vec{k})\cos\omega t\right)|v_{\vec{k}}\rangle = 0 \quad , \tag{77}$$

where $\vec{p}$ is the momentum operator. The principle of the method is then to solve this latter equation perturbatively, with respect to the last term inside the braces:

--to zeroth order one recovers the unperturbed Coulomb wave function:

$$|v_{\vec{k}}\rangle = |\phi_k^{(-)}\rangle \quad , \tag{77}$$

i.e. the Jain and Tzoar ansatz.

--to first order one gets the following, more general expression[23,49,50]:

$$|v_{\vec{k}}\rangle = |\phi_{\vec{k}}^{(-)}\rangle + \frac{1}{2\omega}\left[e^{-i\omega t}\, G(\epsilon_k+\omega)\vec{E}_0\cdot(\vec{p}-\vec{k})|\phi_{\vec{k}}^{(-)}\rangle\right.$$

$$\left. + e^{+i\omega t}\, G(\epsilon_k-\omega)\vec{E}_0\cdot(\vec{p}-\vec{k})|\phi_k^{(-)}\rangle\right] \quad . \tag{78}$$

This result, in fact, generalizes that of Banerji and Mittleman, who made the additional approximation of taking the low-frequency limit of Eq. (78). It also appears that this procedure can be further iterated, leading to the inclusion of higher-order corrections, see, for instance Refs. 52 and 53.

The approximate expression, Eq. (78), of the continuum wave function has been recently used in a calculation of laser assisted (e,2e) collisions.[23,49,50] It has, in particular, the advantage of making clear the connection with the lowest order perturbative results, in the low laser intensity limit: compare Eqs. (75) and (78) with (66).

Coming back to the discussion of the general S-matrix approach described in this section, it should be kept in mind that we have described so far the

first order Born treatment of the collision. The formalism developed here can be extended so as to include a second order Born treatment of the collision. This has been done for multiphoton FPTs in Ref. 54: such a generalization allows one to deal with less energetic collisions while taking into account exchange effects.

Let us now turn to the description of some computational techniques we have recently used in the calculation of the above [illegible] amplitudes.

IV. TOOLS FOR THE COMPUTATION

Either perturbative or not, the transition amplitudes obtained in the preceding Sections share the property of being expressed in terms of typical atomic matrix elements of the following forms:

--generalized atomic form factors:

$$f_{1,n} = \langle n|e^{i\vec{\Delta}\cdot\vec{r}}|1s\rangle \quad . \tag{79}$$

--second order matrix elements:

$$M_A = \langle n|e^{i\vec{\Delta}\cdot\vec{r}}\, G(\Omega)\vec{r}\cdot\vec{\epsilon}|1s\rangle \quad , \tag{80a}$$

and

$$M_B = \langle n|\vec{r}\cdot\vec{\epsilon}\, G(\Omega)e^{i\vec{\Delta}\cdot\vec{r}}|1s\rangle \quad . \tag{80b}$$

We note that depending on the particular process considered, the final state can belong to the discrete or continuous hydrogen spectrum.

The calculation of the generalized form factors for the ground state, Eq. (79), is straightforward. The crux of the calculation lies in fact in the computation of the second order matrix elements M_A and M_B which contain an infinite summation over the whole atomic spectrum. For example, the matrix element M_A reads explicitly:

$$M_A = \sum_{n'} \frac{\langle n|e^{i\vec{\Delta}\cdot\vec{r}}|n'\rangle\langle n'|\vec{r}\cdot\vec{\epsilon}|1s\rangle}{\Omega - \epsilon_{n'}} \quad . \tag{81}$$

Though a direct summation can be considered, since the first order matrix elements entering this expression are in principle known, some technical (numerical) difficulties arise when performing the integration over the

continuous spectrum. This is particularly so when the final state $|n\rangle$ itself belongs to the continuum or when the argument Ω of the Coulomb Green's function is positive, i.e. is located on its cut in the complex plane. It is thus worthwhile to utilize approximate treatments or to more powerful implicit summation techniques, which we will describe next.

A. The Closure Approximation

To illustrate the basis of this approximation let us consider first a typical form of the matrix element M_A with $\Omega = \epsilon_{1s} \pm \omega$. Note that such a matrix element enters the expressions Eq. (60) or Eqs. (70) of the amplitudes considered in the preceding sections. The closure approximation[55] consists of, starting from the exact expansion of the Coulomb Green's function,

$$G(\epsilon_{1s} \pm \omega) = \sum_{n'} \frac{|n'\rangle\langle n'|}{\epsilon_{1s} - \epsilon_{n'} \pm \omega} \quad , \tag{82}$$

to replace the difference $\epsilon_{1s} - \epsilon_{n'}$ by a mean excitation energy Ω_{1s}, independent of $\epsilon_{n'}$. Indeed, by replacing $\epsilon_{1s} - \epsilon_{n'} = \Omega_{1s}$ in the above expansion, one has:

$$G(\epsilon_{1s} \pm \omega) \sim \sum_{n'} \frac{|n'\rangle\langle n'|}{\Omega_{1s} \pm \omega} = (\Omega_{1s} \pm \omega)^{-1} 1 \tag{82'}$$

where 1 is the unit operator.

The crucial step, when using such an approximation, is obviously to choose the "best" value for the mean excitation energy Ω_{1s}. This can be achieved by comparing with known results for similar problems. Let us only mention here that, so far as the hydrdogen ground state is concerned, a popular method is to compare with the known static polarizability of H(1s) whose expression is given in terms of the known second order element:

$$M_0 = \sum_{n'} \frac{\langle 1s|\vec{r}\cdot\vec{\epsilon}|n'\rangle\langle n'|\vec{r}\cdot\vec{\epsilon}|1s\rangle}{\epsilon_{1s} - \epsilon_{n'}} = -\frac{9}{4} \tag{83}$$

which can be compared with:

$$M_0 = \sum_{n'} \frac{\langle 1s|\vec{r}\cdot\vec{\epsilon}|n'\rangle\langle n'|\vec{r}\cdot\vec{\epsilon}|1s\rangle}{\Omega_{1s}} = \frac{\langle 1s|(\vec{r}\cdot\vec{\epsilon})^2|1s\rangle}{\Omega_{1s}} \tag{84}$$

and leads to the (exact) value $\Omega_{1s} = -4/9$.

It appears that this value of Ω_{1s} provides excellent approximations for FFT matrix elements, in the low frequency limit.[40,41] It should be pointed out, however that much care must be exercised when dealing with excited atomic states or when the laser frequency becomes comparable to excitation frequencies of the atom.[41,42] It appears moreover, that this approximation cannot be safely used to compute transition amplitudes involving states pertaining to the continuous spectrum.

We will turn now to the description of more powerful techniques using a compact representation of the Coulomb Green's function or the so-called Dalgarno method.

B. The Coulomb Green's Function (CGF)

Compact representations of the CGF have been widely used in calculations related to multiphoton processes, see Refs. 5 (Chapters 4, 5) and 56. We will only mention here two of them which have been recently used in the context of laser-assisted collisions.[22,23,27-31]

1) Free-free transitions

In the case of FFTs one may have to deal with second order matrix elements of the particular form [see Eqs. (62) or (70)]:

$$M_{1s} = \langle 1s|e^{i\vec{\Delta}\cdot\vec{r}}\, G(\Omega)\vec{r}\cdot\vec{\epsilon}|1s\rangle \quad , \quad \Omega = \epsilon_{1s} \pm \omega \quad , \tag{85}$$

and similar expressions, symmetric with respect to the interchange of the operators $e^{i\vec{\Delta}\cdot\vec{r}}$ and $\vec{r}\cdot\vec{\epsilon}$. By using Hostler's integral representation of $G(\Omega)$,[57] Klarsfeld has been able to derive the following useful formula for the closely related matrix element:[58]

$$M_{\mu,\mu'} = \int d^3\vec{r} \int d^3\vec{r}'\, \frac{1}{r} e^{-\mu' r} e^{-i\vec{\Delta}'\cdot\vec{r}} G_c(r,r';\Omega) e^{+i\vec{\Delta}\cdot\vec{r}'} e^{-\mu r'} \frac{1}{r'} \tag{86a}$$

$$= \frac{-16\pi x}{[(\mu+x)^2+\Delta^2][(\mu'+x)^2+\Delta'^2]} \int_0^1 du\, u^{-1/x}(1-2\beta u+\gamma^2 u^2)^{-1} \quad . \tag{86b}$$

Here $x = (-2\Omega)^{1/2}$ and

$$\beta = \frac{(\mu^2-x^2+\Delta^2)(\mu'^2-x^2+\Delta'^2) + 4x^2\vec{\Delta}\cdot\vec{\Delta}'}{[(\mu+x^2)+\Delta^2][(\mu'+x)^2+\Delta'^2]} \quad , \tag{87a}$$

$$\gamma^2 = \frac{[(\mu-x)^2+\Delta^2][(\mu'-x)^2+\Delta'^2]}{[(\mu+x)^2+\Delta^2][(\mu'+x)^2+\Delta'^2]} \quad . \tag{87b}$$

It appears that M_{1s} can be obtained from $M_{\mu,\mu'}$, via the following transformation:

$$M_{1s} = \frac{1}{\pi}\frac{\partial^2}{\partial\mu\partial\mu'}\lim_{\Delta'\to 0}(i\epsilon\cdot\frac{\partial}{\partial\Delta'}\tau)M_{\mu,\mu'}\Big|_{\mu=\mu'=1} \quad . \tag{88}$$

Using the integral representation of Appell's hypergeometric functions of two variables F_1,[59,60] one can recast the sought-after matrix element M_{1s} in the following compact form:[29,30]

$$M_{1s} = i2^7(\vec{\Delta}\cdot\vec{\epsilon})[A+B(+\Delta)+B(-\Delta)] \tag{89}$$

where

$$A = \frac{1}{2}(1-x^2)^{-1}(\Delta^2+4)^{-3} \times (2 - \frac{(1+x)(\Delta^2+4)}{(2x-1)[(1+x)^2+\Delta^2]}) \; ; \tag{90}$$

and

$$B(\pm\Delta) = \frac{8x^5}{(1+x)^6[(1+x)^2+\Delta^2]^3(2x-1)(3x-1)} \times (2 - \frac{1+x\pm i\Delta}{1+x\pm i\Delta})$$

$$\times F_1(3-1/x;3,2;4-1/x;u^{\pm},u^{\mp}) \quad . \tag{90b}$$

It should be mentioned that, as the F_1 function involved here can be expressed as a finite sum of Gauss hypergeometric functions ${}_2F_1$, even more compact forms of this matrix element can be derived.[29] However, we have found that the form displayed here is suitable for analytical as well as numerical purposes. It is indeed an easy matter to investigate the analytical properties of M_{1s} in some limits of physical interest (low frequency limit, Bethe-Born approximation,...). On the other hand, numerical computations of this formula can be easily done on a micro computer (our own computations were run on a Macintosh Plus).[29,30]

The main limitation of Hostler's representation of the CGF is that it leads to cumbersome calculations when excited or continuum atomic states are involved. This limitation can be removed by using the so-called sturmian expansion of the CGF, which we will introduce next.

2) Laser-assisted atomic excitation and (e,2e) collisions

The Coulomb "sturmian" functions, systematically studied by Rotenberg,[61] provide a natural basis to obtain a very useful expansion for the CGF.[62] Indeed, when specialized to a given partial wave component with angular momentum λ, one obtains the following expansion of the radial part of the CGF

$$G_\lambda(r,r';\Omega) = \sum_{\nu=\lambda+1}^{\infty} \frac{S_{\nu,\lambda}(xr)S_{\nu,\lambda}(xr')}{1-n\chi} \quad , \tag{91}$$

where $x = (-2\Omega)^{1/2}$ and $S_{\nu,\lambda}(xr)$ are Coulomb sturmian functions:

$$S_{\nu,\lambda}(xr) = N_{\nu,\lambda}(x)e^{-xr}r^{\lambda}{}_1F_1(\lambda+1-\nu;2\lambda+2;2xr) \quad , \tag{92}$$

with

$$N_{\nu,\lambda}(x) = \frac{(2x)^{\lambda+1}}{(2\lambda+1)!}\left[\frac{(\nu+\lambda)!}{(\nu-\lambda-1)!}\right]^{1/2} \quad .$$

The main features of this representation can be summarized as follows:

i) The infinite sum over the index ν is <u>discrete</u>.

ii) Although they don't verify the same orthogonality and closure relations,[61] the sturmian functions are formally identical to discrete state hydrogenic wave functions. Accordingly, closed form (polynomial) expressions are available for the matrix elements between sturmian and hydrogenic wave functions.

iii) The infinite sum over the sturmian spectrum contained in the representation, Eq. (91), displays good convergence properties, so long as $\Omega < 0$, i.e. the argument of the CGF is negative. The case $\Omega > 0$ presents some difficulties, which we will discuss below.

For the sake of illustration of the method let us consider again a typical matrix element entering the expression of the transition amplitudes Eqs. (62) or (70):

$$M_n = \langle n|\vec{r}\cdot\vec{\epsilon}\; G(\Omega)e^{i\vec{\Delta}\cdot\vec{r}}|1s\rangle, \quad \Omega = \epsilon_n \pm \omega \quad . \tag{93}$$

Using standard partial wave expansion for the CGF and the retardation term $\exp(i\vec{\Delta}\cdot\vec{r})$, we are left with radial matrix elements of the form:

$$M_\lambda = \langle R_{n\lambda}|r\; G_\lambda(\Omega)j_\lambda(\Delta r)|R_{10}\rangle \quad , \tag{94}$$

where $|R_{10}\rangle$ and $|R_{n\ell}\rangle$ are respectively the radial wave functions of the ground state and of the final excited state and $j_\lambda(Ar)$ is a spherical Bessel function. Note that, as a result of the angular momentum coupling rules, one has: $\ell = \lambda \pm 1$. These or similar matrix elements were computed by replacing $G_\lambda(\Omega)$ by its sturmian expansion, Eq. (91), and the Bessel function by its power series expansion.[63] All the needed integrals are then expressed analytically in terms of hypergeometric polynomials.[22] As already mentioned, the sturmian sum, Eq. (91), has good convergence properties, provided $\Omega < 0$.

The physically important cases in which $\Omega > 0$ need special attention. Indeed, if $\Omega > 0$, the sturmian parameter $\varkappa = (-2\Omega)^{1/2}$ becomes imaginary and the sturmian functions are no longer bounded at infinity, which entails the use of convergence acceleration techniques,[64] or analytic continuation procedures,[65] especially when dealing with physical processes taking place into the continuum.

This latter limitation has revealed itself to be relatively harmless in applications involving transitions towards a bound state, i.e. in the case of laser-assisted atomic excitation.[27,28,31] However, numerical instabilities can show up in some matrix elements entering (e,2e) transition amplitudes, in which case the final atomic state belongs to the continuum.[22,23] This difficulty can be overcome by using the so-called Dalgarno method, we will present next.

C. The So-called Dalgarno Method[66]

This technique is based on the observation that a typical second order matrix element such as M_n, Eq. (93), can be rewritten:

$$M_n = \langle n|\vec{r}\cdot\vec{\varepsilon}\; G(\Omega)e^{i\vec{A}\cdot\vec{r}}|1s\rangle = \langle U|e^{i\vec{A}\cdot\vec{r}}|1s\rangle \quad , \tag{95}$$

where the auxiliary ket:

$$|U\rangle = G(\Omega)\vec{r}\cdot\vec{\varepsilon}|n\rangle \quad , \tag{96}$$

is the solution of the following inhomogeneous differential equation [remember that $G(\Omega) = (\Omega - H_{at})^{-1}$]:

$$(\Omega - H_{at})|U\rangle = \vec{r}\cdot\vec{\varepsilon}|n\rangle \quad . \tag{97}$$

This latter equation can always be solved by quadrature and once $|U\rangle$ is known one can, at least in principle, compute the matrix element M_n.

In hydrogen the analytical calculation can be pushed further and M_n can be ultimately expressed as a Taylor-like expansion in terms of Laplace transforms of the auxiliary ket $|U\rangle$. This technique has been implemented for two-photon ionization processes by Zernik,[67] and generalised to higher order processes (up to 9-photon ionization) by Gontier and Trahin.[68] Note however, that multiphoton transition matrix elements only contain dipole interaction operators and that one has to extend Zernik's approach to deal with the retardation factor $\exp(i\vec{A}\cdot\vec{r})$ contained in the transition amplitudes M_n.[22,23]

We will turn now to a brief discussion of the results which have been obtained by using the above described techniques.

V. RESULTS AND DISCUSSION

A. FFTs and Laser-assisted Electron-impact Atomic Excitation

These processes have the common characteristic of involving initial and final atomic bound states. Accordingly the physics of this class of laser-assisted processes presents strong similarities. We will thus only discuss here the case of FFTs in some detail, the extension to excitation processes presenting no further conceptual difficulties.

Let us now summarize in what ways the FFT differential cross sections, Eqs. (61) or (71), depend on the parameters governing the collision dynamics, i.e. the laser frequency, polarization and intensity and the momenta of the incoming and outgoing electron.[29,30] Note that, as our analysis is restricted to somewhat "large" values of the projectile kinetic energy, we will not discuss directly its influence on the energetics of the process.

We will first address the role of the laser frequency and consider two situations of physical interest:

1) The low-frequency (soft photon) limit.

One easily demonstrates that, in this limit, the last term in the perturbative amplitude, Eq. (62), becomes dominant, except for very small scattering angle; see Fig. 4. This was expected from the general discussion in Sec. IIIB. The same conclusion holds in the nonperturbative regime, in which case the amplitude f_I, Eq. (70a), is dominant.[41]

ii) Optical and VUV frequencies.

At higher laser frequencies, the dominance of the last term of Eq. (62) or f_1 in Eq. (70a) becomes less marked, except for large momentum transfer A or equivalently at large scattering angles. One notes the occurrence of deep minima in the angular distribution of the scattered electron, Fig. 5. Those minima have two distinct origins:

--the minimum denoted (a), Fig. 5, corresponds to an angle such that the scalar product $\vec{A}\cdot\vec{\epsilon} = 0$, i.e. scattering geometries such that the laser polarization is perpendicular to the momentum transfer. This effect demonstrates the role of the laser polarization orientation, with respect to the momenta of the projectile in its initial and final states.

--the minimum denoted (b), Fig. 5, stems from a destructive interference between the components of the global transition amplitude. One observes indeed that their contributions can cancel each other, for some particular values of the collision parameters, as they are of opposite signs.

These minima also appear in the dispersion curves for the variation of the differential cross section in terms of the frequency (Fig. 6). This curve displays also the resonant structure of the cross section when ω matches an atomic excitation energy.

These general features of the cross section survive even if one allows the laser intensity to increase and also if several photons ($|s| > 1$) are exchanged in the course of the process.[41] The onset of nonperturbative effects depends on the laser intensity and also on the momentum transfer $\vec{A}$, through the argument $\lambda = \vec{A}\cdot\vec{\alpha}_0$ of the Bessel functions, entering the nonperturbative expression, Eq. (71), of the transition amplitude. As a matter of fact, at fixed laser frequency and scattering geometry, the ratio R:

$$R = \left(\frac{d\sigma}{d\Omega(\vec{k}_A)}\right)_{\text{nonperturbative}} \Big/ \left(\frac{d\sigma}{d\Omega(\vec{k}_A)}\right)_{\text{perturbative}} \qquad (98)$$

departs from unity at different laser intensities, depending on the incoming electron energy, i.e. on the magnitude of A, Fig. 7.

These results clearly demonstrate that the dressing of the atom by the field plays a dominant role in the dynamics of this kind of laser-assisted process. This is particularly so at small scattering angles and for optical and VUV laser frequencies. Simplified models disregarding the role of the atomic spectrum are only valid in the low frequency limit and they cannot

account for the resonances and destructive interference effects predicted by the theory. More details on the dynamics of FFTs and laser-assisted electron-impact atomic excitation, in the regime of relatively high projectile kinetic energies discussed here, can be found in the references (29-31, 40-42, 49 and 51-54).

B. Laser-assisted Atomic Ionization: (e,2e) Collisions

In the so-called "asymmetric coplanar geometry"[20,21] we have chosen to discuss here, a fast electron of momentum $\vec{k}_I$ is incident on the H-atom target and a fast ("scattered") electron of momentum $\vec{k}_A$ is detected in coincidence with a slow ("ejected") electron of momentum $\vec{k}_B$, the three momenta being in the same plane. Moreover the scattering angle θ_A for the fast electron is kept fixed and small, while one detects the angular distribution θ_B of the ejected electron $\vec{k}_B$, see Fig. 8. The theory of the field-free process is well established and the agreement with experimental results is excellent, see Fig. 9. An interesting feature of this kind of "asymmetric geometry" collisions is that the first Born approximation provides a fair description of the process. It is, in particular, able to correctly predict the presence of a "recoil" and a "binary" peaks, approximately centered on the momentum transfer direction $\vec{A}$, in the angular distribution of the ejected electron. See Fig. 9.

We shall discuss below such angular distributions one would obtain in the presence of a laser field. For the sake of illustration we will present here some results concerning collisions accompanied by the absorption of one photon from the laser field, the conservation of energy relation being then:

$$\frac{k_I^2}{2} + \epsilon_{1s} + \omega = \frac{k_A^2}{2} + \frac{k_B^2}{2} \qquad . \qquad (99)$$

The main predictions of the theoretical analysis presented in the preceding sections can be summarized as follows: The angular distribution of the ejected electron can be dramatically changed in the presence of the laser, with respect to the field-free case. As expected, the magnitude and the form of the modifications depend critically on the laser parameters (frequency, polarization orientation, intensity) and on the kinematics of the collision.

Let us first consider the effects of the laser field strength intensity. The onset of nonperturbative effects is shown in Fig. 10, where the angular variations of the triple differential cross section (TDCS), in the

presence of a Neodymium laser (ω = 0.043 a.u.) are shown for two different field strength intensities: $E_0 = 10^6$ V cm^{-1} ($E_0 \sim 1.9 \times 10^{-4}$ a.u., $\alpha_0 \sim 0.1$), Fig. 10a, and $E_0 = 5 \times 10^7$ V cm^{-1} ($E_0 \sim 9.7 \times 10^{-3}$ a.u., $\alpha_0 \sim 5.3$), Fig. 10b. Besides the expected difference in magnitude, the angular distribution is strongly modified, the relative heights of the peaks being changed. Displayed also are the data obtained from the simplified treatment of Cavaliere et al.[44] which, though we are in its supposed domain of validity since we are considering a low frequency IR laser, underestimate the TDCS. This clearly shows the importance of atomic dressing effects, even in this low frequency regime.

Varying the laser frequency can lead to even more dramatic changes in the TDCS. In particular, if ω matches or nearly matches an atomic excitation excitation frequency:

$$\omega = \epsilon_n - \epsilon_{1s} \qquad (100)$$

the process is resonant, the cross section becomes very large and the angular distribution is completely modified. In Fig. 11a we show the angular variations of the TDCS for the same geometry as in Fig. 10, and for ω = 10.2 eV, which corresponds to a resonance on the 2p atomic state. One observes that the binary peak is split in two parts and that the recoil peak almost disappears.

We note that the process also becomes resonant if:

$$\omega = \frac{k_B^2}{2} - \epsilon_n \qquad (101)$$

(see Fig. 11b for ω = 8.4 eV, $k_B^2/2$ = 5 eV, which makes the process resonant in the state n = 2). Although the TDCS is of the same order of magnitude as before, the angular distribution is again modified and both the binary and recoil peaks are split.

We illustrate the effect of varying the laser polarization orientation in Fig. 12, where the angular variations of the TDCS are shown for two orientations. Displayed are the angular distribution for $\hat{\epsilon} \parallel \hat{k}$ and $\hat{\epsilon} \perp \hat{k}$ corresponding respectively to maximizing or minimizing the laser-projectile coupling. Again, the angular distribution is completely different in either case.

Eventually we want to mention that there is an easy way to scan the atomic resonances in these processes, even with a fixed laser frequency. It is enough to vary the detection energy of the slow electron, everything else being kept fixed. Resonances are observed if the condition, Eq. (100), is fulfilled, i.e. if:

$$\frac{k_B^2}{2} = \omega + \epsilon_n \; ; \quad \epsilon_n = -\frac{1}{2n^2} \; , \quad n = 2,3,\ldots \; .$$

See Fig. 13. We note that ω must be larger than $k_B^2/2$.

This set of results clearly shows that TDCS for laser-assisted (e,2e) collisions is extremely sensitive to the modifications of the various parameters characterizing the laser and the kinematics of the collision. The theoretical analysis conducted here demonstrates the importance of the "dressing" of the target by the laser field and shows that simplified treatments, neglecting the role of the atomic spectrum, are not reliable, even in the infrared domain. More details on the physics of these laser-assisted (e,2e) collisions can be found in Refs. 22, 23, 49 and 50.

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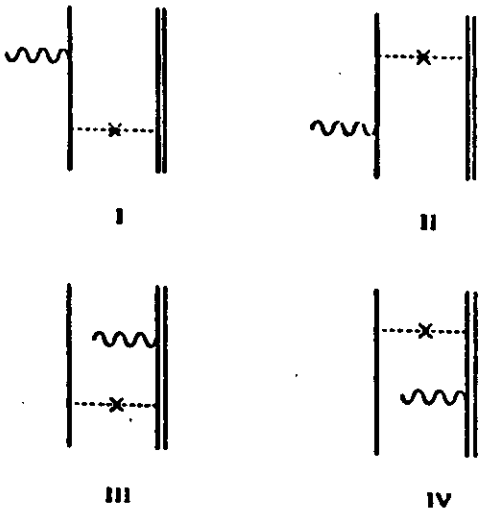

Fig. 1. Second-order diagrams contributing to one-photon transitions occurring in the course of an e-atom collision.

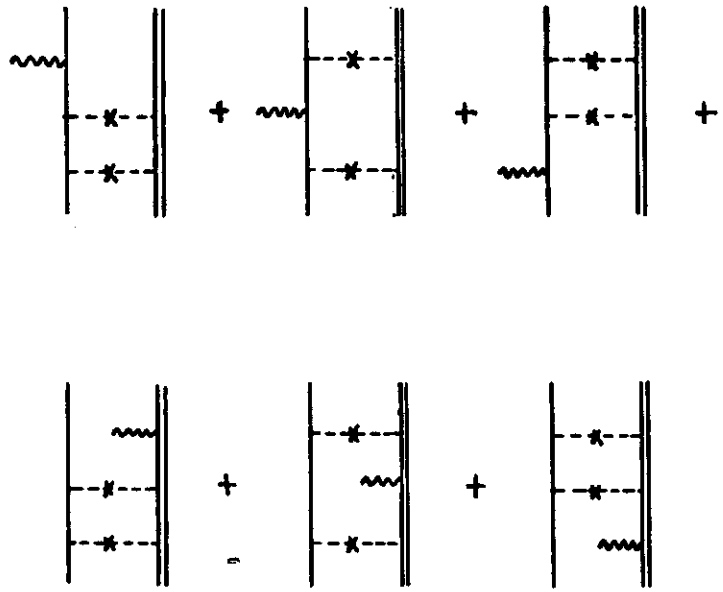

Fig. 2. Third-order diagrams contributing to one-photon transitions occurring during an e-atom collision. The collision step is described within the second-order Born approximation.

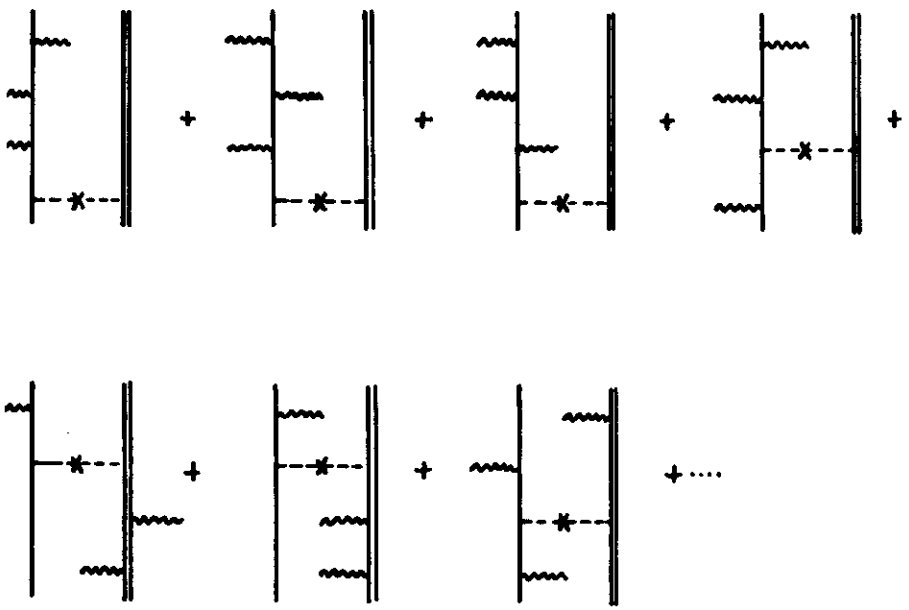

Fig. 3. Some of the fourth-order diagrams contributing to one-photon transitions occurring during an e-atom collision. The lowest-order radiative correction to one-photon absorption is included and the first Born approximation is used to describe the collision step.

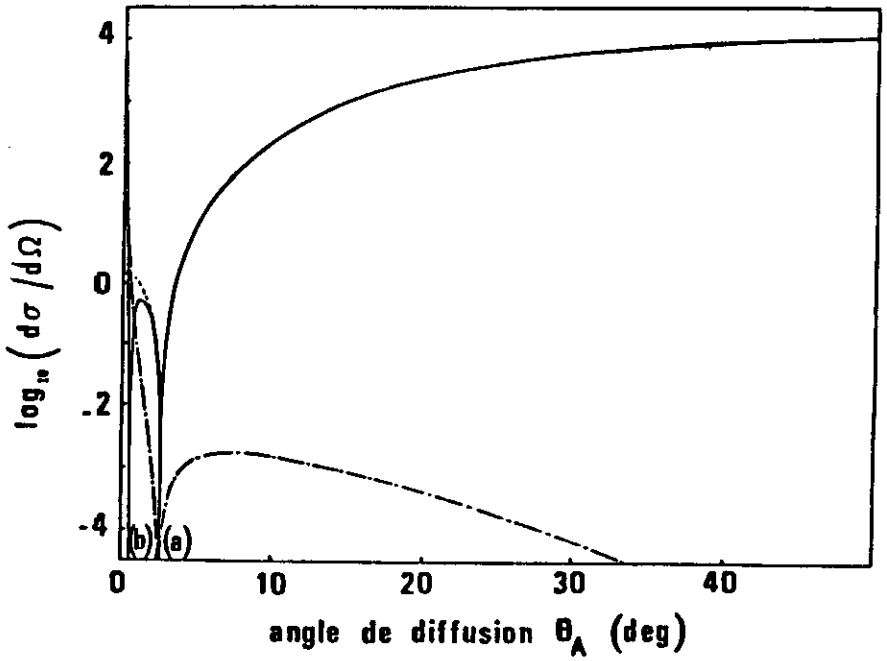

Fig. 4. Variations of $\log_{10}\,[d\sigma/d\Omega(\hat{k}_A)]$ with the scattering angle θ_A for one-photon absorption FFT. $\hat{\varepsilon}\|\hat{k}_i$, E_{k_i} = 5.0 a.u. and ω = 0.01 a.u. (low frequency regime). Dashed-line: contribution of the last term, Eq. (62); dot-dashed line: contribution of the terms containing the CGF, Eq. (62). Solid line: overall cross section (from Ref. 30).

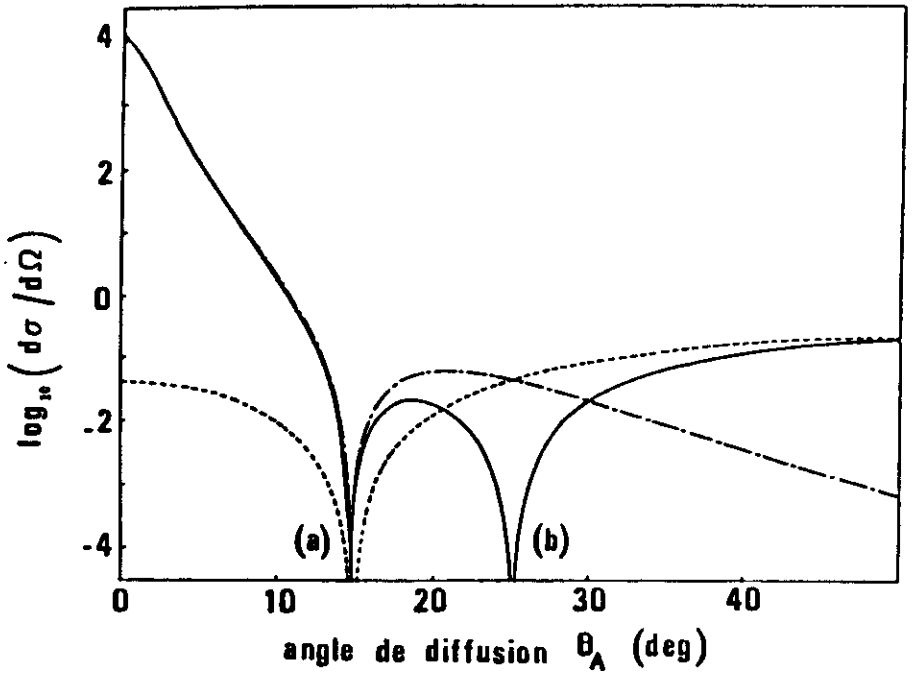

Fig. 5. Variations of $\log_{10}\,[d\sigma/d\Omega(\vec{k}_A)]$ with the scattering angle of θ_A for one-photon absorption FFT. $\hat{\epsilon} \parallel \vec{k}_i$, E_{k_i} = 5.0 a.u. and ω = 0.35 a.u. (low frequency regime). Dashed-line: contribution of the last term, Eq. (62); dot-dashed line: contribution of the terms containing the CGF, Eq. (62). Solid line: overall cross section (from Ref. 30).

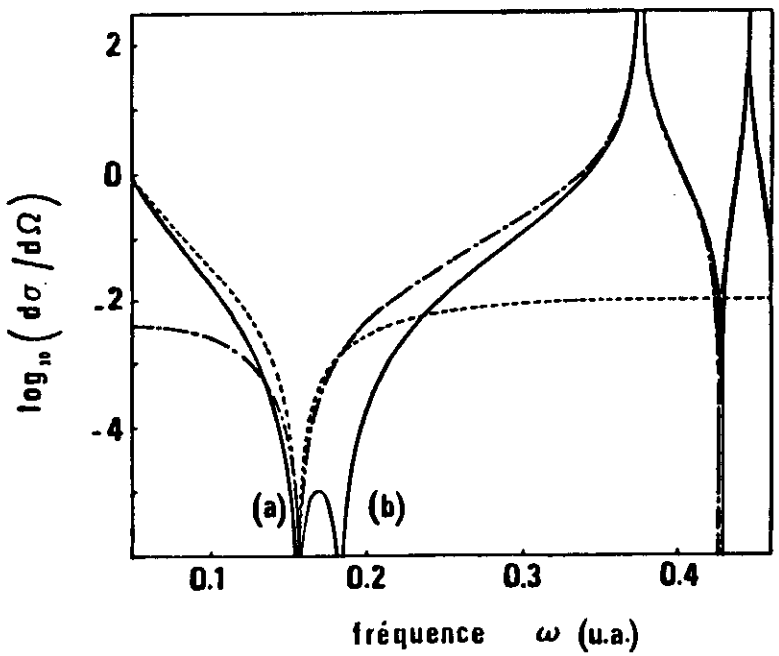

Fig. 6. Variations of $\log_{10}\,[d\sigma/d\Omega(\vec{k}_A)]$ in terms of the laser frequency ω for one-photon absorption FFT. $\hat{\epsilon} \parallel \vec{k}_i$, θ_A = 10°, fixed; E_i = 5.0 a.u. Dashed line: contribution of the last term, Eq. (62); dot-dashed line: contribution of the terms containing the CGF, Eq. (62). Solid line: overall cross section (from Ref. 30).

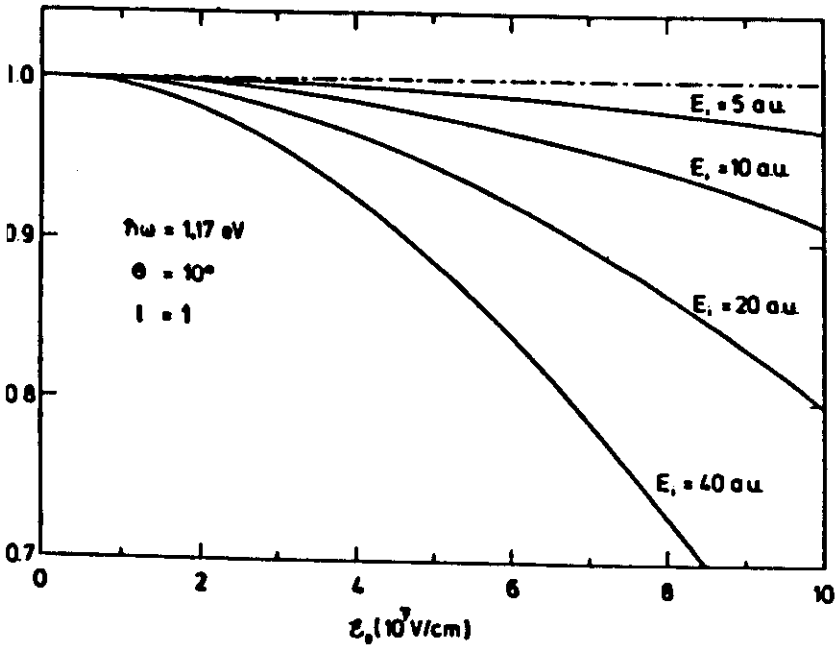

Fig. 7. The ratio R of Eq. (98) as a function of the laser electric field strength E_0 for various values of the incident electron energies for one-photon absorption. The laser photon energy is $\omega = 0.043$ a.u., the scattering angle is $\theta_A = 10°$ and $\hat{\varepsilon} \perp \hat{k}_i$ (from Ref. 41).

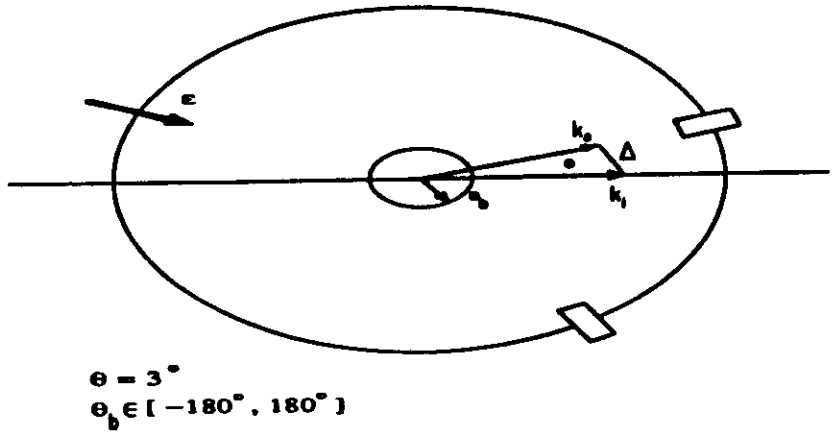

Fig. 8. Typical "asymmetric coplanar geometry" for (e,2e) collisions.

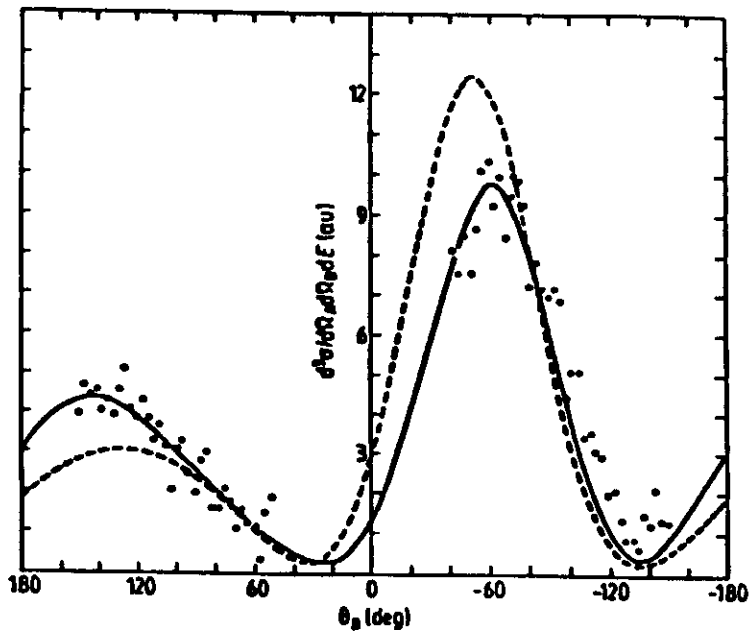

Fig. 9. Angular variations of the (field free) TDCS for (e,2e) collisions. Dots: experimental results (H. Ehrhardt et al., Ref. 21); dashed line: first Born calculation; solid line: results from a second Born EBS calculation (F. W. Byron, Jr. et al., Ref. 21).

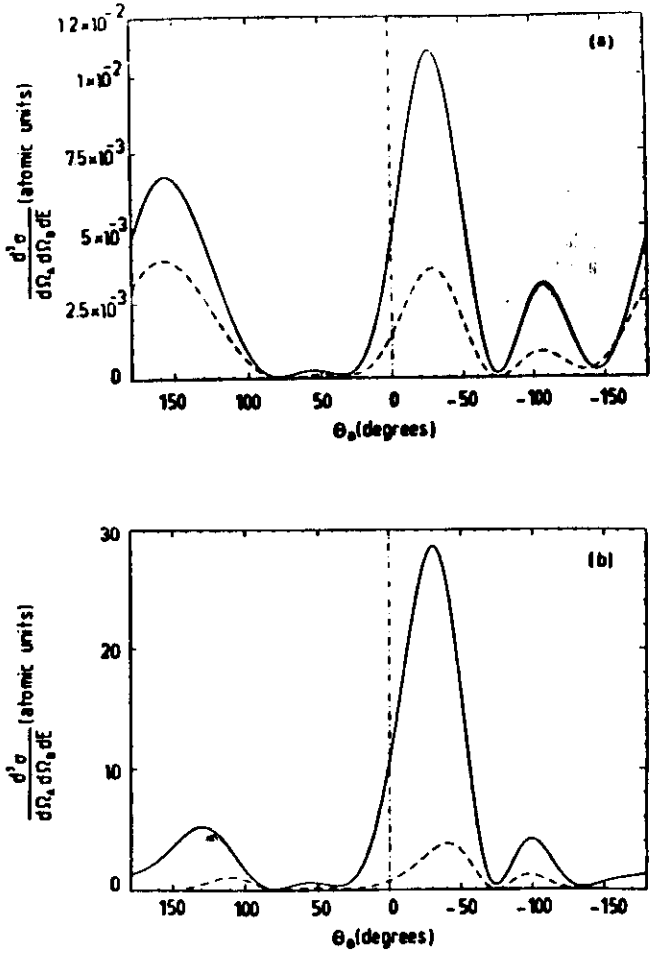

Fig. 10. Angular variations of the TDCS for laser-assisted (e,2e) collisions. E_{k_1} = 250 eV, θ_s = 3°, E_{k_B} = 5 eV, ω = 1.17 eV. (a) Perturbative regime, $E_0 = 10^6$ V cm^{-1} (b) non-perturbative regime, $E_0 = 5 \times 10^7$ V cm^{-1}. Dashed line: results obtained by using the simplified approach of Cavaliere et al., Ref. 44. Solid line: complete calculation.

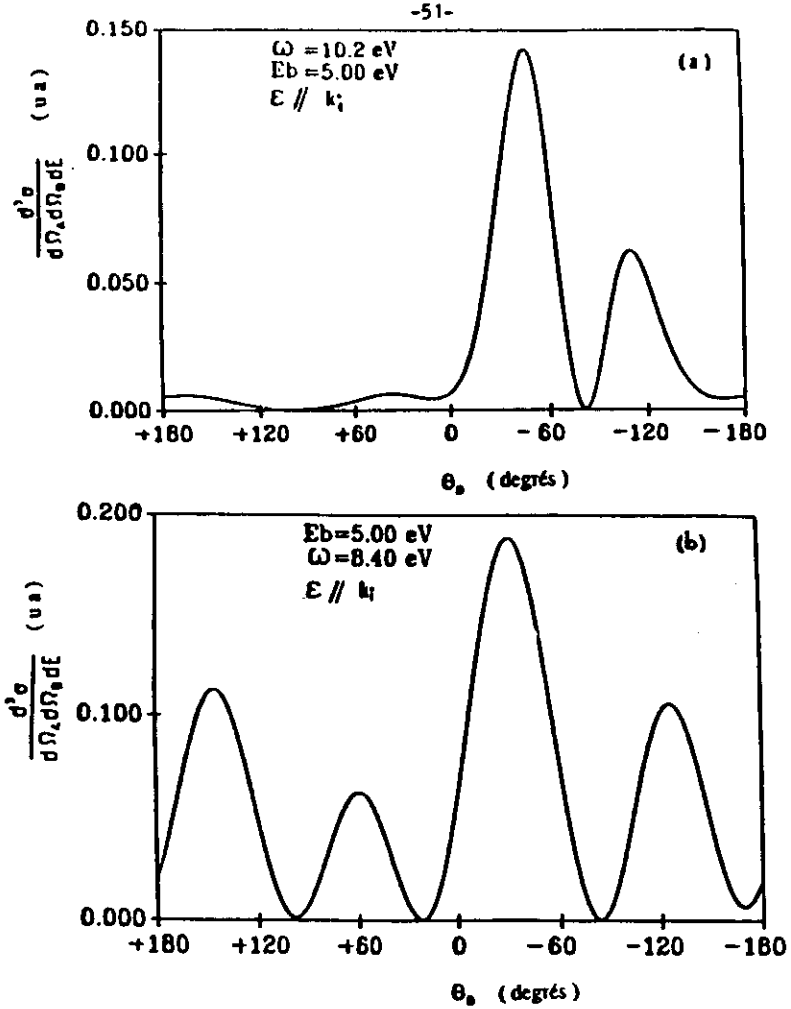

Fig. 11. The TDCS for laser-assisted (e,2e) collisions as a function of the ejected electron angle θ_B. E_{k_i} = 250 eV, E_{k_B} = 5 eV, θ_A = 3°, $\hat{\varepsilon} \parallel \hat{k}_i$, $E_0 = 10^7$ V cm^{-1}. (a) The laser photon energy is ω = 10.2 eV and corresponds to the resonance condition of Eq. (100) for the 2p state. (b) The laser photon energy is ω = 8.4 eV and corresponds to the resonance condition of Eq. (101) for the n = 2 manifold (from Ref. 23).

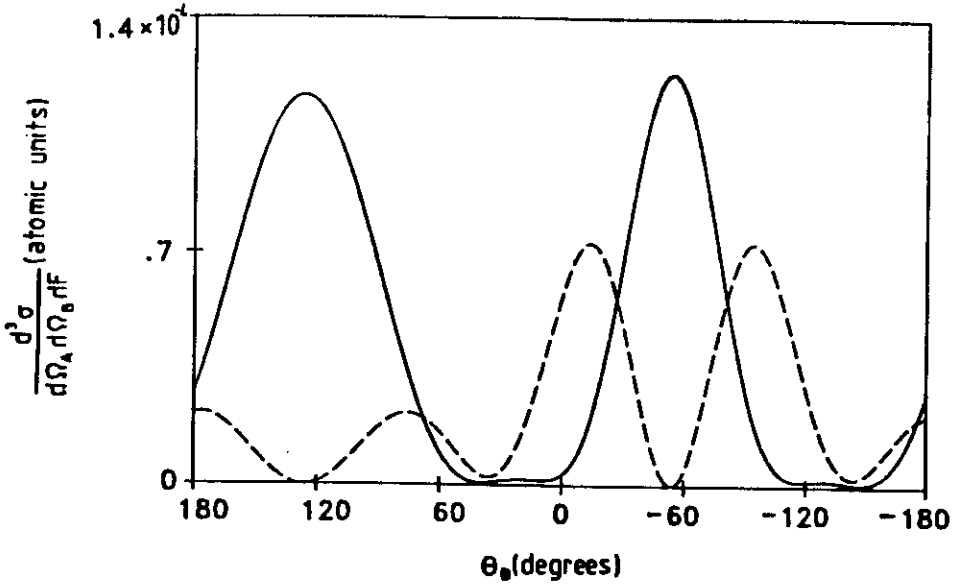

Fig. 12. The TDCS for laser-assisted (e,2e) collision as a function of the ejected electron angle θ_B. E_{k_i} = 250 eV, E_{k_B} = 5 eV, θ_A = 3°, $E_0 = 10^5$ V cm^{-1}, ω = 1.17 eV. Solid line: the laser polarization $\hat{\varepsilon} \parallel \hat{\Delta}$ where $\hat{\Delta} = \hat{k}_i - \hat{k}_A$ is the momentum transfer. Dashed line: $\hat{\varepsilon} \perp \hat{\Delta}$ (from Ref. 22).

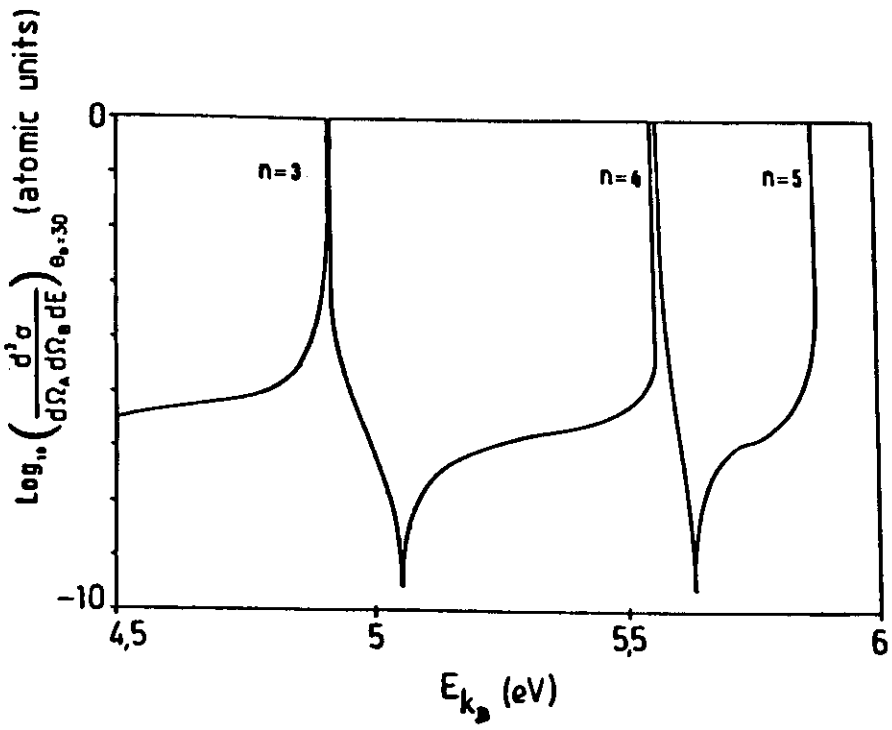

Fig. 13. Logarithm (in base 10) of the TDCS as a function of E_{k_B}. The angle of the ejected electron is fixed at $\theta_B = 30°$, $E_{k_1} = 250$ eV, $\theta_A = 3°$, $E_0 = 10^5$ V cm^{-1}, $\omega = 6.42$ eV (Ar-F laser), $\hat{z} \| \vec{k}_1$ (from Ref. 22).