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UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR/382- 26

WORKSHOP ON SPACE PHYSICS:
"Materials in Micorgravity"
27 February - 17 March 1989

"Marangoni Flows"

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Please note: These are preliminary notes intended for internal distribution only.

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
TRIESTE - ITALY

WORKSHOP ON SPACE PHYSICS:

MATERIALS in MICROGRAVITY

LECTURES ON:

MARANGONI FLOWS

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7-8 March 1989

INTRODUCTION

- MARANGONI FLOWS ARE **FREE CONVECTION FLOWS**: i.e. THEY ARE NOT SUSTAINED BY PRESSURE GRADIENTS AND/OR IMPOSED VELOCITY AT BOUNDARIES.
- IN MARANGONI FLOWS THE EFFECTS OF THE GRADIENTS OF SURFACE TENSIONS ARE OF THE SAME ORDER OF MAGNITUDE OF OTHER "NATURAL" EFFECTS (i.e. BUOYANCY).
- THE MARANGONI EFFECT CAN BE "THERMAL" AND/OR "SOLUTAL"
- WE SHALL CONSIDER THERMAL MARANGONI EFFECT.

- SINCE MARANGONI FLOWS ARE "NATURAL" CONVECTION FLOWS THERE IS NOT A REFERENCE VELOCITY, V_R, KNOWN "A PRIORI", THEN THE NON-DIMENSIONAL NUMBERS:

$$R_e = \frac{V_R L}{\nu} \; ; \; P_e = \frac{V_R L}{\alpha} \; ; \; M_a = \frac{\sigma_T \Delta T}{\mu V_R}$$

ARE TO BE CONSIDERED AS **CONDITIONAL NUMBERS**

R_e	P_e	M_a	V_R	t_R
1	$\nu/\alpha = P_r$	$\frac{\sigma_T \Delta T L}{\mu \nu}$	$\frac{\nu}{L} = V_\nu$	L^2/ν
$\frac{\alpha}{\nu} = \frac{1}{P_r}$	1	$\frac{\sigma_T \Delta T L}{\mu \alpha}$	$\frac{\alpha}{L} = V_\alpha$	$\frac{L^2}{\alpha}$
$\frac{\sigma_T \Delta T L}{\mu \nu}$	$\frac{\sigma_T \Delta T L}{\mu \alpha}$	1	$\frac{\sigma_T \Delta T}{\mu} = V_M$	$\frac{L \mu}{\sigma_T \Delta T}$

THIS CHOICE IS DICTATED BY THE RULE THAT IMPOSES EQUAL TO 1 THE CONDITIONAL No. WHICH CONTAINS THE DRIVING EFFECT

IN SUCH CASE IT HAS BEEN CHOSEN BY NAPOLITANO THAT MARANGONI FLOWS EXHIBIT THE SAME "REGIMES" AS IN EXTERNAL FLUIDYNAMICS, DUE TO THE ORDER OF MAGNITUDE OF THE REYNOLDS NUMBER

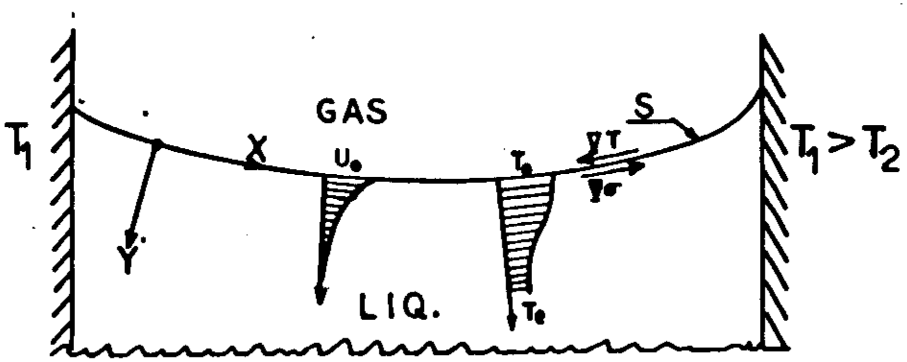
$$R_e = \frac{\sigma_T \Delta T L}{\mu \nu}$$

$R_e \ll 1$ PURELY DIFFUSIVE "STOKES FLOWS"

$R_e \sim O(1)$ NAVIER-STOKES FLOWS

$R_e \gg 1$ BOUNDARY LAYERS

MARANGONI BOUNDARY LAYERS



- MARANGONI BOUNDARY LAYERS ARE THE (SMALL) REGIONS ALONG INTERFACES WHERE THE VISCOUS TERMS ARE OF THE SAME ORDER OF MAGNITUDE OF THE DIFFUSIVE TERMS
- THE THICKNESS OF M.B.L. IS OF THE ORDER

$$\delta \sim R_e^{-1/3}$$

- WITH THE FOLLOWING ADIMENSIONALIZATION:

$$X = L_0 + xL \quad ; \quad Y = \delta L y \quad ; \quad V_2 = V_m \delta = V_m^{2/3} V_\nu^{1/3}$$

$$U = V_2\, u \quad ; \quad V = V_2 \delta v \quad ; \quad T = T_m + (\Delta T)_2\, t$$

THE FOLLOWING EQUATIONS APPLY:

$$\begin{cases} u_x + v_y = 0 \\ u u_x + v u_y + \dfrac{\partial \pi}{\partial x} = u_{yy} \\ u t_x + v t_y = \dfrac{1}{P_r} t_{yy} \end{cases} \qquad \text{B.C.} \begin{cases} v(x,0) = 0 \\ t_y(x,0) = 0 \\ t_x(x,0) = u_y(x,0) \\ \pi(x,0) = -K \\ t(x,\infty) = t_e(x) \\ u_e u_{ex} = -\pi_x(x,\infty) \end{cases}$$

SIMILARITY SOLUTIONS FOR PLANE ISOBARIC MARANGONI BOUNDARY LAYER

LET $\quad u = \psi_y \quad ; \quad v = -\psi_x$

$$\psi(x,y) = p(x)\, f(\eta) \quad ; \quad t(x,y) = h(x)\, g(\eta) \quad ; \quad \eta = y/\ell(x)$$

THE EQUATIONS ARE:

$$f''' + A_4 f f'' - A_3 f'^2 = 0$$

$$g'' + P_r \left[A_4 f g' - A_1 f' g \right] = 0$$

$$f(0) = 0 \; ; \; g'(0) = 0$$
$$f''(0) = -A_2\, g(0)$$
$$\lim_{\eta \to \infty} f'(\eta) \to 0$$
$$\lim_{\eta \to \infty} \eta f''(\eta) \to 0$$
$$\lim_{\eta \to \infty} g'(\eta) \to 0$$

$$A_1 = p \ell h'/h \quad ; \quad A_2 = \ell^2 h/p$$

$$A_3 = \ell^2 (p/\ell)' \quad ; \quad A_4 = p' \ell$$

WE LOOK FOR SIMILARITY SOLUTIONS OF POWER TYPE: LET US ASSUME

$$\ell(x) = \ell_0 x^\alpha \quad ; \quad h(x) = h_0 x^\beta \quad ; \quad p(x) = p_0 x^\gamma$$

AND LET IMPOSE THE CONSTANCY OF A_1, A_2, A_3, A_4.

ASSUMING $\quad A_4 = 1 \quad ; \quad A_1\, g(0) = 1$

WE HAVE

$$\alpha = \frac{2-\beta}{3} \; ; \; \gamma = \frac{1+\beta}{3}$$

$$l_0 = \left\{ (\beta+1)\, h_0\, g(0)/3 \right\}^{-1/3}$$

$$p_0 = \left\{ 9\beta\, h_0\, g(0)/(\beta+1)^2 \right\}^{1/3}$$

β IS ASSUMED AS SIMILARITY PARAMETER

THE PROBLEM IS:

$$f''' + f f'' - \frac{2\beta-1}{\beta+1} f'^2 = 0$$

$$g'' + P_r \left[f g' - \frac{3\beta}{\beta+1} f' g \right] = 0$$

$$f(0) = 0$$
$$f''(0) = -1$$
$$g(0) = 1$$
$$g'(0) = 0$$
$$f'(\infty) \to 0$$

THE SOLUTIONS IN THE SIMILARITY PLANE ARE TRANSFORMED IN THE PHYSICAL PLANE AS FOLLOWS:

$$u(x,y) = \left[\frac{3\beta^2}{\beta+1} \right]^{1/3} x^{\frac{2\beta-1}{3}} f'(\eta)$$

$$v(x,y) = \left[\frac{9\beta}{(\beta+1)^2} \right]^{1/3} x^{\frac{2\beta-1}{3}} \left(f'\eta - \frac{\beta+1}{3} f \right)$$

$$t(x,y) = -x^{\beta} g(\eta)$$

$$\eta = \left[\frac{\beta(\beta+1)}{3} \right]^{1/3} x^{\frac{2-\beta}{3}}$$

NOTE #1

$\beta = 1 \;\to\;$ CONSTANT TEMP. GRADIENT

$\beta = 1/2 \;\to\;$ CONSTANT VELOCITY

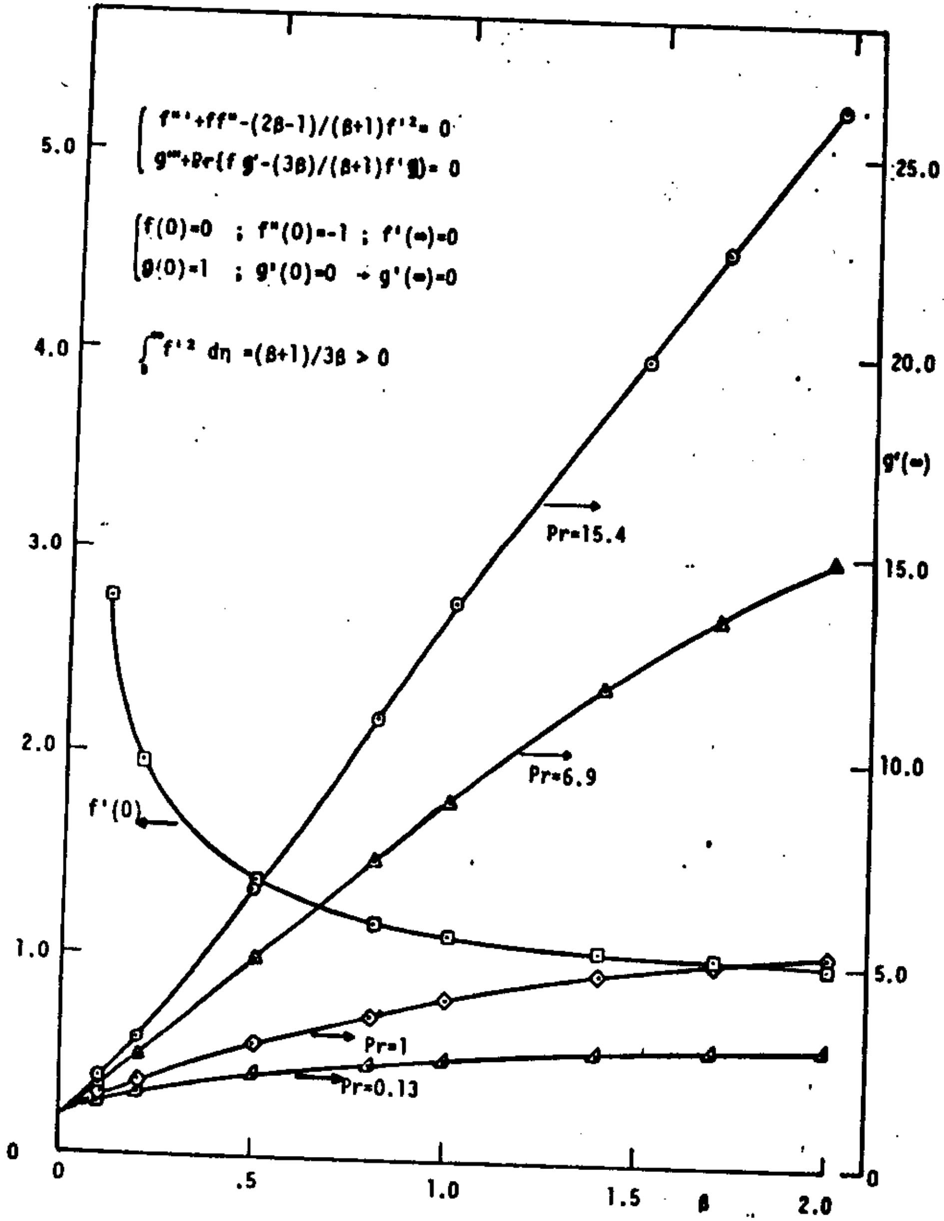
$\beta = 2 \;\to\;$ CONSTANT THYCKNESS

NOTE #2

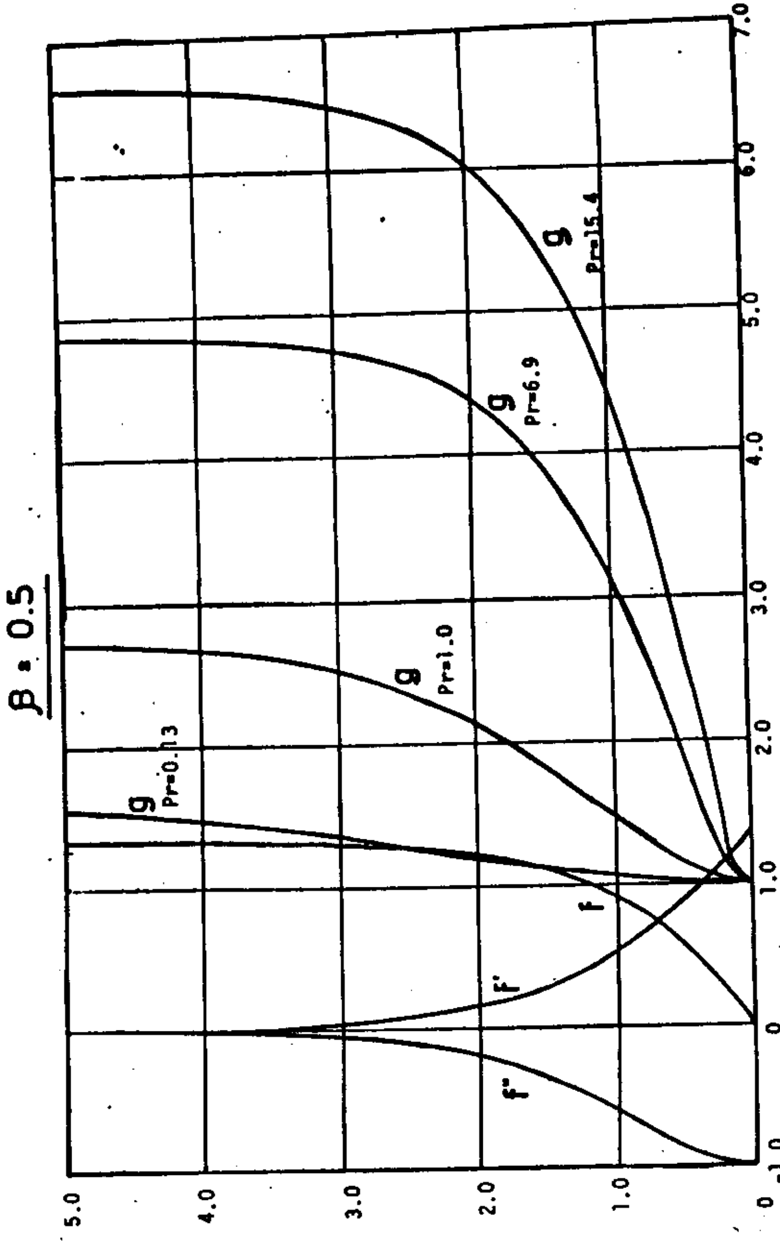
THE SYSTEM OF EQUATION IS NOT COUPLED.

i) THE "f" EQUATION IS A T.P.B.V.P

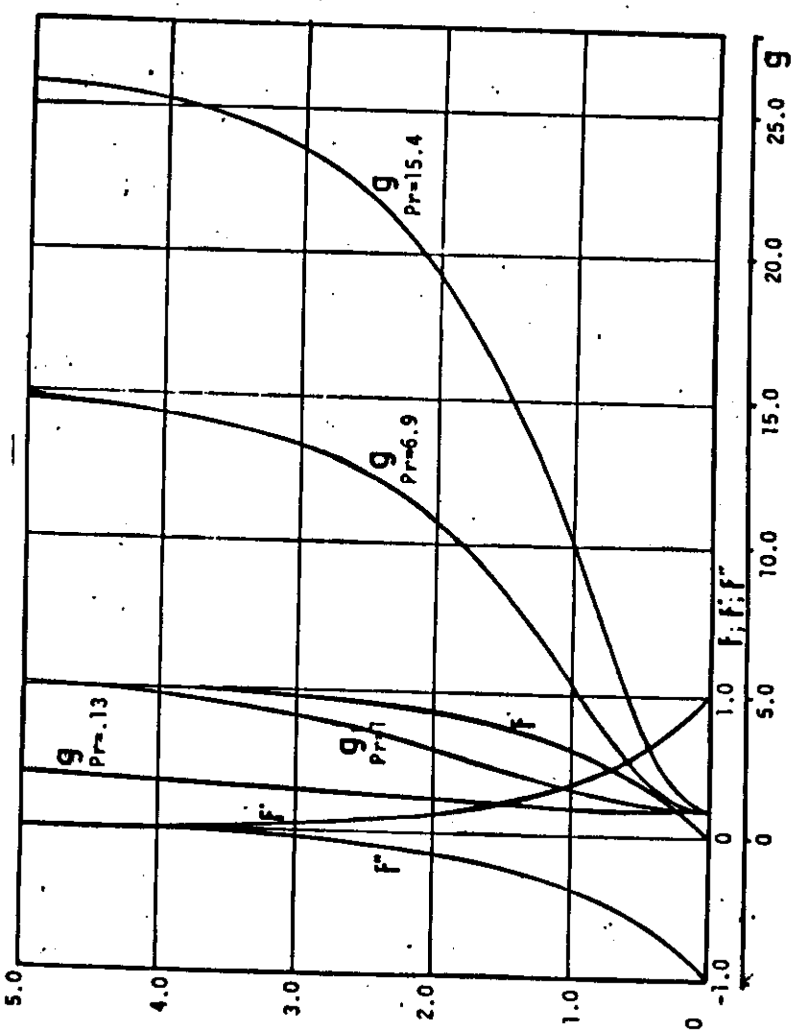
ii) THE "g" EQUATION IS A I.V.P.



Values of $f'(0)$ and $g(\infty)$ versus β for several values of the Prandtl number.



Typical profiles of f, f', f'' and g versus the similarity variable η, for several values of Pr. Case $\beta = 0.5$

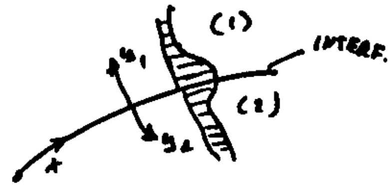


Typical profiles of f, f', f'' and g versus the similarity variable η, for several values of Pr. Case β = 2

DEVELOPMENTS

#1 COUPLED MARANGONI B.L.

TWO FLUIDS $i = 1,2$



$$f_i''' + f_i f_i'' - \frac{2\beta - 1}{\beta + 1} f_i'^2 = 0$$

$$g_i'' + P_i\left[f_i g_i' - \frac{3\beta}{\beta+1} f_i' g_i\right] = 0$$

$$f_1(0) = f_2(0) = 0 \qquad g_1(0) = g_2(0) = g(0)$$

$$f_1'(0) = f_2'(0) \neq 0 \qquad g_1(\infty) = C_1 = \text{CONST.}$$

$$g_2(\infty) = C_2 = \text{CONST.}$$

$$g_1'(0) = -A\, g_2'(0)$$

$$-\beta \frac{l_0}{\mu_0} g(0) = f_1''(0) + B f_2''(0)$$

where

$$u_0 l_0^2 = \frac{3}{\beta+1} \; ; \; A = \frac{\lambda_2}{\lambda_1}\frac{\delta_1}{\delta_2} = \frac{\lambda_2}{\lambda_1}\sqrt{\frac{\nu_1}{\nu_2}} \; ; \; B = \frac{\mu_2}{\mu_1}\frac{\delta_1}{\delta_2} = \frac{\mu_2}{\mu_1}\sqrt{\frac{\nu_1}{\nu_2}}$$

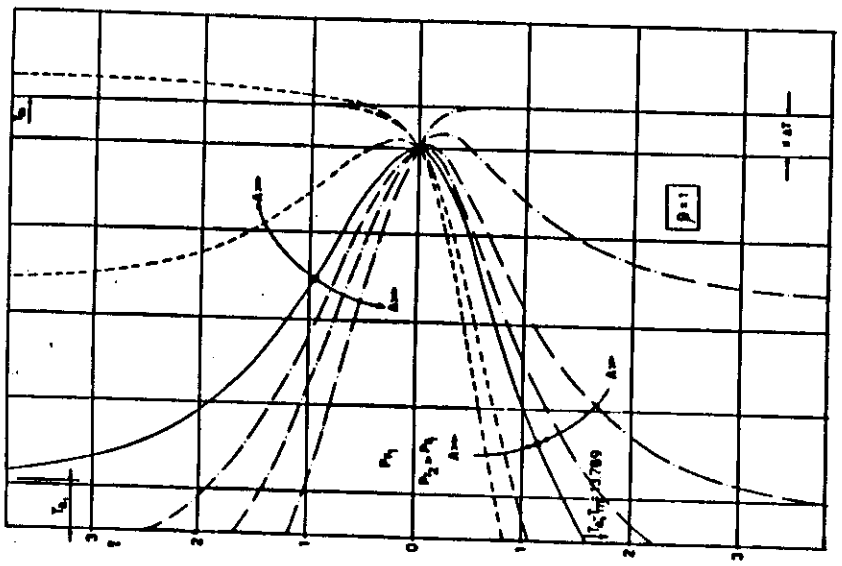
DUE TO THE STILL FREE PARAMETER μ_0 (or l_0) IT IS POSSIBLE TO UNCOUPLE THE PROBLEM AND SOLVE TWO INDEPEN... EQUATIONS

$$F''' + FF'' - \frac{2\beta-1}{\beta+1} F'^2 = 0 \qquad G'' + P_i\left(FG' - \frac{3\beta}{\beta+1} F'G\right) = 0$$

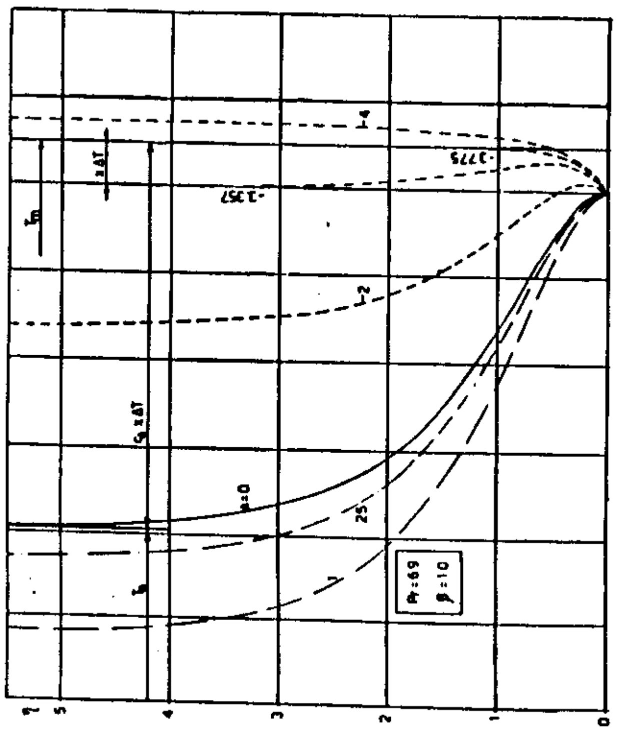
$$F(0) = 0 \quad F''(0) = -1 \quad F'(\infty) \to 0 \qquad G(0) = 1 \quad G'(0) = 2$$

THE COUPLING PARAMETER IS DETERMINED FROM EXTERNAL TEMPERATURE CONDITIONS:

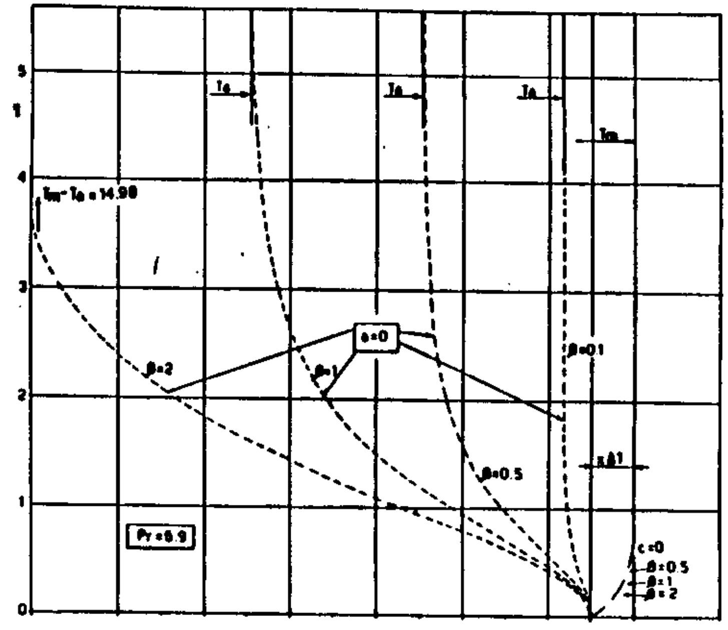
$$C_1 = G(P_1, \beta, -2A, \infty) \; ; \; C_2 = G(P_2, \beta, 2, \infty)$$



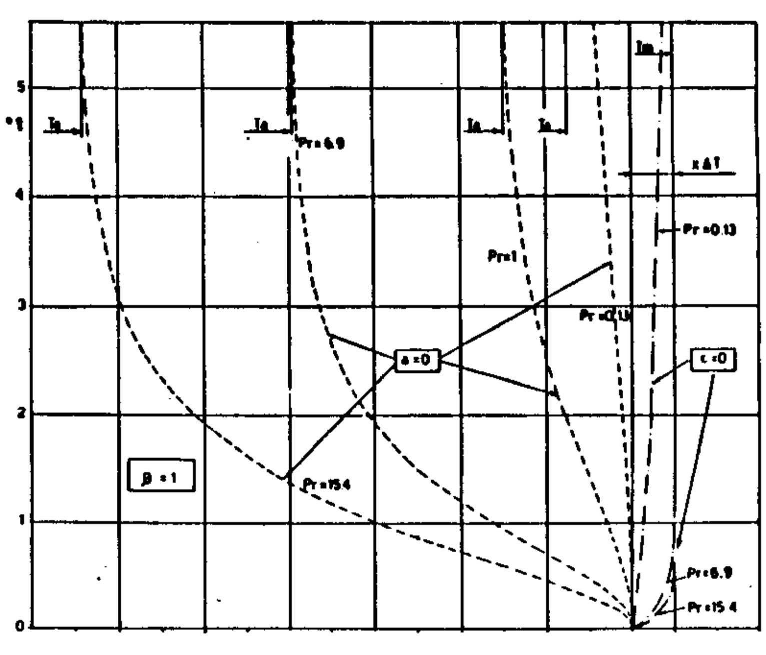
Indicative temperature profiles for different amounts of thermal coupling. Arrows show effect of increasing coupling parameter $A = \lambda_2\sqrt{(\rho_2/\rho_1)}/\lambda_1$ for given profile in one of the two regions.



Temperature profiles in the similarity plane.



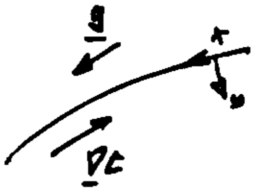
Effect of the similarity parameter β on adiabatic ($T_s = T_a$) and constant outer temperature ($T_s = T_m$ profiles.



Effect of Prandtl number on adiabatic ($T_s = T_a$) and constant outer temperature ($T_s = T_m$) profiles.

#2 MARANGONI-BUOYANT BOUNDARY LAYERS

SIMILARITY SOLUTIONS EXIST ONLY FOR



$$\beta = 5$$

i.e. $t(x,y) = -t_0\, x^5 g(\eta)$

THE EQUATIONS ARE

$$\begin{cases} f''' + f f'' - \frac{3}{2} f'^2 = \Gamma g \\ g'' + Pr\left(f g' - \frac{5}{2} f' g\right) = 0 \end{cases} \qquad \left(\Gamma = G_r \frac{V_\nu}{V_t} = \pm 1\right)$$

$f(0) = 0 \qquad g(0) = 1$

$f'(\infty) \to 0 \qquad g(\infty) \to 0$

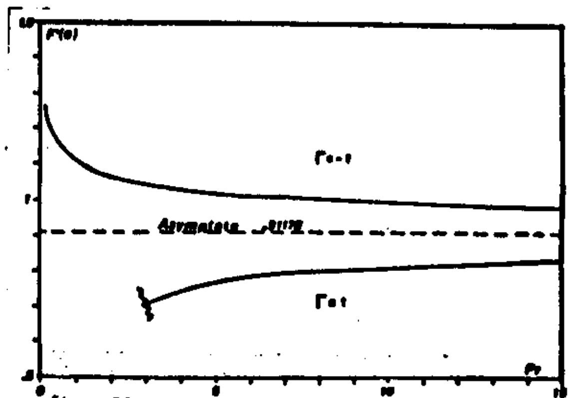
$f''(0) - g(0) = 0$

COUPLED TWO POINTS B.V.P.

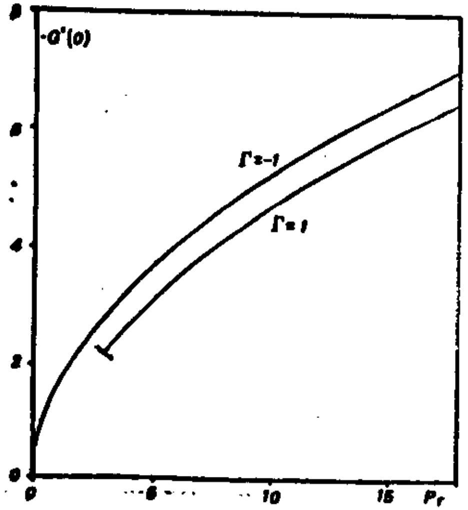
SOLVED VIA A QUASILINEARIZATION METHOD.

KEY POINTS FIND OUT THE VALUES OF

$f'(0) \; ; \; g'(0)$



Plots of f'(0) versus Pr (case Γ=±1)



Plots of g'(0) versus Pr (case Γ=± 1)

Γ = -1			Γ = 1		
Pr	F'(0)	G'(0)	Pr	F'(0)	G'(0)
.13	1.230	-.535			
.25	1.196	-.762			
.5	1.155	-1.106			
.74	1.130	-1.363			
1.	1.111	-1.599			
1.5	1.085	-1.979	2.8	.698	-2.087
2.	1.068	-2.299	3.	.711	-2.201
3.53	1.038	-3.084	3.53	.734	-2.482
5.	1.022	-3.685	5.	.768	-3.109
6.9	1.008	-4.341	6.9	.791	-3.776
8.	1.003	-4.679	8.	.800	-4.075
10.1	.994	-5.265	10.1	.813	-4.706
12.5	.987	-5.864	12.5	.823	-5.305
15.4	.981	-6.514	15.4	.832	-5.956

Values of f'(0,Pr) and g'(0,Pr)

#3 NON-ISOBARIC & BUOYAND MAR. B.L.

STILL A SINGLE SOLUTION $(\beta = 5)$

$$f''' + f f'' - \frac{3}{2}\left((f')^2 - 1\right) = \pm G g$$

$$g'' + P_r \left[f g' - \frac{5}{2} f' g \right] = 0$$

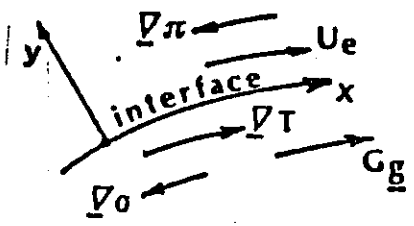
$$f(0) = 0 \qquad g(0) = 1$$

$$f'(\infty) \to 1 \qquad g(\infty) \to 0$$

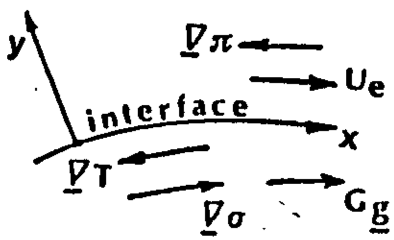
$$f''(0) = \mp g(0)$$

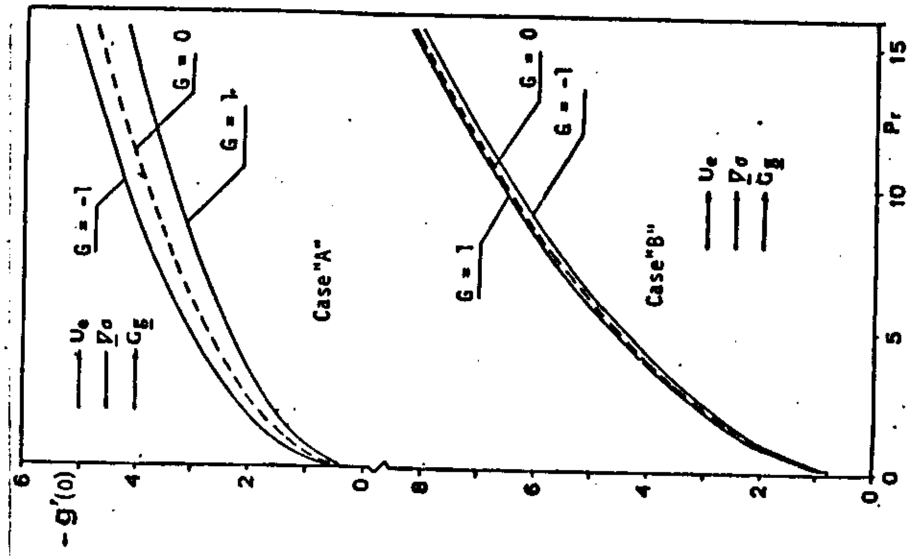
TWO CASES:

Case "A"

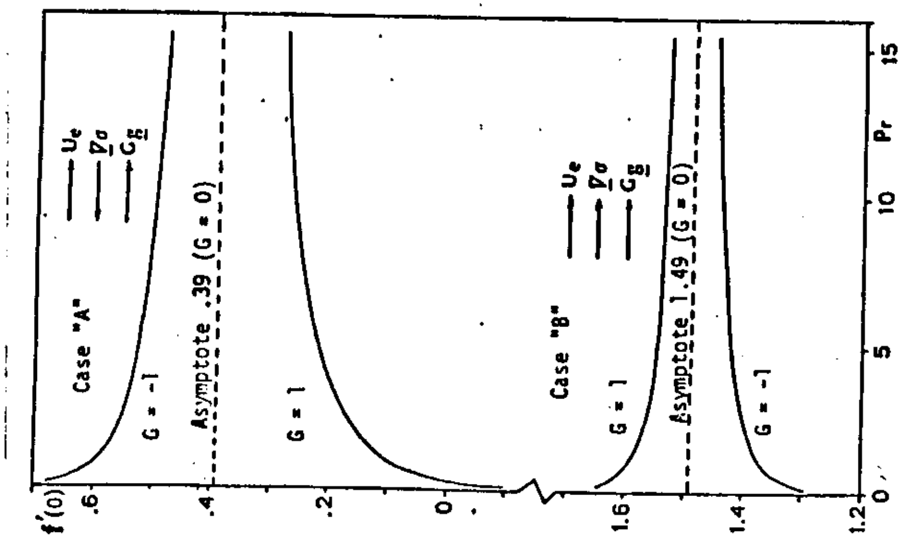


Case "B"



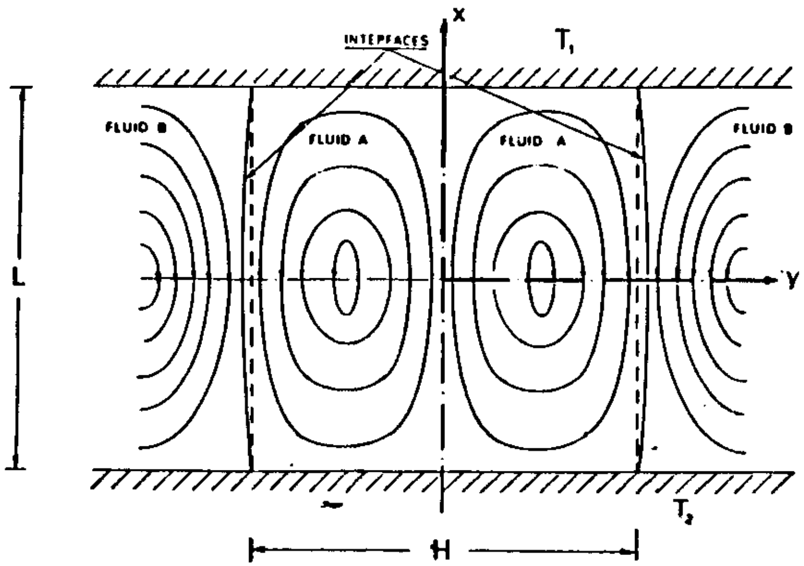


Interface (similar) heat fluxes versus Pr.



Interface (similar) velocities versus Pr.

ANALYTICAL SOLUTIONS OF MARANGONI-STOKES FLOWS



$$\begin{cases} \underline{\nabla} \cdot \underline{V}_i = 0 \\ RE_i \; \underline{V}_i \cdot \underline{\nabla}\, \underline{V}_i + \underline{\nabla}\, \pi_i = \nabla^2 \underline{V}_i \\ PE_i \; \underline{V}_i \cdot \underline{\nabla}\, \theta = \nabla^2 \theta \end{cases}$$

$$\begin{cases} \underline{\nabla}_t \theta + \delta\left[\mu_i/\mu_1 \; \tau_{tn}\right] = 0 \\ \left(1 - (\Delta\sigma/\sigma_m)\,\theta\right) K - K_n + CR\left[\mu_i/\mu_1(\pi - \tau_i)\right] = 0 \\ \delta\left[\lambda_i/\lambda_1 \; \frac{\partial \theta}{\partial n}\right] = 0 \end{cases}$$

$RE_i = V_2 L_1 / \nu_i \qquad PE_i = RE_i \, PR_i \qquad CR = |\Delta\sigma| / \sigma_m$

$L_2 = L/2 \qquad \theta = (T - T_m) / \Delta T \qquad \pi = (P - P_h) / (\mu_2 V_1 / L_1)$

$V_2 = \Delta\sigma / \mu_1 \qquad T_m = (T_1 + T_2)/2 \qquad \Delta T = T_1 - T_2$

$\Delta\sigma = \sigma_T \Delta T$

LET US ASSUME : (CREEPING FLOW)

$$RE_i \, , \, PE_i \, , \, CR = O(\varepsilon) \quad ; \quad \varepsilon \ll 1$$

LINEAR PROBLEM WITH INTERFACE KNOWN AT EACH ITERATION

HYERARCHY OF LINEAR PROBLEMS (i-th order)

i = 0

SOLVE FOR TEMPERATURE FIELD

SOLVE FOR VELOCITY FIELD

SOLVE FOR PRESSURE FIELD

i = i + 1

SOLVE FOR INTERFACE SHAPE (at i+1)

ZERO-TH ORDER PROBLEM:

$$K_0 = K_h = 0$$

$$\begin{cases} \underline{\nabla} \cdot \underline{V}_0 = 0 \\ \underline{\nabla}\, \pi_i = \nabla^2 \underline{V}_{0i} \\ \nabla^2 \theta_{0i} = 0 \end{cases} \qquad \underline{\nabla}_t \theta_0 = \delta\left[\mu/\mu_0 (\tau_{nt})_0\right] \qquad \partial\theta_0/\partial n + \frac{\lambda_2}{\lambda_1}\, \partial\theta_1/\partial n = 0$$

BY TAKING THE CURL OF THE MOMENTUM EQUATION , INTRODUCING THE STREAM FUNCTION IT RESULTS A "STOKES PROBLEM"

$$\nabla^4 \psi = 0$$

THE SHAPE OF THE INTERFACE WILL DERIVE AFTERWARDS FORM THE SOLUTION OF THE BALANCE EQUATION NORMAL TO THE INTERFACE:

$$K_1 + CR\, \delta\left[\mu_i/\mu_1 \; (\pi_0 - \tau_n)\right] = 0$$

THE STOKES PROBLEM MAY BE SOLVED BY USING THE PAPCKOVICH BI-ORTHOGONAL FUNCTIONS

$$\psi^a(x,y) = \lim_{N\to\infty} \sum_{-N}^{N} C_n \sinh(s_n y)\, \varphi_1^n(x)/s_n^2$$

$$\psi^b(x,y) = \lim_{N\to\infty} \sum_{-N}^{N} E_n \exp(-s_n(y-\ell))\, \varphi_1^n(x)/s_n^2$$

WHERE S_n ARE THE EIGENVALUES (COMPLEX CONJUGATED) OF THE EQUATION:

$$2 S_n + \sin(2 S_n) = 0$$

AND:

$$\varphi_1^n(x) = s_n \sin(s_n) \cos(s_n x) - s_n x \cos(s_n) \sin(s_n x)$$

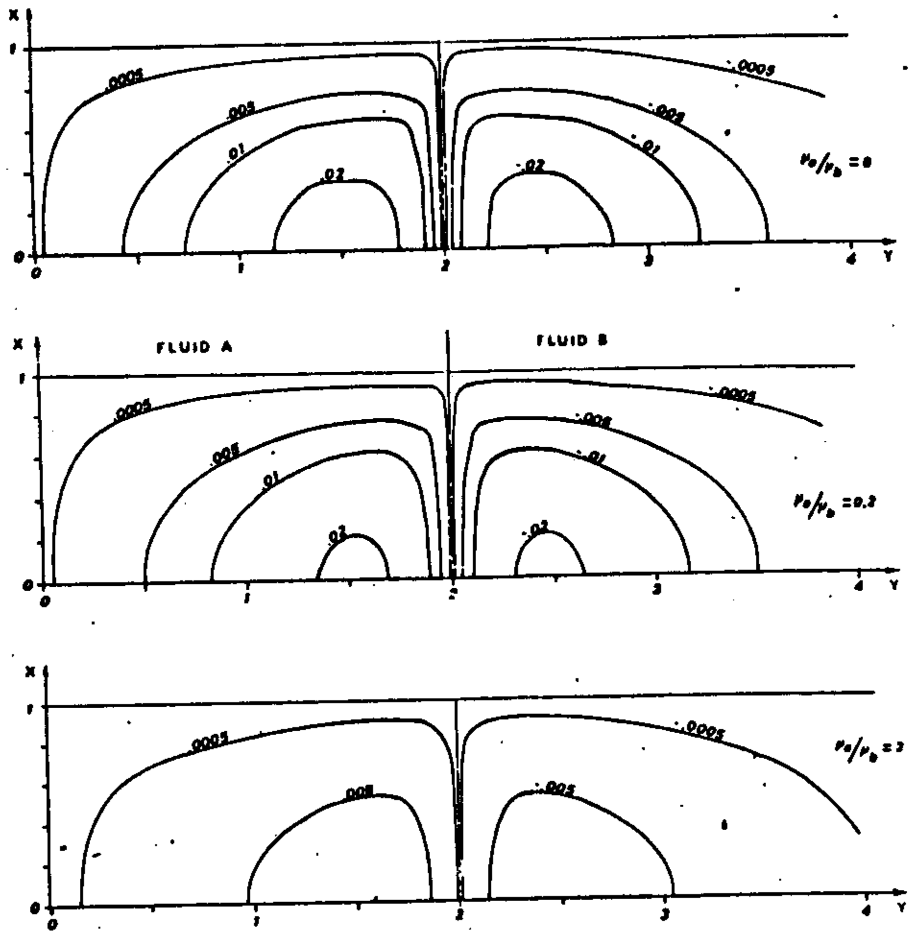
THE COEFFICIENT C AND E ARE COMPUTED BY INVERTING THE SERIES WITH AN ORTHONORMALIZATION ALGORYTHM DUE TO SMITH; IT RESULTS :

$$C_n = \mu_b/\mu_a \; \sinh(s_n \ell) / \cos^4(s_n)$$

WHERE E ARE THE SOLUTIONS OF THE SYSTEM (TRUNCATED AT)

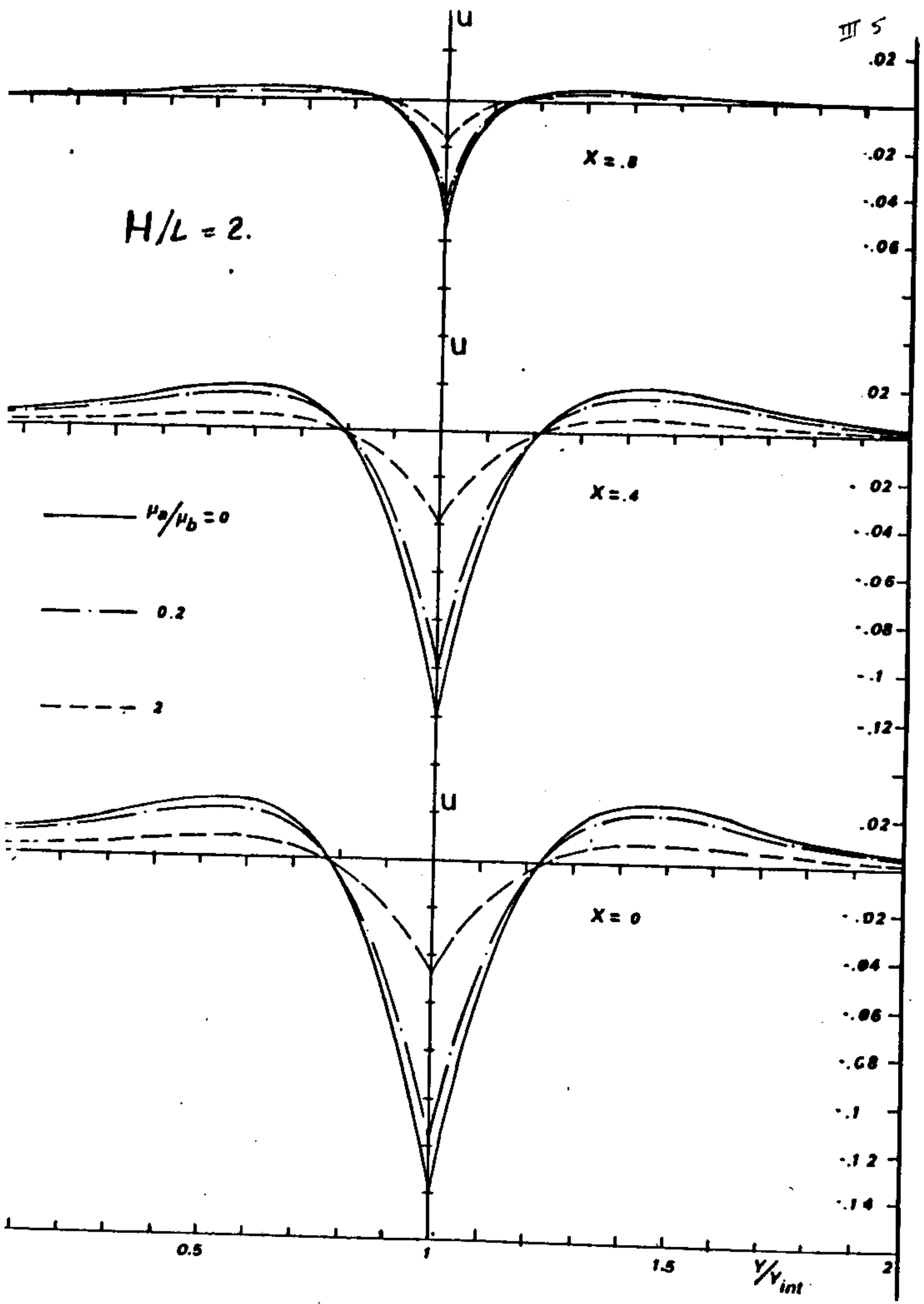
$$\sum_{-N}^{N} \left\{4 \cos^4(s_n)\, E_n\, \delta_{nm} + \left[(1 + S_n)\, E_n - C_n \cosh(s_n \ell)\right] I_{mn}/s_n\right\} = 0$$

with $$I_{mn} = \int_{-1}^{+1} \varphi_1^m(x)\, \varphi_1^n(x)\, dx$$

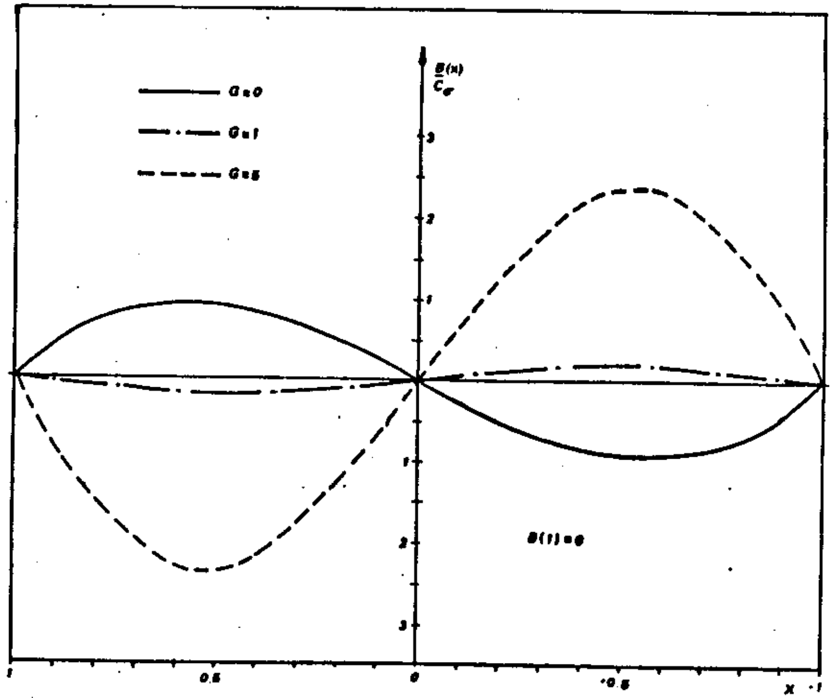


STREAMLINES $H/L = 2$

III 5


u
.02
-.02
-.04
-.06
X = .8
H/L = 2.
u
.02
.02
.04
.06
.08
.1
.12
X = .4
$\mu_a/\mu_b = 0$
0.2
2
u
.02
.02
.04
.06
.08
.1
.12
.14
X = 0
0.5
1
1.5
2
Y/Y_int

III 6


G = 0
G = 1
G = 5
B(x)/C_σ
3
2
1
1
2
3
B(1) = 0
1
0.5
0
0.5
X
1

AXISYMMETRIC LIQUID BRIDGE

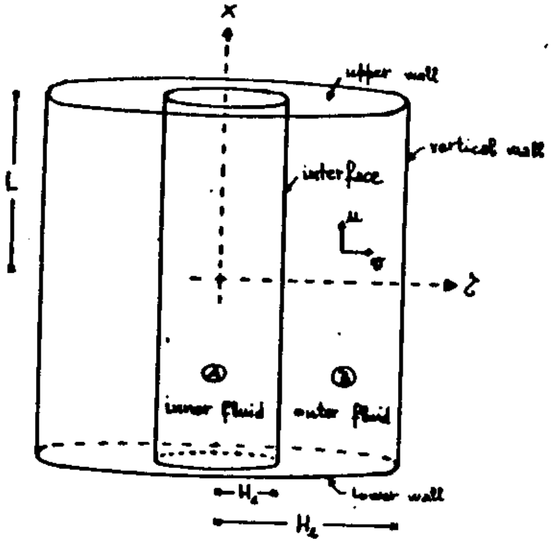


Fig.1 Geometry of the problem

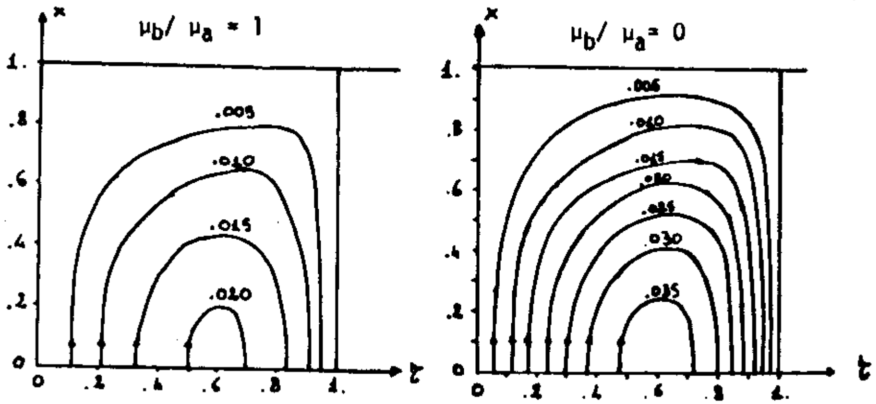


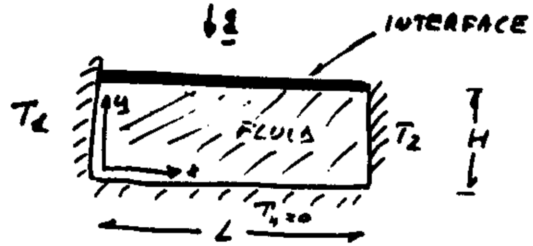
Fig.2 Streamlines : unitary aspect ratio, infinite external region

EXTENSIONS:

- EFFECTS OF GRAVITY LEVELS
- SURFACE TENSION WITH MINIMUM

MARANGONI - NAVIER STOKES FLOWS

A. PLANAR - RIGID INTERFACE



$$(*)\begin{cases} \underline{\nabla}\cdot\underline{V} = 0 \\ Re\left[\dfrac{\partial \underline{V}}{\partial t} + \underline{V}\cdot\underline{\nabla}\underline{V}\right] = G_r\, \underline{i}_g\, \theta + \nabla^2 \underline{V} - \underline{\nabla}\pi \\ Pe\left[\dfrac{\partial \theta}{\partial t} + \underline{V}\cdot\underline{\nabla}\theta\right] = \nabla^2\theta \end{cases}$$

(ψ, ζ, θ) FORMULATION

ψ = STREAM FUNCTION $\left(\underline{V} = \underline{\nabla}\wedge\left(\psi \dfrac{\underline{i}_z}{h_z}\right)\right)$

ζ = VORTICITY $\left(\zeta = (\underline{\nabla}\wedge\underline{V})\cdot\underline{i}_z\right)$

θ = TEMPERATURE

IN CARTESIAN COORDINATES

$$u = \psi_y \quad ; \quad v = -\psi_x \quad ; \quad \zeta = v_x - u_y$$

THE SYSTEM (*) IS TRANSFORMED BY TAKING THE CURL OF THE MOMENTUM EQUATION:

$$Re\left[\frac{\partial \underline{\zeta}}{\partial t} - \underline{\zeta}\cdot\nabla\underline{V} + \underline{V}\cdot\nabla\underline{\zeta}\right] = G_r\, \underline{i}_g \wedge \nabla\vartheta + \nabla^2\underline{\zeta}$$

$$Pe\left[\frac{\partial\vartheta}{\partial t} + \nabla\cdot(\underline{V}\vartheta)\right] = \nabla^2\vartheta$$

$$\underline{\zeta} = \nabla\wedge\left[\nabla\wedge\left(\psi\,\frac{\underline{i}_w}{h_w}\right)\right]$$

$$\nabla^2\pi = -Re\,(\nabla\underline{V}):(\nabla\underline{V}) - G_r\,\underline{i}_g\cdot\nabla\vartheta$$

BOUNDARY CONDITIONS

• ON SOLID BOUNDARIES

$\underline{V} = 0 \longrightarrow \psi = \text{CONST} = 0\ ,\ \psi_n = 0$

$\vartheta = \text{CONST}$ (OR $\vartheta_n = \text{CONST}$)

FOR ζ ? $\quad \zeta = \begin{cases} -\psi_{xx} & \text{IF } y = j \\ -\psi_{yy} & \text{IF } x = i \end{cases}$

• ON THE INTERFACE $(\psi = 0\ ,\ n = j\ ,\ t = i)$

NOTE

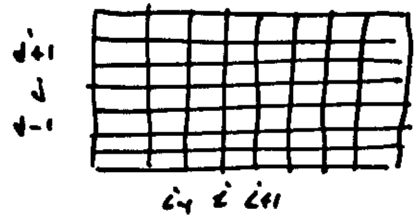
$$\delta(\tau_{nt}) = \nabla_t\vartheta \Rightarrow -\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}\right) = \frac{\partial\vartheta}{\partial x}$$

$$\Rightarrow \cancel{\frac{\partial v}{\partial x}} - \frac{\partial u}{\partial y} = \frac{\partial\vartheta}{\partial x}$$

$$\Rightarrow \boxed{\zeta = \frac{\partial\vartheta}{\partial x}} \qquad (Ma = 1)$$

VERY SIMPLE BOUNDARY CONDITIONS

DISCRETIZE MESH AND TIMES



USE : EXPLICIT METHODS,
IMPLICIT METHODS,

OR **SEMI-IMPLICIT METHODS** :

→ CONVECTIVE TERMS EVALUATED AT t_n TIME LEVEL (SECOND UPWIND DIFF.)

DIFFUSIVE TERMS EVALUATED AT t_{n+1} TIME LEVEL (CENTRAL DIFF.)

t_n

SOLVE TEMPERATURE (GAUSS SEIDEL)

↓

SOLVE VORTICITY (GAUSS SEIDEL)

↓

SOLVE STREAM FUNC. (RELAXATION)

↓

COMPUTE $u_{ij}\ ,\ v_{ij}$

AS AN OPTION COMPUTE PRESSURE FIELD, π:

$$\nabla^2 \pi = -Re\left[\underline{\nabla}\underline{V} : \underline{\nabla}\underline{V}\right] - Gr\, \underline{i}_g \cdot \underline{\nabla}\vartheta$$

THIS IS A POISSON EQUATION WITH NEUMANN CONDITIONS:

B.C. $\dfrac{\partial \pi}{\partial n} = \dfrac{\partial \zeta}{\partial t}$ (n = normal, t = tangential)

NOTE: IN ORDER TO RENDER THIS PROBLEM WELL POSED, THE PRODUCTION TERM MUST BE CORRECTED TO SATISFY INTEGRAL CLOSURE CONDITION (USE, AMONG OTHERS BRILEY METHOD).

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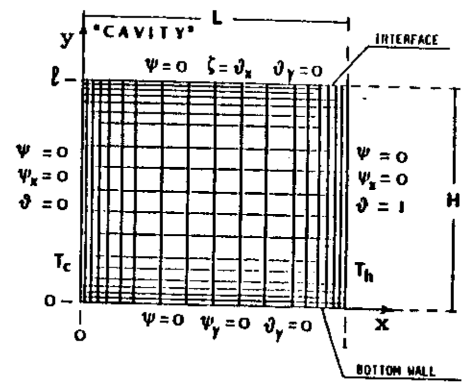
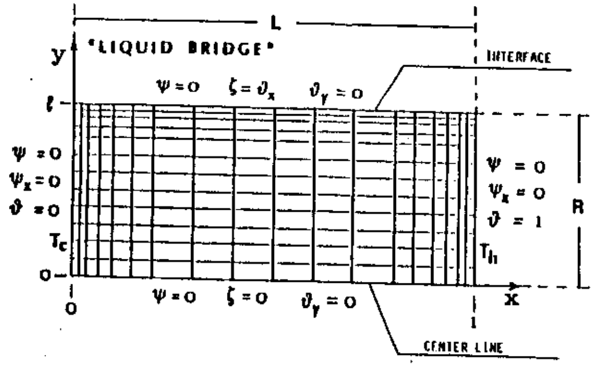




$$Pe\,(\,Str\,\partial\vartheta/\partial\tau + \underline{\nabla}\cdot(\underline{V}\vartheta)) = \nabla^2\vartheta + Pr\,E\,(\underline{\nabla}\underline{V})^S : (\underline{\nabla}\underline{V})^S$$

$$Re\,(\,Str\,\partial\underline{\zeta}/\partial\tau - \underline{\zeta}\cdot\underline{\nabla}\,\underline{V} + \underline{V}\cdot\underline{\nabla}\underline{\zeta}\,) = Gr\,\underline{i}_g \wedge \underline{\nabla}\vartheta + \nabla^2\underline{\zeta}$$

$$\underline{\zeta} = \underline{\nabla}\wedge(\,\underline{\nabla}\wedge(\psi\,\underline{i}_K/h_K))$$

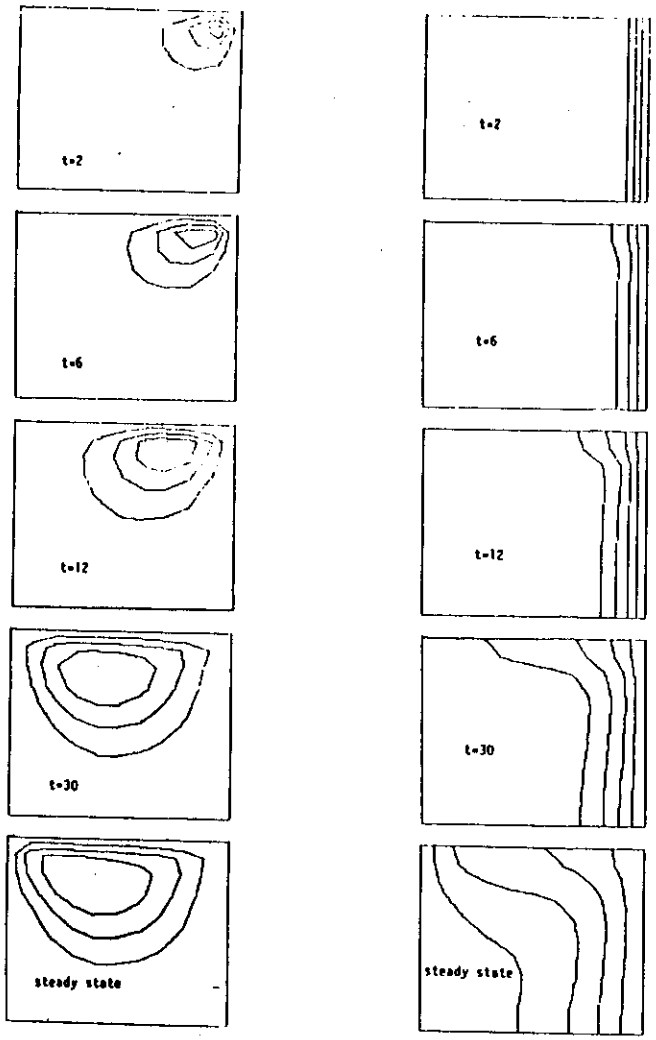
$$\nabla^2\pi = -Re\,(\,\underline{\nabla}\,\underline{V}) : (\underline{\nabla}\,\underline{V}) - Gr\,\underline{i}_g\cdot\underline{\nabla}\vartheta$$

* $\vartheta = (T-Tc)/(Th-Tc)$

* $\underline{V} = \underline{\nabla}\wedge(\psi\,\underline{i}_K/h_K)$

$$\sigma = \sigma_c - \sigma_T\,(T - T_c)\;;\; \sigma_T > 0 \qquad \rho = \rho_c\left[1 - \beta(T - T_c)\right]$$

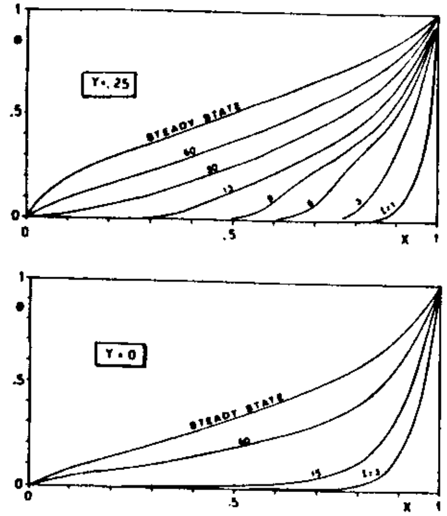
$$V_r = \frac{\sigma_T\,\Delta T}{\mu} \qquad \pi_r = \frac{\mu V_r}{L}$$



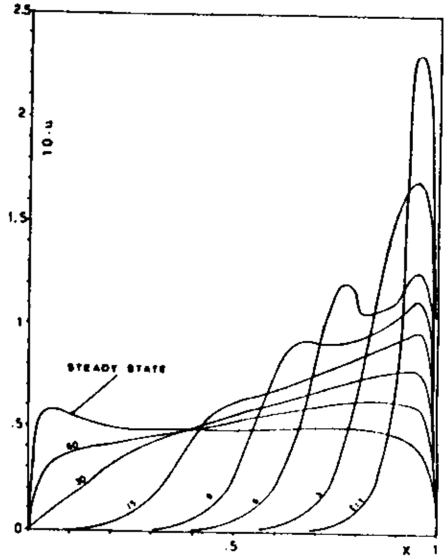
STREAMLINES

Re = 1000
Pr = 1

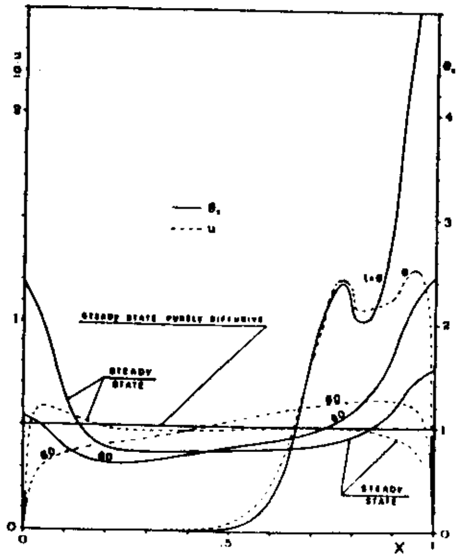
ISOTHERMS



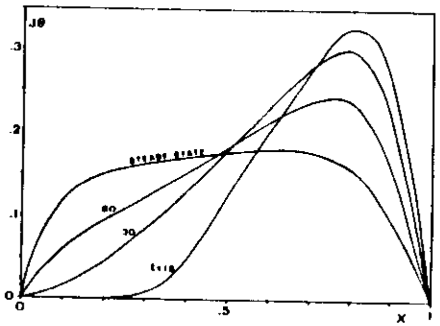
Time evolution of temperature distribution along bottom wall (y = 0) and interface (y = 0.25) Re = 1000 ; Pr = 1 ; ℓ = 0.25



Time evolution of velocity at interface Re = 1000 ; Pr = 1 ; ℓ = 0.25



Velocity and driving temperature gradient at interface. Re = 1000 ; Pr = 1 ; ℓ = 0.25



Time evolution of temperature difference between the interface and the bottom wall Re = 1000 ; Pr = 1 ; ℓ = 0.25

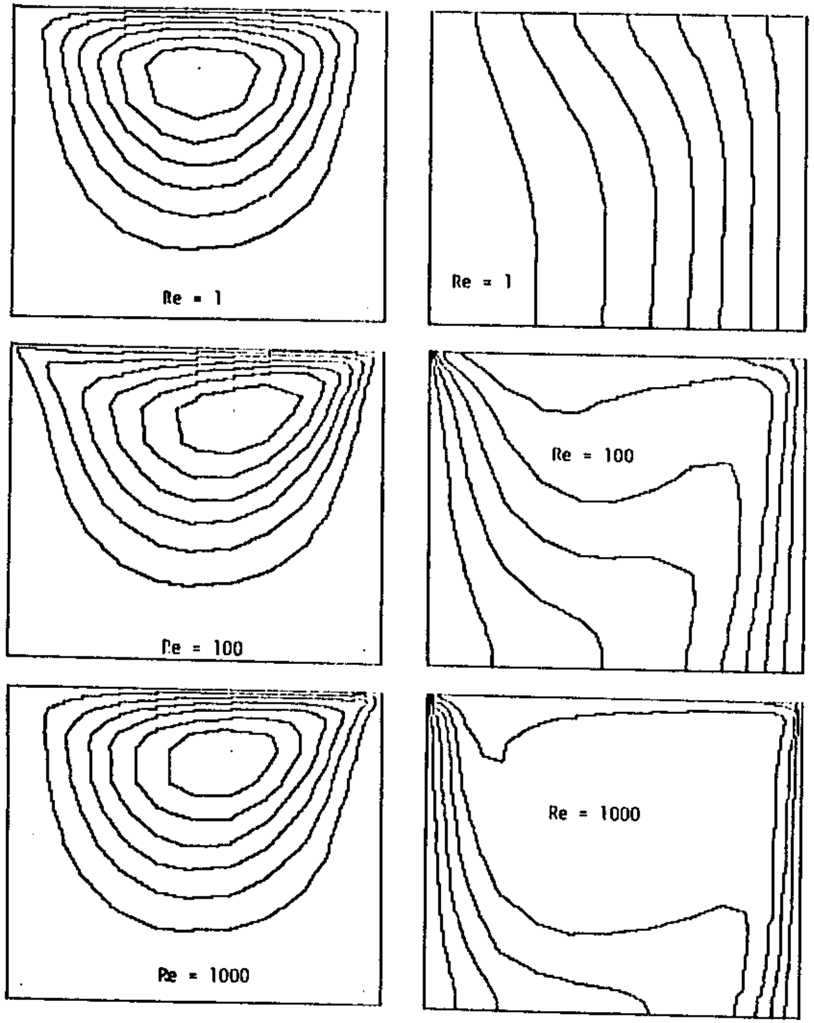


Fig. 2 - Cavity problem. Streamlines (left column) and isotherms. Pr = 79.

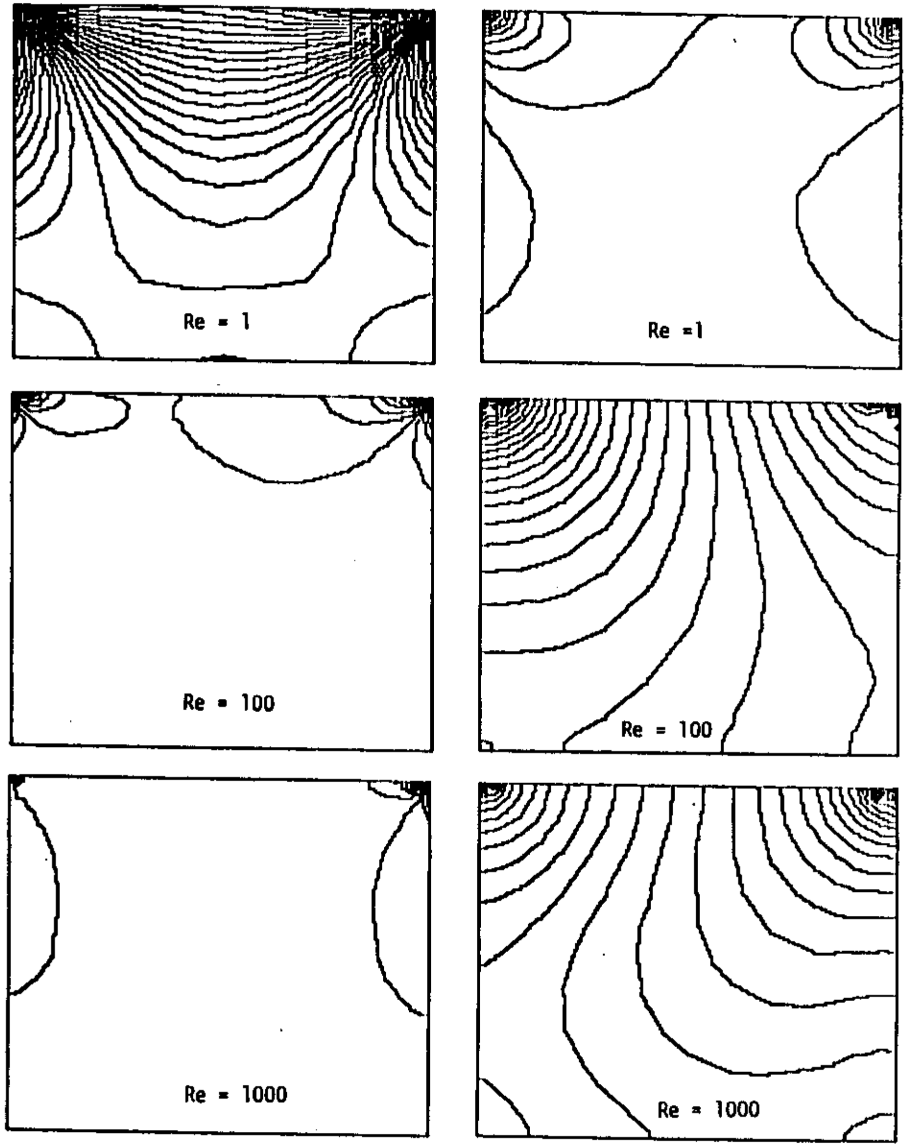
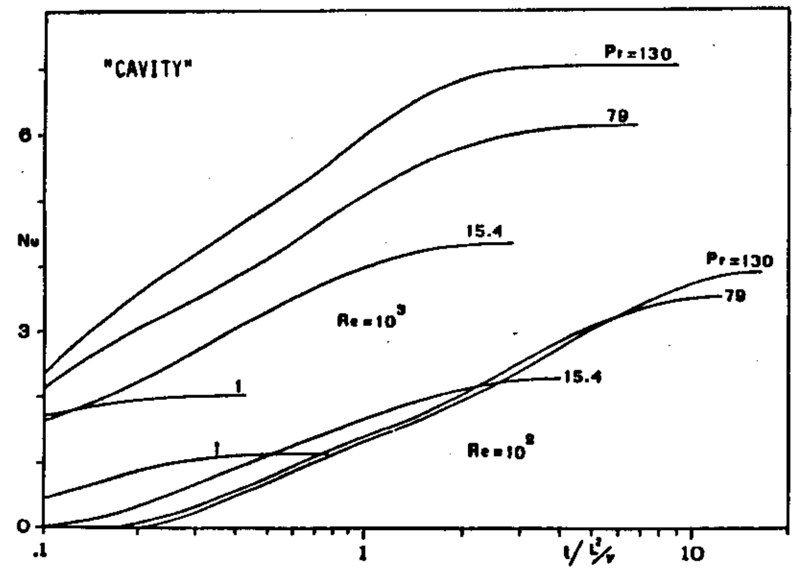
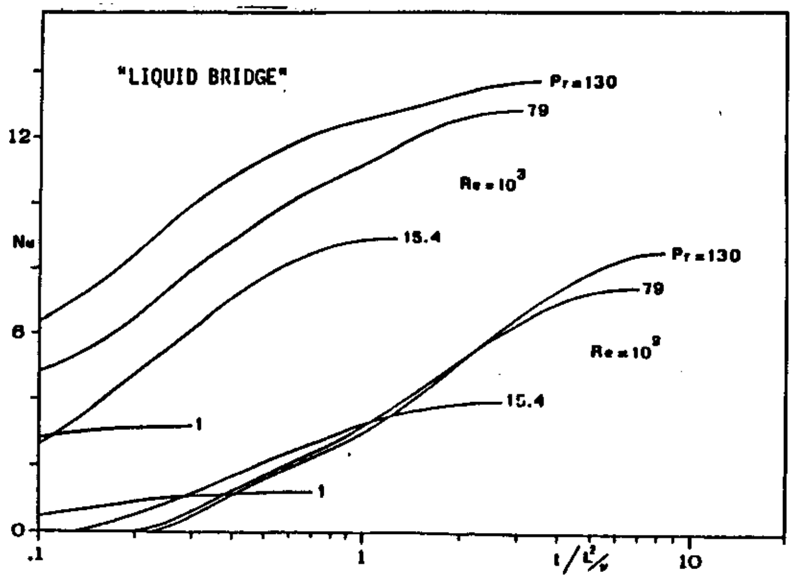
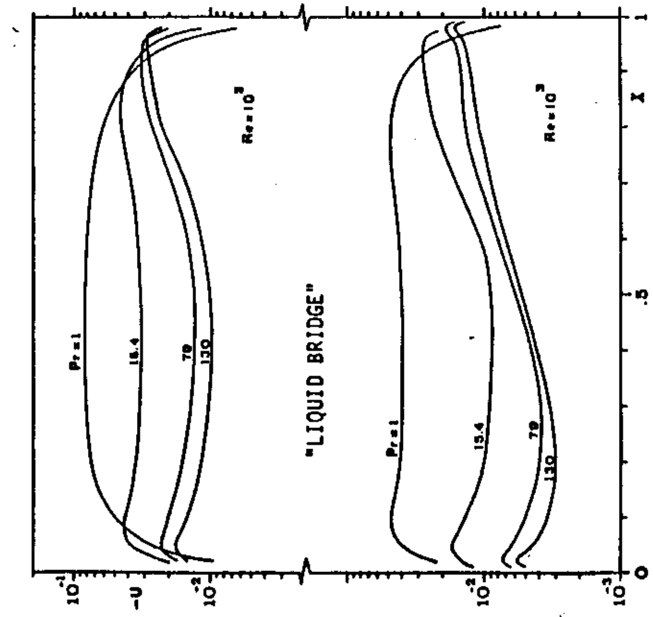


Fig. 3 - Cavity problem. Isovorticy lines (left column) and isobares. Pr = 79.

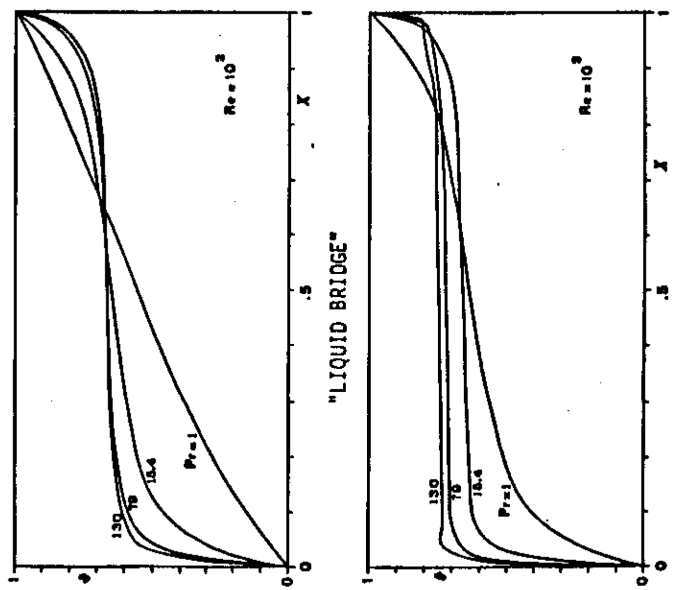




a. B -'Heating curves': Nusselt number on cold wall (disk) versus non-dimensional time for Cavity and Liquid Bridge problems.



- "Liquid Bridge" problem : Axial velocity along the interface.



"Liquid Bridge" problem : Interface temperature distributions.

IAF-85-289

EFFECTS OF VARIABLE TRANSPORT PROPERTIES ON THERMAL MARANGONI FLOWS

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Institute of Aerodynamics "Umberto NOBILE"

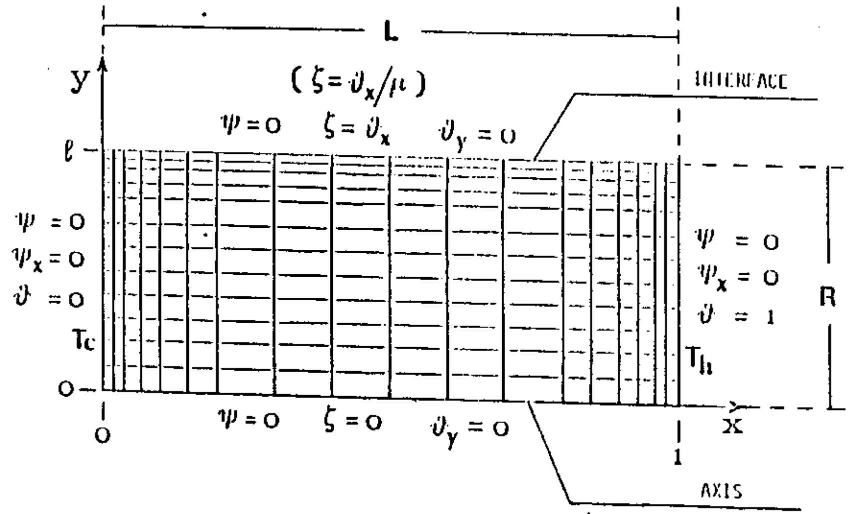


Fig. 1 - Mesh and Problem Geometry

$$Re\left[St\,\frac{\partial \underline{\zeta}}{\partial \tau} - \underline{\zeta}\cdot\nabla\underline{V} + \underline{V}\cdot\nabla\underline{\zeta}\right] = Gr\,\underline{i}_g \wedge \nabla\theta + \mu\nabla^2\underline{\zeta} + \nabla\mu\wedge\nabla^2\underline{V} + \nabla\wedge\left[2\,\nabla\mu\cdot(\nabla\underline{V})^s\right]$$

$$\underline{\zeta} = \nabla\wedge\left[\underline{V}\wedge(\psi\,\underline{i}_K/h_K)\right]$$

* $\theta = (T-Tc)/(Th-Tc)$
* $\underline{V} = \nabla\wedge(\psi\,\underline{i}_K/h_K)$

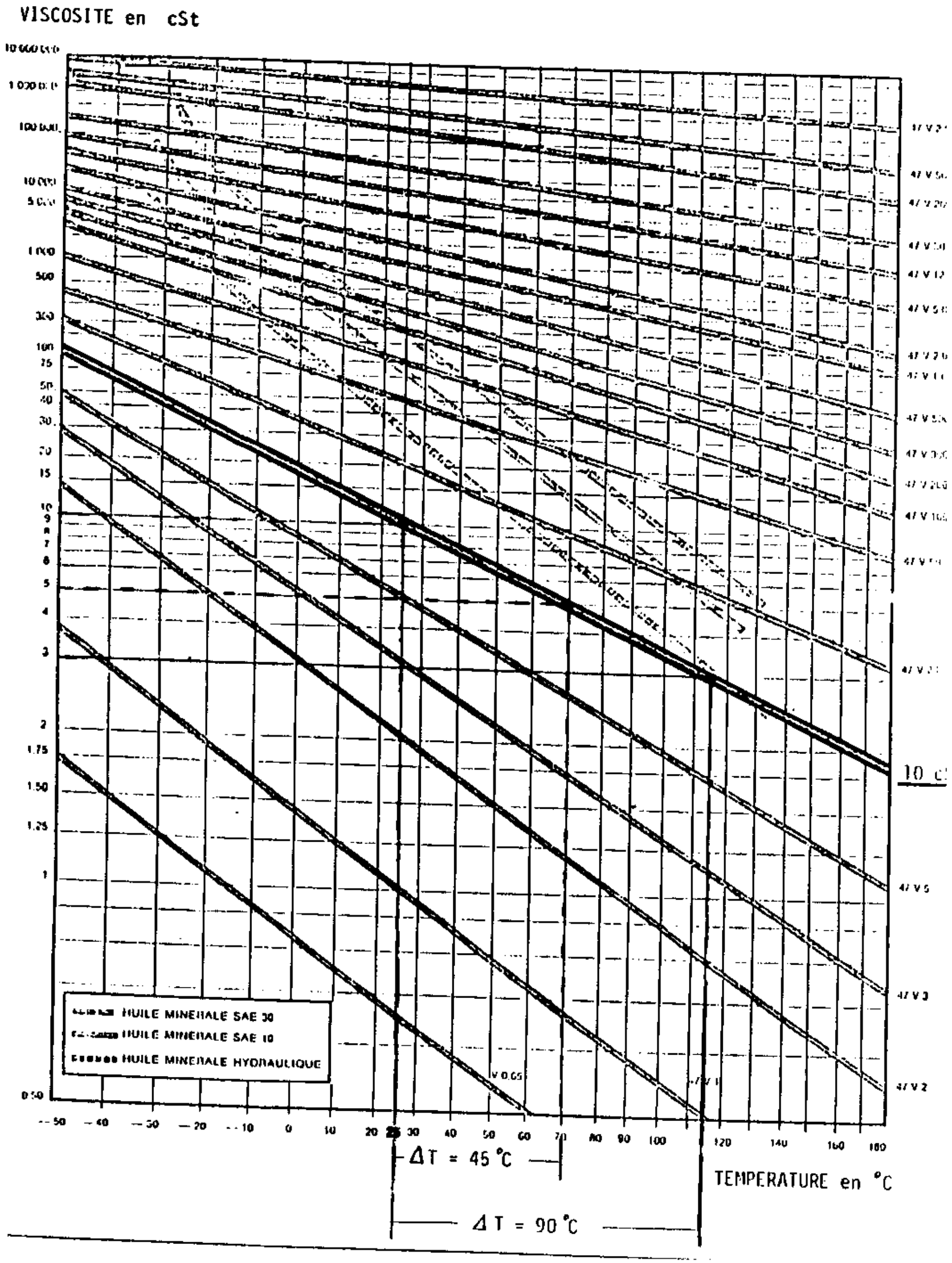
$$Pe\left[St\,\frac{\partial\theta}{\partial\tau} + \nabla\cdot(\theta\underline{V})\right] = \nabla^2\theta + Pr\,E\,\mu\,(\nabla\underline{V})^s:(\nabla\underline{V})^s$$

$$\nabla^2\Pi = -Re(\nabla\underline{V}):(\nabla\underline{V}) - Gr\,\underline{i}_g\cdot\nabla\theta + 2\left[\nabla\mu\cdot\nabla^2\underline{V} + (\nabla\underline{V})^s:(\nabla\nabla\mu)\right]$$

GRAPHIQUE 3

RHODORSIL HUILES 47

VARIATION DE LA VISCOSITE
en fonction de la TEMPERATURE



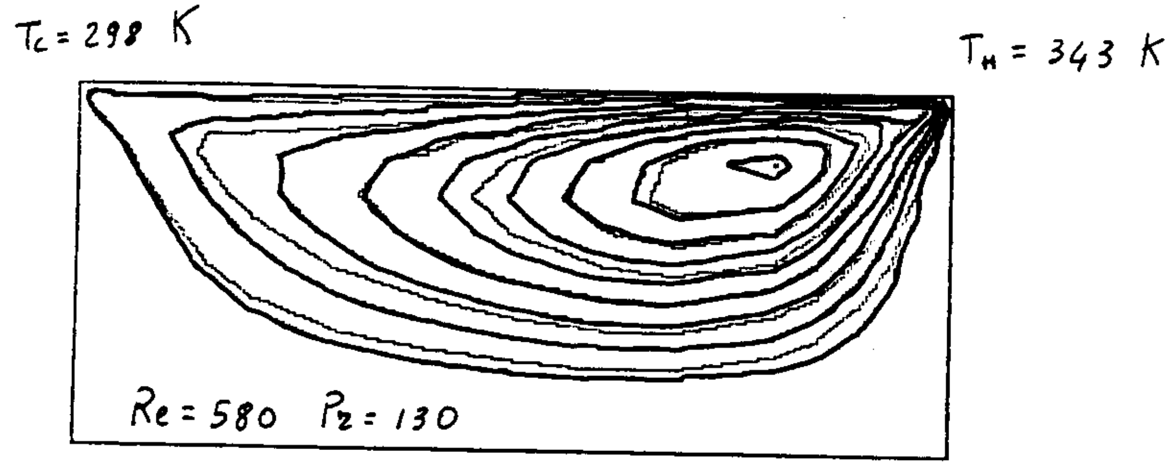


Fig. 2 Steady state Streamlines

— $\Delta\psi = .5E-4$; $\psi_{max} = 3.5E-4$

— $\Delta\psi = .5E-4$; $\psi_{max} = 4.6E-4$ Variable viscosity

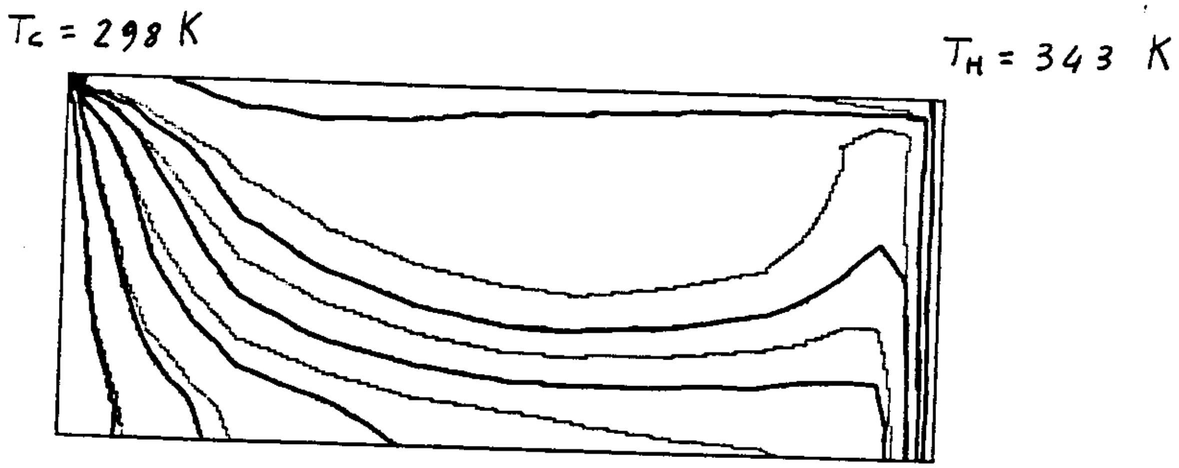


Fig. 3 Steady state Isotherms $\Delta\theta = .125$

$Re = 580$ $P_r = 130$

— Variable viscosity

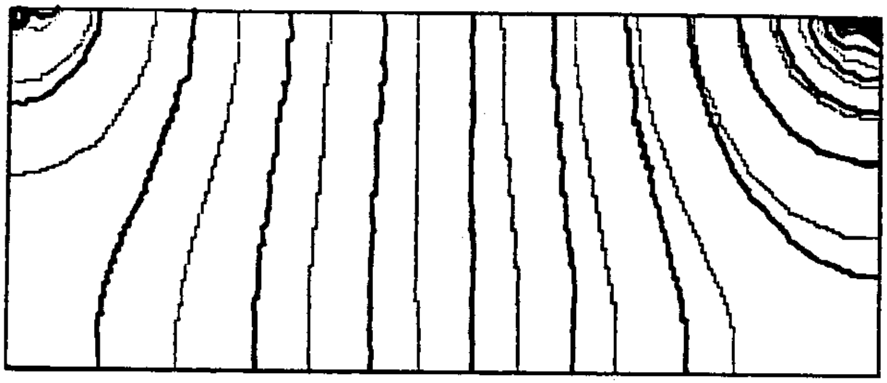


Fig. 4 Steady state isobars. Re = 580 ; Pr = 130
— Variable viscosity

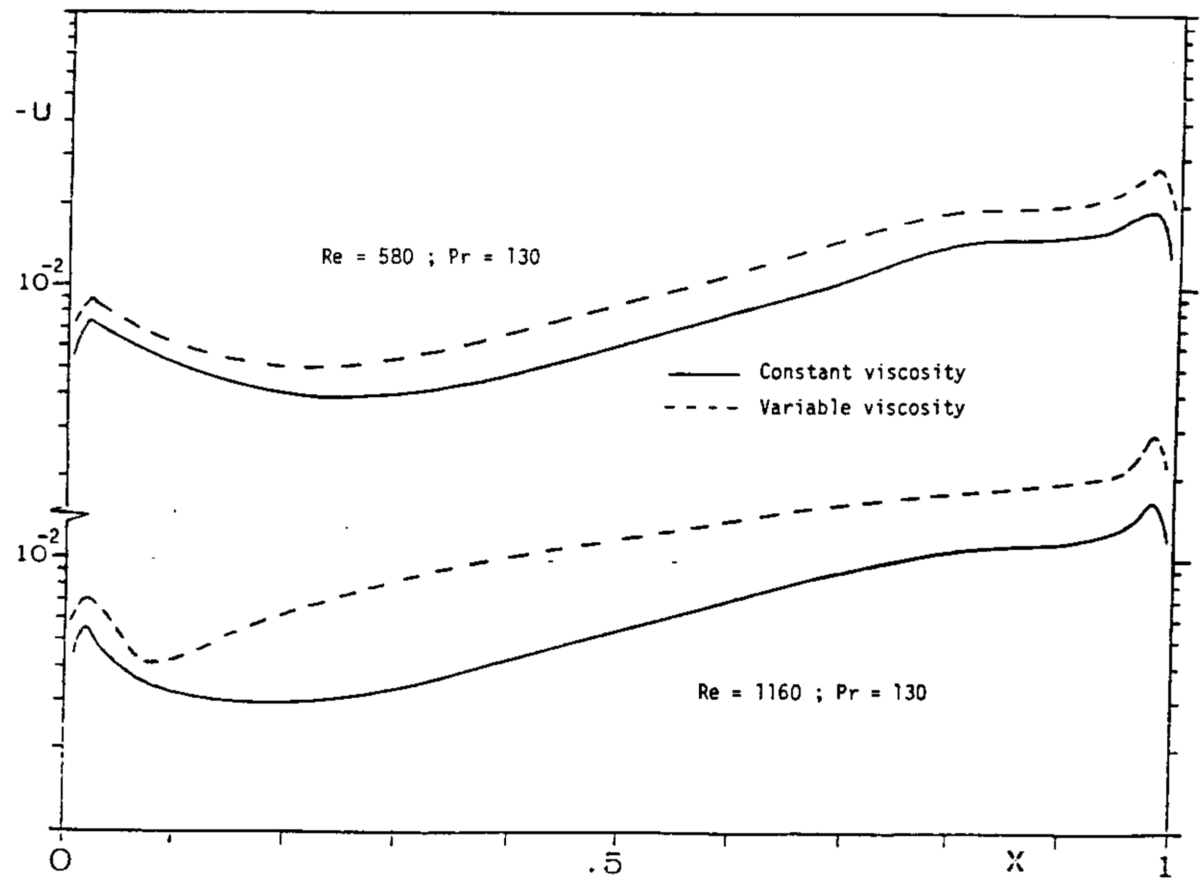


Fig. 6 - Interface velocity distribution.

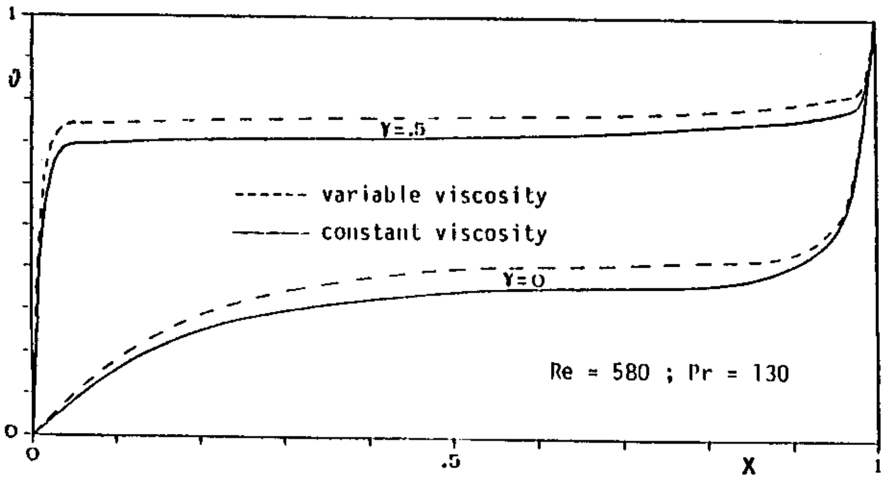


Fig. 7a - Axis (Y = 0) and interface (Y = .5) temperature distribution. Case Re = 580 ; Pr = 130

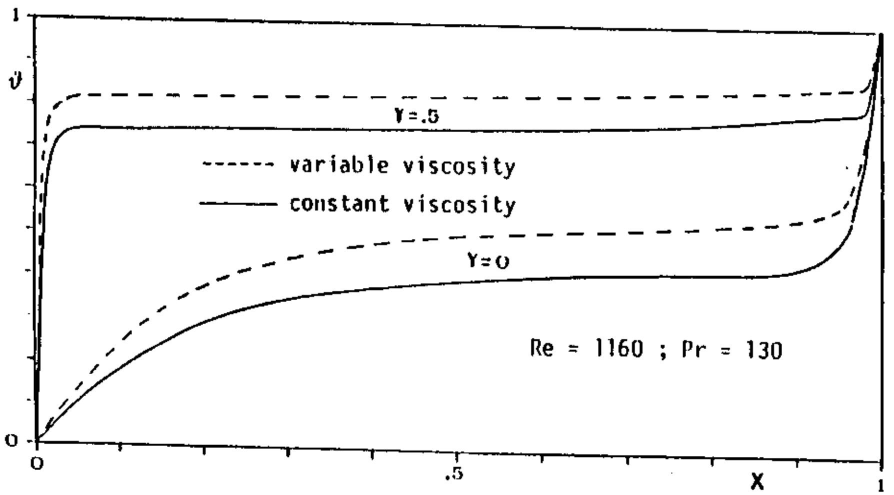


Fig. 7b - Axis (Y = 0) and interface (Y = .5) temperature distribution. Case Re = 1160 ; Pr = 130

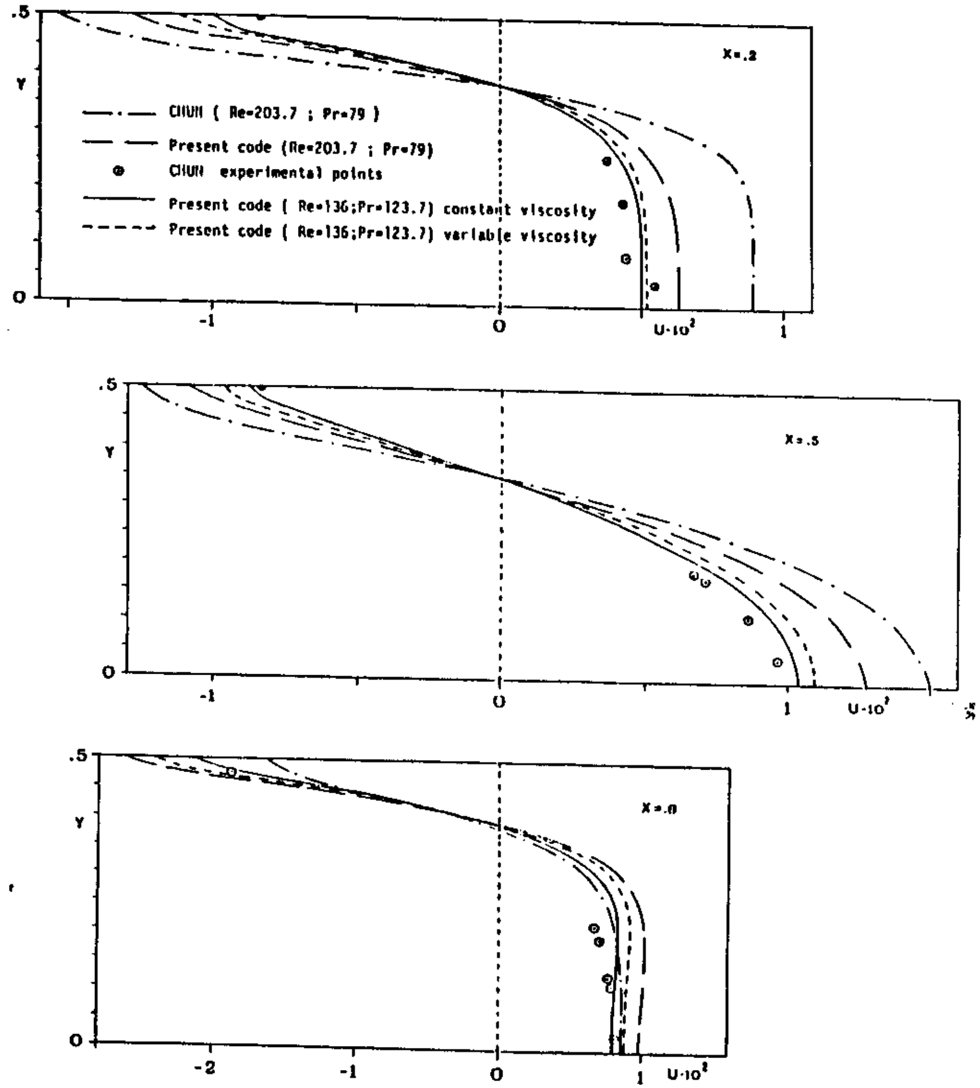
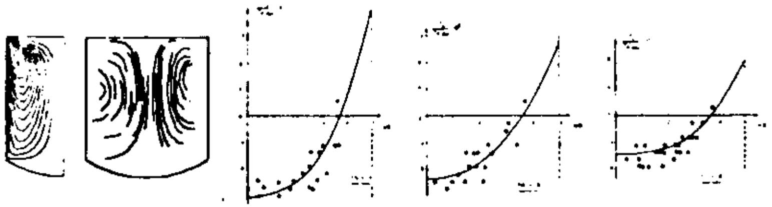


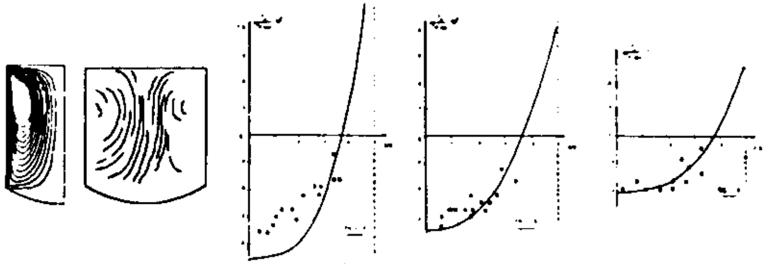
Fig. 9 - Axial velocity profiles over the radius at various stations

t = 30 sec

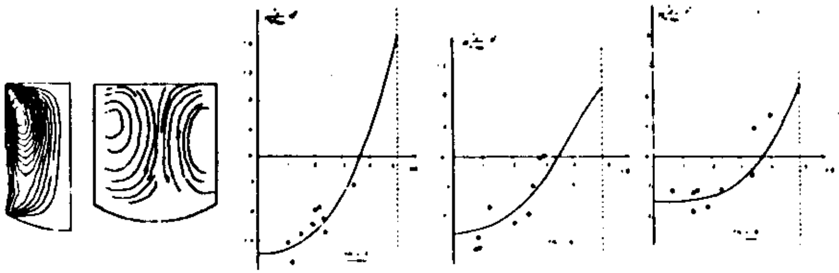
t = 45 sec

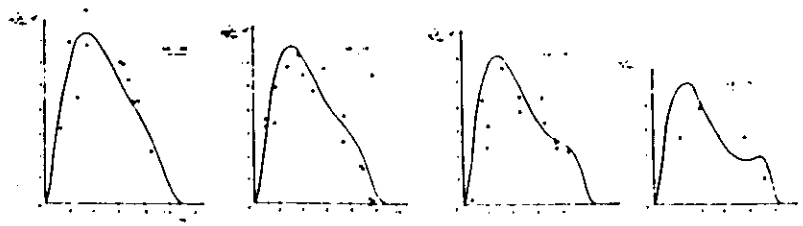


t = 60 sec



t = 560 sec





D1 - EXPERIMENT

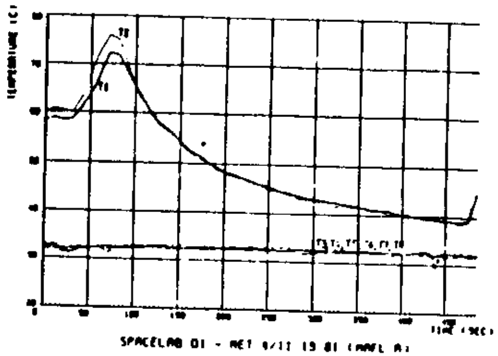


Figure 8. Time histories of the disks temperatures during the natural cooling-down phase

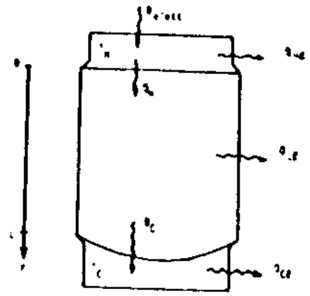
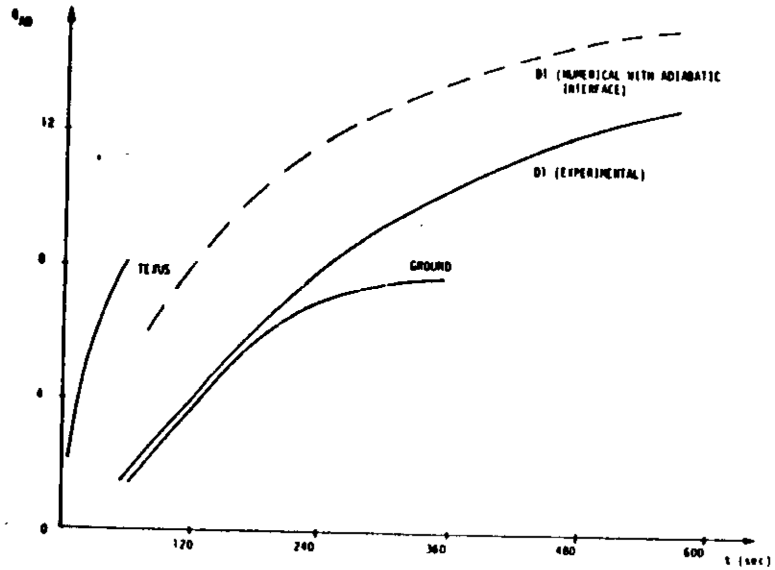
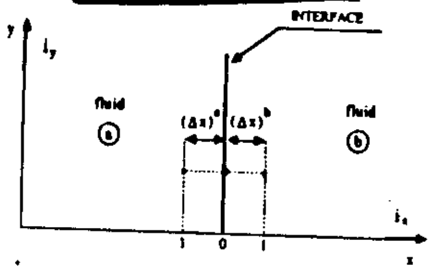


Figure 9. Schematic representation of the thermal problem



Dimensionless convective heat flow entering the cold disk

IV 21



Schematic interface.

$$(\psi)_0^a = (\psi)_0^b,$$
$$\lambda^a/\lambda_r(\theta_x)_0^a = \lambda^b/\lambda_r(\theta_x)_0^b,$$
$$(\theta)_0^a = (\theta)_0^b,$$
$$(\psi_x)_0^a = (\psi_x)_0^b,$$
$$\mathrm{Ma}(\theta_y)_0 = \mu^b/\mu_r(\zeta)_0^b - \mu^a/\mu_r(\zeta)_0^a.$$

$$(\psi_x)_0 = \left\{y'\left(\frac{\Delta x^2}{2}\right)^a\left[(\psi)_1^b\frac{\mu_b}{\mu_r} + y'\left(\frac{\Delta x^2}{2}\right)^b \mathrm{Ma}(\theta_y)_0\right] - (\psi)_1^a\frac{\mu^a}{\mu_r}y'\left(\frac{\Delta x^2}{2}\right)^b\right\}\Big/D,$$

$$(\zeta)_0^a = \left[-(\Delta x)^a(\psi)_1^b\frac{\mu^b}{\mu_r} - (\Delta x)^b(\psi)_1^a\frac{\mu^b}{\mu_r} \quad - \mathrm{Ma}(\theta_y)_0\, y'(\Delta x)^a\left(\frac{\Delta x^2}{2}\right)^b\right]\Big/D,$$

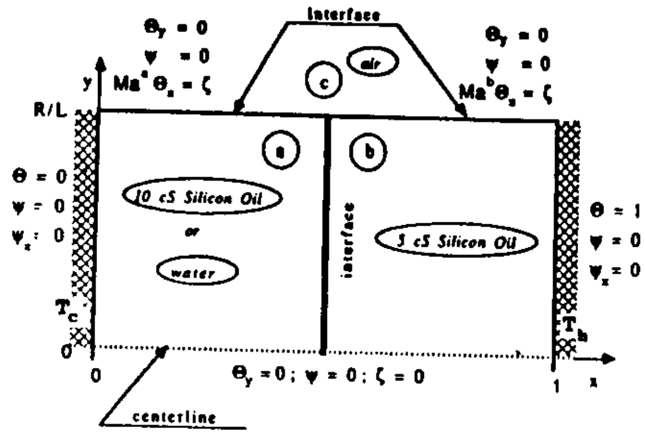
$$(\zeta)_0^b = \left[-(\Delta x)^b(\psi)_1^a\frac{\mu^a}{\mu_r} - (\Delta x)^a(\psi)_1^b\frac{\mu^a}{\mu_r} \quad + \mathrm{Ma}(\theta_y)_0\, y'(\Delta x)^b\left(\frac{\Delta x^2}{2}\right)^a\right]\Big/D,$$

$$D = \tfrac{1}{2}y'(\Delta x)^a(\Delta x)^b\left[\frac{\mu^a}{\mu_r}(\Delta x)^b + \frac{\mu^b}{\mu_r}(\Delta x)^a\right].$$

$$\theta_0 = \left[\frac{\lambda^b}{\lambda_a}\frac{(\Delta x)^a}{(\Delta x)^b}(\theta)_1^b + (\theta)_1^a\right]\Big/\left[1 + \frac{\lambda^b}{\lambda^a}\frac{(\Delta x)^a}{(\Delta x)^b}\right]$$

$$v_0 = -(\psi_x)_0/y'.$$

IV 21



Geometry of the problem.

$$\mathrm{Re}^a = \frac{V_r L}{\nu^a};\quad \mathrm{Pr}^a = \frac{\nu^a}{\alpha^a};\quad \mathrm{Ma}^a = \frac{\sigma_T^a(T_h - T_c)}{\mu^a V_r};$$
$$\mathrm{Re}^b = \frac{V_r L}{\nu^b};\quad \mathrm{Pr}^b = \frac{\nu^b}{\alpha^b};\quad \mathrm{Ma}^b = \frac{\sigma_T^b(T_h - T_c)}{\mu^b V_r};$$
$$\mathrm{Ma} = \frac{\sigma_T^{a/b}(T_h - T_c)}{\mu_r V_r};$$

Case A

Liquid a: 10 cS silicon oil — $\mathrm{Re}^a = 250$; $\mathrm{Pr}^a = 123.7$; $\mathrm{Ma}^a = 1$;

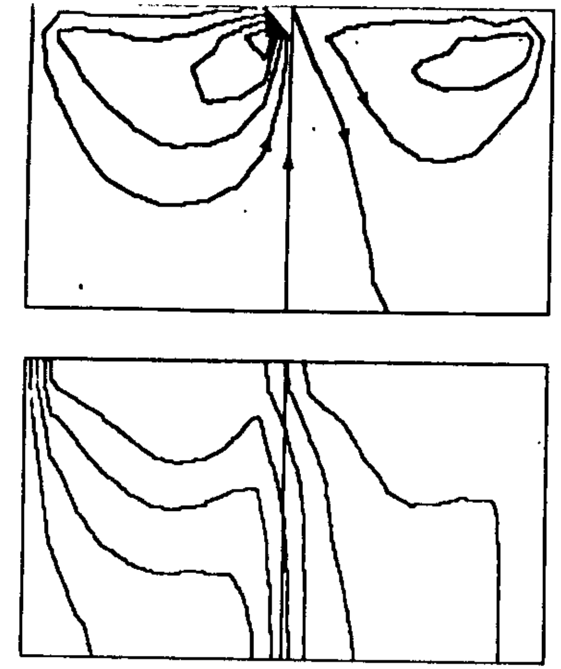
Liquid b: 5 cS silicon oil $\frac{\mu^b}{\mu^a} = 0.5$; $\frac{\lambda^b}{\lambda^a} = 1$. — $\mathrm{Re}^b = 500$; $\mathrm{Pr}^b = 71.4$; $\mathrm{Ma}^b = 2$; $\mathrm{Ma} = 0$.

Case B

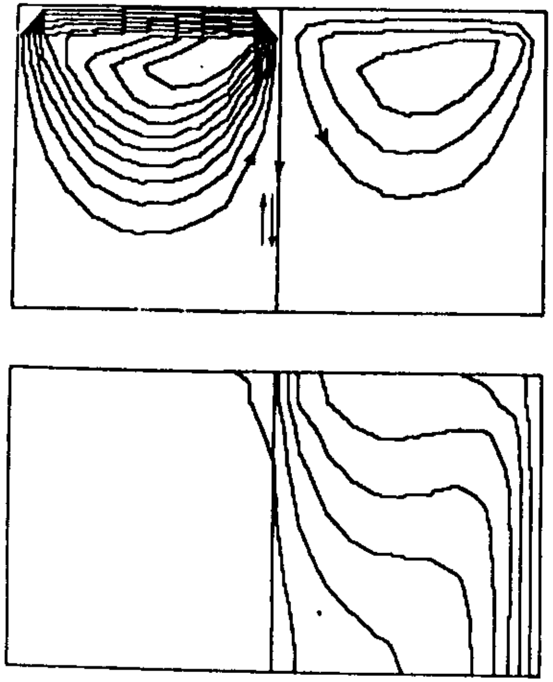
Liquid a: Water — $\mathrm{Re}^a = 300$; $\mathrm{Pr}^a = 6.9$; $\mathrm{Ma}^a = 13.74$;

Liquid b: 5 cS silicon oil $\frac{\mu^b}{\mu^a} = 4.85$; $\frac{\lambda^b}{\lambda^a} = 0.186$. — $\mathrm{Re}^b = 60$; $\mathrm{Pr}^b = 71.4$; $\mathrm{Ma}^b = 1$; $\mathrm{Ma} = 0$.

V 23



Stream-lines and isotherms. Case A: $\mu^b/\mu^a = 0.5$; $\lambda^b/\lambda^a = 1$.



Stream-lines and isotherms. Case B: $\mu^b/\mu^a = 4.85$; $\lambda^b/\lambda^a = 0.186$.

V 1

SOLA-VOF: A Solution Algorithm for Transient Fluid Flow with Multiple Free Boundaries

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$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\xi u}{x} = 0 \; .$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + g_x + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \xi\left(\frac{1}{x}\frac{\partial u}{\partial x} - \frac{u}{x^2}\right)\right]$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + g_y + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\xi}{x}\frac{\partial v}{\partial x}\right] \; .$$

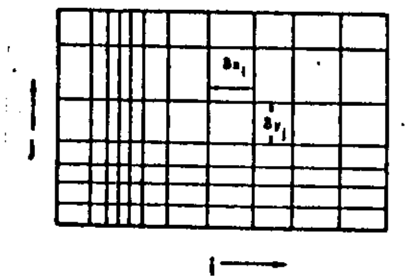


Fig. 1.

A finite-difference mesh with variable rectangular cells.

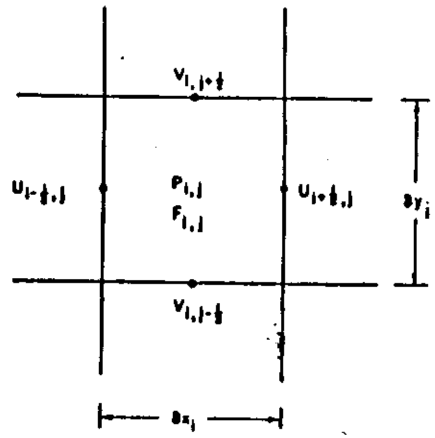


Fig. 2.

Location of variables in a typical mesh cell.

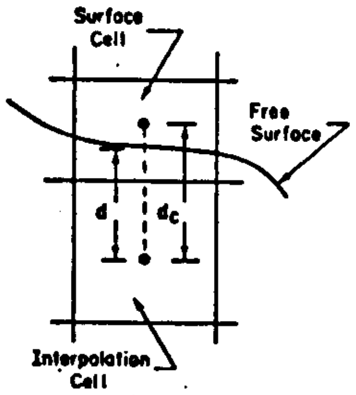


Fig. 4.
Definition of quantities used in defining free-surface pressure boundary condition.

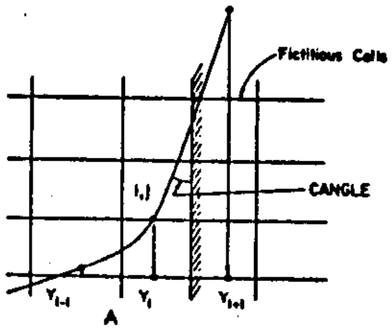


Fig. 6a.
Wall adhesion for the locally near-horizontal interface is modeled by setting the value of Y_{i+1} such that CANGLE is the angle between the wall and the interface.

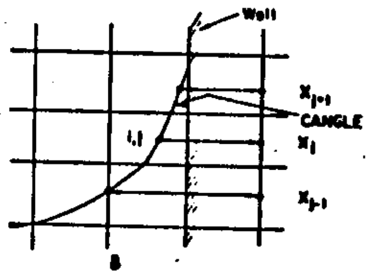


Fig. 6b.
Wall adhesion for the locally near-vertical interface is modeled by setting X_{j+1} such that CANGLE is the angle between the wall and the interface.

$$FUX = [u_{i+1,j} < u_{i+1,j} > - u_{i,j} < u_{i,j} >]/\delta x_{i+\frac{1}{2}} \; .$$

where, e.g.,

$$u_{i,j} = (u_{i-\frac{1}{2},j} + u_{i+\frac{1}{2},j})/2$$

and

$$< u_{i,j} > = \begin{cases} u_{i-\frac{1}{2},j} \,, & \text{if } u_{i,j} > 0 \\ u_{i+\frac{1}{2},j} \,, & \text{if } u_{i,j} < 0 \; . \end{cases}$$

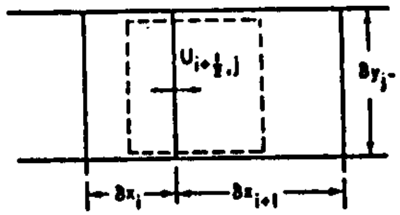
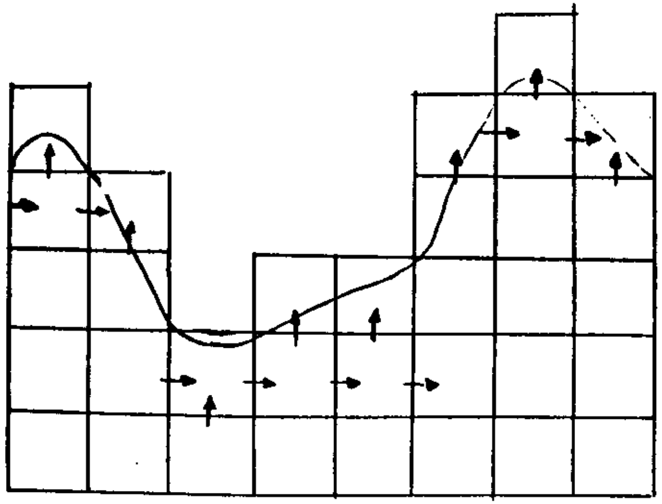
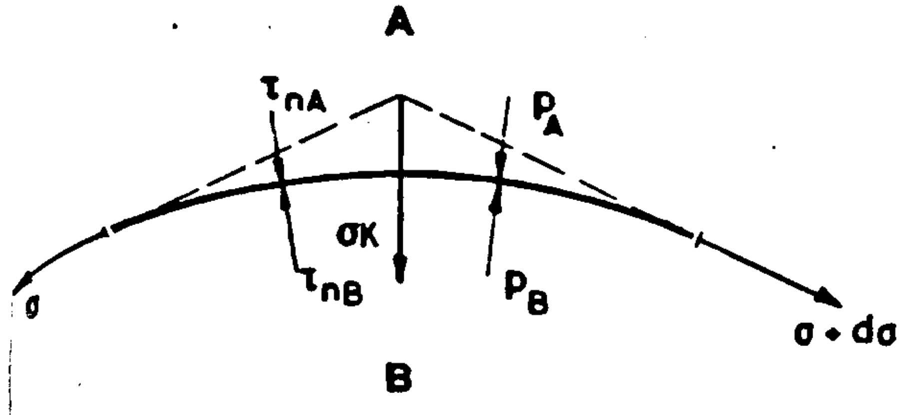


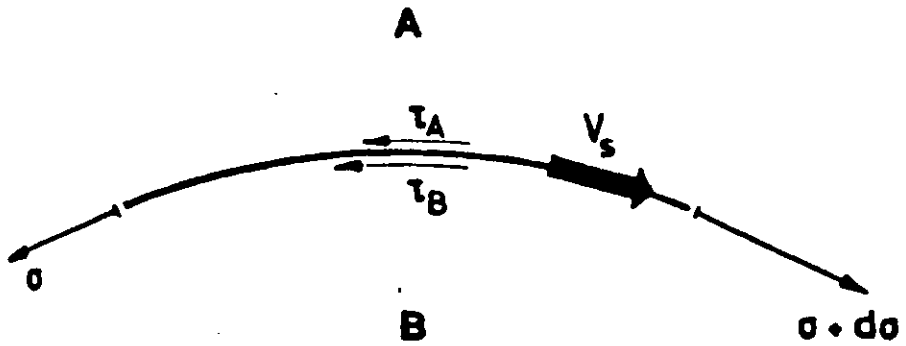
Fig. 3.
Control volume (dashed rectangle) used for constructing a finite-difference approximation for the u momentum equation at location $(i + \frac{1}{2}, j)$.

VOLUME of FLUID function "F":

$$\frac{\partial F}{\partial t} + u \frac{\partial F}{\partial x} + v \frac{\partial F}{\partial y} = 0$$







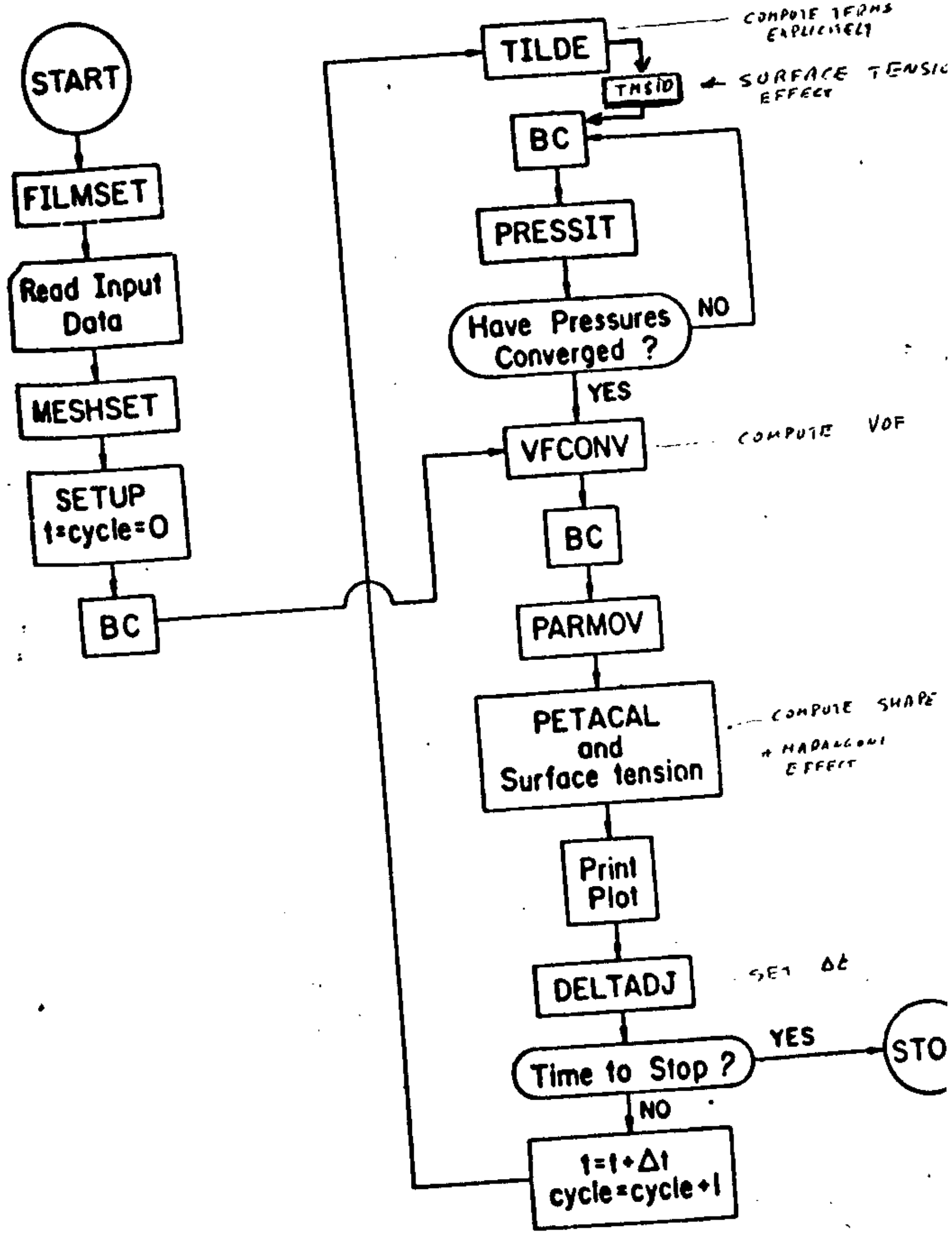
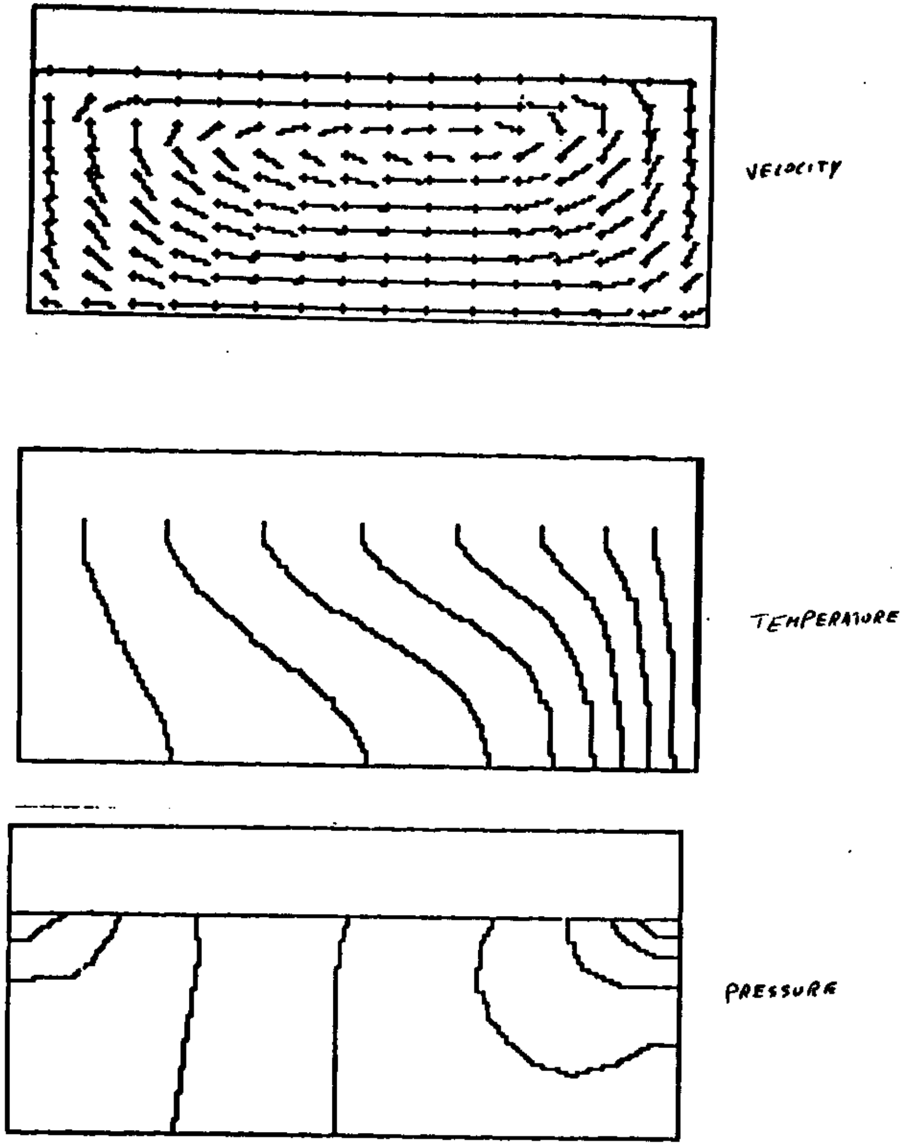
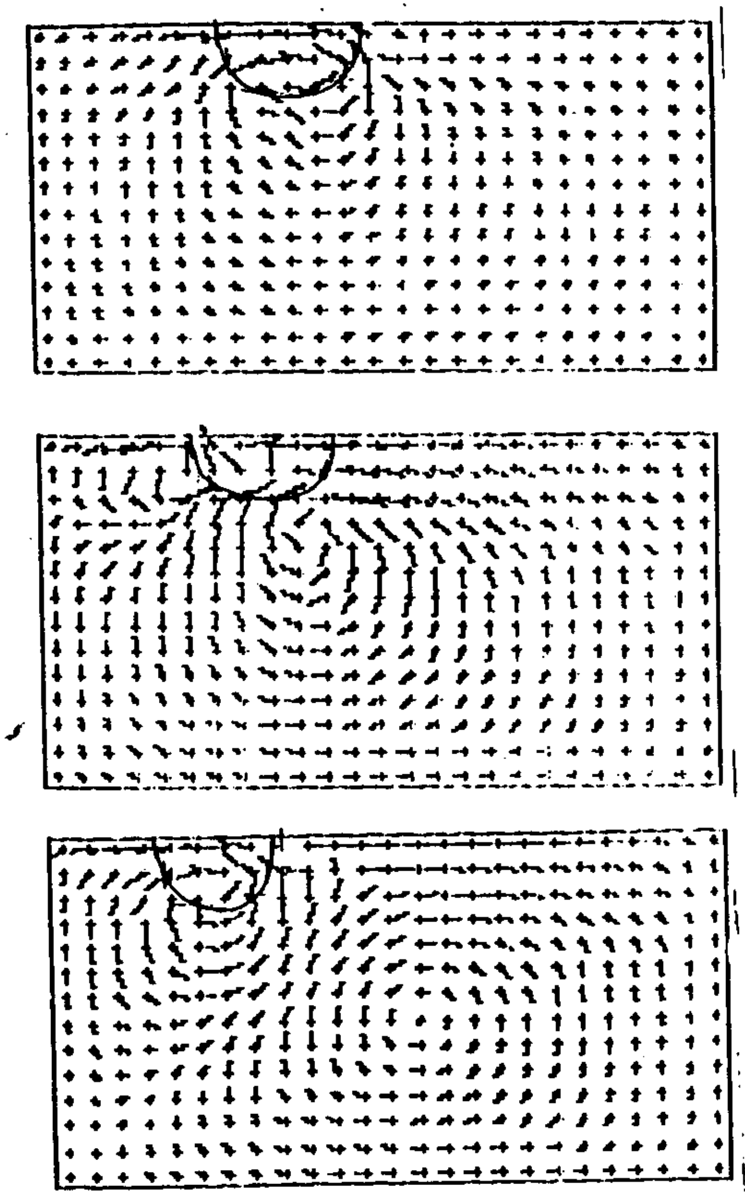


Diagramma di flusso del programma SOLAVOF



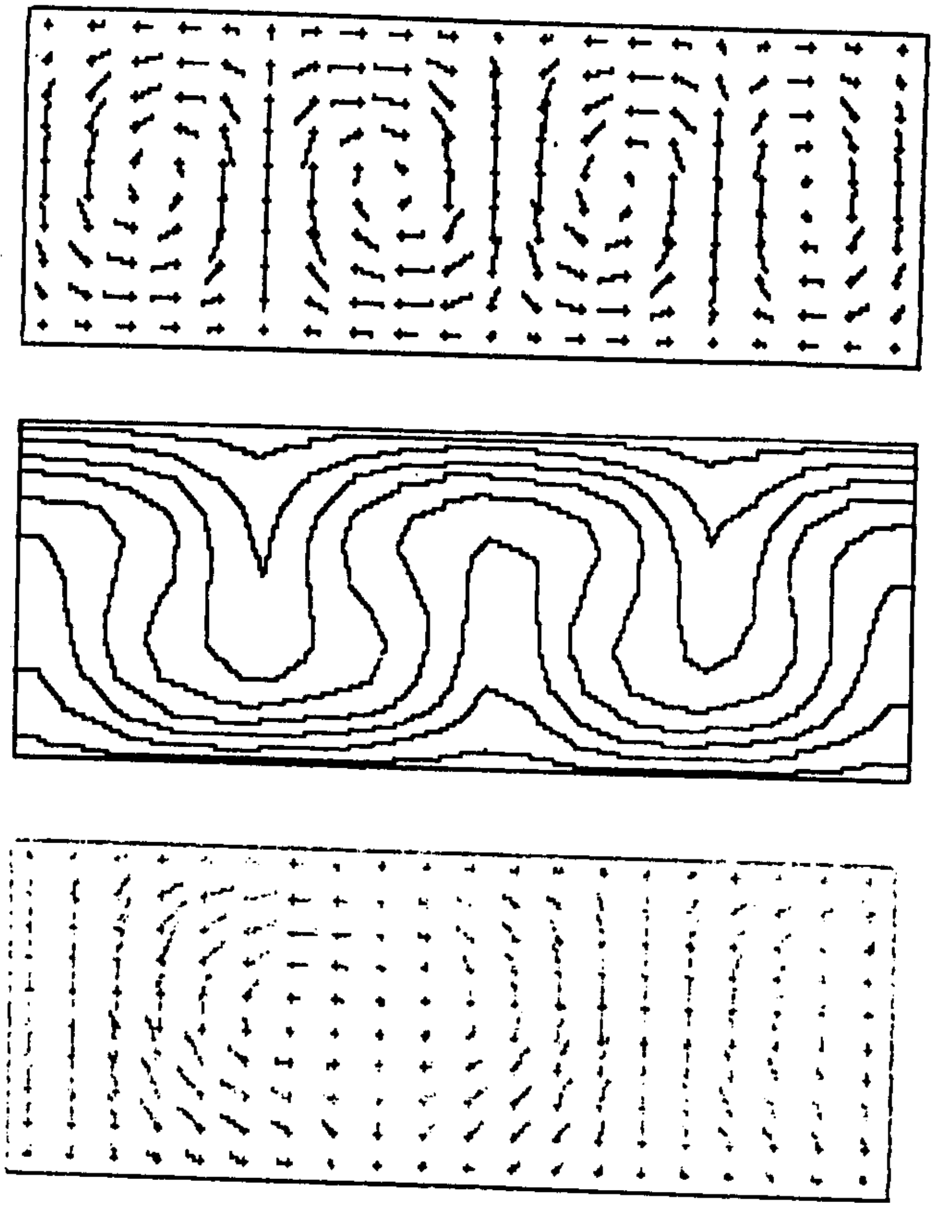
FLUID-BRIDGE.

Campi di Velocità, temperatura e pressione nel caso assial-simmetrico con Re = 100 Pr = 1 Wb = .03
Soluzione stazionaria.



BOLLE.

Moto di una bolla con Re = 20 Pr = 10 in successivi istanti di tempo.
Sopra: tempo = 0.5
Al centro: tempo = 2.0
Sotto: tempo = 7.5

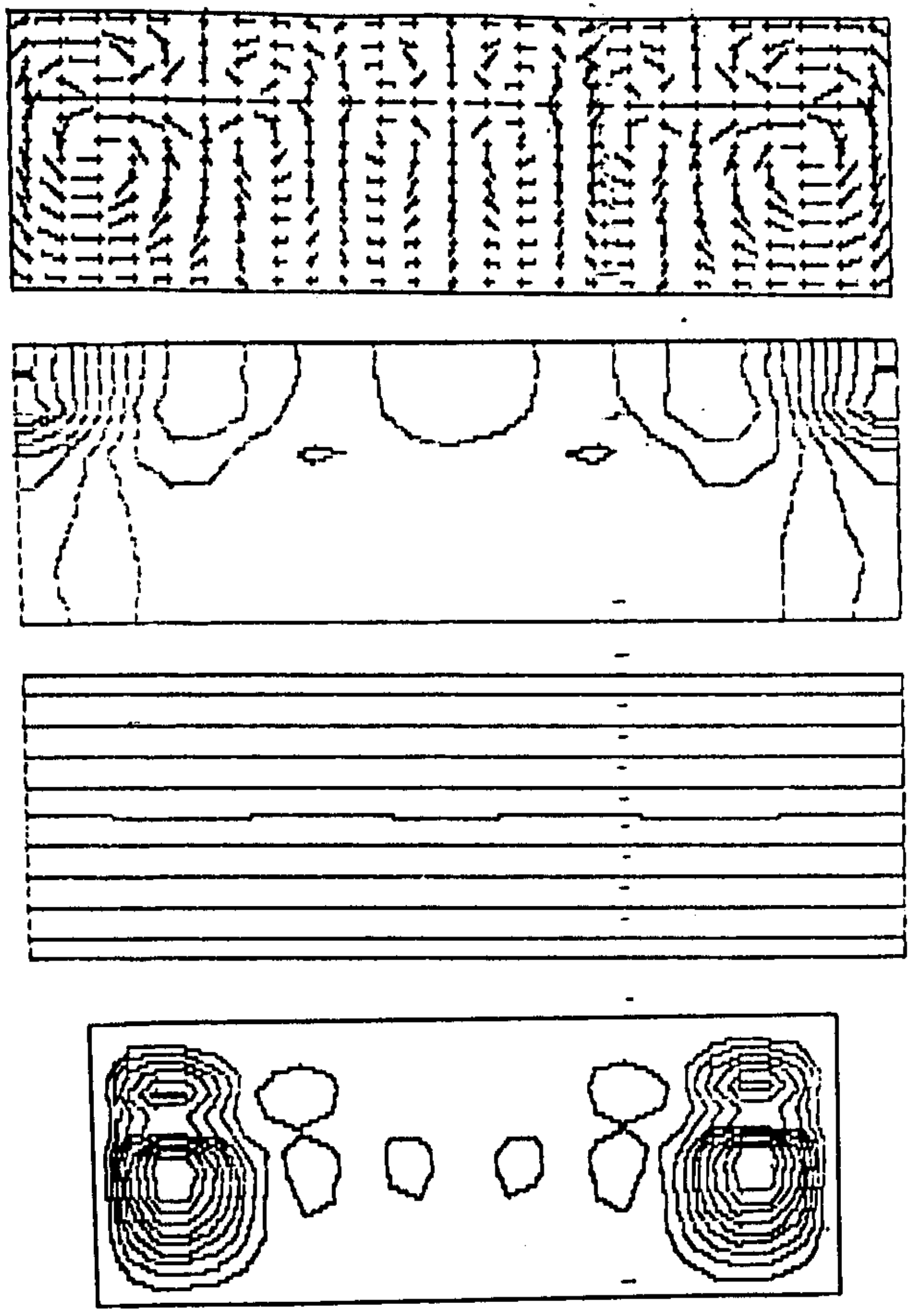


MOTI ALLA BENARD.

Sopra: campo di velocità stazionario per un fluido con Pr = 10
e Ra = 10000

Al centro: campo di temperatura

Sotto: perturbazioni del campo di velocità in un fluido con
Ra subcritico: i disturbi, puramente numerici, in
questo caso non innescano il moto convettivo.



MARANGONI-BENARD.

Campi di velocità, pressione, temperatura e linee di corrente
dopo 2 s.