INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR/382- 27

WORKSHOP ON SPACE PHYSICS:
"Materials in Micorgravity"
27 February - 17 March 1989

"Liquid Menisci and Their Stability"

D. LANGBEIN
Battelle Europe - Battelle Institut E.V.
Frankfurt, Federal Republic of Germany

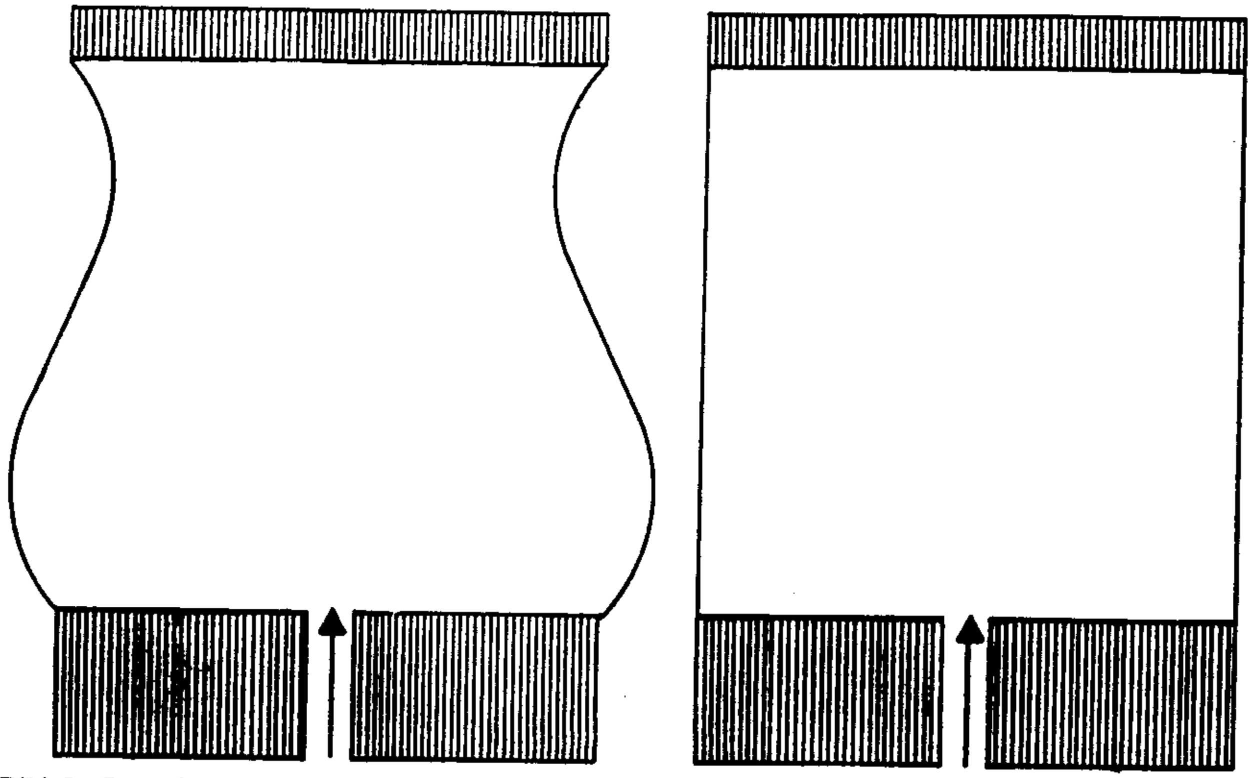
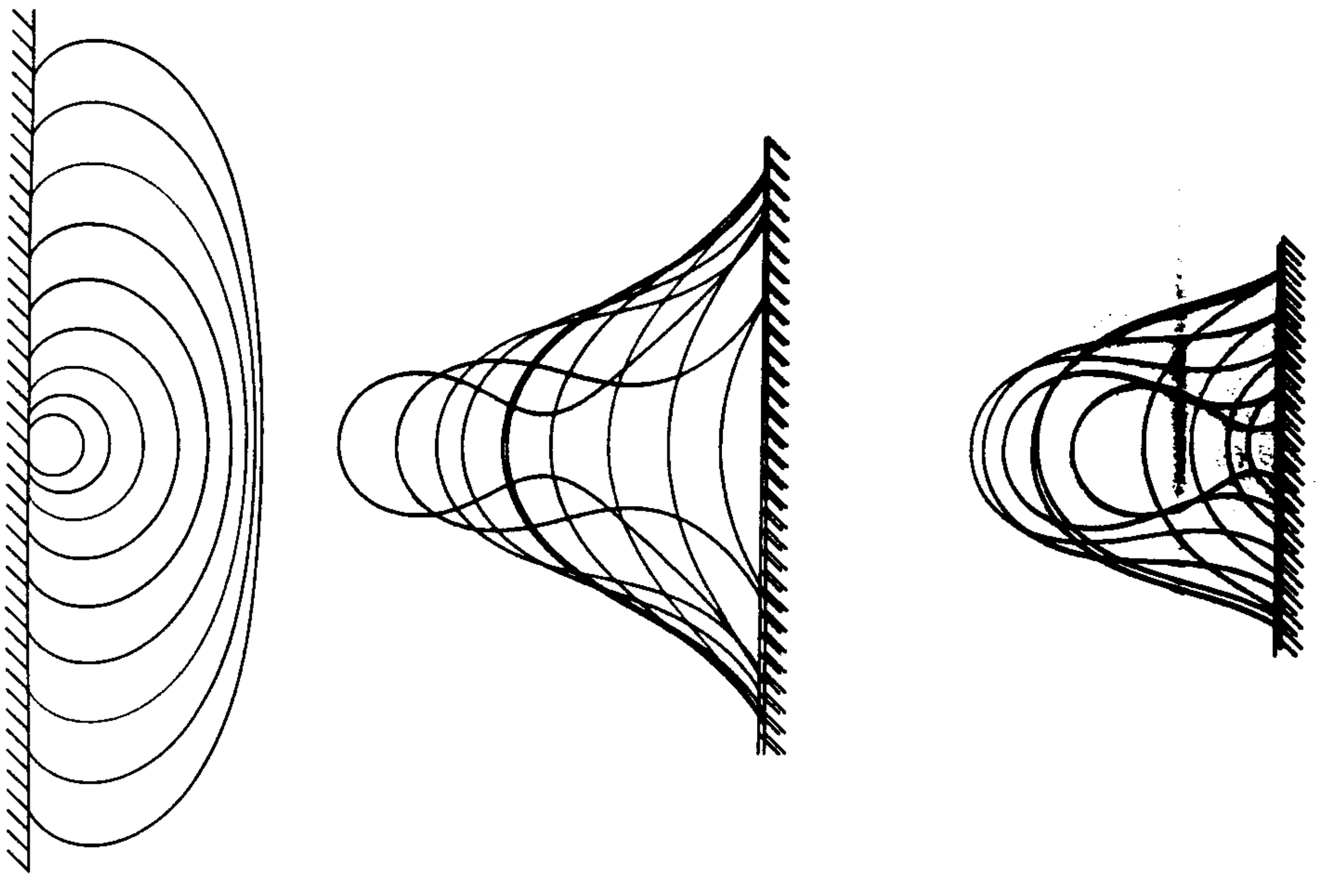
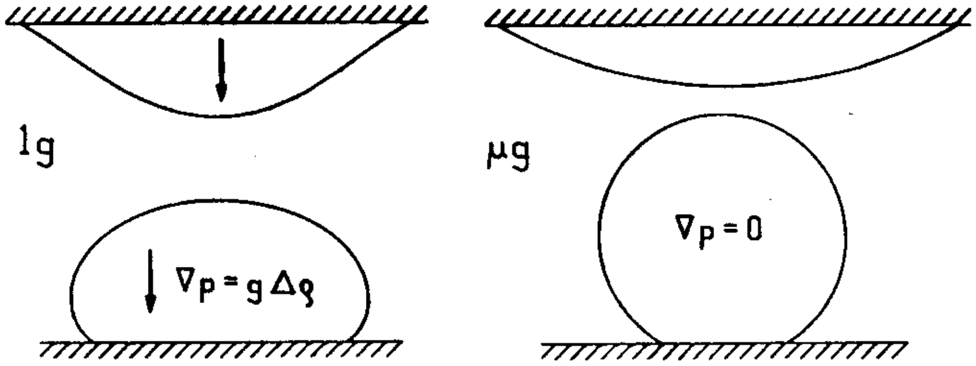
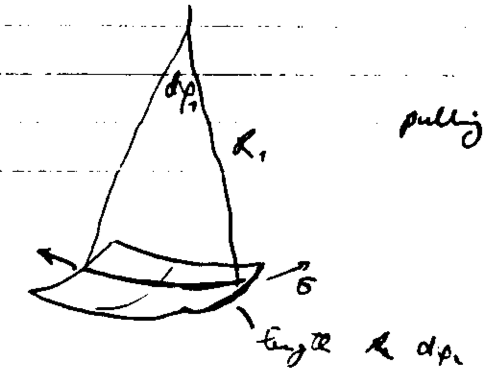


Bild 5: Grenzflächenformen einer Flüssigkeitssäule unter terrestrischen Bedingungen (links) und unter Mikrogravitation (rechts). Das vorgegebene Flüssigkeitsvolumen ist zwischen zwei festen Stempeln durch eine Öffnung im unteren Stempel eingebracht worden. Unter terrestrischen Verhältnissen nimmt der Flüssigkeitsdruck von oben nach unten zu, so daß mit dem Krümmungsdruck auch die Krümmung von oben nach unten ansteigt. Unter Mikrogravitation ist der Druck über die gesamte Flüssigkeitsbrücke konstant. Bei passender Wahl von Flüssigkeitsvolumen und Stempelabstand bildet sich eine zylindrische Grenzfläche aus.



Prof. Langbein: Liquid Menisci and Their Stability



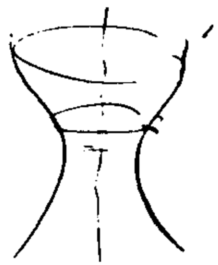


pulling is σ

$$p \cdot R_1 d\varphi_1 \cancel{R_2 d\varphi_2} = \sigma\, d\varphi_1 \cdot R_2 d\varphi_2$$
$$+\ \sigma\, d\varphi_2 \cdot R_1 d\varphi_1$$

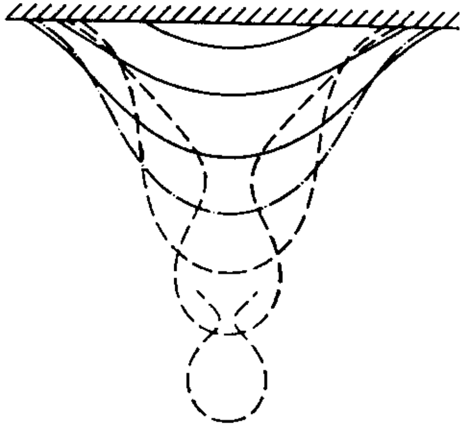
$$p = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$ capillary equation
Laplace equation

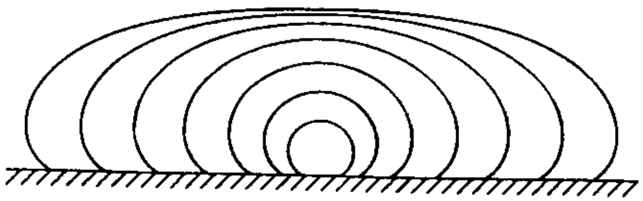
$\sigma A + U_{pot} = \min$ for $V = \text{const.}$



axisymmetric surface

$$A = \int dz \sqrt{1+\left(\frac{dr}{dz}\right)^2}\; 2\pi r$$
$$V = \int dz\; \pi r^2$$
$$U_{pot} = +\int dz\; \pi r^2\, \varrho\, g\, z + \text{Rotation}$$
$$-\int dz\; \pi \varrho\, \omega^2 \frac{r^3}{3}$$





area $A = \int dz\, 2\pi r \sqrt{1+(\frac{dr}{dz})^2}$

volume $V = \int dz\, \pi r^2$ constraint

$U_{grav} = \int dz\, \pi r^2 g \rho z$

$U_{rot} = -\int dz\, \pi \rho \omega^2 \frac{r^3}{3}$

energy $\sigma A + U_{grav} + U_{rot} = \text{Min}$

$V = \text{const.}$

Lagrange formalism   Lagrange parameter p

$$\int dz \left(\sigma 2\pi r \sqrt{1+(\tfrac{dr}{dz})^2} + \pi r^2 g\rho z - \pi\rho\omega^2 \frac{r^3}{3} - p\,\pi r^2 \right) = \text{Min}$$

$$\int dz\, f(z, r, \tfrac{dr}{dz}) = \text{Min}$$

$$\left(\frac{\partial}{\partial r} - \frac{d}{dz}\frac{d}{d(\frac{dr}{dz})}\right) f = 0$$

Euler equation
Laplace
Lagrange

$$2\pi \left\{ \sigma\sqrt{1+(\tfrac{dr}{dz})^2} + g\rho r z - \rho\omega^2\frac{r^2}{2} - p r - \sigma\frac{d}{dz}\, r \frac{\frac{dr}{dz}}{\sqrt{1+(\frac{dr}{dz})^2}} \right\} = 0$$

$$\sigma \left\{ \sqrt{1+(\tfrac{dr}{dz})^2} - \frac{(\frac{dr}{dz})^2 + r\frac{d^2r}{dz^2}}{\sqrt{1+(\frac{dr}{dz})^2}} + \frac{(\frac{dr}{dz})^2\, r\frac{d^2r}{dz^2}}{\sqrt{1+(\frac{dr}{dz})^2}^{\,3}} \right\} = r\{\underbrace{p - g\rho z + \rho\omega^2\frac{r^2}{2}}_{\text{local pressure}}\}$$

$$\frac{\sigma}{\cancel{r}} \left\{ \frac{1}{r\sqrt{1+(\frac{dr}{dz})^2}} - \frac{\cancel{r}\frac{d^2r}{dz^2}}{\sqrt{1+(\frac{dr}{dz})^2}^{\,3}} \right\} = p - g\rho z + \rho\omega^2\frac{r}{2}$$

rotation   force $\rho\omega^2 r$

energy $-\frac{1}{2}\rho\omega r^2$ of ring

→ energy of disk

$$-\int dr\, 2\pi r\, \tfrac{1}{2}\rho\omega^2 r^2 = -\pi\rho\omega^2\frac{r^3}{3}$$

$$-\int dz\, \pi\rho\omega^2 \frac{r^3}{3}$$

Switch from $r(z)$ to $z(r)$

$$\frac{dr}{dz} = \frac{1}{\frac{dz}{dr}} \qquad \frac{d^2r}{dz^2} = -\frac{\frac{d^2z}{dr^2}}{(\frac{dz}{dr})^3}$$

$\boxed{\frac{dz}{dr}}$

$$\sigma\left\{ \frac{\frac{dz}{dr}}{r\sqrt{1+(\frac{dz}{dr})^2}} + \frac{\frac{d^2z}{dr^2}}{\sqrt{1+(\frac{dz}{dr})^2}^{\,3}} \right\} = p - g\rho z + \rho\omega^2\frac{r^2}{2}$$

$$\sigma\,\frac{1}{r}\frac{d}{dr}\left(\frac{r\frac{dz}{dr}}{\sqrt{1+(\frac{dz}{dr})^2}}\right) = p - g\rho z + \rho\omega^2\frac{r^2}{2}$$

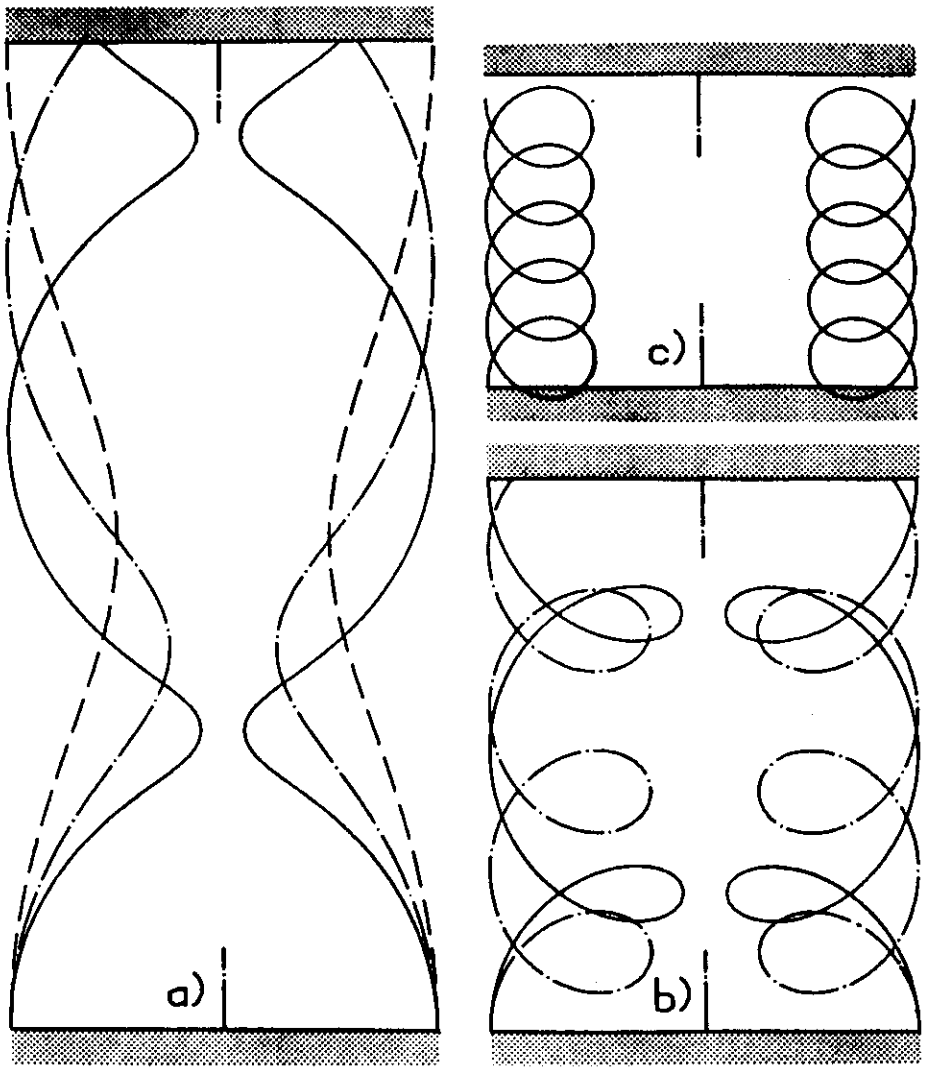
$$\sigma\,\frac{r\frac{dz}{dr}}{\sqrt{1+(\frac{dz}{dr})^2}} = p\frac{r^2}{2} - \int dr\, g\rho z r + \rho\omega^2\frac{r^4}{8} + \text{const}$$

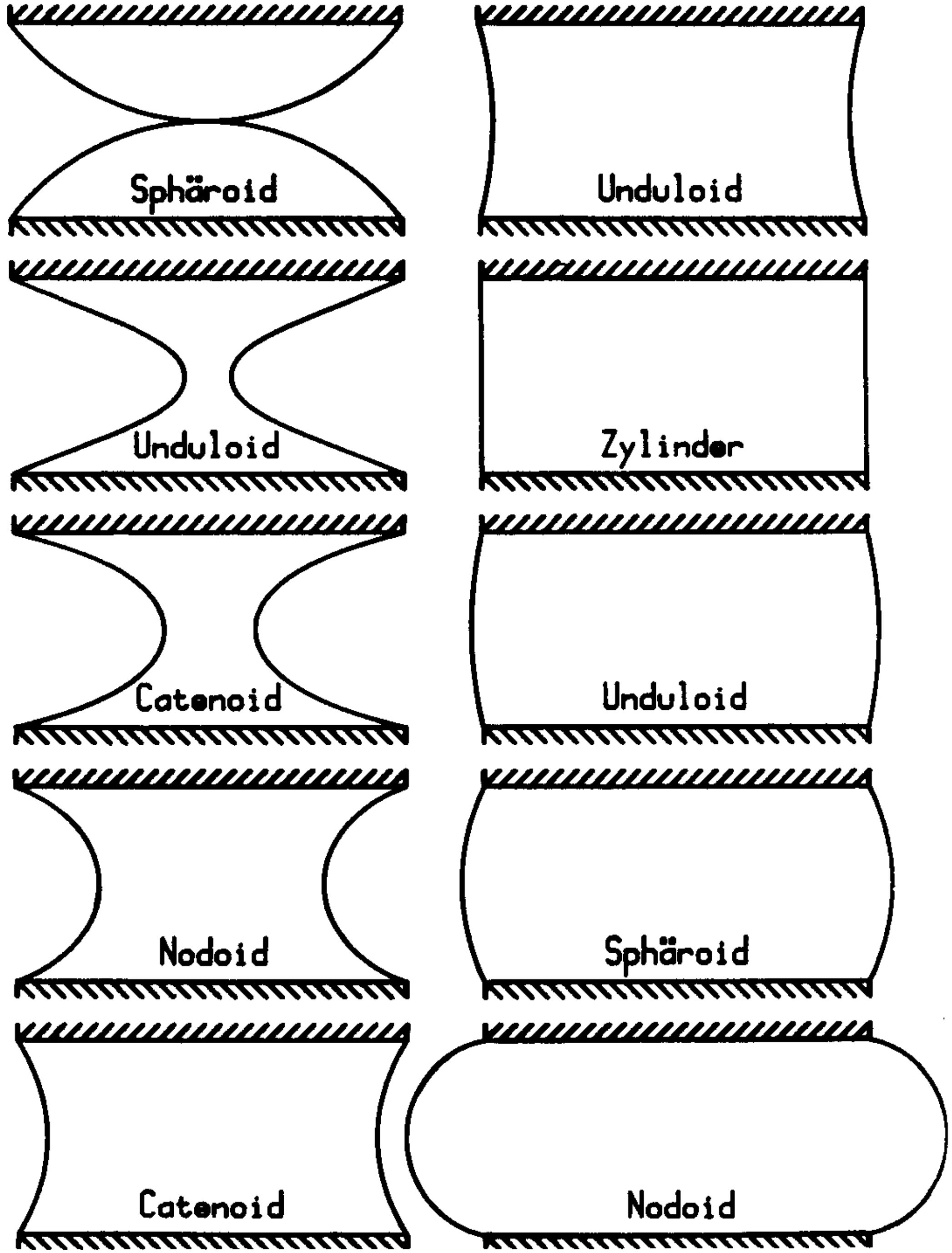
first order differential equation

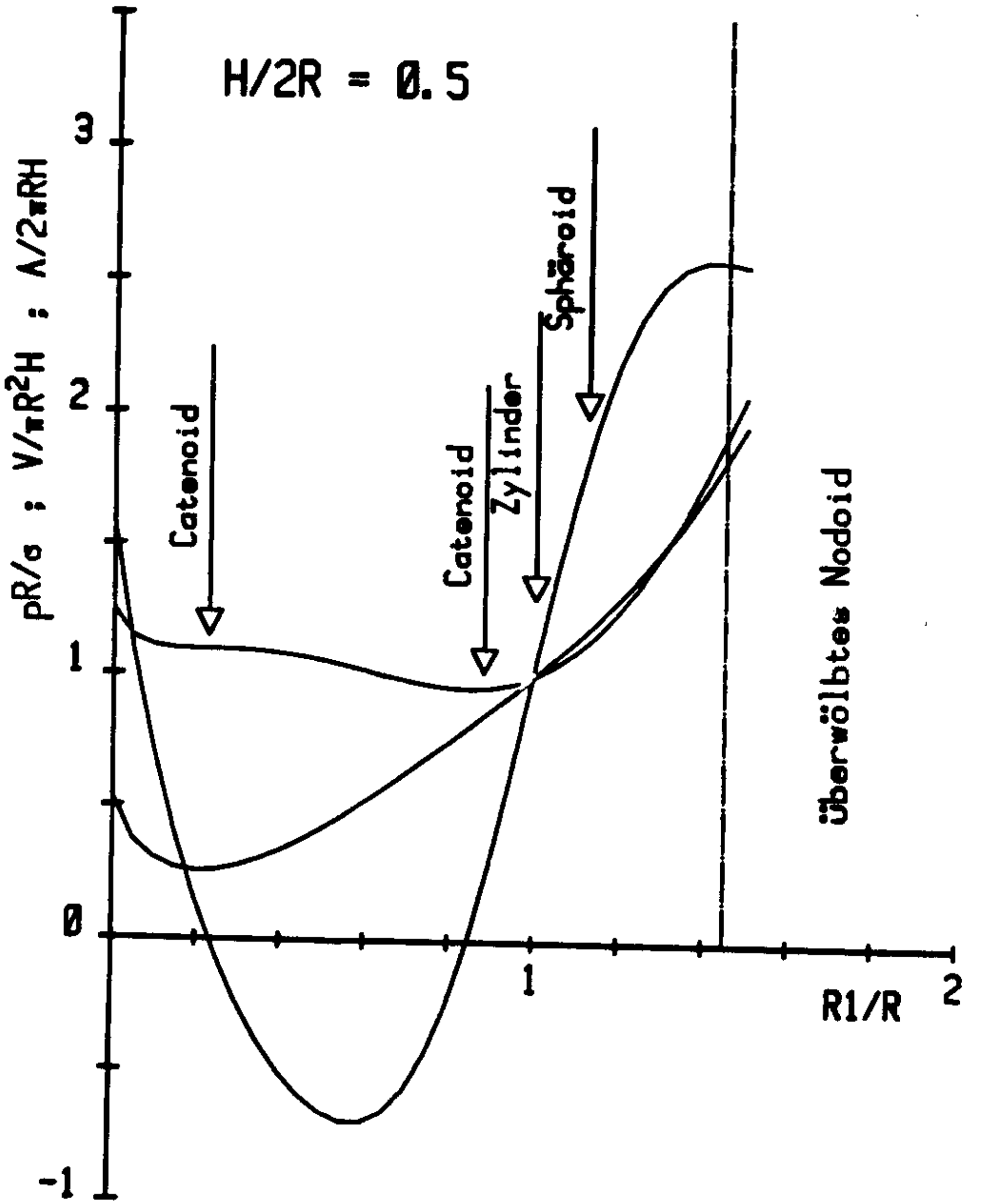
unduloid
nodoid
catenoid —
~~spheroid~~ sphere

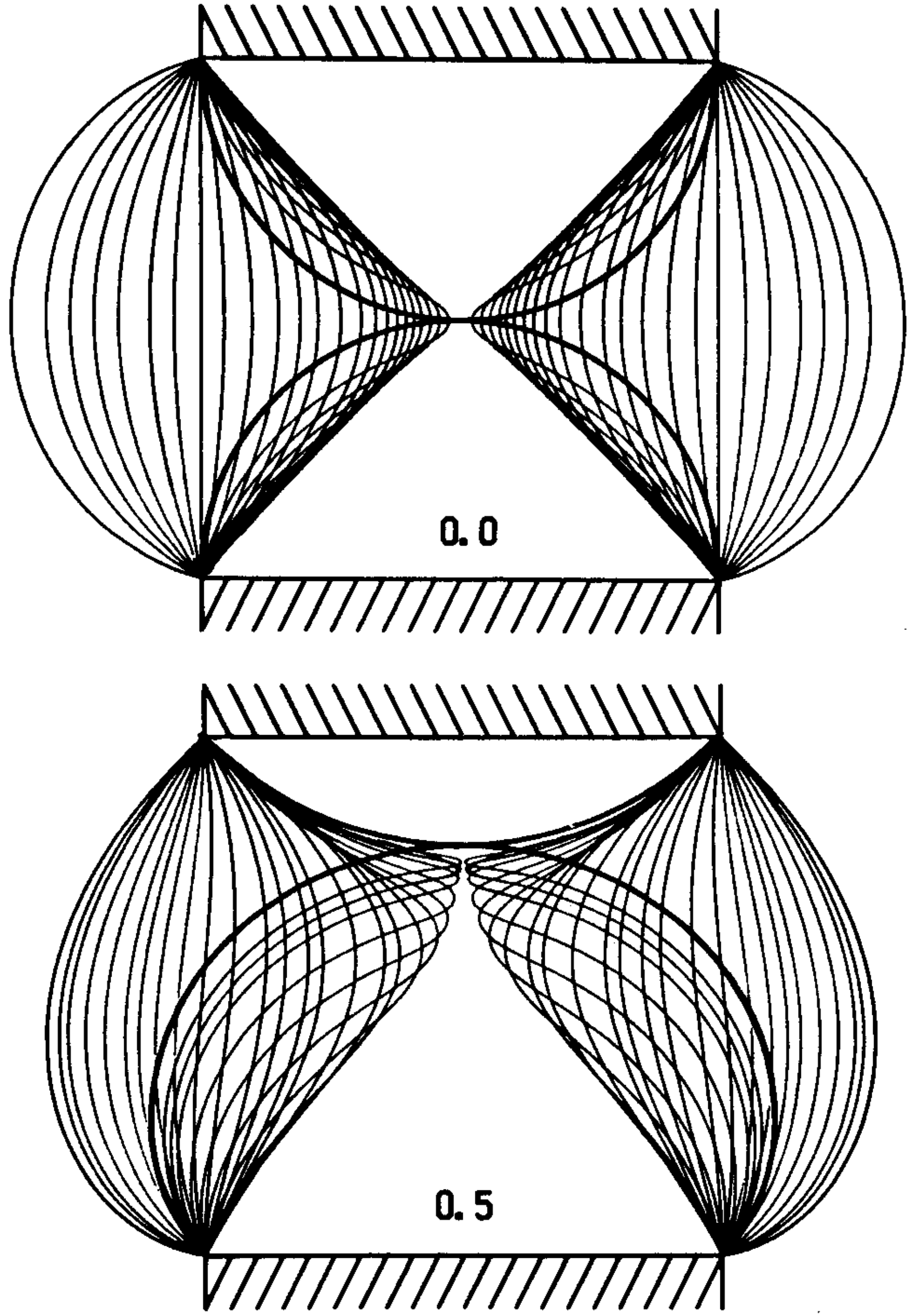
$\frac{dz}{dr} \to 0$

$$\boxed{\begin{aligned} r &= \cosh z \\ \frac{dr}{dz} &= \sinh z \\ \frac{dz}{dr} &= \frac{1}{\sinh z} \end{aligned}}$$


a)
b)
c)


Sphäroid
Unduloid
Unduloid
Zylinder
Catenoid
Unduloid
Nodoid
Sphäroid
Catenoid
Nodoid


H/2R = 0.5
pR/σ ; V/πR²H ; A/2πRH
Catenoid
Catenoid
Zylinder
Sphäroid
Überwölbtes Nodoid
R1/R
3
2
1
0
-1
1
2


0.0
0.5