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SOME 2-D DIGITAL SYSTEMS WITH APPLICATION  
TO IMAGE PROCESSING

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# SOME 2-D DIGITAL SYSTEMS WITH APPLICATION TO IMAGE PROCESSING

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## Abstract

The actual interest of two-dimensional (2-D) digital systems is pointed out, as resulting from several reasons: high efficiency for image processing, great application flexibility, decreasing cost of software and hardware implementation. Some efficient 2-D non recursive digital filters of FIR type using suitable window functions (Cappellini windows) and fast 2-D recursive digital filters obtained through a suitable rotation and stabilization procedure are presented. 2-D local space operators and Kalman filters are also considered. Some applications of the above 2-D systems to digital image processing in biomedicine, remote sensing, recognition of moving objects and robotics are finally presented.

## 1. INTRODUCTION

2-D digital systems are of increasing interest in several application areas such as facsimile-television, sonar-radar, remote sensing, underwater acoustics, biomedicine, moving object recognition, robotics. These digital systems present indeed attracting aspects in comparison with analog ones: high efficiency permitting better image processing and analysis; capability of performing non linear operations, decreasing cost of software or hardware implementations due to the large expansion and evolution of standard computers, minicomputers, microprocessors and high integration digital circuits (VLSI); great application flexibility and adaptivity.

Important operations, which can be performed by 2-D digital systems, are the following ones: 2-D digital filtering, local space processing, data reduction (compression), pattern recognition. Digital filtering and local space processing operations play a relevant role both in pre-processing of images performing smoothing, enhancement, noise reduction and in final processing, before pattern recognition, extracting boundaries and edges. Data compression operations permit to reduce the large amount of data representing the images in digital form, solving transmission or storage problems. Pattern recognition operations permit to extract the significant information data and configurations from the images for final interpretation and utilization.

In this paper 2-D digital systems performing digital filtering and local space processing are essentially considered, pointing out their crucial importance for image processing and analysis. In particular efficient 2-D non recursive digital filters of FIR type, fast 2-D recursive digital filters of IIR type, 2-D local space operators and Kalman filters are presented. Some typical applications of these 2-D systems to digital image processing in biomedicine, remote sensing and recognition of moving objects (robotics) are also shown. In connection with these applications implementation aspects are considered with particular reference to the use of microprocessor and minicomputer systems, suitably equipped with input-output units (to digitize input images and present the processed output images).

## 2. 2-D NON RECURSIVE DIGITAL FILTERS

A 2-D non recursive digital filter can be defined by the following relation [1]

$$g(n_1, n_2) = \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} a(k_1, k_2) f(n_1 - k_1, n_2 - k_2) \quad (1)$$

where  $f(n_1, n_2)$  are the input data,  $g(n_1, n_2)$  are the output data and  $a(k_1, k_2)$  are the coefficients defining the digital filter [1]. By using the z-transform, the transfer function  $H(z_1, z_2)$  can be obtained and hence, by setting

$z_1 = e^{j\omega_1}$ ,  $z_2 = e^{j\omega_2}$  (the case is considered with the space sampling interval  $X=1$ ), the 2-D frequency response can be defined [1]

$$H(e^{j\omega_1}, e^{j\omega_2}) = \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} a(k_1, k_2) e^{-j(k_1\omega_1 + k_2\omega_2)} \quad (2)$$

Of particular interest for image processing are linear phase filters, having a symmetric impulse response, which don't introduce phase distortion in the processed image.

The problem of designing the digital filter in the 2-D frequency domain appears clearly connected to the evaluation of the coefficient matrix  $a(k_1, k_2)$  in such a way that the obtained frequency response satisfies the required characteristics. Several design techniques have been proposed for 2-D digital filters, some of which are a direct generalization of their 1-D counterpart.

However direct generalization of multiple exchange algorithms, as the Remez algorithm for the design of optimal (minimax error) 1-D filters, is not possible because the alternation theorem is not directly generalizable. The design of 2-D optimal filters is possible by means of the linear programming approach, but the number of points where it is necessary to compute the transfer function to obtain constraint relations and the variable number (coefficient matrix) is in general very high. Consequently the time necessary to design the filter tends to become very long, at least using linear programming formulations [2][3].

A more tractable design technique, from the computational point of view, can be obtained by reducing the number of variables in the linear programming problem by means of the frequency sampling approach [2]. A grid of samples in the frequency domain is chosen and most of the samples are fixed as a direct translation of the filter specifications. A linear programming problem can be indeed set up using constraint relations for the interpolated frequency response, where the variables are the frequency

samples in the transition band.

Another suboptimum design method is based on the frequency transformations of 1-D filters to 2-D filters. It can be easily shown [1] that the real part of the frequency response of a 1-D filter can be transformed to the real part of the frequency response of a 2-D filter by means of a transformation of variables of the form

$$\cos \omega = A \cos \omega_1 + B \cos \omega_2 + C \cos \omega_1 \cos \omega_2 + D \quad (3)$$

This means that the filter can be designed in 1-D and then mapped in 2-D. With the choice  $A=B=C=D=1/2$ , the mapping contours are approximately circular, at least for small values of  $\omega$ , and circularly symmetric filters can be designed. This design procedure, which can be generalized to more complex transformation relations, is convenient from the computational point of view, even if some care has to be given in carrying out the transformation, which is sensitive to numerical errors, when the number of coefficients is becoming very high.

Among the different design techniques, the window design method is of interest due to the fact that it assures a good efficiency with a relatively simple procedure [1]. In this method we start from an impulse response, which has to be truncated introducing the minimum error in the frequency response. To this purpose the obtained values of the sampled impulse response,  $h(k_1, k_2)$ , are multiplied by the samples  $w(k_1, k_2)$  of a suitable "window" function having zero value in the region out of the truncation and high concentration in the frequency domain. The obtained frequency response of the finite-impulse-response (FIR) digital filter is therefore the convolution between  $H(\omega_1, \omega_2)$ , Fourier transform of  $h(k_1, k_2)$ , and  $W(\omega_1, \omega_2)$ , Fourier transform of  $w(k_1, k_2)$  (in general the discrete form, DFT, of Fourier transform is considered and in the implementation the fast form FFT is applied).

Many window functions are known for designing digital filters in the 1-D case [1]. For the 2-D design, here considered, extensions to 2-D domain are in general used. In particular, as shown by Huang [4], a 2-D window, having circular symmetry properties, can be defined through a  $w(t)$  1-D window as

$$w(x, y) = w(\sqrt{x^2 + y^2}) \quad (4)$$

Three window functions of particular interest for their properties are: Lanczos-extension window (Cappellini window-1); Kaiser window; Weber-type approximation windows (Cappellini windows-2,3). The Lanczos-extension window,  $w_1(t)$ , is a very simple and good window having the expression in the 1-D form as [1]

$$w_1(t) = \left[ \frac{\sin((\pi t)/\tau)}{(\pi t)/\tau} \right]^m \quad \text{for } |t| \leq \tau \quad (5)$$

while it is zero for  $|t| > \tau$ , where  $m$  is a positive parameter, controlling the correction performance [1]. The Kaiser window is expressed through Bessel functions (modified) of the first kind and zero order. The Weber-type approximations  $w_2(t)$  and  $w_3(t)$  are close representations of a window giving a minimum value of the uncertainty product in a modified form [1], having a complicate expression. The detailed expression of  $w_2(t)$  as a third order polynomial approximation is reported in [1]. In the following the expression of  $w_3(t)$  is given (defined in the time interval 0-1.5)

$$w_3(t) = at^3 + bt^2 + ct + d \quad (6)$$

$0 \leq t \leq 0.75$	$0.75 < t \leq 1.5$
$a = 1.783724$	$a = -0.041165$
$b = -3.604044$	$b = 1.502131$
$c = 0.076450$	$c = -4.591678$
$d = 2.243434$	$d = 3.651582$

Efficiency comparisons were developed for the FIR digital filters using the above different windows: a computer program was prepared which measures the maximum ripple in band  $\Delta_1$ , the maximum level of fluctuations out of the band  $\Delta_2$  and the width  $\Delta\omega_b$  of the transition band, defined as the region in which the frequency response magnitude decreases from 1- $\Delta_1$  value to  $\Delta_2$  value. From these comparisons, the following results were obtained: Cappellini windows  $w_2$  and  $w_3$  have very near efficiency to that of Kaiser window, the first ones resulting better at lower levels of out-band attenuations, the last one having higher efficiency at greater levels of out-band attenuation; Cappellini window  $w_1$  presents good efficiency and is very flexible, by changing the value of the parameter  $m$ .

### 3. 2-D RECURSIVE DIGITAL FILTERS

A 2-D causal recursive digital filter can be described by the following relation

$$g(n_1, n_2) = \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} a(k_1, k_2) f(n_1 - k_1, n_2 - k_2) - \sum_{m_1=0}^{M_1-1} \sum_{m_2=0}^{M_2-1} b(m_1, m_2) g(n_1 - m_1, n_2 - m_2) \quad (7)$$

$m_1 + m_2 \neq 0$

where  $f(n_1, n_2)$  and  $g(n_1, n_2)$  are, respectively, the input and output data (image samples) and  $a(k_1, k_2)$  and  $b(m_1, m_2)$ , defined only for positive values of  $k_1, k_2, m_1, m_2$ , are the coefficients defining the digital filter frequency response [1]. By using the z-transform, the transfer function  $H(z_1, z_2)$  can be obtained, as already shown for 2-D non recursive digital filters: however now this transfer function results to be the ratio of two polynomials, having at numerator the coefficients  $a(k_1, k_2)$  and at the denominator the coefficients  $b(m_1, m_2)$ . 2-D recursive digital filters, as defined by the relation (7), have an infinite impulse response (IIR).

To design the 2-D IIR digital filter in the frequency domain, the coefficients  $a(k_1, k_2)$  and  $b(m_1, m_2)$  have to be chosen to approximate the desired transfer function with a stable recursive realization. The stability is indeed a specific problem of recursive structures as in (7) and in the 2-D domain greater difficulties are than in the 1-D domain, due to the need of factorizing 2-variable transfer functions [1].

The design of 2-D IIR filters is a more difficult task than the design of 1-D IIR filters. In fact the design methods for 1-D IIR filters normally rely on the factorability of one variable polynomials, which result in very simple algorithms for the analysis of the filter behaviour, for the stability test and for the stabilization of unstable filters. These techniques are not directly generalizable to the 2-D case, which makes the 2-D filter design and analysis less easily tractable.

Two main classes of design methods have been proposed in the literature. The first one is based on spectral transformations from 1-D to 2-D [1] [3] and the second one on parameter optimization [1], using some classes of filter structures, as the second order section cascade, where the stability control is easily introduced in the approximation algorithm.

A general design procedure has been recently introduced

ced [5], where a non linear optimization is used to minimize an error expression, where the distance from an ideal frequency response and the distance from a stable implementation, obtained by means of the "cepstrum" (defined as the inverse Fourier Transform of the logarithm of the Fourier Transform of a sequence) decomposition, are present. In this case it is possible to obtain a contemporary control of the frequency domain approximation and of the stability of the filter, with a procedure which is general, but which is rather complex in the implementation and requires some knowledge of the general non linear approximation problems in the case when an acceptable minimum of the error is not automatically reached.

A method, defined in last years [1][3][6] is based on transformations of the squared magnitude function of 2-D digital filter to 2-D domain and uses a suitable decomposition in four stable digital filters. At first we can observe that the causal digital filter (7), having an impulse response  $h(k_1, k_2)$  different from zero only for  $k_1 \geq 0$  and  $k_2 \geq 0$ , is often called a "first quadrant filter". Starting from a first quadrant filter with transfer function  $H_1(z_1, z_2)$  and impulse response  $h_1(k_1, k_2)$ , it is possible to define the corresponding second, third and fourth quadrant filters, according to the relations

$$h_1(k_1, k_2) = h_2(k_1, -k_2) = h_3(-k_1, -k_2) = h_4(-k_1, k_2) \quad (8)$$

with transfer functions as

$$H_1(z_1, z_2) = H_2(z_1, z_2^{-1}) = H_3(z_1^{-1}, z_2^{-1}) = H_4(z_1^{-1}, z_2) \quad (9)$$

The cascade of these four filters is a zero-phase digital filter, whose frequency response  $H_s(\omega_1, \omega_2)$  is defined by coefficients  $p(k_1, k_2)$  and  $q(m_1, m_2)$  determined through the convolution of the coefficients of the four filters [1].

An important design step in this last method corresponds to the consideration that a frequency response as above considered  $H_s(\omega_1, \omega_2)$  can be obtained through a transformation from 1-D domain, avoiding the design in 2-D which takes a great computation time. The squared magnitude frequency response of a 2-D recursive filter can be obtained by means of the McClellan transformation (3) carried out on the numerator and the denominator of the filter. The obtained squared magnitude transfer function has to be factorized to obtain stable recursive filters: the cepstrum properties can be used. Some approximations have however to be introduced to perform the decomposition on the denominator, due to the fact that the cepstrum of the denominator sequence is not in general of finite extent. Windows can be applied to smooth the oscillations which are produced by the truncation of the impulse response of the filter: some windows of exponential type [1] or Gaussian type [3] have been proposed.

Several tests have been performed using the squared magnitude transfer function of a fourth order Chebychev low-pass filter, having a 2% in band ripple, a normalized cutoff frequency  $f=0.25$  and a -20 dB frequency  $f_{att}=0.35$

[3]. The filter in Fig. 1 has a numerator and a denominator with 4 by 4 coefficients obtained using a Gaussian window: the maximum in band error results to be 0.024 and the transition band, defined as the difference between the normalized frequencies where the amplitude of the frequency response is, respectively, 90% and 10% of the in band nominal value, is equal to 0.125.

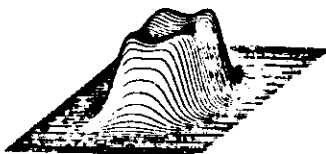


Fig. 1 - Filter frequency response.

#### 4. 2-D LOCAL SPACE OPERATORS

Local space operators correspond in general to low complexity 2-D digital systems: small blocks of data (image samples) are processed in the space domain [6]. By evaluating, for instance, the mean value of the block data, a smoothing is obtained, while by performing the difference of data along lines or columns enhancement effects are resulting.

A very important class of 2-D local space operators is represented by "edge operators", which extract boundaries or edges in the processed image. The most part of these operators are based on the evaluation of the "gradient" through a test on a given image point (pixel) and its close values. In fact if the magnitude and the direction of the gradient in a point are known and if the magnitude is greater than a given threshold, it is assumed that in that point there is an edge or contour whose direction is orthogonal to the gradient direction. The used techniques for this purpose can be divided in two groups: to the first group belong the operators which evaluate two orthogonal components of the gradient; the second group is based on gradient detection by means of a set of "templates" or "masks" of different orientation [6]. Well known operators are: Roberts operator, smoothed gradient operator, Sobel operator, isotropic operator, Prewitt operator, Kirsch operator, Robinson operator, Chen-Frei operator.

Recently a special edge operator was introduced [6] for extracting edges in noisy images. This operator considers a block of 3x3 data: to each one of the 8 pixels surrounding the central one a binary value is given according to the difference among the pixel value and the central value. In this way 256 configurations are resulting: they are divided in 5 classes, having a decreasing probability that the central pixel is a part of an edge or contour. Adaptive criteria can be used to estimate if the central pixel pertains to an edge, depending on the noise characteristics in the processed image. Another advantage of this operator is represented by its speed of implementation: practically to estimate if a pixel is or not part of an edge, after the binary values are obtained, it is sufficient to compare the actual binary configuration with a memorized "decision table".

Of increasing interest are 2-D local space operators performing non linear filtering operations, assuring very fast image processing. An interesting example is represented by the following non linear smoother of noisy images. If we consider a block of 3x3 data and we denote with  $P_0$  the value of the central pixel, while with  $P_1, P_2, \dots, P_8$  the values of the surrounding pixels, the smoother is defined by the following relation

$$P'_0 = \frac{1}{n} \sum_{P_i \in S} P_i \quad (10)$$

where  $S = \{P_i | |P_i - P_0| \leq k\}$  and  $i = 0, 1, 2, \dots, 8$

By means of this smoother, the value of each pixel is replaced by the average of itself and its neighbourhood values, except those which have level differences greater than a fixed threshold in absolute value. In this way small amplitude noise is removed, while no degradation is resulting for edges or boundaries present in the processed image regions. Therefore this operator is very useful to reduce small intensity random noise and its application is indeed convenient before the edge extraction through a usual edge operator (especially in noisy image processing).

#### 5. KALMAN FILTERING FOR IMAGE PROCESSING

To restore degraded images (the available images are the output of some system in presence of noise) Kalman filtering can be usefully applied. Recently an efficient method of applying a 2-D Kalman filter to process noisy de-

graded images was defined [6]. Its efficient implementation is achieved by computing the Kalman gains in the z-transform domain and by evaluating convolutions through recursive equations.

Extended tests of this method have shown that it is very useful in many noisy degraded image processing problems, especially for low signal-to-noise ratios. Therefore Kalman filtering can be an interesting alternative solution to frequency-domain digital filters of FIR or IIR type, as above described in image restoration problems.

## 6. EXAMPLES OF APPLICATION TO DIGITAL IMAGE PROCESSING

In the following some examples of application of the above described 2-D digital systems are shown in three main areas: biomedicine, remote sensing, recognition of moving objects (robotics).

Fig. 2 gives an example of application of a 2-D FIR digital filter of low-pass type to a 64x64 nuclear medicine image (scintigraphy): the effect of smoothing, reducing high space frequency components of noise and disturbances, is clearly resulting, by comparing the original image (at left) and the processed one (at right), in the "iso-contour" display (the contours at some gray levels are presented).

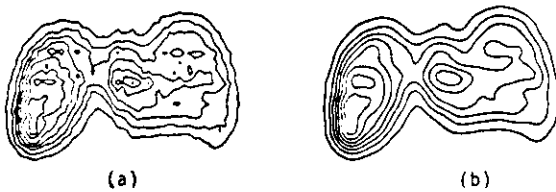


Fig. 2 - Iso-contour display of a nuclear scintigraphy: (a) original; (b) after a 2-D FIR digital filtering of low-pass type.

Fig. 3 shows an example of application to aircraft photo of an agriculture zone near Florence: in (a) the original digitized image is presented; in (b) the edge extraction (through the isotropic operator applied after image enhancement) is shown; in (c) the classification in forest (at left), winegrape-oil land (in the middle), other land (at right).

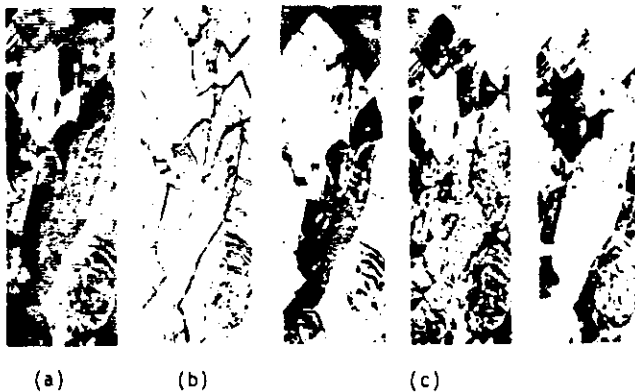


Fig. 3 - Example of application to aircraft image processing.

The last example of application is related to the recognition of moving objects. A special processing procedure was recently proposed and defined at Istituto di Elettrotecnica, University of Florence. The main processing steps performed on a sequence of images taken on the moving object scene are the following ones:

1. Learning phase, including the acquisition and the modelling of the objects which are to be recognized (FFT module of the external boundaries of the ob-

jects are memorized).

2. Pre-processing of the images by means of a non linear smoothing (as given by the relation (10)), reducing low amplitude noise components and disturbances.
3. Edge detection through a modified Sobel operator.
4. Moving object detection: non stationary components of the image sequence are separated from stationary ones ("background" filtering), extracting the useful configurations corresponding to the moving objects.
5. Post-processing, performing a non linear filtering of isolated noise "spikes" or "scintillation pulses".
6. Image segmentation, representing the silhouettes of the moving objects in a two level code (a modified Freeman code is used).
7. Object modelling: only the shape of the silhouettes of the detected objects are considered (interior part is ignored), by performing the FFT of the boundaries (the FFT is applied to the distances of the boundary points from the "centroid").
8. Object recognition: a "matching" is performed between the memorized FFT modules and those actually evaluated on the image sequence (FFT modules are used to obtain "rotational invariance") by means of "minimum distance" criteria.
9. Tracking of moving objects: subsequent positions of the objects are followed through the identification of the external boundaries and of the centroid (a prediction or estimation of the "near future" object centroid position can also be performed).

The above processing method was implemented in software and widely tested by using a PDP 11-34 minicomputer system with a TV camera, digitizing interface and color display. An example of application is shown in Fig. 4 for three moving objects: at left are original digitized images and at right object recognition and tracking (each object is visualized through a particular color). It can be observed how, in the third movement step, a "disturbing object" of cylindrical shape is not recognized.

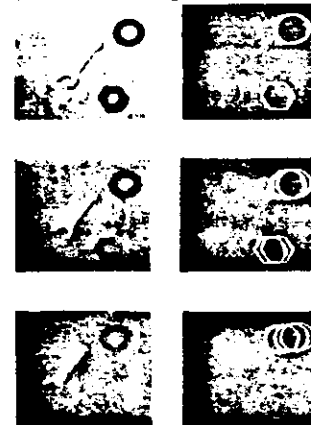


Fig. 4 - Example of application to moving object recognition.

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