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EXPERIMENTAL WORKSHOP ON
"HIGH TEMPERATURE SUPERCONDUCTORS"
(30 March - 14 April 1989)

THEORETICAL DEVELOPMENTS IN Hi-Tc SUPERCONDUCTIVITY

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THEORETICAL DEVELOPMENTS
IN
Hi-Tc SUPERCONDUCTIVITY

* Introduction
    - Basic Experimental Facts
    - Questions to Be Answered
* CuO_2 Plane and Electronic States
* Weak Coupling Theories and Related Models
* Experiments in Favour of Strong-Coupling Models
* One-Band vs Two-Band Hubbard Models
* The Resonance Valence Bond (RVB) States
* Interplay between Quantum Antiferromagnetism and Superconductivity
* Concluding Remarks

Classes of Hi-Tc Superconductors

$La_{2-x}Sr_xCuO_4$ (214) $T_c \sim 40K$

$YBa_2Cu_3O_{7-y}$ (123) $T_c \sim 90K$

$X_m Ba_2 Ca_{n-1} Cu_n O_{2(n+1)+m}$

$X =$ Bi, Tl. $n = 1, 2, 3$, $m = 1$ (Tl), 2

(2212) $T_c = 110K$

(2223) $T_c = 125K$

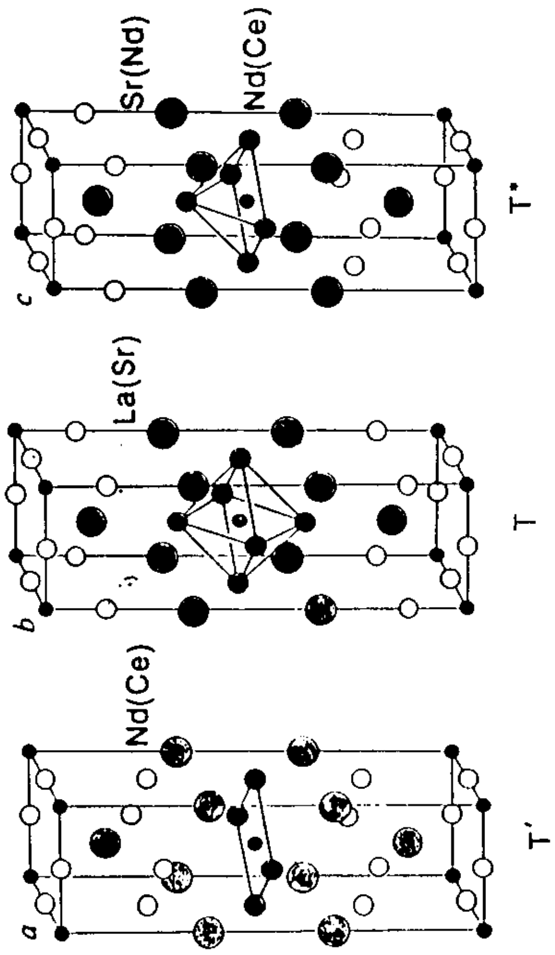
Others $Ln_{2-x}Ce_xCuO_{4-y}$ $T_c = 24K$ $x = 0.15$

$Ln =$ Nd, Pr. Y. Tokura et al.

Cubic

$BaBiO_3$ — $BaPb_xBi_{1-x}O_3$ 13K

$BaBiO_3$ — $Ba_xK_{1-x}BiO_3$ 30K



$Nd_{2-x-z}Ce_xSr_zO_4$

$La_{2-x}Sr_xCuO_4$

$Nd_{2-x}Ce_xCuO_4$

Y. Tokura et al. Nature 337, 345 (89)

Basic facts

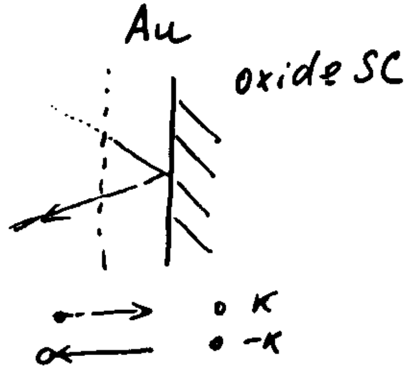
What is common?

* Zero resistance (?)
* Meissner effect (complete?)
* Flux quantization 2e
* Energy gap in spectrum

What is different?

* sensitivity to sample preparation and soon
* poor conductors and "strange" normal state properties
* interplay with magnetism
* "almost no" isotope effect
* very short correlation length extreme type II superconductor
* Non-exponential low-temperature specific heat

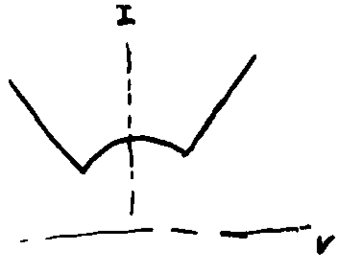
Andreev Scattering



$eV < E_g$

ballistic scattering

anti gap. current is enhanced. additional "hole" current.



In favor of $K, -K$ pairing

no direct evidence. s, p, d?

Questions to be answered

1. Scenario for SC

   BCS pairing? Bose condensation? Bipolarons? Something entirely new?

2. What is the interaction responsible for SC?

   Electron-phonon? Coulomb interaction in terms of charge or magnetic fluctuations

3. What is the appropriate model?

   Charge transfer? Hubbard? one, two, or three bands?

4. What is the theoretical basis?

   Weak-coupling theory based on Landau Fermi-Liquid theory
   or
   Strong-coupling theory with entirely new concepts?

Attempts in constructing Hi-Tc theory

- Phonon
  - Strong coupling
  - Bipolaron
  - Jahn-Teller
- Electronic
  - charge fluctuations
    - charge transfer exciton $[Cu^{++}O^{-}] \to [Cu^{+}O^{-}]$
    - plasmon
    - two band (Localized vs extended) model
  - Magnetic fluctuations
    - weak coupling SDW
    - strong coupling two (three) band models
    - One-band Hubbard model ⇒ RVB; t-J model ⇒ Spin wave theory

CuO_2 plane is responsible for SC

1) "Common" structure

2) anisotropy

ρ , $\rho_{\parallel} \sim T$

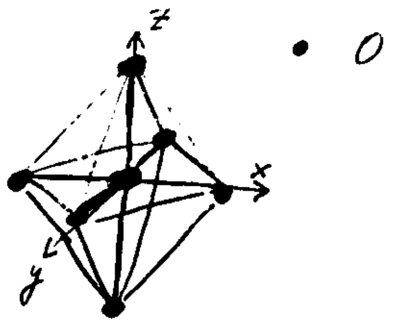
$\rho_{\perp}/\rho_{\parallel} \sim 10^{-5}$, $\rho_{\perp} \sim 1/T$

3) strong dependence of Tc upon interlayer distance of CuO_2

4) Band structure calculation

5) 2D behavior . like Kosterlitz - Thouless transition

$La_2^{6+} Cu^{2+} O_4^{8-}$



Cu^{2+}, $3d^9$

Cubic harmonics.

T_{2g}, xy, xz, yz

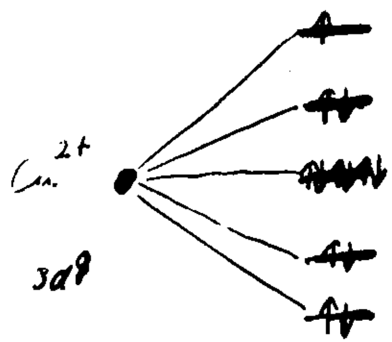
E_g, x^2-y^2, $3z^2-r^2$

Jahn-Teller splitting

Cu-O 2.4Å ∥z

1.9Å xy plane

Cu-O hybridization



E_g^* (antibonding) $x^2-y^2 \Leftrightarrow O_{2p}$, $3z^2-r^2 \Leftrightarrow O_{2p}$

T_{2g} (nonbonding) xy, xz, yz

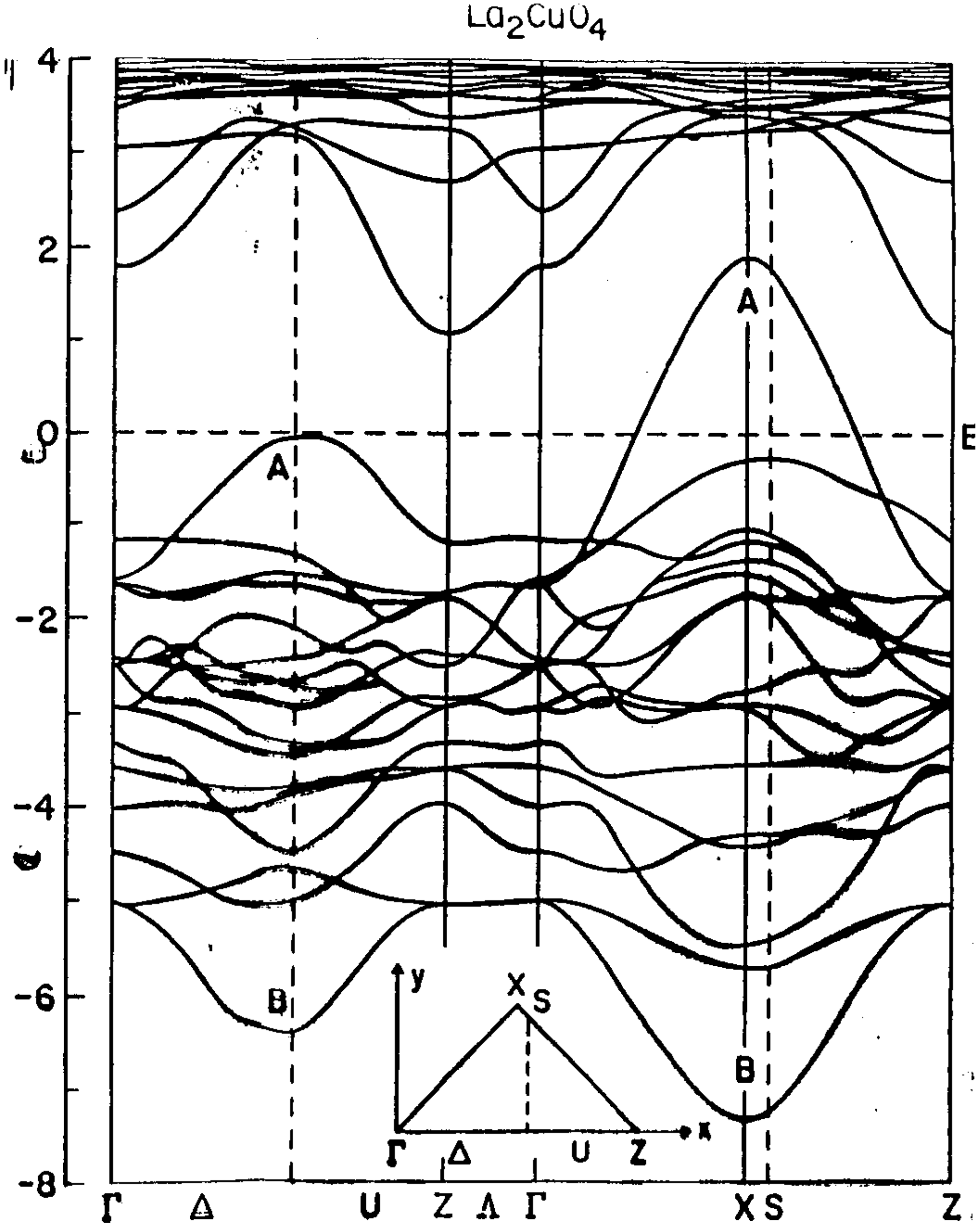
E_g (bonding) $O_{2p} \Leftrightarrow 3z^2-r^2$, $O_{2p} \Leftrightarrow x^2-y^2$

Half-filled, should be a METAL

but it is not!

lower symmetry! AF order? Mott insulator,

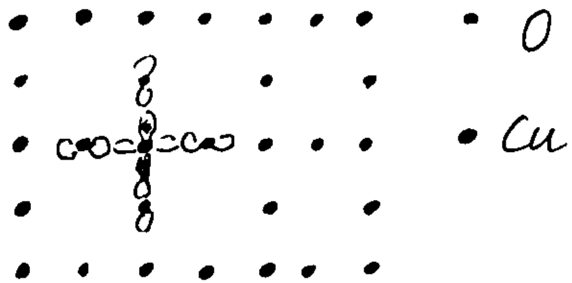
Mattheiss. Phys. Rev. Lett. 58, 1028 (1987)



36

CuO_2 layers

½-filled n.n. tight binding band.



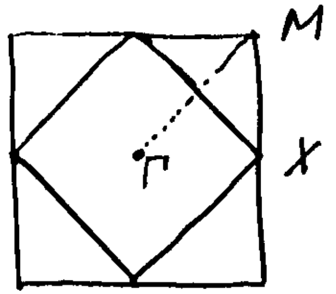
$$\varepsilon(\vec{k}) = 2t(\cos k_x a + \cos k_y a)$$

$t \simeq 0.5$ eV

Fermi Surface

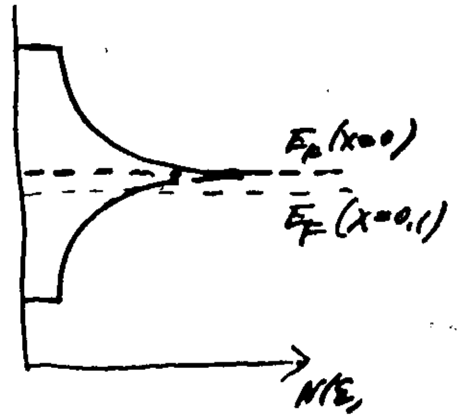
a) Saddle Points

b) Perfect Nesting



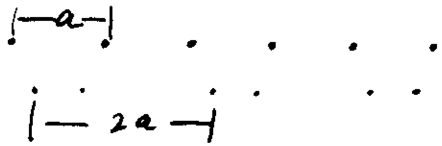
favor distortion with wavevector ΓM

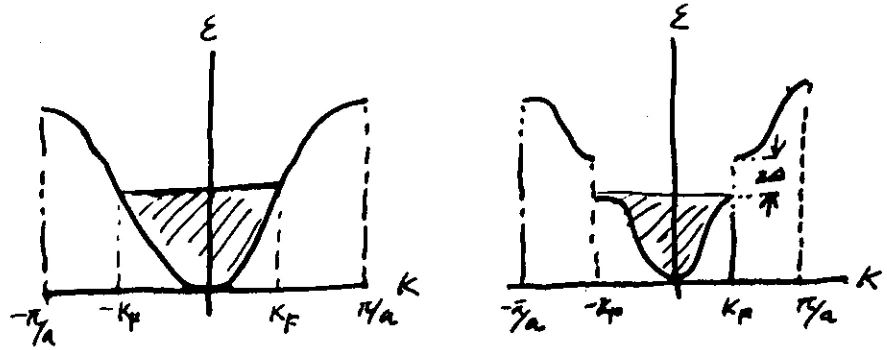
Saddle-points give a van Hove logar. Singularity at $\varepsilon = E_F$



Peierls instability

One-dimensional metal is unstable — due to coupling to lattice, a charge modulation (CDW) is formed and an energy gap opens at Fermi level

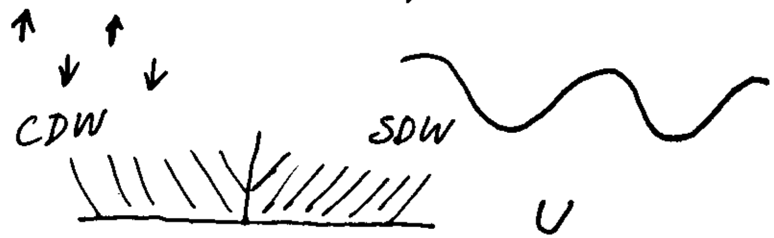




In general, the distortion is incommensurate.

* Spin density wave due to Coulomb repulsion

spin modulation



General Analysis in the weak-coupling limit.

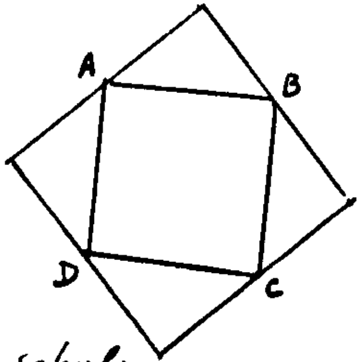
1D: Nesting: $g \ln \frac{t}{T} \sim 1$

$T_c \sim e^{-1/g}$

2D:

Neighbourhood of four points

A(C), B(D)



Renormalization Group (scaling) Schulz

Parquette diagram Dzyaloshinsky

$g \ln^2 t/T \sim 1$, $T_c \sim e^{-\frac{1}{\sqrt{g}}}$

PP → SS TS

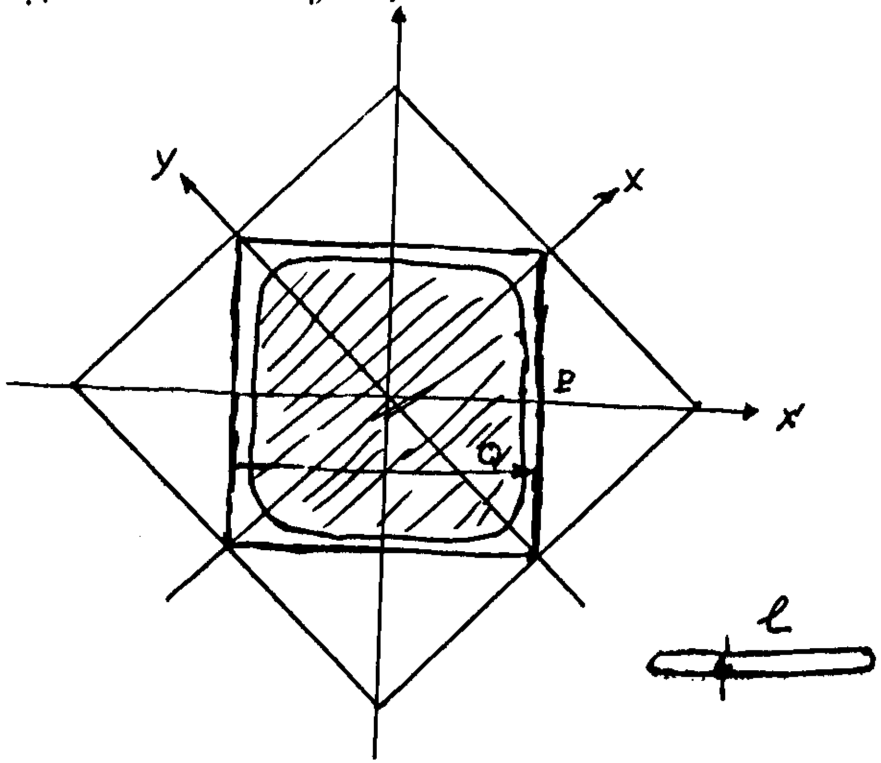
Ph → CDW SDW

1/2 filling SDW

away — d-wave SC.? Phase transition?

"Spin Bag" model of J.R. Schrieffer, X.G. Wen, S.C. Zhang

1. SDW background, Δ_{SDW}
2. Nesting vector is fixed
3. Doping reduces local gap Δ_B
4. "Sharing Bag" lowers energy

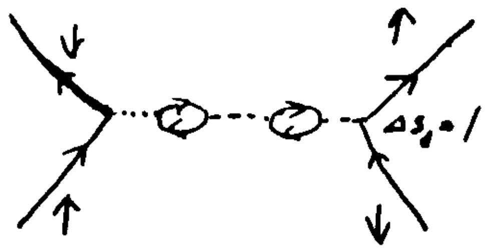


$l \sim \xi_{SDW} = \frac{\hbar v_F}{\pi \Delta_{SDW}}$, $\Delta_0 \approx \Delta_{SDW} e^{-\frac{t}{\alpha U}}$

Hole Pairing induced by AF Fluctuations

Z.B. Su, L. Yu, J.M. Dong & E. Tosatti

1) AF or SDW background
2) AF coupling of hole to SDW
3) Separation of longitudinal and transverse modes, RPA
4) Effective attraction for triplet state



$$V(\vec{q}) \propto J \frac{1}{q^2}$$

5) P- wave pairing SC.

Strong Electron-phonon coupling

Eliashberg equation — Phenomenological

W. Weber — phonon spectrum

$La_{2-x}Sr_xCuO_4$ — 30-40K

marginal.

Consequences of very strong coupling $\lambda \gg 1$

Bulaevskii et al. — J. Rammer et al.

J.P. Carbotte et al

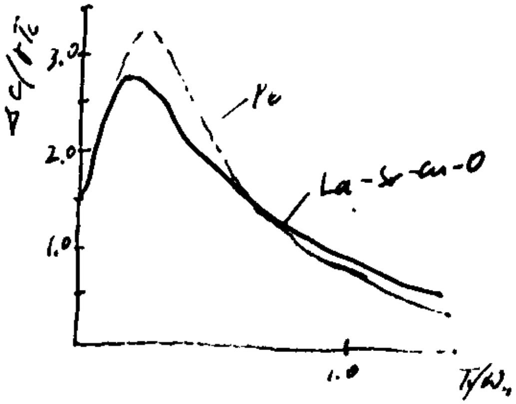
$\Delta C/\gamma(T_c)$ — non-monotonic, and other thermodynamic quantities

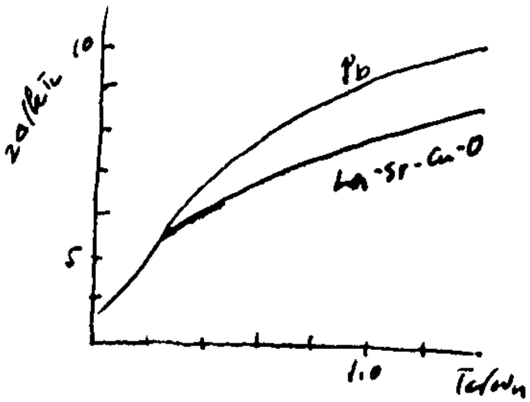
$2\Delta/kT_c$ — increasing with λ

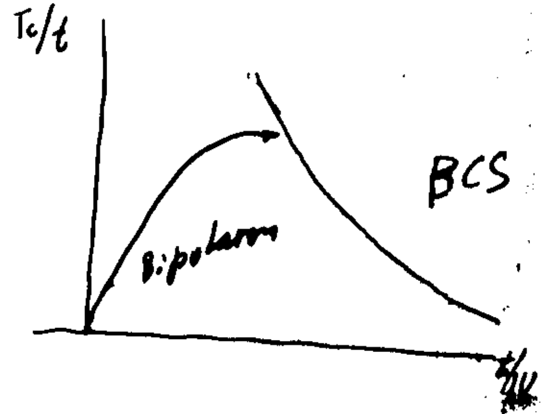
Check of the theory.

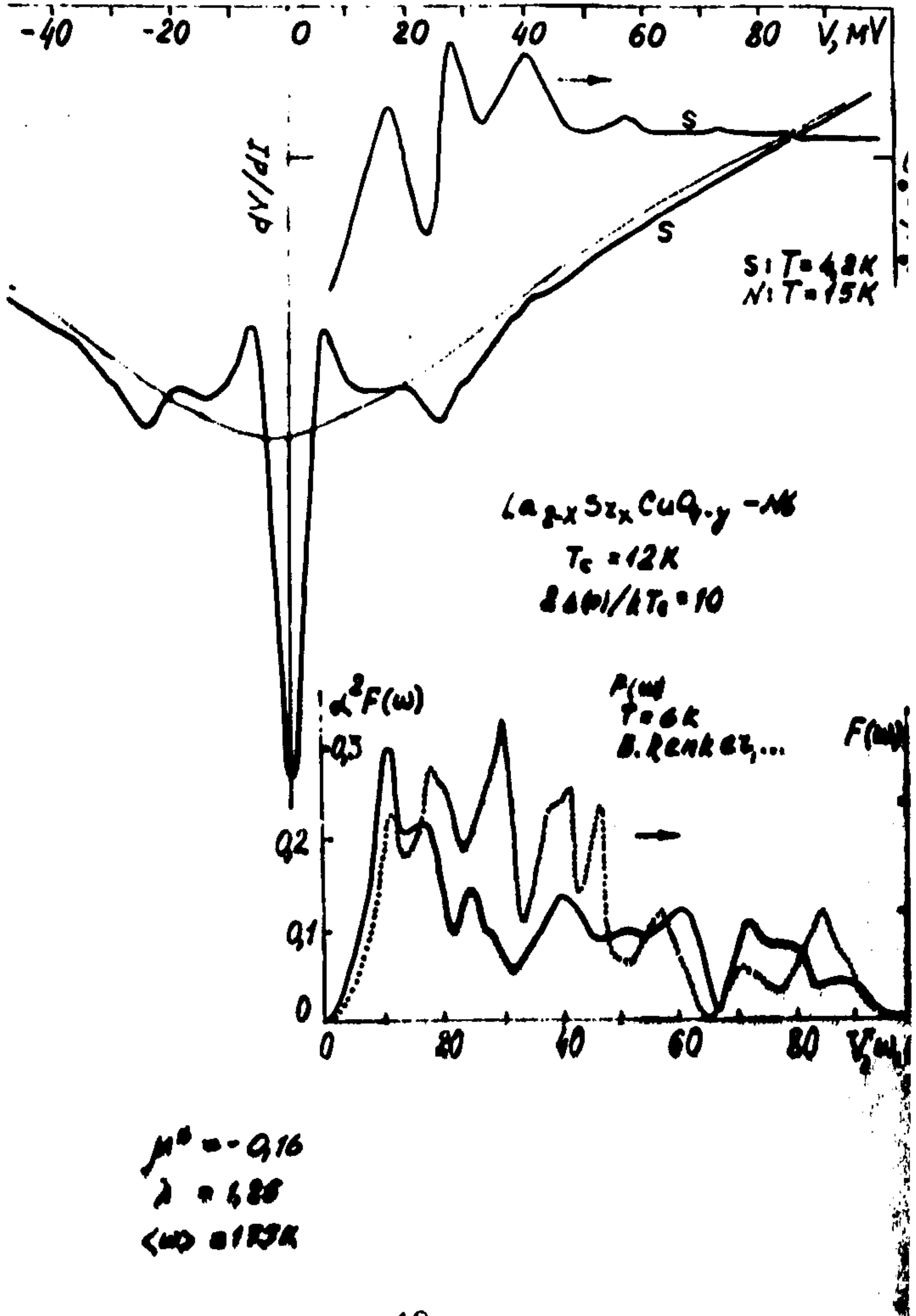
1) Measuring $\alpha^2F(\omega)$ from tunneling
2) Thermodynamic quantities
3) First-principle calculation of $\alpha^2F(\omega)$

F. Marsiglio et al
PR B 36, 5245, 87.









Bipolaron model

Possible relevance to $Ba_x K_{1-x} BiO_3$

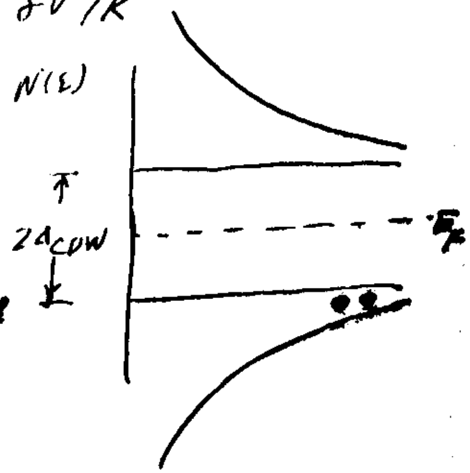
B.K. Chakraverty ...

A. Alexandrov & J. Ranninger

Prelovsek, Rice, Zhang J. Phys C Lett 20, L 229 (87)

coexistence with CDW

$U < U_c = 8V^2/K$



$\Delta_{CDW} = \begin{cases} t\, e^{-\pi/8\sqrt{\lambda}}, & t \text{ large} \\ 8V^2/K - U/2, & t \text{ small} \end{cases}$

$\lambda = (V^2/K - U/16)/t$

Energy to remove 1 el — Δ_{CDW} rigid lattice; — $\Delta_{CDW} - \varepsilon_p$ distorted lattice; — $(\Delta_{CDW} - \varepsilon_p) - \frac{1}{2}\varepsilon_{pp}$ Bipolaron state

Superconductivity as Bose condensation

$m_{eff} \approx 20\, m_e \Rightarrow T_c \simeq 80K$ for $x = .1$

Evidence in favor of large U

in Cu oxides

1) La_2CuO_4 is an Antiferromagnet

magnetic moment $\sim 0.5\,\mu_B/Cu$

$T_N \sim 200K$

2) No CDW distortion was found.

Ortho - Tetra transition is not M-I

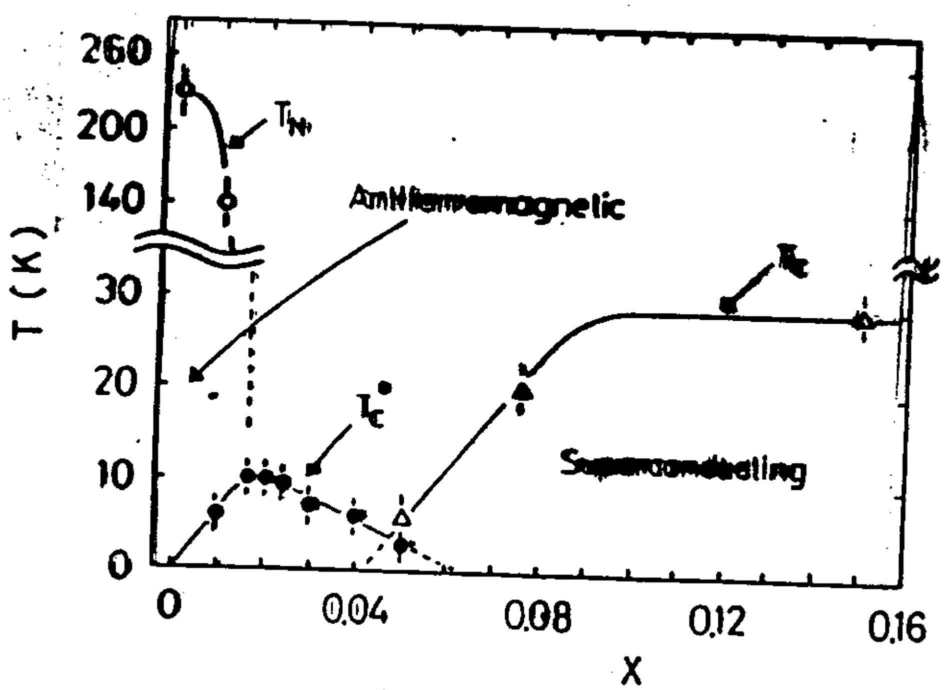
3) La_2CuO_4 has optical gap $\gtrsim 3$ eV

4) Spectroscopic data — no states at Fermi Level

5) $YBa_2Cu_3O_{7-\delta}$ has no isotope effect

6) Hall conductivity - Holes! ~ carrier concentration

Electrons! for new $Nd_{2-x}Ce_xCuO_4$



Low Temp. Phase diagram for $(La_{1-x}Ba_x)_2CuO_4$
Kitagawa et al. Physica 151-153, 12/1988

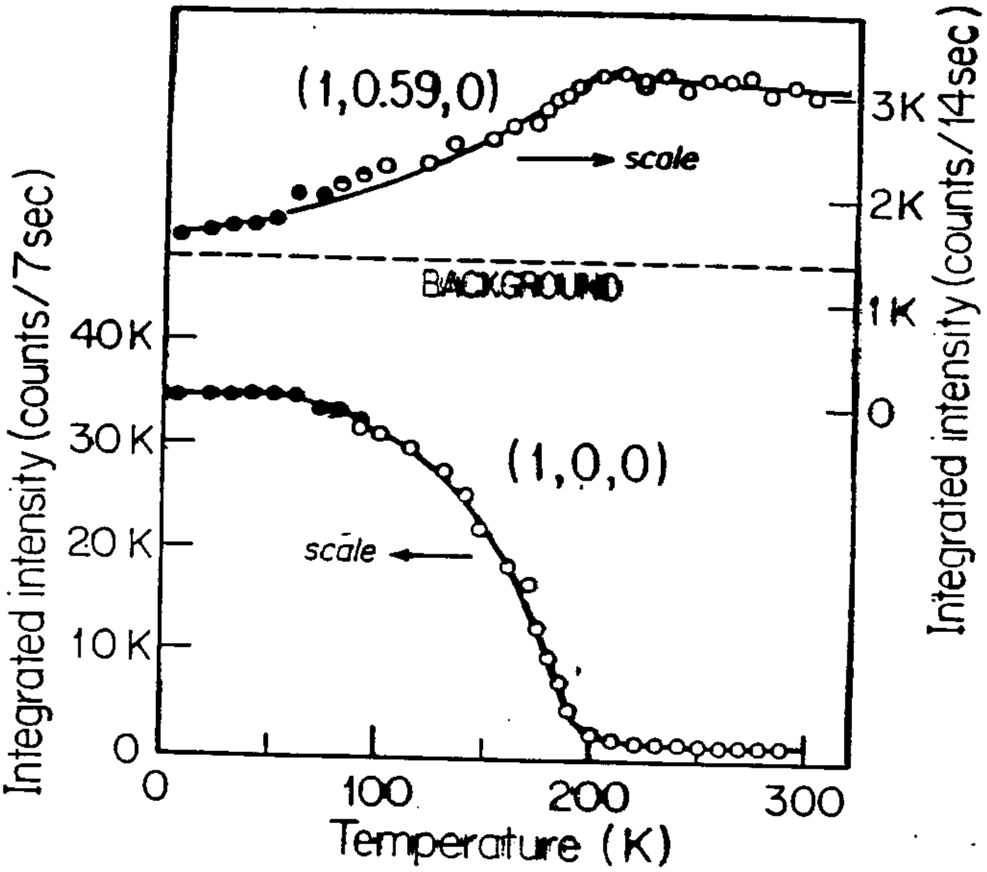
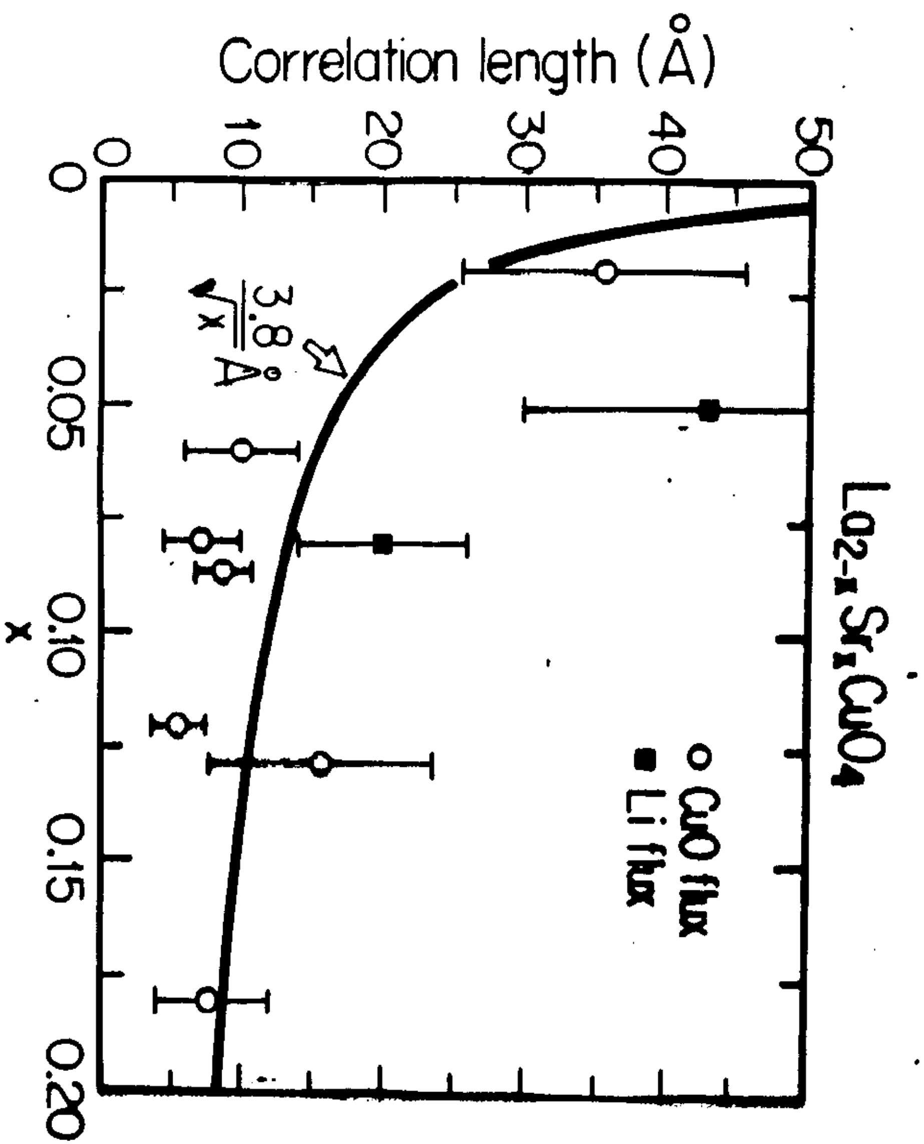
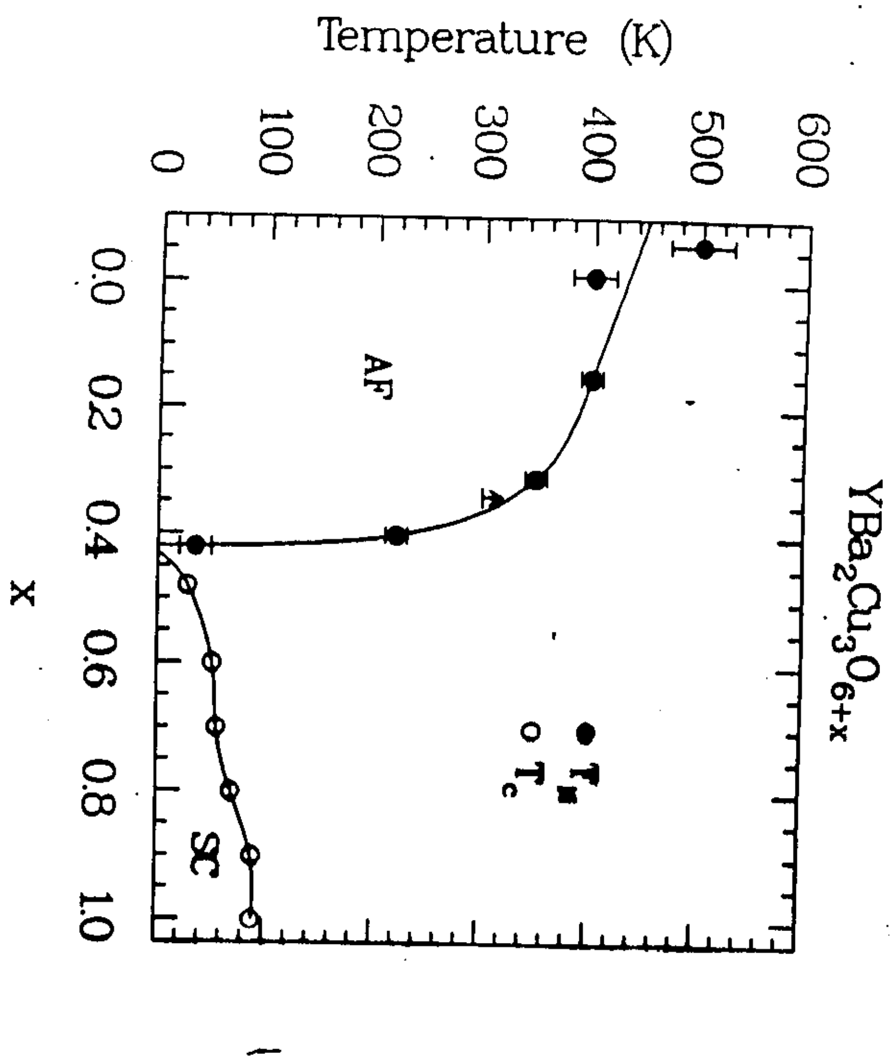


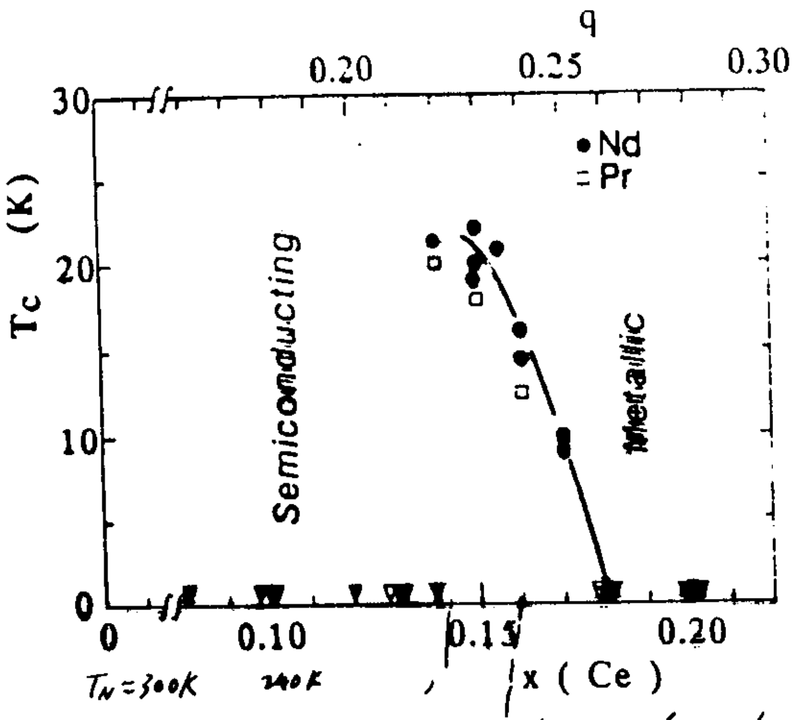
FIGURE 2



Correlation length (Å)
x
$La_{2-x}Sr_xCuO_4$
○ CuO flux
■ Li flux
$\frac{3.8}{\sqrt{x}}$ Å



Temperature (K)
x
$YBa_2Cu_3O_{6+x}$
● T_N
○ T_c
AF
SC



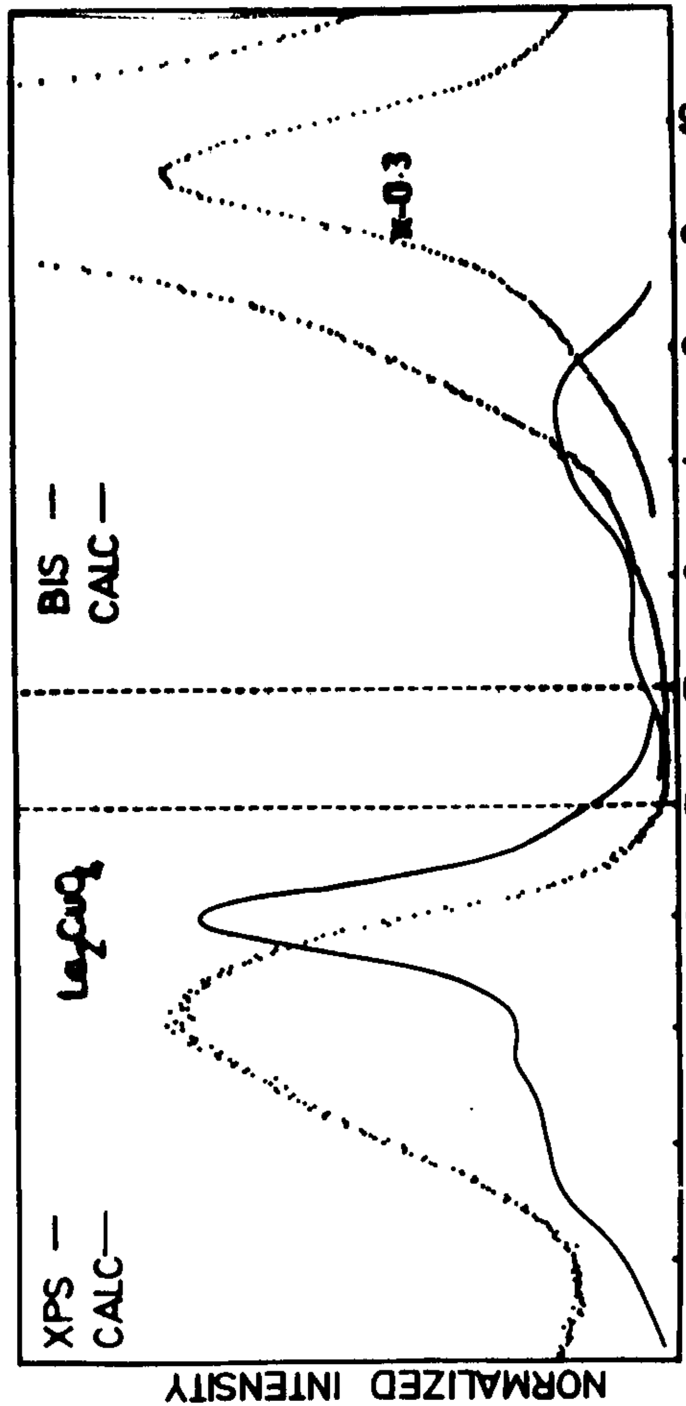
$T_N = 300K$ 240K

Not magnetic above 4K

$Ln_{2-x}Ce_x Cu O_{4-y}$

H. Takagi et al. PRL 62, 1197(89)

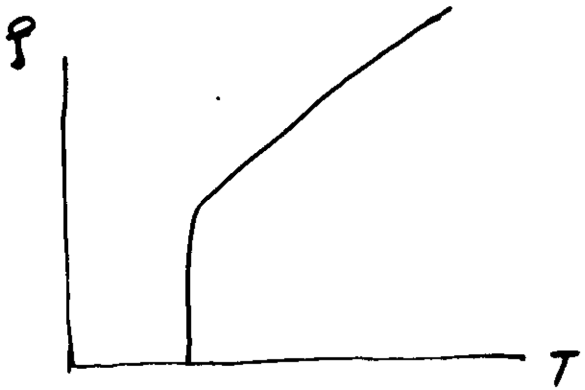
⑥



Photoemission and inverse photoemission spectra at 1486.7 eV for La_2CuO_4

J.C. Fuggle et al. P.R.B

linear Temperature dependence of resistivity



El-ph. interaction ?

But why down to 0K (for some compounds with very low T_c)

N.P. Ong et al.

Hubbard Model

Anderson's initiative

Strong on-site Coulomb repulsion

$$H_H = -t \sum_{\langle ij \rangle, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + h.c. + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

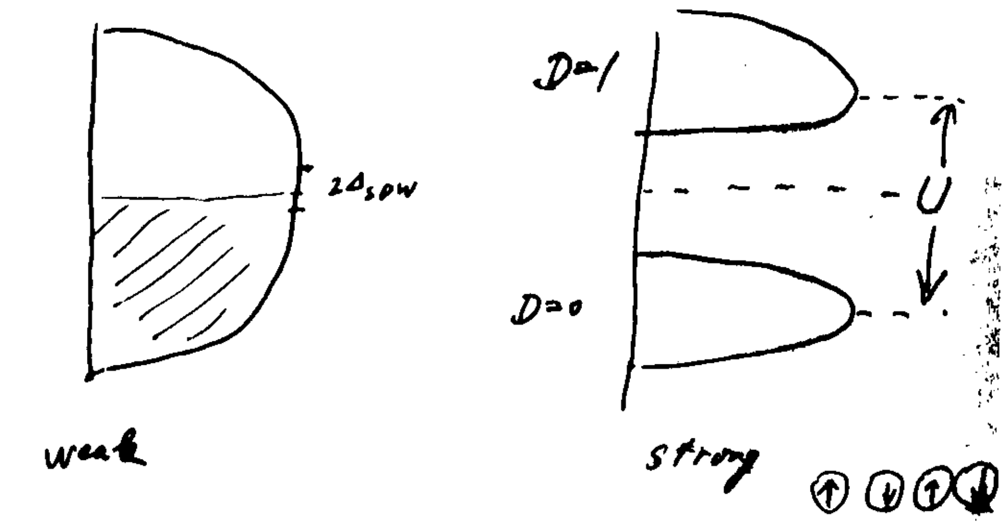
Exactly solvable only in 1D (Lieb & Wu)

Strong Coupling $U \gg W = 2zt$

Mott insulator. localized states

Weak coupling $U \ll W = 2zt$

Bloch states are good starting point

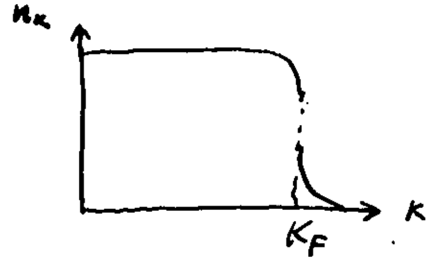


$U=0$ fixed point, weak coupling

* Landau Fermi-Liquid theory is valid

well-defined quasiparticles

Luttinger theorem – jump in the momentum distribution at Fermi Level

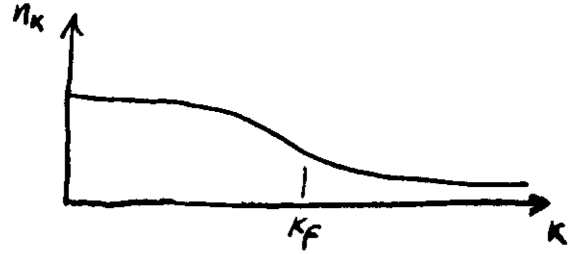


$U=\infty$ fixed point, strong coupling

* Fermi-liquid behavior breaks down

No well-defined quasiparticles

No jump in the momentum distribution



Effective Hamiltonian in strong coupling limit

$U \gg t$, $\delta = 1 - n \ll 1$

Canon. transform. $H_H \rightarrow H_{eff} = e^{iS} H_H e^{-iS}$

$T \rightarrow T_{hole} + T_{mix} + T_{double\ occ.}$

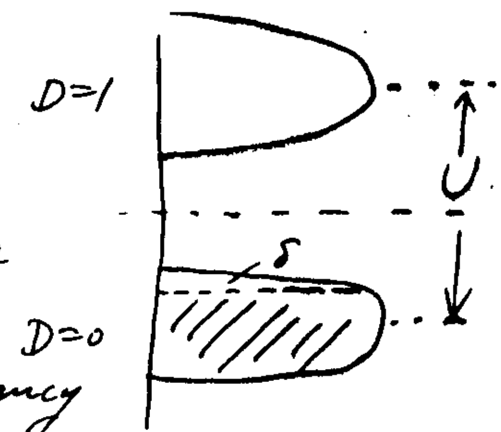
($T_{mix} = 0$; $T_{double\ occ.}$: project out)

$$H_{eff} = P_{D=0}(-t)\sum_{\langle ij\rangle \sigma} c^\dagger_{i\sigma} c_{j\sigma} P_{D=0} + h.c. + J\sum_{\langle ij\rangle}\left(\vec{S}_i\cdot\vec{S}_j - \tfrac{1}{4}\right) + O(\delta^2, t^3/U^2, \delta t/U)$$

$t-J$ model

$J = 4t^2/U$

H_{eff} operates in Hilbert subspace without double occupancy



Hubbard Model

1 D   Lieb-Wu   Bethe ansatz exact solution

n=1 . singlet state, no LRO

n≠1   non-Fermi Liquid behavior
⇒ Luttinger theorem is not valid

2 D   No exact solution

Large U → Heisenberg model

n=1 . LRO ?   Yes, probably.

3 D   LRO   rigorous proof

What happens if one dopes?
M-I transition?

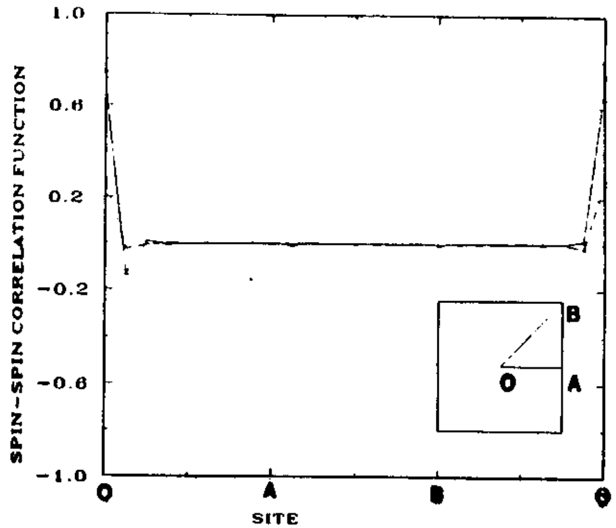


Figure 5. Spin-spin correlation function of a 2D 16×16 Hubbard model for $U=4$, $\nu=$[illegible] (continuous line) and $\nu=$[illegible] (dashed line). The path in the unit cell is shown in the inset.

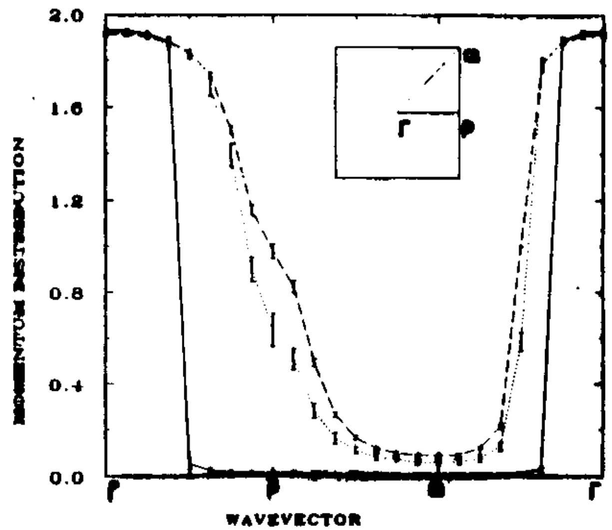


Figure 6. Momentum distribution of the 2D 16×16 Hubbard model with $U=4$, for $\nu=$[illegible] (dashed line), $\nu=$[illegible] (dotted line), and $\nu=$[illegible] (continuous line). The path in the Brillouin zone is shown in the inset. The imaginary time was $T=13$ (half filled) and $T=$[illegible] (less than half filled).

the momentum distribution $n(\mathbf{k})$ for the three values of δ considered above. Our

S. Sorella et al.

One band vs Two band Hubbard models

Anderson: One band H. model contains the basic Hi-Tc Physics

However: Holes on oxygen.

High-Energy spectroscopy,

Hall measurements, wet chemistry

. . .

different NMR relaxation & Knight shift results

. . .

Emery et al:

At least two bands

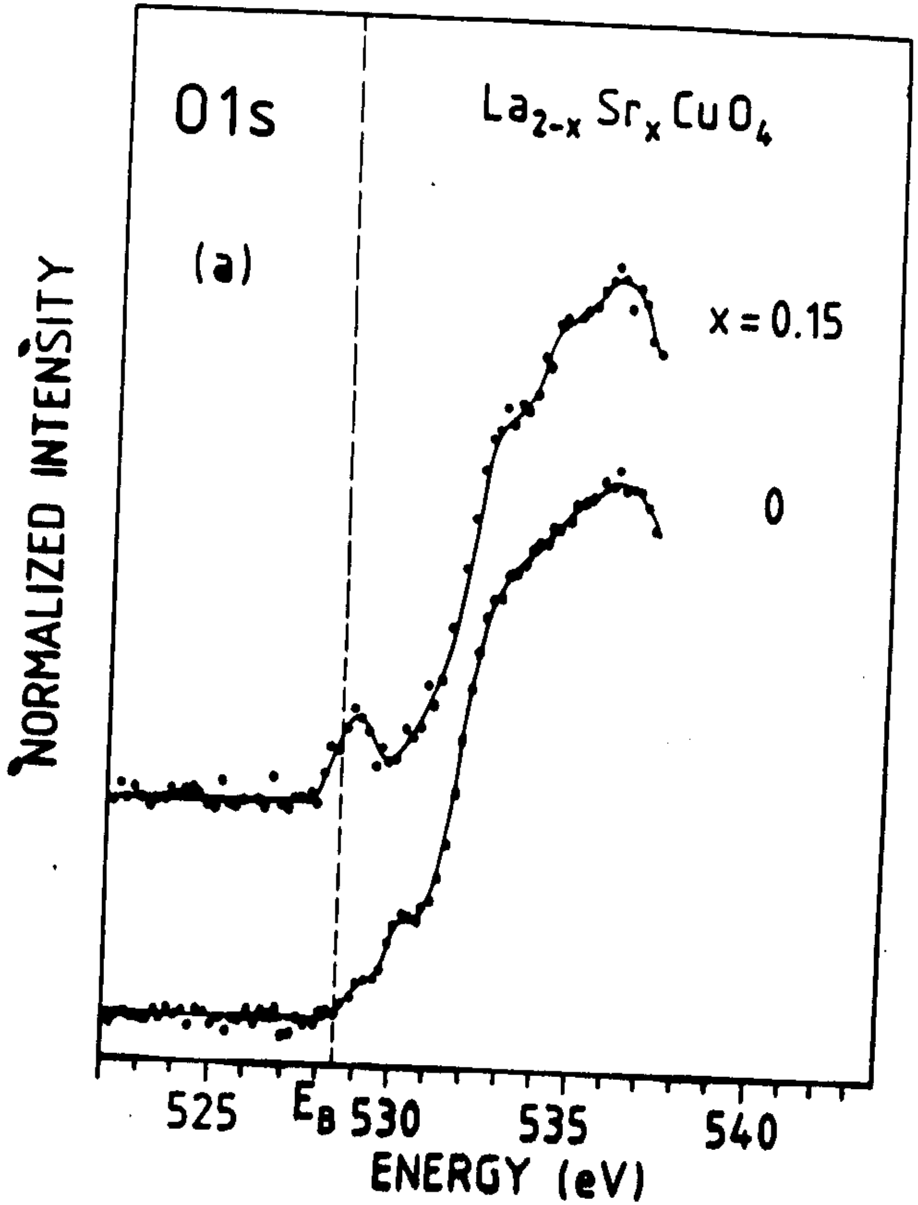
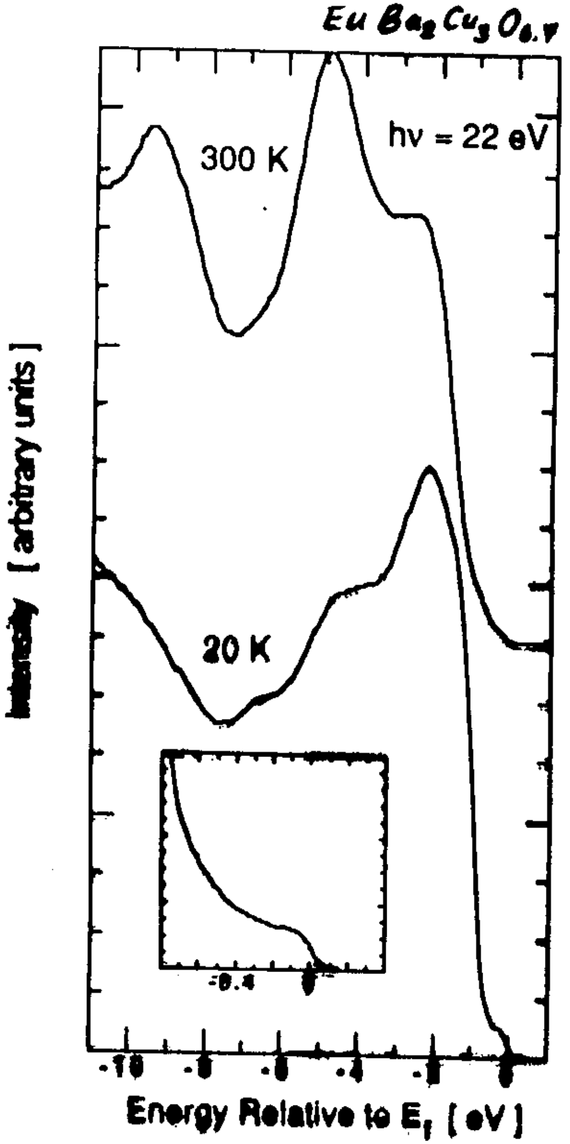


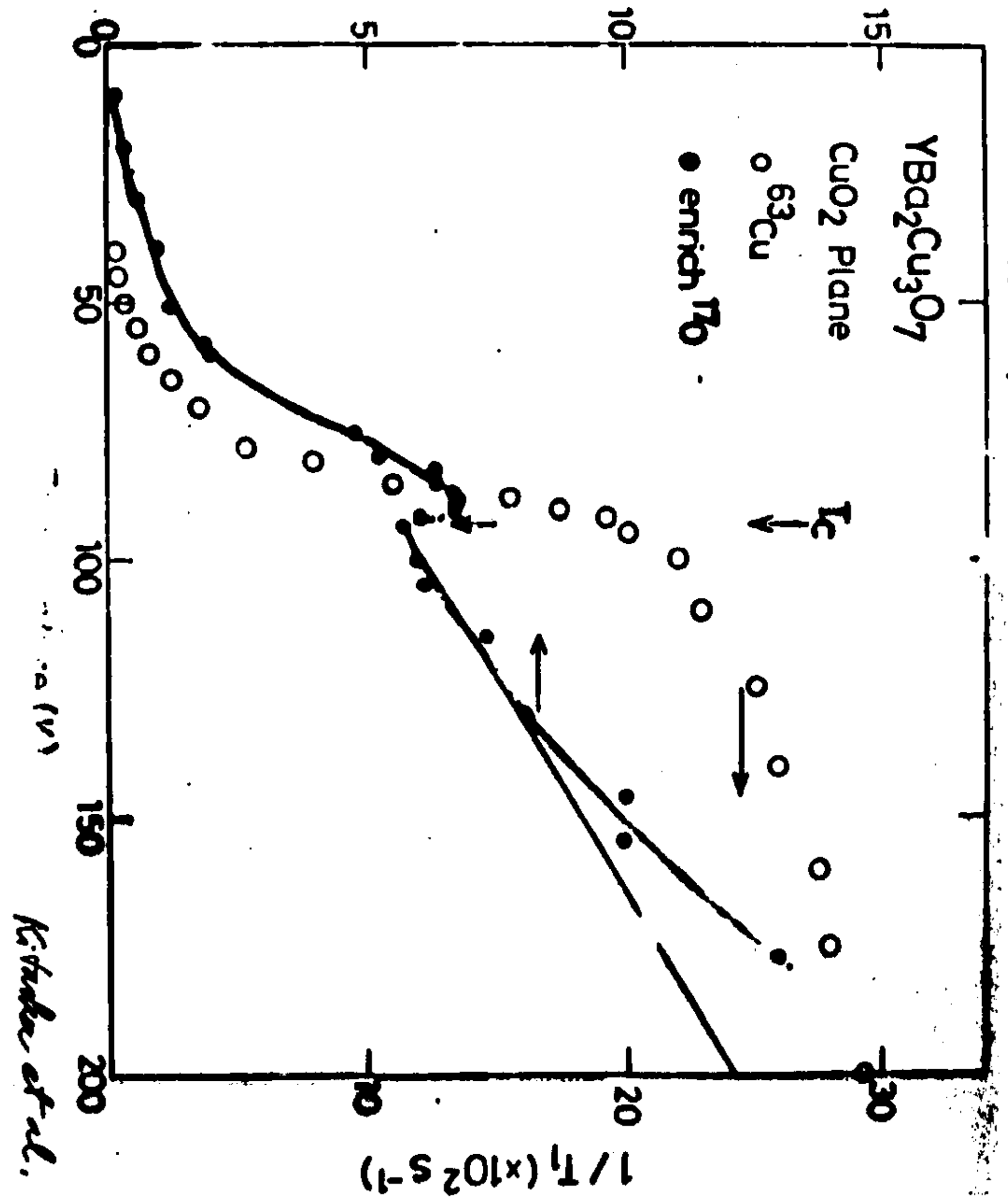
Fig. 1a

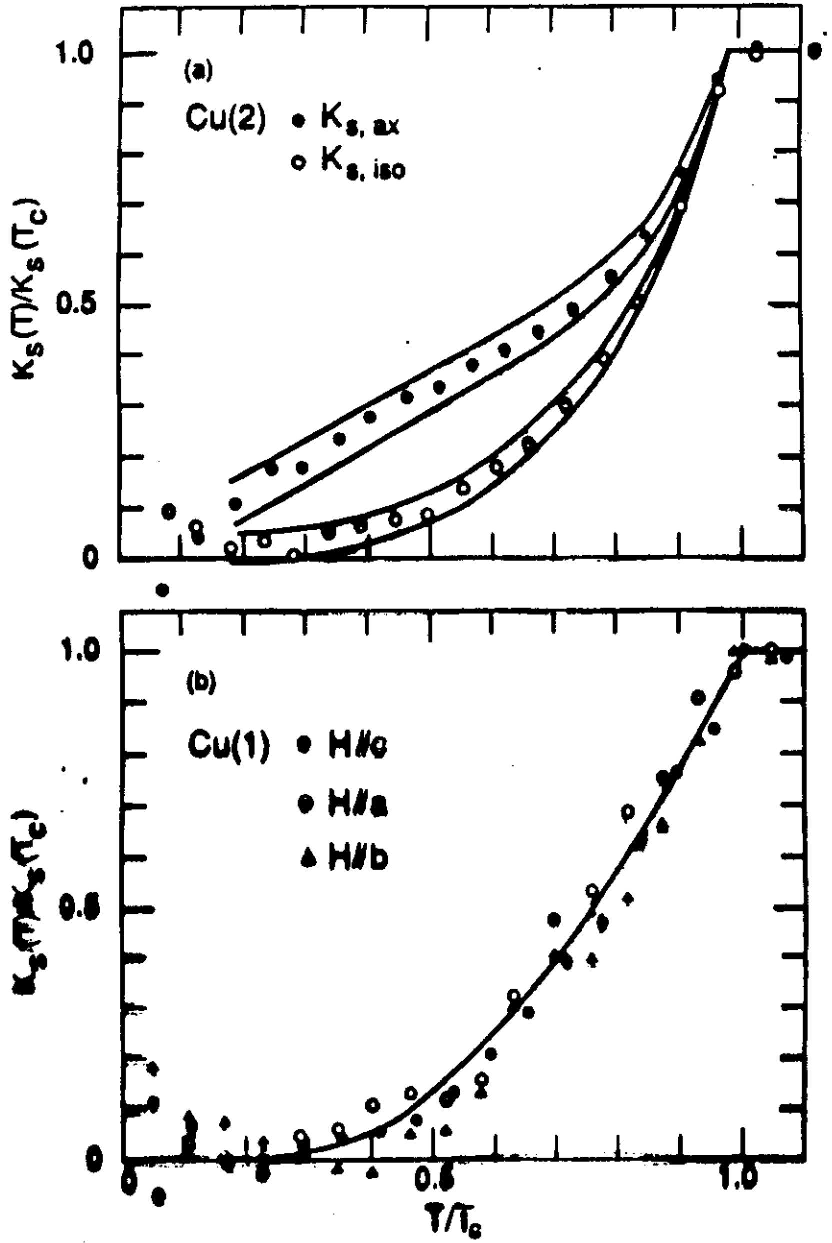
N. Nücker et al. EELS

Photoemission data: evidence of Fermi edge
R. S. List et al, Los Alamos



Nuclear Relaxation Rate $1/T_1 (S^{-1})$





M. Tanigawa
P.C. Hammel
R.H. Heffner
Z. Fisk
Los Alamos

Knight shift (spin susceptibility) in superconducting $YBa_2Cu_3O_7$. Axial part shows T-linear behavior at low temperatures, suggesting nodes in the gap realized for example in d-wave pairing

Model Hamiltonian for CuO_2 plane

$$H = -\sum_{\langle i,l\rangle\sigma} t_{il}\left(d^{+}_{i\sigma} p_{l\sigma} + c.c.\right) - \sum_{\langle ll'\rangle\sigma} t'_{ll'}\left(p^{+}_{l\sigma} p_{l'\sigma} + c.c.\right) + \epsilon_d \sum_{i,\sigma} d^{+}_{i\sigma} d_{i\sigma} + \epsilon_p \sum_{l\sigma} p^{+}_{l\sigma} p_{l\sigma} + U_d \sum_{i} n_{di\uparrow} n_{di\downarrow} + U_p \sum_{l} n_{pl\uparrow} n_{pl\downarrow} + V \sum_{\langle il\rangle,\sigma,\sigma'} n_{di\sigma} n_{pl\sigma'}$$

o
o •Cu o
o

$U_d = 5 \div 10$ eV, $U_p = 3 \div 6$ eV, $t = 1 \div 1.5$ eV

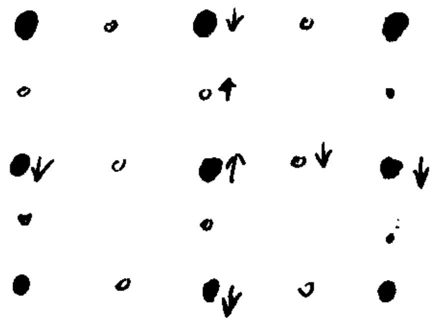
$t' = 0.5$ eV, $V \leq 1.5$ eV, $\Delta = \epsilon_p - \epsilon_d \approx 3$ eV

Energy model and effective Hamiltonian
P.R.L. 58, 2794 (1987)

Exp. hole on O.

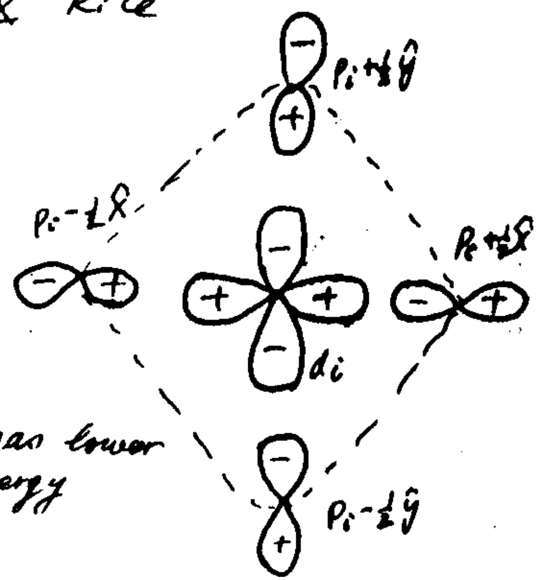
Single band model is OK ?

d-like pairing of O(2p) hole: induced by strong coupling to local sp. configuration on Cu



Zhang & Rice

Start from 2 bands. Strong coupling to Cu^{++} to form a singlet. Effective Ham. as for a single band



* Sym. comb. of O has lower energy
* Singlet state

Symmetrized orbit

$$\tilde{p}_i = \frac{1}{2}\sum_l S_{il} p_l \qquad S_{il} = \begin{cases} 1 \\ -1 \end{cases}$$

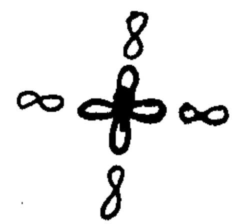
$$\phi^s = \frac{1}{\sqrt{2}}(d_{i\uparrow}\tilde{p}_{i\downarrow} - d_{i\downarrow}\tilde{p}_{i\uparrow})$$

$$\phi^t_\alpha = \begin{cases} d_{i\uparrow}\tilde{p}_{i\uparrow}, \\ \frac{1}{\sqrt{2}}(d_{i\uparrow}\tilde{p}_{i\downarrow} - d_{i\downarrow}\tilde{p}_{i\uparrow}), \\ d_{i\downarrow}\tilde{p}_{i\downarrow} \end{cases}$$

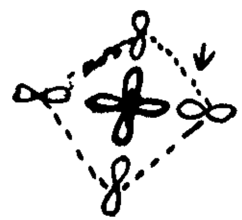
$$E^t - E^s \sim 16\, t^2/U$$

Discovery of "electronic" superconductors — electron-hole symmetry — in favor of t-J model

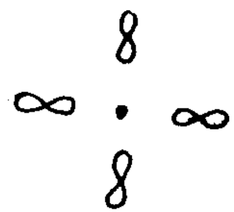
One hole per square



additional hole forming singlet



One hole missing



Composite vs constituent particle aspects of hole motion in CuO_2 plane

Z.B. Su & L. Yu

Singlet state is a composite particle with binding energy $E_b \sim 1$ eV.

* It appears as a single particle in "Low" energy phenomena, when $\hbar\omega << E_b$, like transport, IR absorption, but virtual excitation $\phi^s \rightarrow d$ plays an important role. In particular, $\rho \sim T$

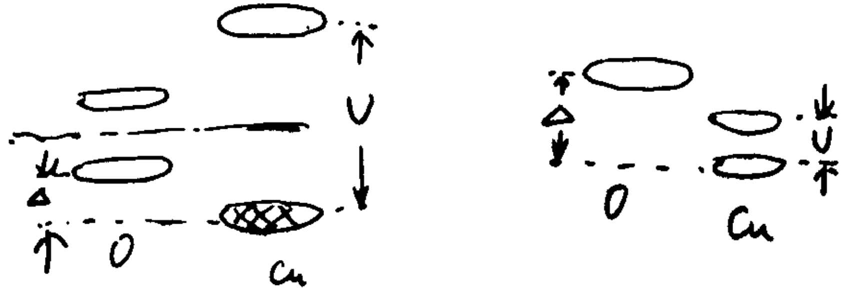
* It appears as constituent particles in "High" energy phenomena, when $\hbar\omega >> E_b$, like EELS etc.

* Consistent with available experiments.

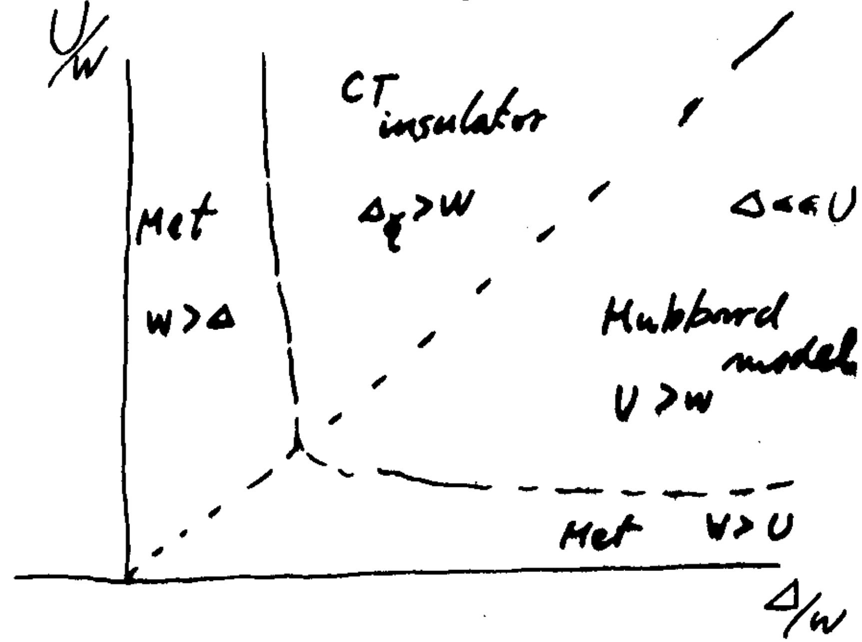
Hubbard vs Charge-transfer

$U >> \Delta = \varepsilon_p - \varepsilon_d$ CT

$U << \Delta$ H



$U >> \Delta$



Sawatzky et al. 1984

Resonant Valence Bond RVB

P.W. Anderson
1973, 1987

No (or very few) $S=1/2$ AF

Maybe Neel state is not the ground state.

Heisenberg AF

$$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \quad , \quad J>0$$

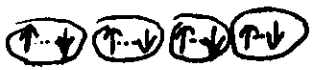
1D case:

Néel state, ↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓

$$\langle \mathcal{H} \rangle = -\frac{1}{4} NJ$$

Singlet pairs:

$$| \rangle = \frac{1}{\sqrt{2}} (\alpha_i \beta_j - \alpha_j \beta_i)$$



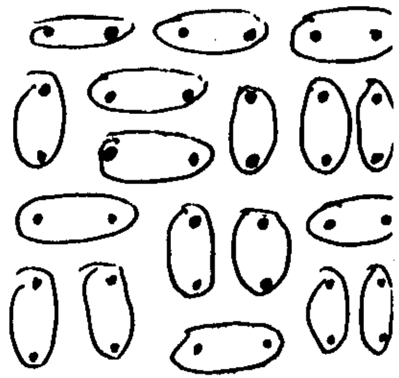
$$\langle | \mathcal{H} | \rangle = -\frac{3}{4} J \cdot \frac{N}{2} = -\frac{3}{8} NJ$$

Exact Bethe Ansatz solution

$$\boxed{E_g = -0.443 \; NJ}$$ 15% better

Linear combination of all possible singlet pairs

$$|\Psi\rangle = \sum_P (ij)(kl)(mn) \cdots$$



No spontaneous magnetization

All possible (including well-separated) pairs

Quantum Spin Liquid as opposed to "spin crystal" – Néel state

Triangular lattice. Yes

Square lattice. ? frustration

What happens upon doping

Excitations:

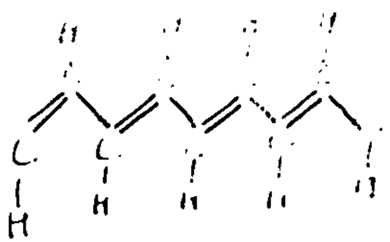
SPINON, $S = 1/2$, $Q = 0$

Holon, $S = 0$, $Q = e$

Hole = spinon + holon

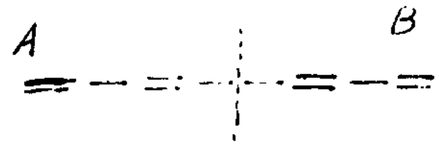
Polyacetylene $(CH)_x$

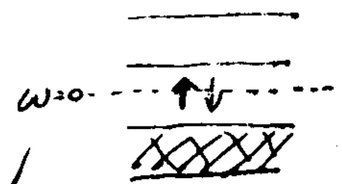
... Schrieffer, Heeger



alternating short and long bonds

Mismatch – defect – domain wall – Soliton





non bonding state

DIMER PICTURE

Neutral soliton $S = 1/2$, $Q = 0$

charged soliton $S = 0$, $Q = \pm e$

There is a topological constraint in this problem $A \to B$

Is this necessary?

Fractional Quantum Hall effect

Elementary excitations in RVB

One approach: Kivelson et al.

Spinons (fermion) spin ½ charge 0

holons (boson) spin 0 charge e

Another approach: Laughlin et al.

spinon neutral

holon charged e

both of fractional statistics

$$\psi(1,2) = e^{i\theta/\pi}\,\psi(2,1)$$

$$\theta = \pi/2$$

As in FQHE fractional charge $e/3$

here fractional spin ½

Mean Field Approximation

Baskaran, Zou, Anderson
Ruckenstein, Hirschfeld, Appel
Kotliar ...
Zhang, Rice, Gros, Shiba

1) No double occupancy constraints

Tholes → $-t\delta \sum_{\langle ij\rangle} c_{i\sigma}^{\dagger} c_{j\sigma}$

Gutzwiller Approximation

$H = g_t H_t + g_s H_s$

$g_t = 2\delta/(1+\delta)$, $g_s = 4/(1+\delta)^2$

2) PP and ph condensation

$b_{ij}^{\dagger} = \frac{1}{\sqrt{2}}(c_{i\uparrow}^{\dagger} c_{j\downarrow}^{\dagger} - c_{i\downarrow}^{\dagger} c_{j\uparrow}^{\dagger})$

$\xi_{ij} = \sum_{\sigma} c_{i\sigma}^{\dagger} c_{j\sigma}$

$\Delta_x = \langle b_{i,i+\hat{x}} \rangle$, $\Delta_y = \langle b_{i,i+\hat{y}} \rangle$

$\xi_x = \langle \xi_{i,i+\hat{x}} \rangle$, $\xi_y = \langle \xi_{i,i+\hat{y}} \rangle$

s-wave pairing $\Delta_x = \Delta_y = C$, $\xi_x = \xi_y = 0$
d-wave pairing $\Delta_x = -\Delta_y = C$, $\xi_x = \xi_y = C$
d-wave density matrix $\Delta_x = \Delta_y = \epsilon$, $\xi_x = -\xi_y = C$
chiral state $\Delta_x = -i\Delta_y = C$, $\xi_x = \xi_y = 0$

For 1/2 filling all these states are degenerate (apart from the first).

There is a local SU(2) gauge symmetry.

$c_{i\uparrow}^{\dagger} \to \alpha_i c_{i\uparrow}^{\dagger} + \beta_i c_{i\downarrow}$

$c_{i\downarrow} \to -\beta_i^{*} c_{i\uparrow}^{\dagger} + \alpha_i^{*} c_{i\downarrow}$

$|\alpha_i|^2 + |\beta_i|^2 = 1$

electron-hole transformation with conservation of spin.

Physical origin: redundancy in fermion representation.

Spin: ↑, ↓

fermion: 0 ↑ ↓ ↑↓

After Gutzwiller projection, they are the same state.

This symmetry is broken by doping.

(The d-wave pairing has the lowest energy)

Numerical Simulations

1) Q. M. C. J.E. Hirsch et al.

Hubbard model itself does not give rise to SC. This question is still open!

spin - phonon ! extended Hubbard n.n. repulsion V

2) Variational M.C. Yokoyama, Shiba

Gros, Joynt, Rice.

"BCS" type var. function with Gutzwiller proj.

$$\psi = P_{D=0} P_{N=N} \prod_k (u_k + v_k a_{k\uparrow}^+ a_{-k\downarrow}^+)|0\rangle$$

* AF and RVB have almost the same energy. Regar, Young; Liang, Doucot, Anderson
* d-wave pair "condensation"

$\Delta \sim t$

* Is there true ODLRO (off-diagonal long-range order)?

Gros: Yes . open

Possible coexistence of AF & SC (or RVB)

SHEN, SU, DONG, YU

INUI, DONIACH, Hirschfeld, Ruckenstein

CHEN, SU, YU

In the earlier MF studies two order parameters: $\Delta \sim \langle b_{ij} \rangle = \langle C_{i\uparrow} C_{j\downarrow} - C_{i\downarrow} C_{j\uparrow} \rangle$

$$\xi_{ij} \sim \langle C_{i\sigma}^+ C_{j\sigma} \rangle$$

What happens, if one includes spontaneous magnetization.

$$S = \langle S_i^z \rangle (-1)^i$$

The absolute value of Δ, ξ is reduced, but the free energy is lower.

* Confirmation by VMC calculation T.K. Lee & S.P. Feng

$E_g = -0.319\ J/\text{bond}$ without S

$E_g = -0.332\ J/\text{bond}$ $S \neq 0$

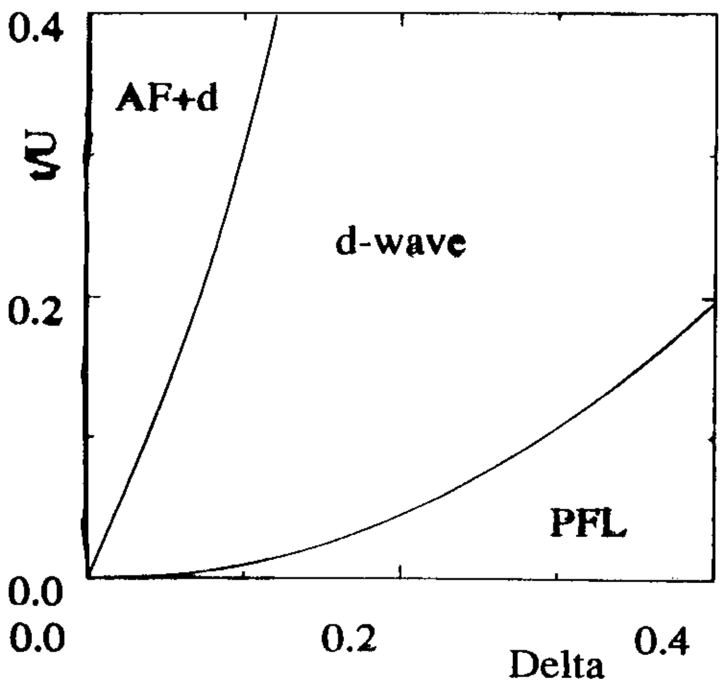
* μSR experiments.

Weidlinger et al.

AF order below 5K in

$La_{1.92} Sr_{0.08} CuO_{4-\delta}$

Fig.1

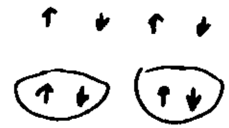


Spin-phonon coupling

$H \propto J_0 (1 + \alpha(u_i - u_j)) \vec{S}_i \cdot \vec{S}_j$

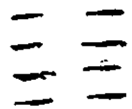
Anderson
Kivelson, Rokhsar, Sethna
Zhang, Prelovsek
Tang, Hirsch

* What is the groundstate? AF, SP
* Will this coupling favor RVB or SC?



1) 1 D always SP

2 D α_c, $\alpha > \alpha_c$ SP
$\alpha < \alpha_c$ AF
first order transition

 not 

d-like p-like

2) Kivelson et al. Yes
Anderson negligible
second order in polarization
Tosatti & Yu probably disfavor!

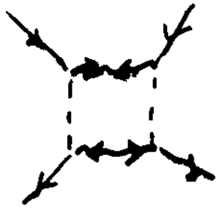
Possible scenarios for SC

* Doping releases pre-existing pairs (Anderson)
* "Holon" condensation
* Holon pair condensation

  in-plane

  inter plane

  (Anderson, Hsu, Wheatley)



* Holon condensation due to long-range gauge field (Laughlin)

An Alternative Description

— 2D Quantum Antiferromagnet

First to understand the motion of single holes on a QAFM background.

Néel state.

"Wrong" spin alignment

Confinement potential $\propto L$

Bulaevskii et al

Brinkman & Rice

Shraiman & Siggia

Retraceable path approximation

$J/t \to 0$

No quasiparticle propagation

diffusive type motion

Spin wave theory — Kane, Lee, Read; Schmitt-Rink, Varma, Ruckenstein

t-J model

$$H = -t\sum_{\langle ij\rangle}\left(C_{i\sigma}^{+}C_{j\sigma} + h.c\right) + J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j$$

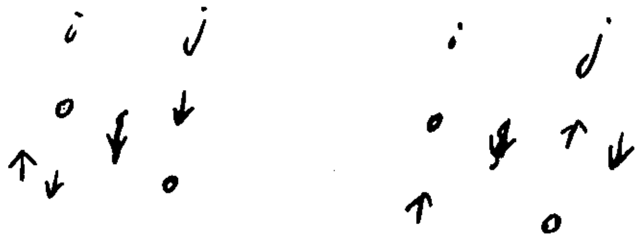
Sublattice A $\uparrow$ $\quad C_{\uparrow} \to h^{+}$

" B $\downarrow$ $\quad C_{\downarrow} \to h^{+}$

Hard-core Boson (spin wave)

$b_i|N\rangle = 0 \qquad b_i^{+} = S_i^{-}$ on A

$b_i^{+} = S_i^{+}$ on B



$$H_t = t\sum_{\langle ij\rangle} h_j^{+} b_i \left(b_i^{+} + b_j\right)$$

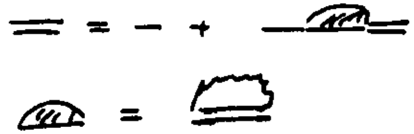
$$H_J = -\frac{NJ}{2} + \frac{J}{2}\sum_{\langle ij\rangle}\left[\left(b_i^{+}b_j^{+} + b_i b_j\right) + 2b_i^{+}b_i\right]$$

$t \ll J$ perturbation works

$t \gg J$ diverges

QAFM $\quad |\tilde{0}\rangle = \prod e^{-\frac{1}{2}\lambda_k b_k^{+} b_{-k}^{+}}|N\rangle$

Renormalization:



$$G = \frac{a_k}{\omega - \omega_k} + G_{inc}(\vec{k},\omega)$$

coherent part $\qquad$ incoherent part

$a_k \sim J/t \qquad$ effective mass enhanced

* Self-consistent Spin Polaron motion

Quantum Bogoliubov de-Gennes formalism. Z.B. Su, Y.M. Li, W.Y. Lai & L. Yu

Analogy with polarons

electron + phonon cloud (lattice distortion)

Here spin polaron

hole + spin wave cloud (spin background distortion)

Generalized model: $\langle b\rangle \neq 0$

$$H_J = \frac{J}{2}\sum_{\langle ij\rangle}\left(b_i^{+}b_i + \frac{\alpha}{2}\left(b_i^{+}b_j^{+} + b_i b_j\right)\right)$$

$\alpha = 0$ Ising, $\quad \alpha = 1$. Heisenberg

Some results:

1. Hole propagation is possible even in the Ising limit. $m_{eff} \neq \infty$
2. The propagating state is also well defined in the Heisenberg limit $\alpha = 1$. Although there is no gap in the spin wave spectrum

$\omega \propto K$

3. The wave function and effective mass can be calculated explicitly.

$m_{eff} \to \infty$ in both limits

$t/J \to 0$ and ∞.

Concluding Remarks:

1) The theoretical understanding of HTc SC is still lacking
2) More experimental clues are emerging and the range of acceptable theoretical models is narrowing
3) Probably the generalized pairing picture is valid.
4) Corrections due to strong correlations are needed.