INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONE: 2240-1
CABLE: CENTRATOM - TELEX 460392 - I

SMR/388 - 48

SPRING COLLEGE IN MATERIALS SCIENCE
ON
'CERAMICS AND COMPOSITE MATERIALS'
(17 April - 26 May 1989)

MECHANICAL BEHAVIOUR OF COMPOSITES
(Lectures I & II)

O. SBAIZERO
Istituto di Chimica Applicata ed Industriale
Cerimacs
Via Valerio, 2
34127 Trieste
Italy

MECHANICAL BEHAVIOR OF COMPOSITES

\# PARTICULATE COMPOSITES

\# WHISKERS COMPOSITES

\# FIBERS COMPOSITES

P.M.C – M.M.C – C.M.C

UNIVERSITA' DEGLI STUDI DI TRIESTE

I was asked a few days ago to talk about . . . – juste for a couple of hours.

I will try to do my best, but after 1 month of working a lot of stuff about ceramics and composite ceramics has been already presented; so please be patient if there will be some kind of overlapping between my presentation and what has been already presented by other teachers

The idea to increase the mechanical performance of materials is an old one; more than $\sqrt{3}$ thousand years ago egyptian ~~used the~~ used to incorporate straw in clay bricks ~~to in order to increase their~~

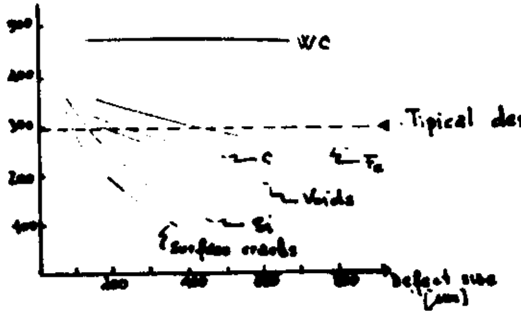
Other examples are of course steel in concrete and for istance C → cement/concrete (3% Volume) change the failure behaviour

But it ~~was~~ has been already said that is still difficult to ~~there are still a~~ produce good fibers and also is difficult to process the composite in the presence of fibers so the utilization of particles and whiskers for increasing toughness and strength appears to be ~~more~~ easier and more appealing from an economical point of view due to simpler processing

MMC → stiffness – strength – creep – fatigue / CMC more concerned with K_c

Weight reduction

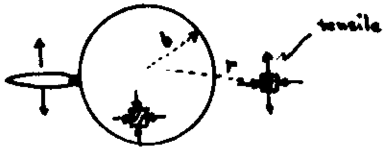
HP-Si_3N_4



Inclusions lead to stress concentrations due to different elastic constans. In addition, thermal stresses can develop during heating/cooling after processing of the material as a result of different thermal expansions.

Different elastic properties ⇒ stress concentration during loadi
Residual stresses due to $\Delta\alpha$ ⇒ " " during cooling/heati
Phase transformation ⇒ " " " " "
(ZrO_2 ; SiO_2 ; HfO_2)

$\boxed{\alpha_I < \alpha_m}$ | Si in Si_3N_4 | most severe cond.



$$\sigma_r = \sigma_i^{max} \left(\frac{b}{r}\right)^3 \qquad r \geq b$$

$$\sigma_\varphi = -\frac{\sigma_i^{max}}{2}\left(\frac{b}{r}\right)^3$$

$$\sigma_i^{max} = \frac{2 E_i E_m \cdot \Delta T \cdot \Delta\alpha}{E_i(1+2\nu_m) + 2E_m(1-\nu_i)}$$

During cooling there are compression stresses in the inclusions

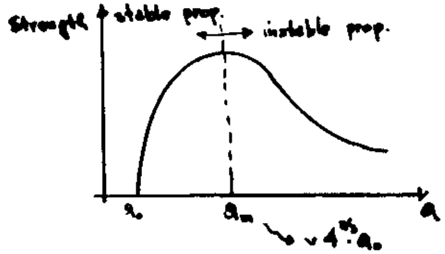
⇒ there is a stress concentration

$$K_R = \frac{3\sqrt{\pi}\cdot E\cdot \Delta\alpha \cdot \Delta T \cdot b^3}{a^{3/2}}$$

during loading:

$$K_A = \frac{2}{\sqrt{\pi}}\,\sigma\,\sqrt{a}$$

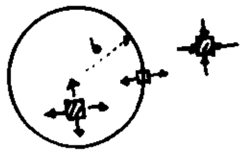
$\boxed{\text{NB. } K \sim Y \cdot \sigma \cdot \sqrt{a}}$



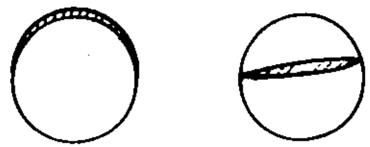
$$\sigma = \frac{(3\sqrt{\pi})^3}{8\cdot 4^{4/3}}\cdot\left(\frac{K_c^{4/3}}{E\cdot b^3 \Delta\alpha \cdot \Delta T}\right) \sim \frac{K_c^{4/3}}{b^{4/3}}$$

⇒ if the inclusions are small the strength degradation is negligible; also the matrix toughness is very import.

$\boxed{\alpha_I > \alpha_m}$ | Fe in Si_3N_4 | is less dangerous



- inclusions are in tension
- radial tension at the interface



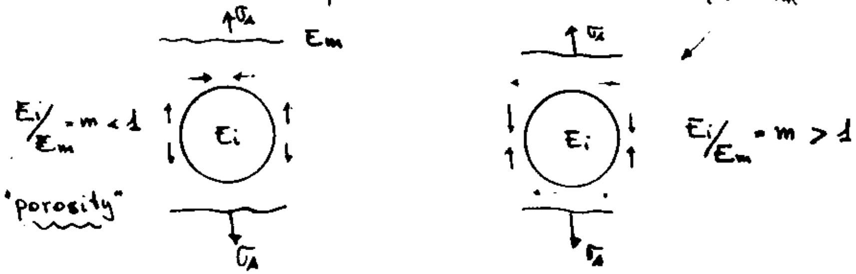
$$\# \; K_R = \sigma_R \frac{2}{\sqrt{\pi}} \sqrt{b}$$

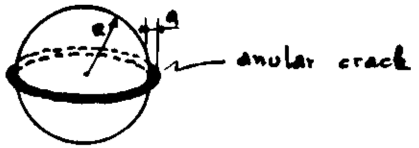
$$\Rightarrow K_c = (\sigma_R + \sigma_A) \frac{2}{\sqrt{\pi}} \sqrt{b}$$

$$\# \; K_A = \sigma_A \cdot \frac{2}{\sqrt{\pi}} \sqrt{b}$$

$$S = \frac{\sqrt{\pi} \cdot K_c^m}{2\sqrt{b}} - \sigma_R$$

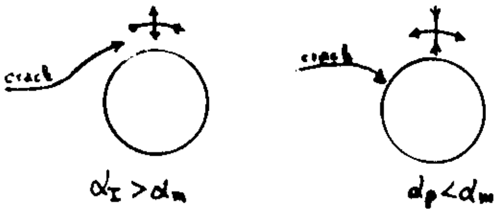
For the inclusions of WC in Si_3N_4 the overall situation is even better, because $\alpha_I > \alpha_m$ $E_i < E_m$





D.J. Green. J. Amer. Ceram. 62(5-6) [illegible], 1980

the "best" inclusion will have $\alpha_I > \alpha_m$, $E_i \gg E_m$



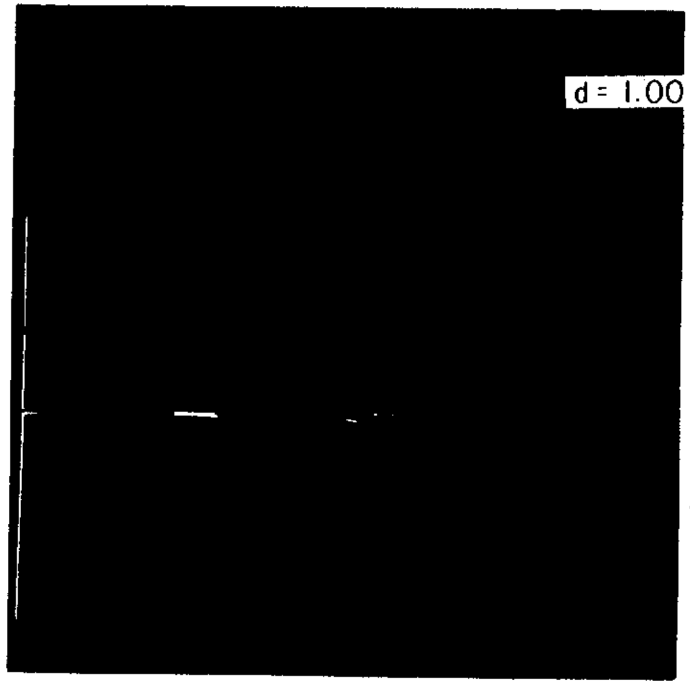


Figure 9. Trajectory of a Crack Approaching An Inclusion of Higher Elastic Determined in a Mechanical Test with Plexiglass.

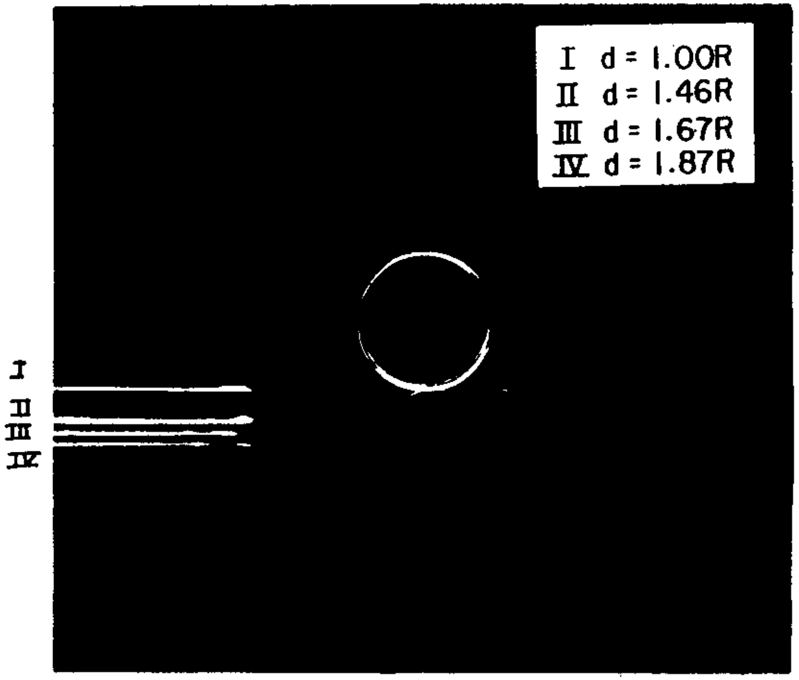


Figure 8. Trajectories of Cracks Approaching a Hole As Determined in the Mechanical Test on Plexiglass.

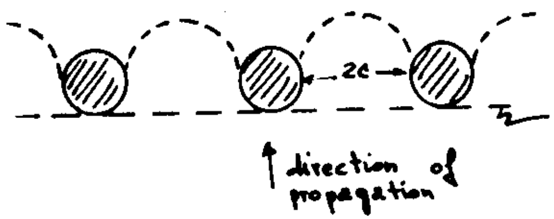
PARTICULATE COMPOSITES

Second phase particles located in the near tip field of a propagating crack perturb the front, causing a reduction in the stress intensity.
This reduction depends on the character of the particle and the nature of the crack interaction.
A second phase particle can only give toughening if the particle is more resistant to crack propagation than the matrix. (⇒ high particle toughness)

Two dominant perturbations exist:

- CRACK BOWING → non-linear crack front
- CRACK DEFLECTION → non-planar crack

CRACK BOWING



The stress needed to propagate the bowed segment of the crack front is greater than that needed to extend the primary, unbowed crack.

γ_0 fracture surface energy of the matrix

LANGE 1970 $\quad \sigma_A = \left\{ 2E/\pi c' \cdot \left(\gamma_0 + \frac{T}{2c} \right) \right\}^{1/2}$

In particulate composites the matrix material is the primary load-bearing element.

The dispersion of reinforcing particles gives either one of two possible purpose:

*1. if the matrix is <u>ductile</u> and has large plastic deformation the stronger reinforcing particles block dislocation motion or slip and therefore riducing plastic deformation

⇒ ~~block dislocation motion or slip → ↓ plastic deformation~~

2. if the matrix is brittle the tough particles can reduce the initial crack size ⇒ controlling stang

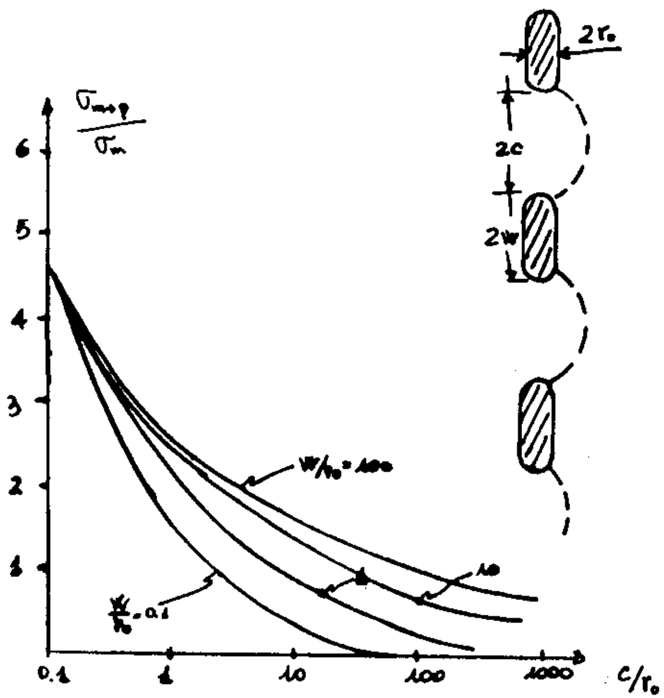
⇒ ↓ grain size ⇒ reduce initial crack size ⇒ ↑ strength

The fracture stress of the composite is:

$$\sigma_f = \left(\frac{2 E \Gamma_i}{\pi c'} \right)^{1/2} \qquad \text{(Griffith)}$$

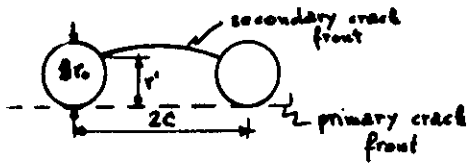
$$\Gamma_i = \Gamma_0 + \frac{T}{2c} + \gamma_{plast.}$$

provides an additional energy term for fracture through the particles.



T 'line tension' should be indipendent by particle size instead $T \cong 0.2 \div 7.5\ \mu J/m$

There are other processes during these crack-particle interactions e.g. increases in surface roughness, obstacle failure etc.



Green introduced the concept of 'penetrability'

$= (r'/r_o) \rightarrow = 1$ maximum penetrability

$\rightarrow = 0$ minimum "

strong particles $r'/r_o = 1$ (secondary crack ~ semicircular)

weak " $r'/r_o \rightarrow 0$ (0.64 more reasonable) (secondary crack ~ linear)

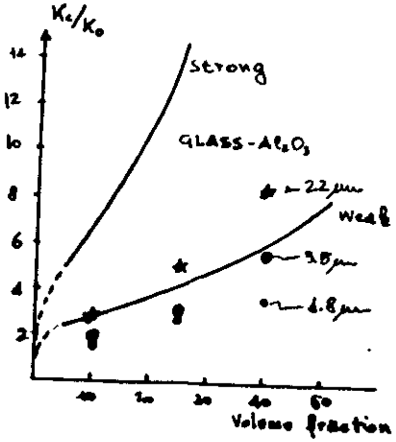
$$\boxed{\Delta K = \sum_{n=0}^{n=7} A \cdot \left(\frac{r'}{c}\right)^n}$$

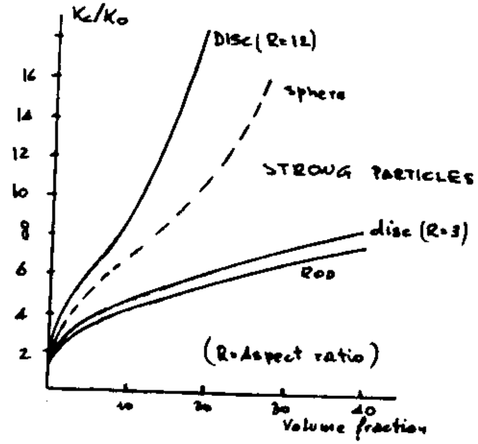
$C = f$ (Volume fraction, size, strength of particles)

<u>strong particles</u> $\Rightarrow 2C = N_A^{-1/2} - 2r_o$ $\quad \left(N_A^{-1/2} = \left(\frac{2\pi}{3V_v}\right)^{1/2} \cdot r_o\right)$ random array of spherical particles

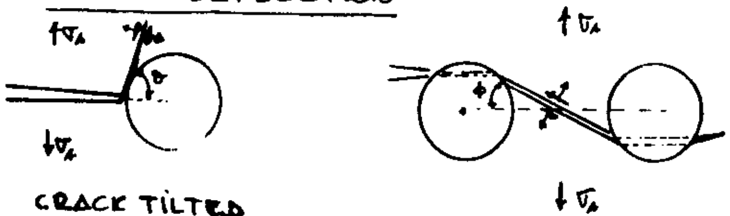
<u>weak particles</u> $\Rightarrow 2C = \dfrac{4 r_o (1-V_v)}{3 V_v}$

crack front will remain relatively straight





CRACK DEFLECTION



CRACK TILTED

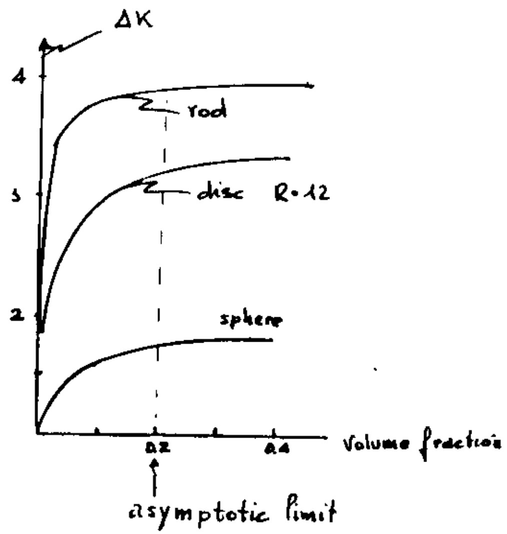
CRACK TWISTED

$\alpha_I > \alpha_m$

$E_I > E_m$

Tilted and twisted crack fronts are subject to mixed mode loading

- * tilted ⇒ mode I and II
- * twisted ⇒ mode I " III



CRACK DEFLECTION:

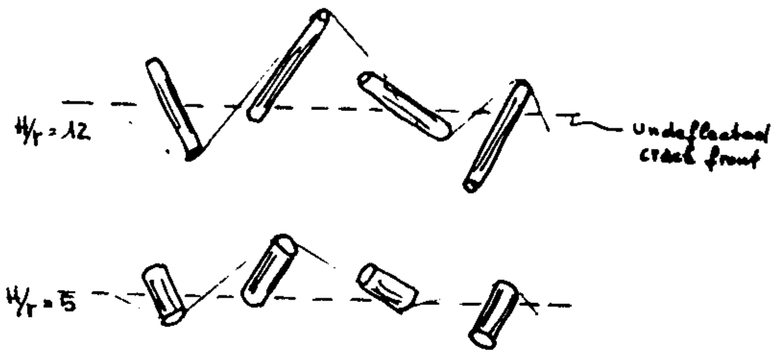
Insensitive of temperature
Independent of size

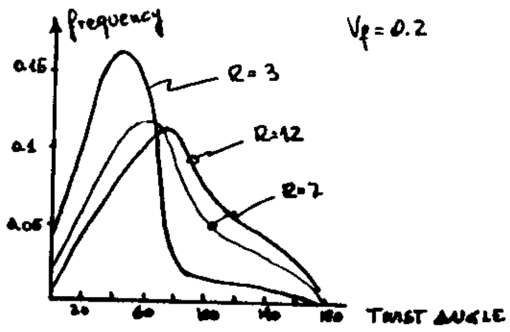
A.G. EVANS
K.T. FABER

$$\mathcal{G}^t_{sphere} = (1 + 0.87\, V_f)\, \mathcal{G}_m$$

$$\mathcal{G}^t_{rod} \simeq \left[1 + V_f \left(0.6 + 0.007\,(H/r)\right) - 0.0001\,(H/r)^2\right] \mathcal{G}_m$$

$$\mathcal{G}^t_{disc} \simeq \left[1 + 0.56\, V_f\, (r/t)\right] \mathcal{G}_m$$





The twist component overrides the tilt derived toughening

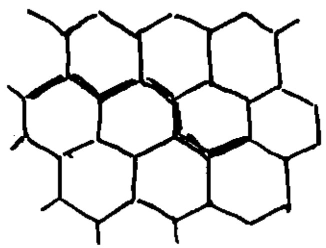
⇒ High toughness two-phase ceramic material will have:

* Chemical compatibility
* $V_f \sim 20\%$
* Particles with high aspect ratio (rod-shaped morphologies)

Experimental evidence

60% increase in K_{Ic} was observed as the grain morphology of a Si_3N_4 moved toward more elongated forms
(starting powder with large amount of β-Si_3N_4 ⇒ equiaxed)
(" " " " " " α-Si_3N_4 ⇒ elongated)

S-SiC no B along grain boundaries ⇒ $K_{Ic} \sim 2.3$ MPa$\sqrt{m}$
HP-SiC (2% Al_2O_3) ⇒ Al along grain boundaries ⇒ $K_{Ic} \sim 3.5$ MPa$\sqrt{m}$



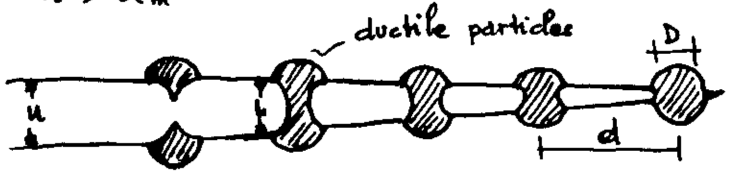
* DUCTILE DISPERSION TOUGHENING *
* CRACK BRIDGING *

Ductile particles are effective only if they attract and locally entrap the matrix crack.

⇒ Particles should have elastic modulus $\leq E_{matrix}$

Residual compression stresses in the matrix will also increase toughness because they must be exceeded before crack opening will start

⇒ $\alpha_i \geq \alpha_m$



Toughening by:

* Extensive plastic stretching in the crack wake
* Inhibiting crack opening (residual stresses)

Problems of oxidation resistance!

compressive strese σ_c imposed by the ligaments:

$$\sigma_c \simeq \frac{\pi \sigma_y}{4\,(1+d/D)^2}$$

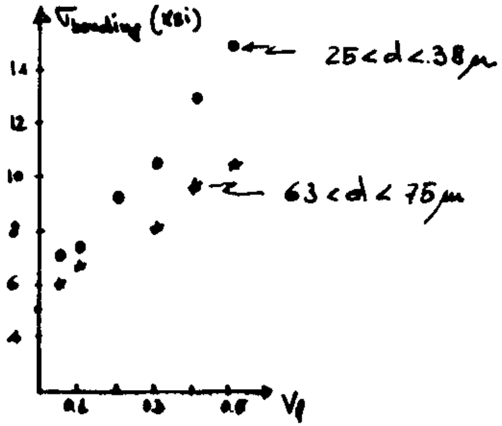
$$K_c \simeq K_m + \sqrt{K_m + 8\pi \cdot u \cdot E \cdot \sigma_y/(1+d/D)}$$

- good ceramic/metal bonding
- large particle size
- high yield strength

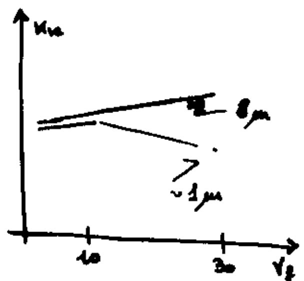
Al_2O_3/Mo Si_3N_4/TiC
WC/Co
Al_2O_3/Al
Al_2O_3/Nb
Al_2O_3/Co

{ Soda-borosilicate glasses → different α
{ W spheres → different size

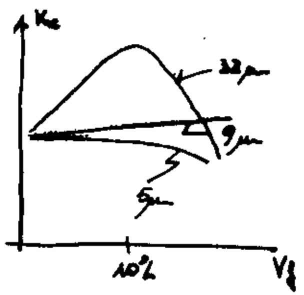
V_p 1÷50%



* no great influence of Δd *



$Si_3N_4 + SiC_p$



WHISKERS COMPOSITES

Obtained by vapor phase growth (SiC, Si_3N_4) (BN, BC)
Monocrystalline short fibers
Extremely high strength

SiC (monolithic) $\sigma^T \sim 300 \div 400$ MPa

SiC (fiber) $\sigma^T \sim 2$ GPa

SiC (whisker) $\sigma^T \sim 8.5$ GPa (24 GPa) $E \sim 580$ GPa $\sigma/\rho \approx 600$

Vapour feed gases
Liquid catalyst
Solid crystalline whiskers

... only laboratory scale product !!

Length $\sim 1 - 100\,\mu m$
$\phi\ 0.1 \div 1\,\mu m$

\# Processing is a critical step in matrix/whiskers compo...
(Whiskers act as a rigid network depleting sinter...

To understand toughening by whiskers
★ * nature of bonding whisker/matrix
* residual stress state of the interface whisker/matrix
($Al_2O_3/SiC_w \Rightarrow SiC$ radial compressive stress ~ 1900 MPa)

★ $SiC_w\ (\uparrow SiO_2) \Rightarrow K_{Ic} \sim 4.2$ (after HF) $\Rightarrow K_{Ic} \sim 6$
$SiC_w\ (\downarrow SiO_2) \Rightarrow K_{Ic} \sim 8.3$ (after heating) $\Rightarrow K_{Ic} \sim 6.5$

The role of the catalyst is to form a liquid solution interface with the crystalline material to be grown and fed from the vapour through the liquid-vapour interface.

The liquid become supersaturated

Crystal growth occurs by precipitation from the supersaturated liquid at the solid-liquid interface.

The catalyst must ~~display~~ display the affinity, when molten, to take into solution the constituent of the whiskers therefore Si and C

Now ~ 80 \$/lb

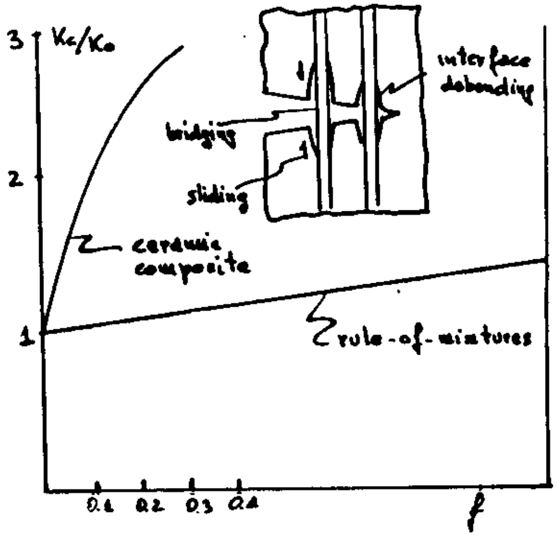
in the future 40 → 10 \$/lb — 20.000 lb/year — 3 milions of cutting tools

40% more expensive

20% more expensive

Al_2O_3 / SiC ~ 150 \$/lb

Al_2O_3 ~ 5 \$/lb

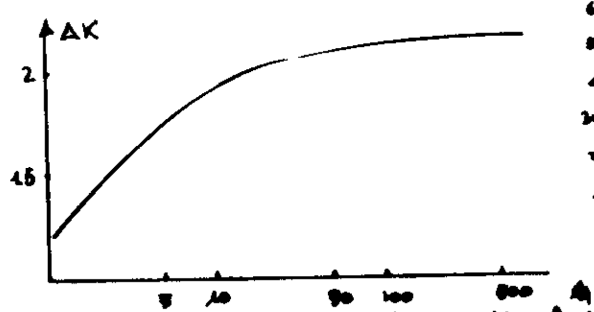


Variables:

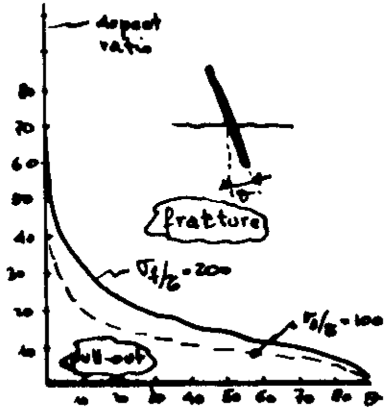
S = fiber strength

m = Weibull modulus

- τ = sliding resistance
- orientation



Aspect ratio of deflecting whiskers



Using whiskers (≥ 20% v.f.) ⇒ ~ 40% ↑ Kc ↑ σ ~ 25%

A distinction must be made between long fibers and discontinuous fibers (whiskers).

The latter must be of a critical aspect ratio:

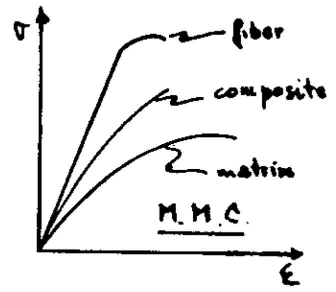
$$l_c/d \simeq \sigma^f / 2\tau$$

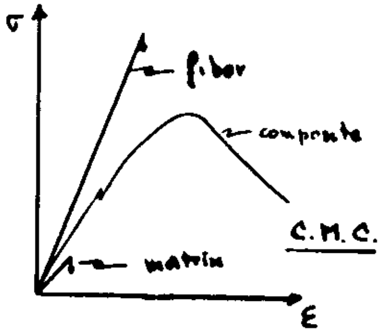
MECHANICAL BEHAVIOUR OF COMPOSITES

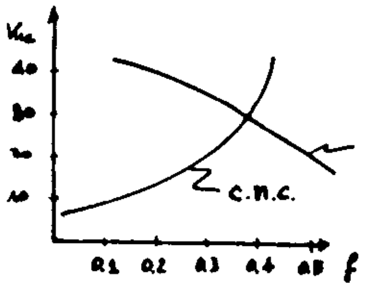
B. HARRIS – "Engineering composite materials" 1986
ISBN - 0-901462-28-4

K. K. CHAWLA "Composite Materials; science and engineering"
MRE - Spring and Verlag - 1987
ISBN - 0-387-96478-9

D. HULL "An introduction to composite materials"
Cambridge Univ. Press 1981
ISBN 0-521-23991-2



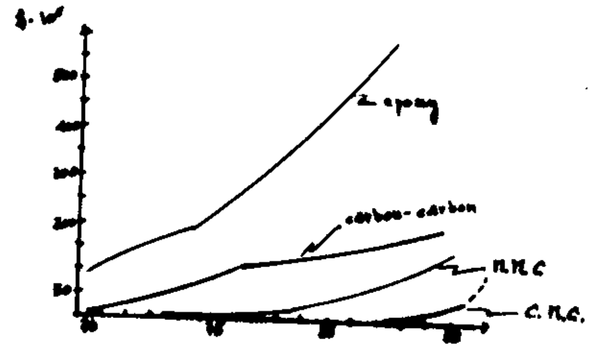




Among the most important M.M.C systems, we can include:

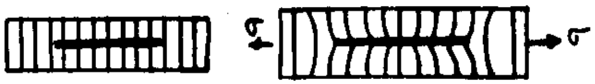
* B/Aluminum (~ 1960)
* C/Aluminum (~ 1970)
* Al_2O_3/Aluminum ; Al_2O_3/Mg
* SiC/Al
* Eutectic or in situ composites (Ni-C-Ta)

These and other MMC systems afford us high specific strength, specific modulus, plus high temperature capability, excellent toughness.



Mechanics of load transfer from matrix to fiber

The matrix holds the fibers together and trasmits the applied load to the fibers.



- High modulus fiber
- Low modulus matrix
- Perfectly bonded ⇒ no sliding !!
- $\nu_f = \nu_m$ (⇒ no transverse stresses when load is applied)

When loaded there will be locally different axial displacement because of the different elastic moduli ($E_m \neq E_f$)
⇒ shear strains are being produced in the matrix ⇒ the applied load will be transfer to the fibers by means of these shear strains in the matrix.

There are two important cases:

a) both matrix and fibers are elastic

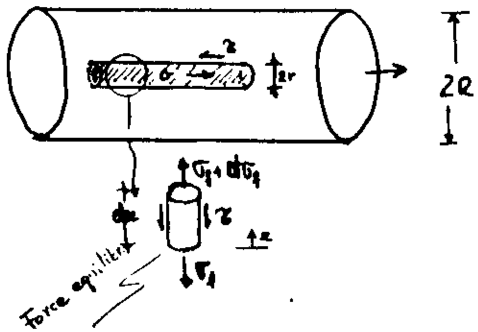
b) matrix is plastic and fibers elastic (B, C, Al_2O_3 ....)

H.L. Cox - Br. J. Appl. Phys 3, 72-79 - 1952

G. Holister & Thomas C. - Fiber Reinforced Materials - Elsevier 1966

* how is the shear stress at interface

* how is the variation of the tensile stress in the fiber

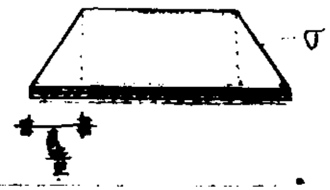


$$\left(\frac{\pi}{4} d_f^2\right)(\sigma_f + d\sigma_f) - \left(\frac{\pi}{4} d_f^2 \sigma_f\right) - \pi d_f \, dx \, \tau = 0$$

$$\frac{d\sigma_f}{dx} = \frac{4\tau}{d_f} \qquad \text{if } \sigma_f = 0 \text{ at } x = 0$$

$$\sigma_f = \frac{4}{d_f}\int_0^x \tau \, dx \qquad \text{if } \tau = \text{const.}$$

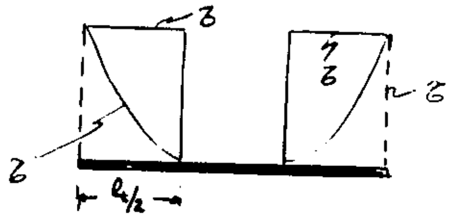
if τ = const. $\boxed{\sigma_f = \frac{4\tau}{d_f} \cdot x}$

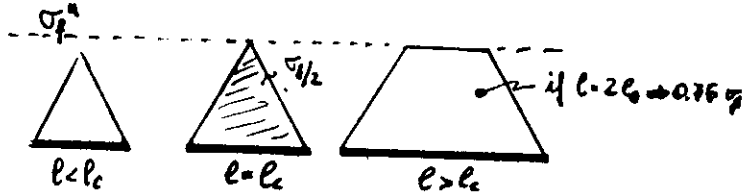


$$\boxed{\sigma_f = E_f \cdot \varepsilon_m \left\{ \frac{1 - \cosh \beta \left(\frac{1}{2} l - x\right)}{\cosh \frac{1}{2} \beta \cdot l} \right\}} \qquad \beta = \left[\frac{2 G_m}{E_f \cdot r^2 \ln(R/r)}\right]$$

$$(\sigma_f)_{max} = 2 \cdot \tau \cdot \frac{l_c}{d_f} \qquad \text{and} \qquad \boxed{l_c = \frac{\sigma_f^u}{2 \cdot \tau} d_f}$$

$$\boxed{\tau = E_f \cdot \varepsilon_m \left\{\frac{G_m}{2 E_f \ln(R/r)}\right\}^{1/2} \cdot \left(\frac{\sinh \beta \left(\frac{1}{2} l - x\right)}{\cosh \frac{1}{2} \cdot \beta \cdot l}\right)}$$





$$\bar{\sigma}_f = \frac{1}{l_f}\int_0^{l_f}\sigma_f\,dx \Rightarrow \bar{\sigma}_f = \sigma_f^u\left(1-\frac{l_c}{2l}\right)$$

$$\sigma_c = \sigma_f^u V_f\left(1-\frac{l_c}{2l}\right)+\sigma_m'(1-V_f)$$

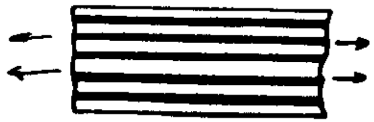
(All the fibers fail at the same strength level!)

Discontinuous fibers strengthen a matrix to a lesser degree than continuous fibers.
But for $l_f > 5\,l_c$ 90% of strengthening can be achieved

* Orientation effect *

	Fiber efficiency	
Random, discontinuous	~42%	$V_f \sim 45\%$
Cross ply, continuous	~72%	
Unidirectional, cont.	~82%	

Strength of the composite



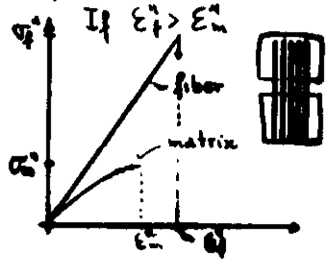
- Aligned fibers
- Continuous
- Strong
- Rigid
- Well bonded $\varepsilon_m = \varepsilon_f$

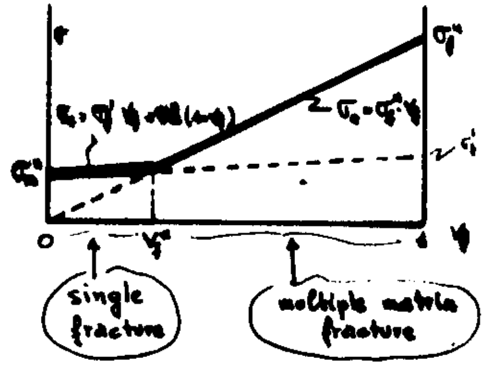
The load on the composite is shared between matrix and fibers in proportion to their cross-sectional areas.

$$\sigma_c = \sigma_f \cdot V_f + \sigma_m(1-V_f)$$

if both remain elastic $\Rightarrow \sigma_f = E_f\cdot\varepsilon_f \quad \sigma_m = E_m\cdot\varepsilon_m$

If $\varepsilon_f^u > \varepsilon_m^u$



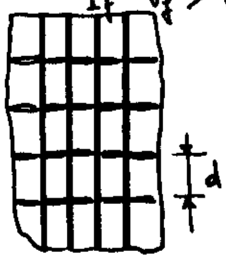


The matrix fails before the fibers and all the load is transferred to the fibers, if V_f is not enough, fibers are unable to bear the load and a single fracture will take place

$$\boxed{V_f^* = \sigma_m^u/(\sigma_f^u - \sigma_f' + \sigma_m^u)}$$

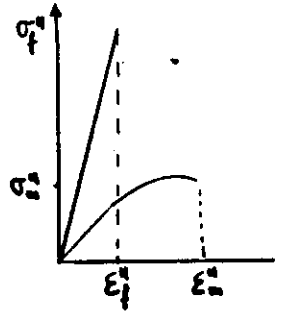
stress carried on the fibers when the matrix cracks

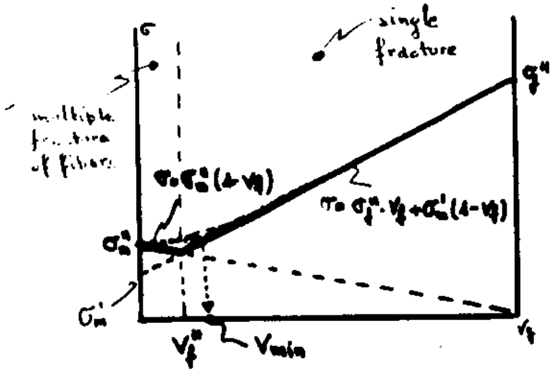
If $V_f > V_f^u$ multiple matrix crack takes place because after the first matrix crack still matrix and fibers are bonded



$d = f(E_f/E_m\,;\ \text{bond}\,;\ \varepsilon_f/\varepsilon_m)$

If $\varepsilon_m^u > \varepsilon_f^u$





When fibers fail ($V_f < V_f^u$) a work hardened matrix can counterbalance the loss of carrying capacity as a result of fiber fracture.

$$V_f^u = \frac{\sigma_m^u - \sigma_m'}{\sigma_f^u + \sigma_m^u - \sigma_m'}$$

$\sigma_m^u - \sigma_m'$ = work hardening

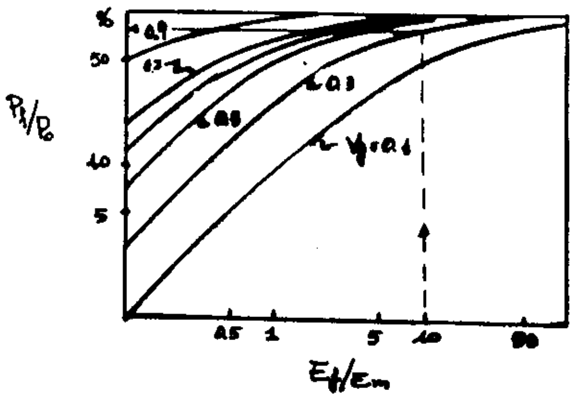
$$V_{min} = V_{crit} = \frac{\sigma_m^u - \sigma_m'}{\sigma_f^u - \sigma_m'}$$

Glass fibers – polyester matrix	C fibers – epoxy resin
$\varepsilon_m^u = 0.02$ $\varepsilon_f^u = 0.025$	$\varepsilon_f^u = 0.005$ $\varepsilon_m^u = 0.02$
$\sigma_m^u = 72\,MPa$ $\sigma_f^u = 2100\,MPa$	$\sigma_m^u = 80\,MPa$ $\sigma_f^u = 2\,GPa$
$\sigma_f' = 1520\,MPa$	$\sigma_m' = 26.5\,MPa$ $V_f' = 0.026$
$V_f^u \sim 0.11$	

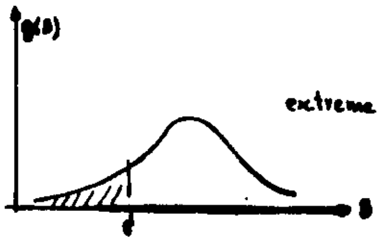
σ_m': matrix stress at the strain corresponding to the fiber failure

The fraction of load carried by fibers

$$\frac{P_f}{P_c} = \frac{\sigma_f V_f}{\sigma_f V_f + \sigma_m (1-V_f)} = \frac{E_f V_f}{E_f V_f + E_m (1-V_f)}$$



STATISTICAL ANALYSIS OF COMP. FAILURE



extreme value distribution.

NB. flaws belong to a single population

$g(s)\,dS$ = n° of flaws in a unit surface area of fiber with strength between S and S+dS

failure probability $= \delta\phi = 2\pi R \cdot \delta\ell \int_0^{\sigma} g(s)\,ds$

survival prob $= 1 - \delta\phi = 1 - \left[2\pi R \cdot \delta\ell \int_0^{\sigma} g(s)\,ds\right]$

Weakest link assumption

⇒ survival probability of fiber ≡ product of survival prob. of all elements

$$1-\phi = \prod (1-\delta\phi) = \prod \left(1 - 2\pi R \cdot \delta\ell \cdot \int_0^{\sigma} g(s)\,ds\right)$$

NB $1-x = e^{-x}$ if $x \ll 1$ ⇒ $1-\delta\phi = e^{-\delta\phi}$

also $\prod (1-x) \to e^{-\Sigma x}$

$$\Rightarrow 1-\phi = \exp\left[-2\pi R \cdot \Sigma\delta\ell \cdot \int_0^{\sigma} g(s)\,ds\right]$$

$$= \exp\left[-2\pi R \cdot \ell \cdot \int_0^{\sigma} g(s)\,ds\right]$$

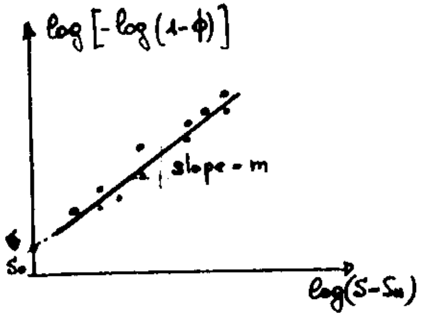
Weibull distribution function ⇒ $\int g(s)\,ds = \left(\frac{S-S_u}{S_0}\right)^m$

S_0 = scale parameter
S_u = offset "
m = shape "

flexible function but is not the only suitable function

$$1-\phi = \exp\left[-\frac{2\pi R \cdot \ell}{A_0}\left(\frac{S-S_u}{S_0}\right)^m\right]$$

if $\ell \uparrow \Rightarrow (1-\phi) \downarrow$
if $R \uparrow \Rightarrow (1-\phi) \downarrow$



N = n° of fibers tested

n = order of strength

$\phi = n/N+1$

n	S	φ
1	30	20/1
2	80	20/2
3	100	20/3
4	120	20/4
⋮	⋮	⋮
20		

$$\boxed{\bar{S} = S_0 \cdot \omega^{-1/m} \cdot \Gamma\left(\frac{m+1}{m}\right) + S_u}$$

There is another important statistical point : ~~the~~ aligned fibers act in a bundle and the strength of a bundle of fibers without uniform strenght <u>is not</u> the average strength of the fibers. (Non trivial problem)

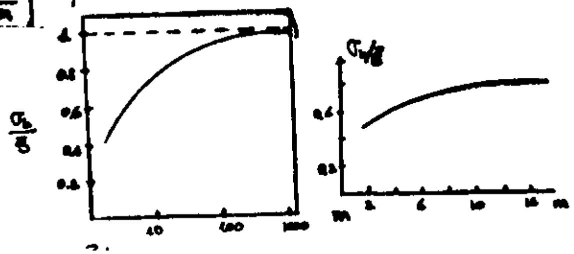
\# All the fibers have the same cross section

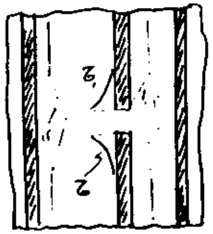
\# " " " " " " σ-ε curve but with different σ_f^u and ε_f^u

$$\sigma_b = S_0 \, (m \cdot \omega)^{-1/m} \cdot e^{-1/m} \qquad \left(\omega = \frac{2\pi\, \ell \cdot \ell}{A_0}\right) \simeq \ell/d$$

$$\boxed{\frac{\sigma_b}{\bar{S}} = \frac{(m \cdot e)^{-1/m}}{\Gamma\left(\frac{m+1}{m}\right)}}$$

Γ = gamma fonction $= \int_0^\infty \exp(-x)\, x^{m-1}\, dx$

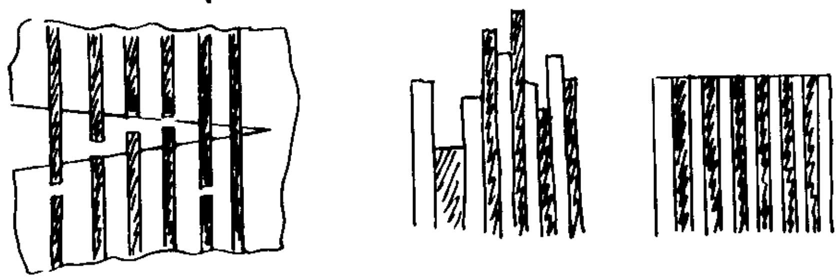




The stress state in a region close to the broken ends contain :

1. High shear stress conc. in the matrix
2. An increased σ in adjacent fibers

⇒ 1. Partial debonding of the broken fiber 
2. Microcracks in the matrix 
3. Matrix yielding → blunting the crack
4. Failure of other fibers
5. Bridging and pull-out



$$W_{deb} = N \pi d^2 \sigma_f^2 \, y / 8 E_f$$

[illegible]

$$W_{sliding} = N \tau \pi d y^2 \varepsilon_f / 2$$

$$W_{pull\text{-}out} = N \tau \pi d \, \ell_c^2 / 12$$

— o —

$\Gamma_i / \Gamma_f < 1/4$

$\Delta\alpha \geq 3 \cdot 10^{-6}$

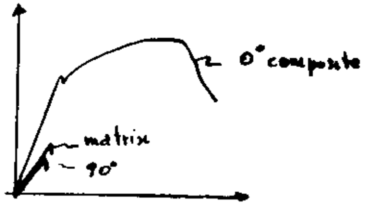
$2 < \tau \leq 40$ MPa

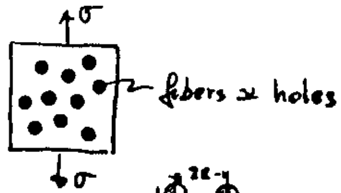
TRANSVERSE TENSILE STRENGTH

It is almost impossible to avoid transverse stresses which may lead to composite failure.

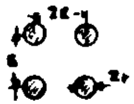
Transverse strength will depend on:

* properties of matrix and fibers
* interface bond strength
* presence and distribution of voids
* internal stresses due to the interaction between fibers

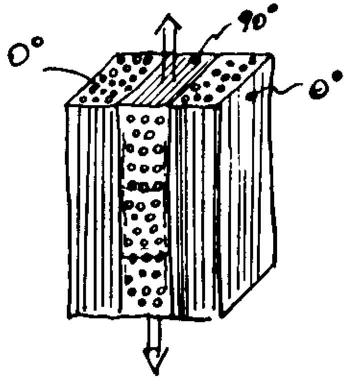


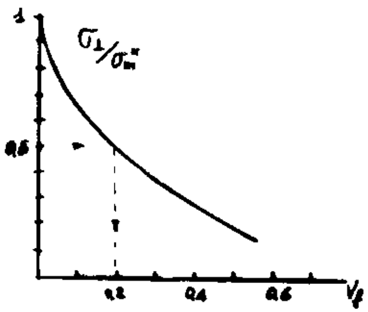


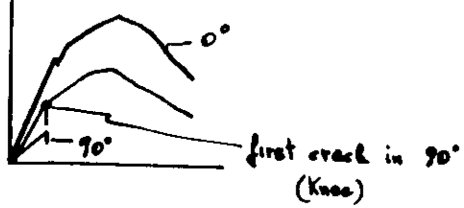
$$\frac{S}{2R} = \left[1 - 2\left(\frac{V_f}{\pi}\right)^{1/2}\right]$$

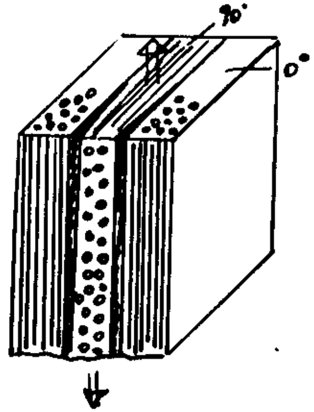


$$\Rightarrow \sigma_c^{\perp} = \sigma_m^{*}\left[1 - \left(V_f/\pi\right)^{1/2}\right]$$





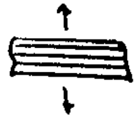




Delamination cracks

* elastic anisotropy
* residual stresses

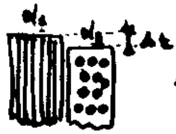
$$\nu_L \simeq f\,\nu_f + (1-f)\,\nu_m$$



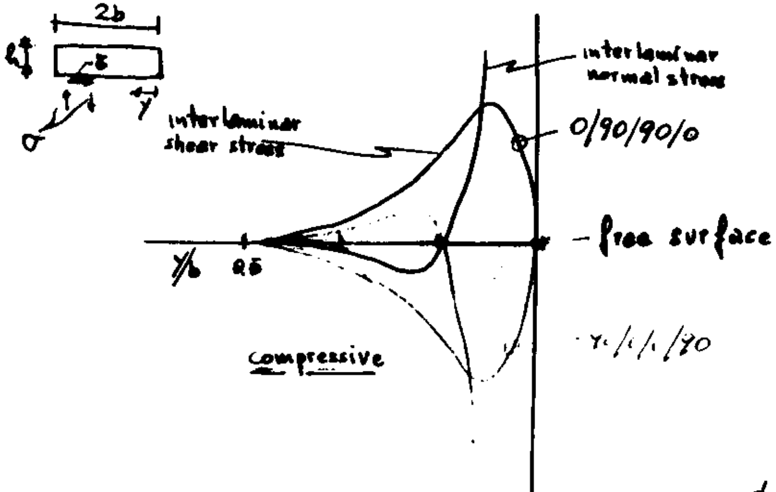
$$\nu_T \simeq \nu_m$$

If the laminas were not bonded we would end up with dissimilar transverse strain in various laminas due to the difference in their Poisson's ratios.

But ε_x must be identical ⇒ normal stresses and shear

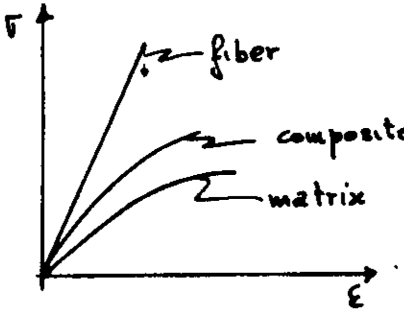


⟹ also residual stresses due to Δα give rise to interlaminar stresses.



Ø

CERAMIC MATRIX COMPOSITES



M. M. C.

M. M. C. → first damage = fiber failure

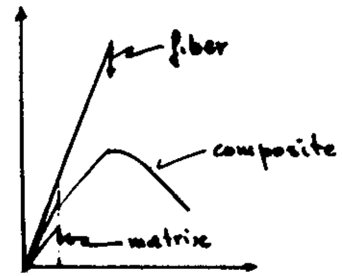
⇒ # high E_f (the load partitioning depends on E_f/E_m)
$E_f/E_m > 3 \sim 4$

- # good bonding fiber/matrix
- # tough matrix

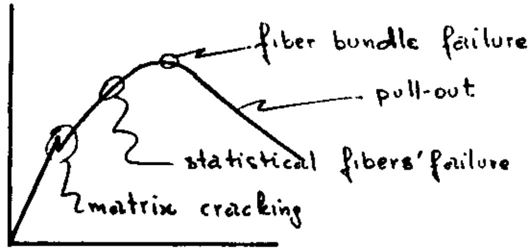
} good delamination resistance

- # good fibers alignment
- # dispersion homogeneity

} good compression and shear strength

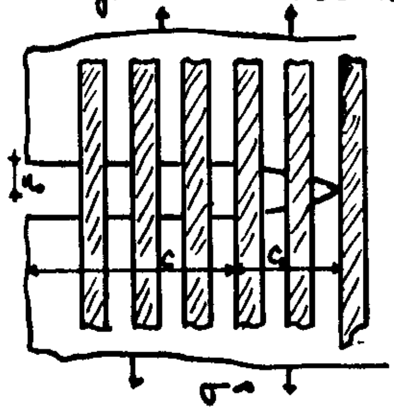


NB. two brittle systems but "ductile" behaviour



matrix cracking = design stress !!

the first matrix crack is of prime concern because matrix fracture means the onset of permanent damage the loss of protection provided by the matrix against corrosion and oxidation of the fibers, and the likelihood of an enhanced susceptibility to degradation due to cyclic loading.

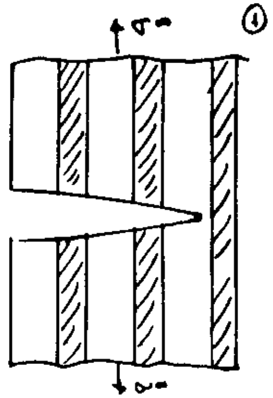


"Large cracks" = crack opening u that asymptotically approaches the equilibrium separation "u_0". Crack growth in this region is referred to as "steady state growth"

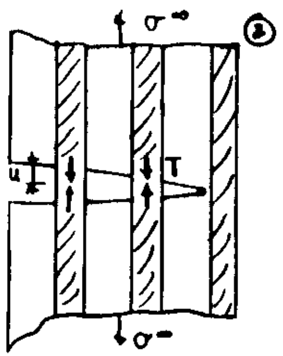
"Short cracks" ($c < c_0$) → the stress required to propagate a crack is sensitive to the crack lenght

Separation of the surfaces of a matrix crack which is bridged by fibers requires some sliding of the matrix over the fibers. (Debonding followed by sliding against frictional forces)

The restraining effect of the fibers gives rise to a reduction in both crack opening and the crack tip stresses



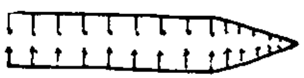
1. All of the bonds across the crack plane are cut and the stress σ^∞ is applied, causing the crack to open.



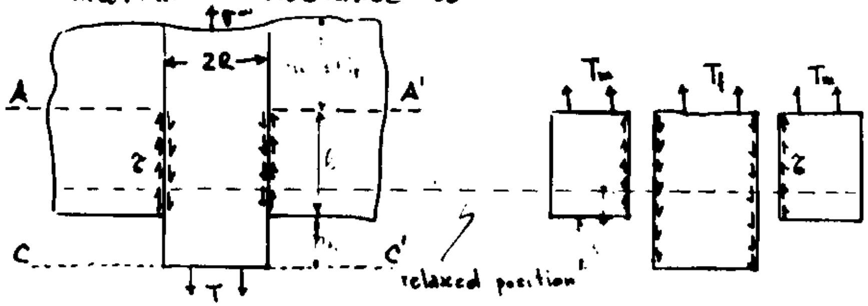
2. Tractions, T, are applied to the end of each fiber ⇒ the fiber ends displace relative to the matrix and allow the fibers to be rejoined

This procedure is equivalent to applying a distribution of closing pressure $p(x)$ to the crack surfaces
$$p(x) = T(x)\cdot V_f \quad (x<c)$$
$$p(x) = 0 \quad (x>c^*)$$

$$K^t = 2\left(c/\pi\right)^{1/2}\int_0^1 \frac{[\sigma^\infty - p(x)]\,x\,dx}{\sqrt{1-x^2}} \qquad x = x/c$$



The application of tractions T to the end of the fiber gives sliding between the matrix and fiber over a distance "ℓ", and allows the fiber to pull-out of the matrix a distance u



τ = shear resistance of interface
ℓ = slip length

For $z>\ell$ $\quad \varepsilon_m = \varepsilon_f \Rightarrow \boxed{\sigma_m/E_m = \sigma_f/E_f}$ (1)

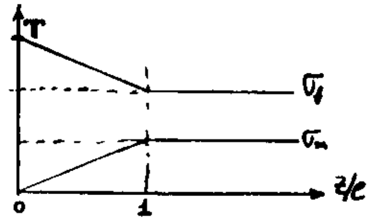
MATRIX $\quad 2\pi R\cdot\ell\cdot\tau = \sigma_m\cdot A_m$

$$\Rightarrow \boxed{\sigma_m = 2\pi R\cdot\ell\cdot\tau/A_m} \quad (2)$$

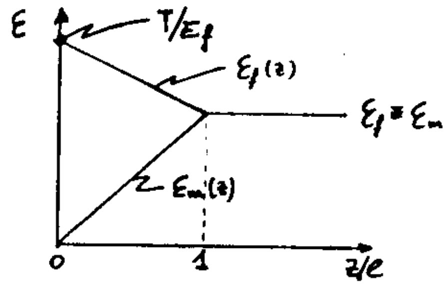
fiber cross-section area
$A_f = \pi R^2$

FIBER $\quad \boxed{2\pi R\cdot\ell\cdot\tau = (T-\sigma_f)\cdot A_f} \quad (3)$

$A_m = \pi R^2(1-f)/f$
Area of the matrix per fiber



Using strain distribution



$$\varepsilon(z) = \varepsilon_o\,(z/\ell)$$

$$\Rightarrow u(\ell) = \varepsilon_o \int_0^{\ell} z \cdot dz \Rightarrow = \varepsilon_o \cdot \ell/2$$

<u>matrix</u>

$$\delta = \varepsilon_m \cdot \ell^2/2 = \sigma_m \cdot \ell/2 \cdot E_m$$

using eq. 2

$$\Rightarrow \boxed{\delta = \frac{\pi \cdot R \cdot \ell^2 \cdot \tau}{E_m \cdot A_m}} \quad ④$$

<u>Fiber</u>

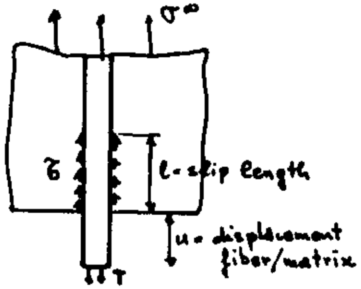
$$u + \delta = \left(\frac{T}{E_f} + \varepsilon_f\right) \cdot (\ell/2)$$

$$\Rightarrow \boxed{u + \delta = \frac{\pi R \ell^2 \tau}{A_f \cdot E_f} + \frac{\sigma_f}{E_f}} \quad ⑤$$

using eq. 1 → 5 we get

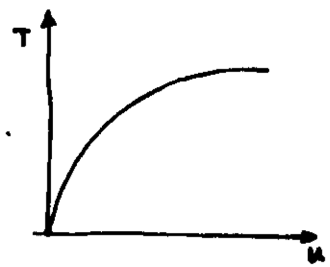
$$T = 2\left[\frac{u \cdot E_f \cdot \tau}{R}\left(1 + \frac{E_f \cdot f}{E_m(1-f)}\right)\right]^{1/2}$$

$$\ell^2 = \left[u \cdot R \cdot E_f / \tau\left(1 + \frac{E_f \cdot f}{E_m(1-f)}\right)\right]$$

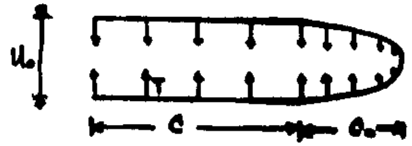


$$T = 2\left[\frac{u \cdot E_f \cdot \tau}{R}(1+\phi)\right]^{1/2} \quad \text{(fibers' tract.)}$$

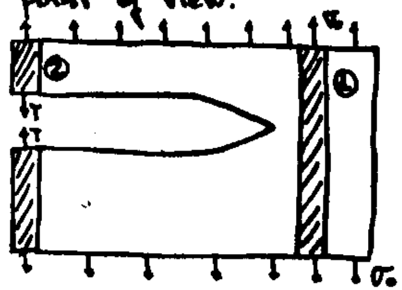
$$\phi = E_f \cdot f/(1-f)\,E_m$$



$$\ell^2 = \frac{u \cdot R \cdot E_f}{\tau(1+\phi)} \quad \text{(slip length)}$$



⇒ in c there is a 'steady state' propagation and σ_o (matrix cracking) is indipendent of the total crack length ⇒ this is very important from a technological point of view.



The energy changes occuring in the specimen and loading system during an crack extension

When the cut is made in the matrix, the matrix must slide back over the fibers while the fibers also extend.
When this occurs, work, dU_s, is done against frictional forces, the strain energy in the matrix decreases by dU_m, the strain energy in the fibers increases by dU_f, and the potential energy of the loading system decreases by dU_L; there is also a variation in the surface energy of the matrix U_Γ

$$\frac{dU_s}{dc} = \left[\sigma_0^2 \cdot R/6 \cdot \tau \cdot E_f \cdot f^2 (1+\phi)^2\right] \qquad \phi = E_f \cdot f/(1-f) E_m$$

$$\frac{dU_m}{dc} = \left[\sigma_0^3 \cdot R \cdot \phi / 3 \cdot \tau \cdot E_f \cdot f^2 (1+\phi)^3\right]$$

$$\frac{dU_f}{dc} = \left[\sigma_0^3 \cdot R \cdot (3\phi+1)/6 \cdot \tau \cdot E_f \cdot f^2 (1+\phi)^3\right]$$

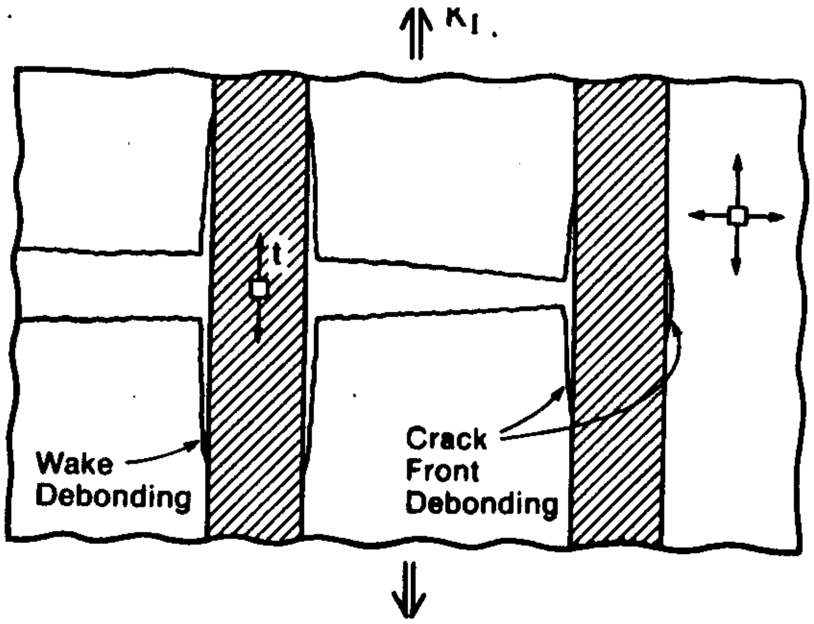
$$\frac{dU_L}{dc} = \left[\sigma_0 \cdot R / 2 \cdot \tau \cdot E_f \cdot f^2 (1+\phi)^2\right]$$

$$\frac{dU_\Gamma}{dc} = 2 \cdot \Gamma_m \cdot (1-f)$$

Griffith criterion $\partial G/\partial c = 0$

$$\Rightarrow \boxed{\sigma_0^3 = 6\,(1-\nu_m^2) \cdot K_m^2 \cdot \tau \cdot E_f \cdot f^2 (1-f)(1+\phi)^2 / E_m \cdot R}$$

NB. $2\,\Gamma_m = K_m \cdot \cancel{K_m} \cdot (1-\nu_m^2)/E_m$

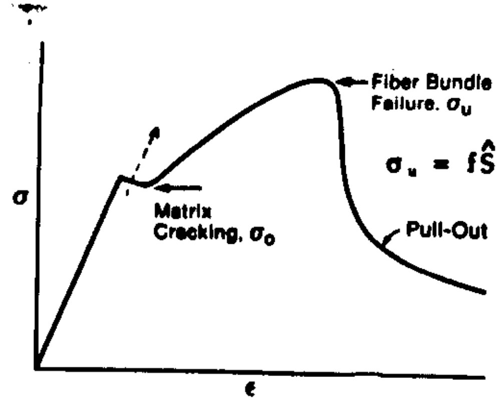


1. DEBONDING
   - → residual tensile stress at interface $\Delta\alpha \leq 3 \cdot 10^{-6}\ ^\circ C^{-1}$
   - → $\Gamma_i/\Gamma_f \leq 1/4$ $(E_f = E_m)$

2. NO KINKING $\Rightarrow$ high E_f

3. STATISTIC FIBER FAILURE

4. PULL-OUT $\Rightarrow$ low m

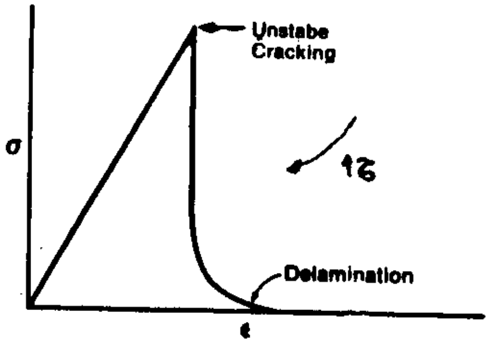


a) 'Tough' Composite

$$\sigma_u = f\hat{S}\exp\left[-\frac{\left[1-(1-\tau D/R\hat{S})^{m+1}\right]}{(m+1)\left[1-(1-\tau D/R\hat{S})^{m}\right]}\right]$$

$$\frac{\sigma_o}{E} = \left(\frac{6 f^2 E^f \tau \mathcal{G}_M}{(1-f)E(E^M)^2 R}\right)^{\frac{1}{3}} - \frac{p}{E^M}$$

$$\sim\left(\tau/R\right)^{1/3}$$



b) 'Brittle' Composite

$\uparrow\tau$

$$\Delta G \sim R\,\hat{S}^2/\tau$$

$$\boxed{B\,(\text{when } \sigma_o = \sigma_u) = \tau\cdot E\cdot \Gamma_m/S_o^2\cdot R}$$