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DISLOCATION SHIELDING (Part I)

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These are preliminary lecture notes, intended only for distribution to participants.

DISLOCATION SHIELDING

I. Introduction -

The elastic field of cracks

II. 1-D distribution of dislocations

1. BCS model
2. Ohr's experiment and modification
3. Dislocation system under crack-tip-like stress field.
4. Dislocation distribution near a semi-infinite crack
5. Dislocation distribution near a finite length crack
6. The formation of DFZ.

II. Dislocation Emission from Cracks and the Role of External Sources

1. The image force of finite length crack
2. Dislocation emission from crack
3. The role of external sources
4. The path independent integral
5. Applications

The Elastic Fields of Cracks

$$\sigma_{ij} = \frac{K_\alpha}{(2\pi r)^{3/2}} f_{\alpha,ij}(\theta)$$

Mode I: K_I

$$f_{I,yy}(\theta) = \cos(\theta/2) [1 + \sin(\theta/2) \sin(3\theta/2)]$$

Mode II: K_{II}

$$f_{II,xy}(\theta) = \cos(\theta/2) [1 - \sin(\theta/2) \sin(3\theta/2)]$$

Mode III: K_{III}

$$f_{III,yz}(\theta) = \cos(\theta/2)$$

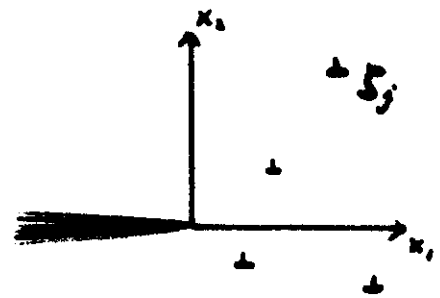
with dislocations

$$\sigma_{zz} \approx k_\alpha / \sqrt{2\pi r}$$

$$k_\alpha = K_\alpha - \frac{4}{3} \sum_j \left(\frac{b_j}{\sqrt{2\pi r_j}} + \frac{b_j}{\sqrt{2\pi r_j}} \right)$$

$b_j > 0$, dislocation shielding

$b_j < 0$, dislocation antishielding



Elastic theory of dislocation-crack interaction

$$\sigma_{xx} = \frac{\mu b_j}{2\pi(z-s_j)} \left(\frac{z}{z-s_j} \right)^{1/2}$$

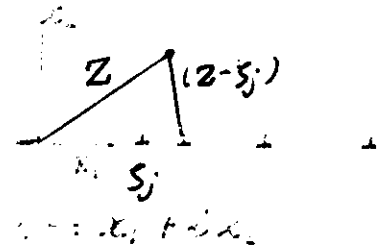
for $z \gg s_j$

$$\frac{\mu b_j}{2\pi(z-s_j)} \left(\frac{z}{z-s_j} \right)^{1/2} \approx \frac{-\mu b_j}{\sqrt{2\pi z} \cdot \sqrt{2\pi s_j}}$$

$$\sigma_{xx} = \frac{1}{(2\pi z)^{1/2}} \left[K - \sum_j \frac{\mu b_j}{(2\pi s_j)^{1/2}} \right] =: \frac{k}{(2\pi z)^{1/2}}$$

$$k = K - \sum_j \frac{\mu b_j}{(2\pi s_j)^{1/2}}$$

$$G_c = \frac{k_i}{2\mu}$$



dislocation shielding (for $b_j > 0$)

dislocation antishielding (for $b_j < 0$)

for continuous distribution dislocation:

$$\sum_j \rightarrow \frac{\mu b}{(2\pi)^{1/2}} \int_0^{s^*} \frac{D(s)}{s^{1/2}} ds$$

R. Thomson in:

«Physical Met.» Eds. R.W. Cahn and P. Haasen,

1980, p. 1512.

For 1-D distribution

$$\sigma_{x \rightarrow 0} \cong k_a / \sqrt{2\pi x}$$

$$k_a = K_a - \sum_{i,j} \frac{L}{2} \left(\frac{b_i}{\sqrt{2\pi x_i}} - \frac{-b_j}{\sqrt{2\pi x_j}} \right)$$

Continuous distribution dislocations

$$\sum_{i,j} \rightarrow \frac{\mu b}{2} \int \frac{D^i(x_i) dx_i}{\sqrt{2\pi x_i}} + \frac{\mu b}{2} \int \frac{D^j(x_j) dx_j}{\sqrt{2\pi x_j}}$$

The dislocation distribution function is quite important !

$D^i(x_i)$ dislocations in the plastic zone.

$D^j(x_j)$ image dislocations due to the crack.

$$D^i(x_i) \rightleftharpoons D^j(x_j)$$

We put $P_0 = R - \sigma_0$ and $P_1 = \sigma_1 - R$. Thus $P_1 > 0$ and $P_0 > 0$. Let there be $f(x_1) dx_1$ dislocations each of strength $b > 0$ in any distance dx_1 ; with the *RH/FS* convention (Bilby, Bullough & Smith 1955), $f(x_1)$ is positive for positive edge or right-hand screw dislocations. We wish to determine $f(x_1)$ and the relation between c and a for various values of the physical parameters R , σ_0 and σ_1 . This problem in the theory of continuous distributions of dislocations is most easily solved by setting up the integral equation which expresses the requirement that the resultant

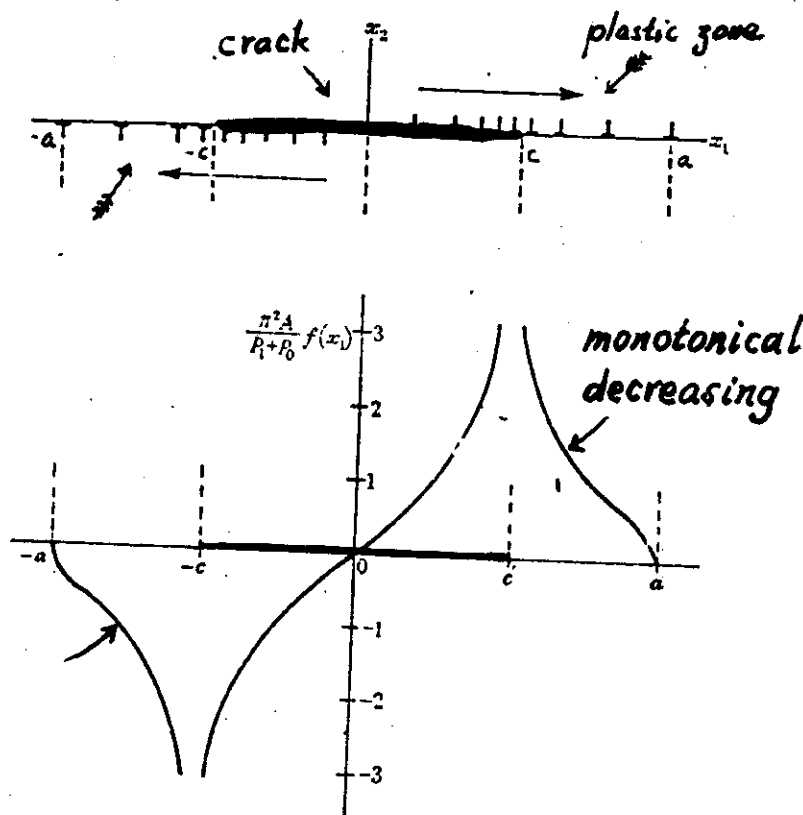


FIGURE 1. Distribution of dislocations along a sheared slit ($|x_1| < c$) and its associated yield zones ($c < |x_1| < a$).

shear stress on any dislocation in the distribution is zero when the system is in equilibrium (Leibfried 1951; Head & Louat 1955). The shear stress at x_1 due to the dislocations at x'_1 is

$$p_{23}(x_1) = A \int f(x'_1) dx'_1 / (x_1 - x'_1)$$

and hence, for equilibrium

$$\int_D \frac{f(x'_1) dx'_1}{x_1 - x'_1} = P(x_1)/A \quad (1)$$

where D is the region of the x_1 axis over which the dislocations are distributed and $P(x_1)$ is the resultant external shear stress (p_{23}) at x_1 . We have $P = P_0$ for $|x| < c$, and $P = -P_1$ for $c < |x| < a$.

The solutions of integral equations of this type have been discussed in some detail by Muskhelishvili (1946) and Head & Louat (1955). In the present problem D is

Proc. Roy. Soc. London, A272, 306 (1963).

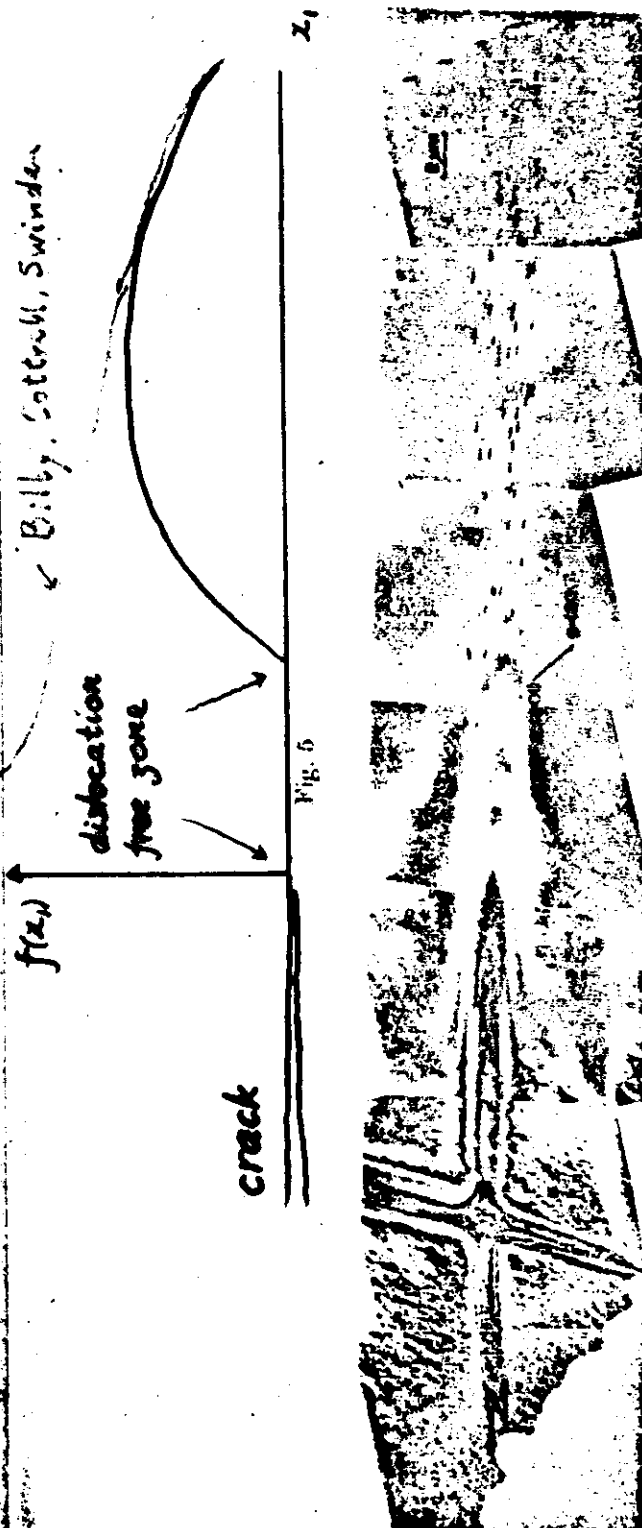


Fig. 5

Micrograph showing plastic zones in tungsten single crystal. Note that the plastic zones near the crack are free of dislocations.

$$F = \frac{Ab}{2x} - \frac{Kb}{(2\pi x)^{1/2}} > \sigma_i$$

$$\left(A = \frac{\mu b}{2\pi} \right)$$

S. Kobayashi and S. M. Ohr,
 Philo. Mag. A, Vol. 42, No. 6,
 763 (1980).

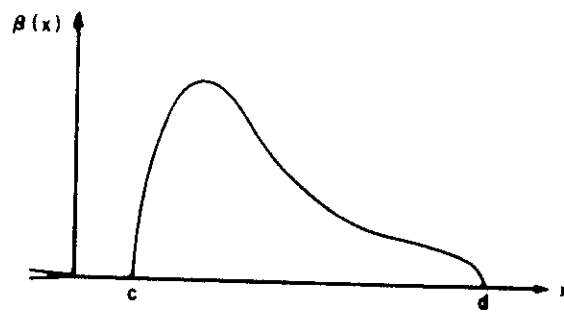
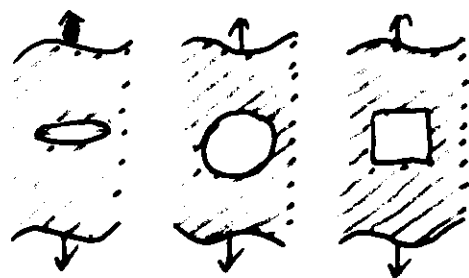
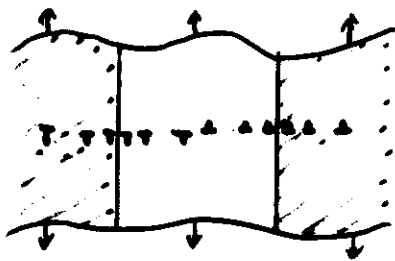


Fig. 20. Solution of eq. (50) for the continuous dislocation distribution, $\beta(x)$. The crack tip is at the origin, and the distribution begins at c and drops to zero at d . There is a strong maximum near c .

BCS - type Crack model

1. not consistent with the new DFZ expts.*
2. Dislocations with same Burgers vectors distribute as monotonical decreasing curve not consistent with expts.
3. Systems are not continuous and homogeneous.
 - (1) $\sigma_0 \neq \sigma_1$ (two kinds of materials)
 - (2) not continuous at boundaries.
4. Does not reflect the characteristic of the crack tip.

$$P_0 = R - \sigma_0 \text{ (const.)}, \quad P_1 = \sigma_1 - R \text{ (const.)}$$



- * Chang and Orr made their calculation consistent with DFZ expts. by firstly assuming a DFZ ($D(x)|_{x=0} = 0$) as a boundary condition for solving their singular integral equation.

How can DFZ form ?

BCS-type Model

This work

$$1. \quad \int_D = \int_{-a}^{-c} + \int_{-c}^c + \int_c^a \quad \int_0^a$$

$$2. \quad P(x_1) = \text{Const.}$$

$$P(x_1) = \text{Const.} + f(x_1)$$

$$\int_0^a \frac{f(x') dx'}{x_1 - x'} = \text{const.} + \frac{f(x_1)}{A}$$



Dislocation system under crack tip-like stress field.

Stress Field

$$A \int_0^a \frac{D(x') dx'}{x - x'} - \sigma_i + \sigma^\infty + \underline{\underline{\sigma^c}} = 0$$

$$A = Gb / 2\pi(1-\nu)$$

Small-Scale Yielding

$$\underline{\underline{\sigma^c}} = \frac{K}{(2\pi x)^{\frac{1}{2}}} = \frac{\sigma^\infty \sqrt{c}}{\sqrt{2x}}$$

$$K = \alpha \sigma^\infty \sqrt{\pi c}$$

Change variables

$$x' \rightarrow u (= \frac{x'}{a}) \rightarrow \zeta (= 2u - 1)$$

$$x \rightarrow w (= \frac{x}{a}) \rightarrow \eta (= 2w - 1)$$

$$A \int_{-1}^{+1} \frac{f(\zeta)}{\zeta - \eta} d\zeta = -P + \frac{\sigma^\infty \sqrt{\frac{c}{a}}}{(\eta + 1)^{\frac{1}{2}}}$$

$$\underline{\underline{f(\zeta) = -\frac{1}{\pi^{\frac{1}{2}} A} \left(\frac{1-\zeta}{1+\zeta} \right)^{\frac{1}{2}} \int_{-1}^{+1} \left(\frac{1+\eta}{1-\eta} \right)^{\frac{1}{2}} \frac{1}{\eta - \zeta} \left[-P + \frac{\sigma^\infty \sqrt{\frac{c}{a}}}{(\eta + 1)^{\frac{1}{2}}} \right] d\eta}}$$

The conditions for it to exist reduce to

$$\int_{-1}^{+1} \left(\frac{1+\eta}{1-\eta} \right)^{\frac{1}{2}} \left[P - \frac{\sigma^\infty \sqrt{c/a}}{(\eta + 1)^{\frac{1}{2}}} \right] d\eta = 0$$

$$\boxed{\frac{a}{c} = \left[\frac{2\sqrt{2} \alpha}{\pi(1-\alpha)} \right]^2}$$

$$\text{Here, } \alpha = \sigma^\infty / \sigma_i$$

$$D(u) = \frac{\sigma_1}{\pi^2 A} \left(\frac{1-u}{u} \right)^{\frac{1}{2}} (1-\alpha) \pi \left[1 - \frac{1}{4\sqrt{1-u}} \ln \left(\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}} \right) \right]$$

— * — * — * —

Elastic-Plastic Stress Fields

$$\sigma^e = \frac{\sigma^\infty}{\sqrt{1 - \left(\frac{c}{x+c} \right)^2}} - \sigma^\infty \begin{cases} (1) x \rightarrow 0 \\ \sigma^e \approx \frac{\sigma^\infty \sqrt{c}}{\sqrt{2x}} = \frac{K}{\sqrt{2\pi x}} \\ (2) x \rightarrow \infty \\ \sigma^e \rightarrow 0 \end{cases}$$

$$\alpha = \frac{\pi}{2\sqrt{2n} + \arccos \left(\frac{2n-1}{2n+1} \right)} \quad \left(n = \frac{c}{a} \right)$$

$$D(u) = E(\alpha) \sqrt{\frac{1-u}{u}} - F(\alpha) \frac{u+n}{\sqrt{u(u+2n)}} \ln \frac{[\sqrt{2n} + \sqrt{(2n+u)(1-u)}]^2 + u^2}{u(2n+1)}$$

where, $E(\alpha) = \frac{\sigma_1}{\pi^2 A} \left\{ 2\alpha \left[\arctg \sqrt{2n} - \frac{\pi}{2} \right] + \pi \right\}$

$$F(\alpha) = \frac{\alpha \sigma_1}{\pi^2 A}$$

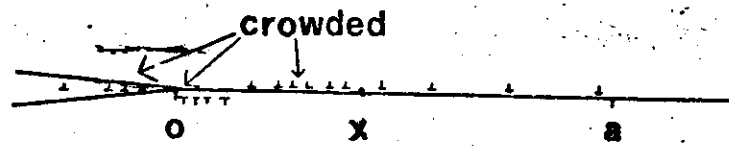


Fig.1

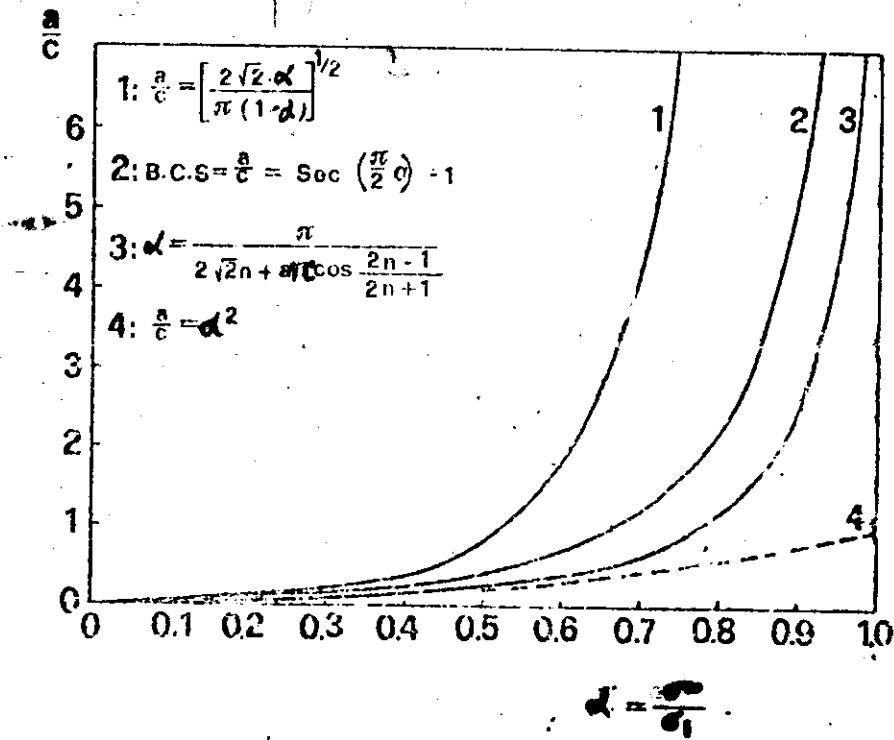


Fig.2

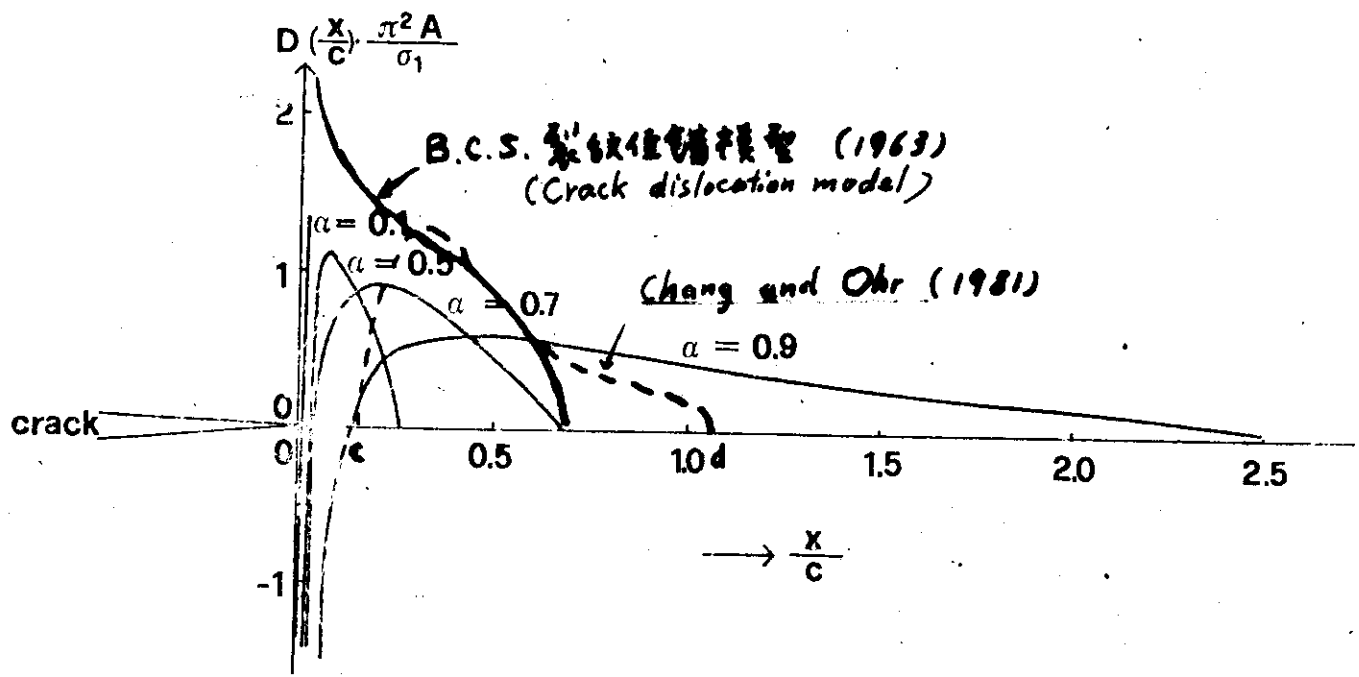


Fig.4(b)

phys. stat. sol. (a) 77, (1983), 81

(龙, 等.)

Chang and Ohr assumed a boundary condition, $D(x)|_{x=c} = 0$, on the basis of their dislocation free zone investigation. It is empirical.

How can DFZ form?

When a large number of mobile† sources S exist near the tip of a stopped crack (i.e., $\sigma < \sigma_F$), these sources will emit loops, some part C of which, having opposite sign from those of the crack dislocations, are attracted to the crack tip (Fig. 5.6a). The loops C move into the region near the tip until their total Burgers vector just compensates for that of the crack tip D ; this neutralizes the large stress field at the tip, both the back stress acting on the center of the crack and the forward stress that acted ahead of the crack. Because the back stresses are relaxed, dislocations can move from the center of the crack to the crack tip, and this blunts the crack (Fig. 5.6b). The crack, therefore, is no longer elliptical but has parallel sides, separated

† Unpinned or lightly pinned.

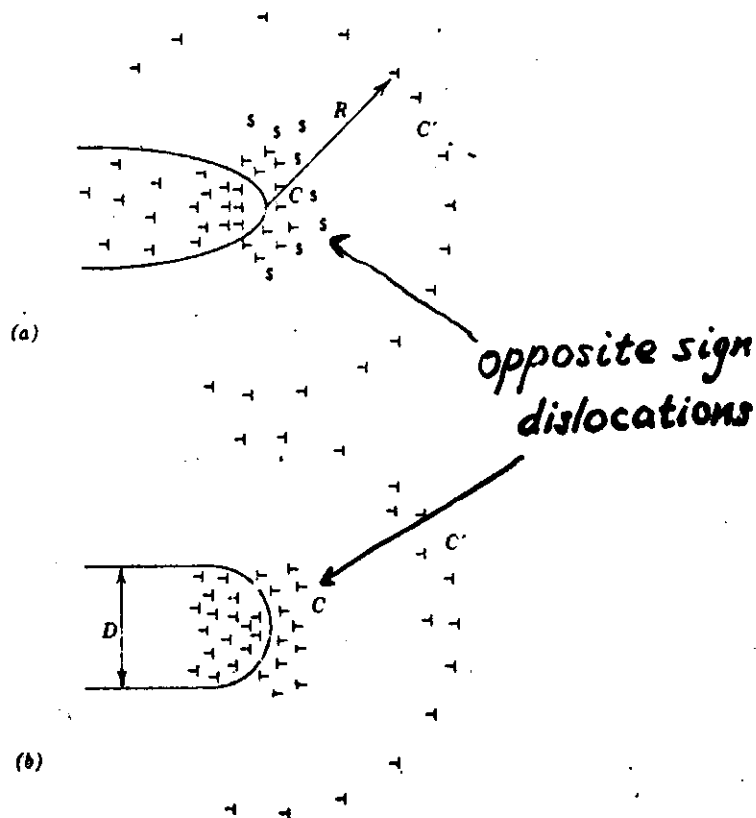


Fig. 5.6. Dislocation representation of crack blunting [2].

A. S. Tetelman (1969)

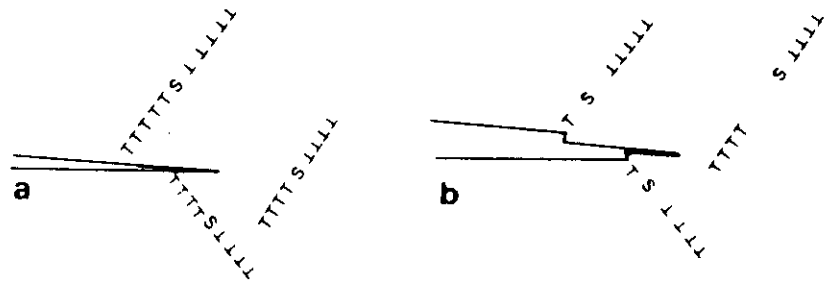


Fig. 19. Dislocation sources operate external to a crack and cause a crack-opening displacement as shown.

R.M. Thomson, in *Physics of Metallurgy*, II
ed. Cohn and Haasen, 1972, pp. 1514.

"pairs of opposite Burgers vectors."

"The one with shielding Burgers vectors
will be repelled from the crack and the other,
with antishielding Burgers vectors, will be attracted
to the crack. It will be absorbed and
annihilated by the open cleavage surface,
producing steps on these surfaces."

Semi-infinite Length Crack

We treated this problem within a homogeneous and continuous system. We only consider the plastic zone and let the K field to be an applied stress (external).

We use the image force expression for a semi-infinite length crack as the upper limit of the effect.*

1) Dislocations moving toward the crack tip:

$$A' \int_0^{\infty} \frac{D^I(x') dx'}{x+x'} - A' \int_0^{\infty} \frac{D^II(x') dx'}{x+x'} + \sigma_I + \sigma^{II} + \sigma^c = 0 \quad (1)$$

2) Dislocations moving away from the crack tip:

$$A \int_0^{\infty} \frac{D^I(x') dx'}{x+x'} - A \int_0^{\infty} \frac{D^II(x') dx'}{x+x'} - \sigma_I + \sigma^{II} + \sigma^c = 0 \quad (2)$$

where $A' = -A$.

* Lung and Wang, Phil. Mag. A, 1984, Vol 50, 5, L19

Zhou and Lung, J. Phys. F, 1988.

For small scale yielding case,

$$\sigma^c = \frac{K}{\sqrt{2\pi x}}, \quad K = \sigma^\infty (\pi c)^{\frac{1}{2}}$$

Let $u = x^2/a^2$, $w = x^2/a^2$, where a is the plastic zone size,

then from (1)

$$A \int_0^1 \frac{D^I(u) du}{w-u} = \sigma_1 + \sigma^\infty + \sigma^c$$

This singular integral equation can be solved by a finite Hilbert transformation. with the help of elliptic integrals.

$$D^I(x) = \frac{1}{\pi^{\frac{1}{2}} A} \left(\frac{a^2 - x^2}{x^2} \right)^{\frac{1}{2}} \left\{ (\sigma_1 + \sigma^\infty) \pi + \sigma^\infty \sqrt{\frac{\pi}{a}} \left[2F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right) - \sqrt{\frac{2a-x}{a^2-x^2}} \left[KZ\left(\sin^{-1}\sqrt{1-\frac{x}{a}}, \frac{1}{\sqrt{2}}\right) + \frac{\pi}{2} \Lambda_0\left(\sin^{-1}\sqrt{1-\frac{x}{a}}, \frac{1}{\sqrt{2}}\right) \right] \right] \right\}$$

The condition for it to exist leads to the relation

$$\int_0^{a^2} \left(\frac{x^2}{a^2 - x^2} \right)^{\frac{1}{2}} (\sigma_1 + \sigma^\infty + \sigma^c) dx^2 = 0$$

we obtain

$$\alpha = \frac{-\pi}{\pi + \frac{\pi}{3} \sqrt{\frac{\pi}{a}} F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right)}, \quad \alpha = \sigma^\infty / \sigma_1$$

Substitute c/a relation to $D^I(x)$ and let $\eta = \frac{x}{a}$

$$D^I(\eta) = \frac{\sigma_1}{\pi^2 A} \left(\frac{1-\eta^2}{\eta^2} \right)^{\frac{1}{2}} (1+\alpha) \left\{ \pi - \frac{3\pi}{4F(\frac{\pi}{2}, \frac{1}{\sqrt{2}})} \left[2F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right) - \sqrt{\frac{2\eta}{1-\eta^2}} \left(KZ(\sin^{-1}\sqrt{1-\eta}, \frac{1}{\sqrt{2}}) + \frac{\pi}{2} \Lambda_0(\sin^{-1}\sqrt{1-\eta}, \frac{1}{\sqrt{2}}) \right) \right] \right\}$$

where $F(\frac{\pi}{2}, \frac{1}{\sqrt{2}})$ is the complete elliptic integral of first kind.

The definition of $KZ(\beta, k)$ and $\Lambda_0(\beta, k)$ may be found in the Handbook.

Similarly, in the second case,

$$D^II(\eta) = \frac{\sigma_1}{\pi^2 A} \left(\frac{1-\eta^2}{\eta^2} \right)^{\frac{1}{2}} (1-\alpha) \left\{ \pi - \frac{3\pi}{4F(\frac{\pi}{2}, \frac{1}{\sqrt{2}})} \left[2F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right) - \sqrt{\frac{2\eta}{1-\eta^2}} \left(KZ(\sin^{-1}\sqrt{1-\eta}, \frac{1}{\sqrt{2}}) + \frac{\pi}{2} \Lambda_0(\sin^{-1}\sqrt{1-\eta}, \frac{1}{\sqrt{2}}) \right) \right] \right\}$$

The relationship of $D^I(\eta)$ with $D^II(\eta)$ is

$$D^II(\eta) = \frac{1-\alpha}{1+\alpha} D^I(\eta)$$

Usually, we use $\frac{x}{c}$ as unit.

$$D\left(\frac{x}{c}\right) = D\left(\frac{a}{c}\eta\right)$$

Summary:

1. Curves are qualitatively similar, but $D^I(x) > D^R(x)$.
2. The negative dislocation zone is enlarged by the image force.

The large repellent forces of a large number of negative image dislocations of the positive dislocations in the plastic zone.

3. The effect is overestimated due to the expression of image forces for a semi-infinite length crack.

Dislocation Distribution near a Finite Length Crack

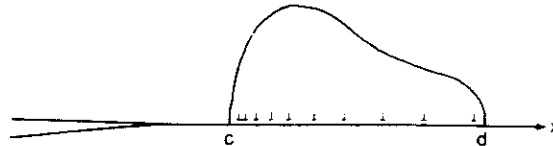
The expression of image forces is more complicated.

Numerical solutions are needed. Calculations are in progress.



Kobayashi and Ohr,
Scripta Metall. 15, (1981)
343.

FIG. 13. (a) Dislocations emitted from the tip of a crack in a foil of copper. The crack has grown from the left. Immediately ahead of the crack is a region free of dislocations. The pile-up of dislocations is seen extending on the cleavage-slip plane to the right.



S. Chang and Ohr S. M.
J. Appl. Phys., 52, 7174 (1981).

FIG. 45. Dislocation shielding configurations. (a) A Mode-III slit crack is depicted with a distribution of screw dislocations on its cleavage plane. The solid curve represents a continuous distribution function of the same set of dislocations, on the interval from $x_1 = c$ to $x_1 = d$.

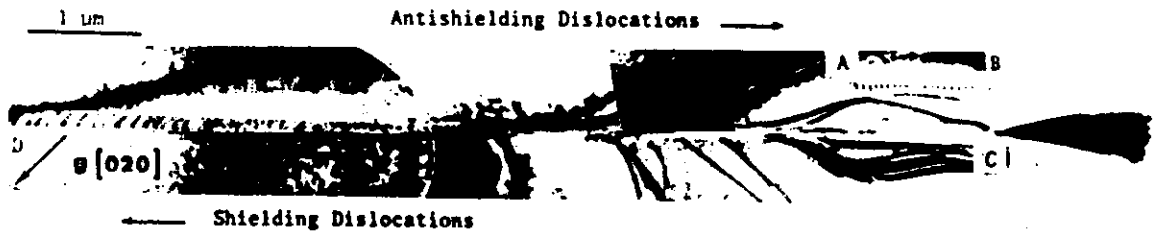


Fig. 1 Distribution of antishielding dislocations (AB) near a crack tip observed during in situ TEM studies of crack propagation in stainless steel. Shielding dislocations emitted from the crack tip are located between C and D.

Ohr S. M., 1987,
Scripta Metall. 21, 1681

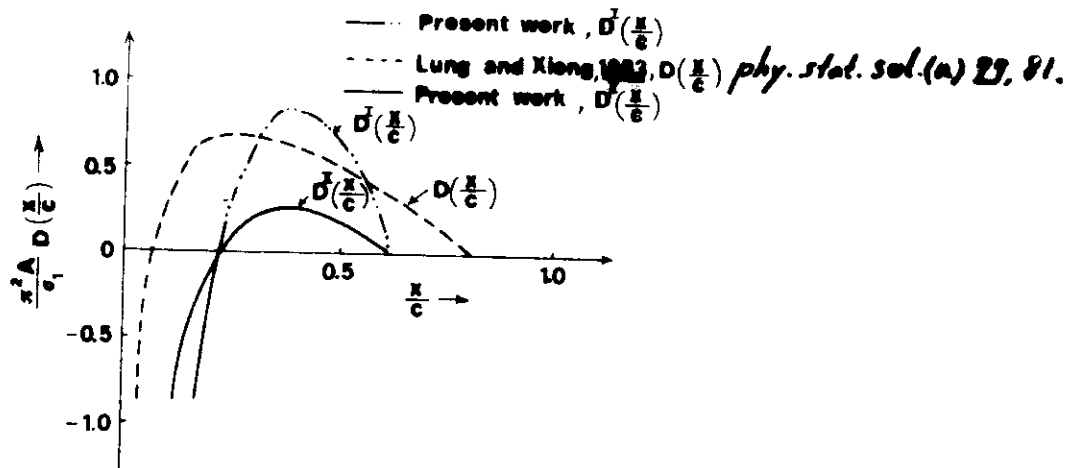
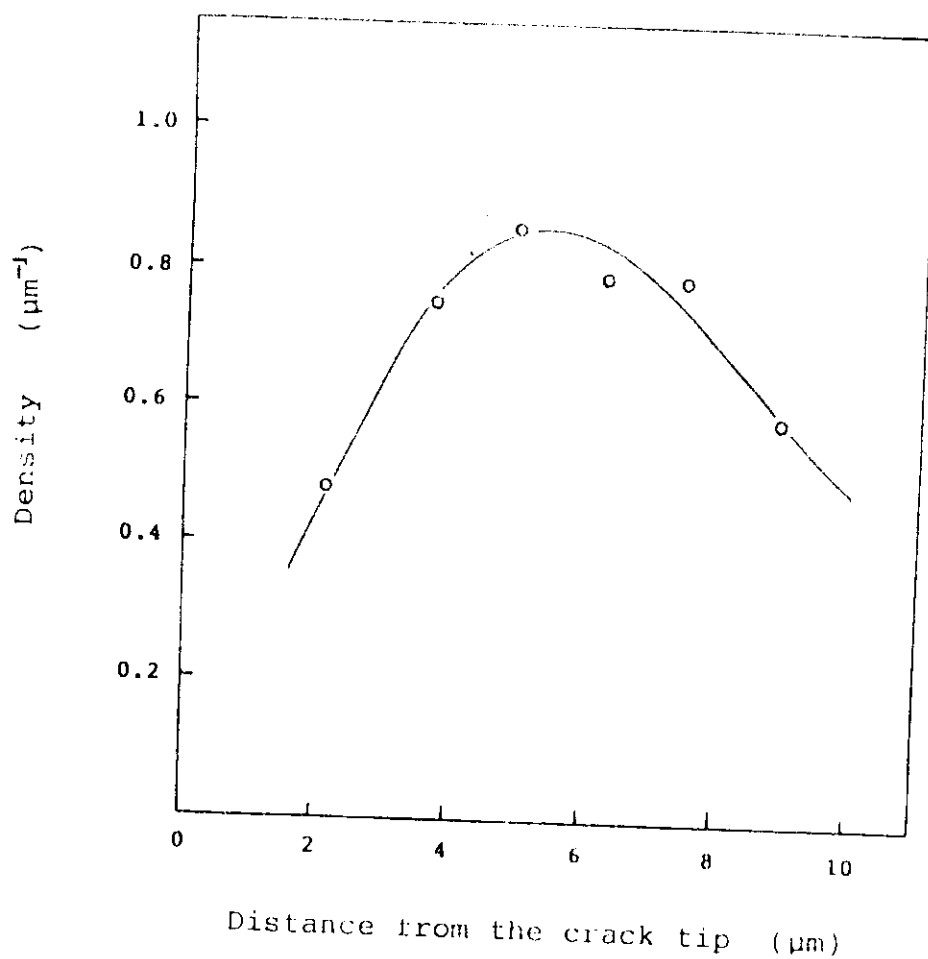


Fig. 3

Dislocation distribution curves calculated by various methods ($\alpha = 0.5$).

- a) $D^I(x/c)$,
- b) Lung and Xiong (1983), $D(x/c)$
- c) $D^{II}(x/c)$.

Lung and Deng: IC/88/130
(1988)



Zn Single crystal

The curve of dislocation density versus the distance from the crack tip. The density is plotted with $1/(X_{n+1} - X_n)$, and the distance with $(1/2)(X_{n+1} + X_n)$, here X_n is the distance of N -th dislocation from the crack tip.

FU Ran, Q. Y. LONG et al. (1989)

The formation of Dislocation Free Zone

The role of image force

The force on a dislocation in the presence of the crack and other dislocations

$$\bar{f}_d = \frac{K_{III}b}{\sqrt{2\pi x}} - \frac{\mu b^2}{4\pi x} + \sum_j' \frac{\mu b b_j}{2\pi(x-x_j)} \left(\frac{x_j}{x}\right)^{3/2} \quad (15.11)$$

(SFR/392 - 1)

$$K_{II} = k_{II} + \sum_j \frac{\mu b_j}{\sqrt{2\pi x_j}}$$

Assuming only one dislocation for simplicity and semi-infinite crack

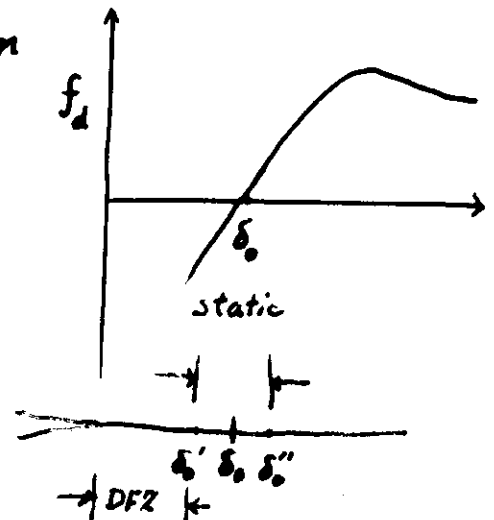
$$\bar{f}_d = \frac{Kb}{(2\pi x)^{3/2}} - \frac{Ab}{2x} \lesseqgtr 0 \quad \left(A = \frac{\mu b}{2\pi}\right)$$

$$\bar{f}_d > \sigma_1 b \quad \text{repulsive motion}$$

$$\bar{f}_d < -\sigma_1 b \quad \text{attractive motion}$$

$$\sigma_1 b > \bar{f}_d > -\sigma_1 b \quad \text{static}$$

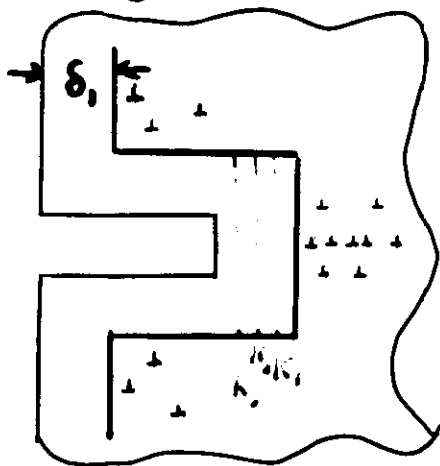
Let $\bar{f}_d = -\sigma_1 b$ we have $\delta_0(K) = x_0$



$$\underline{\underline{\delta_0'(K) = \left(\frac{A^2}{2}\right) \left[\left(A\sigma_1 + \frac{K^2}{2\pi}\right) + \sqrt{\left(A\sigma_1 + \frac{K^2}{2\pi}\right)^2 - A^2\sigma_1^2} \right]^{-1}}}$$

DFZ due to image forces

The image force theory for the formation
of dislocation free zone.



$$\frac{A}{2x_1} > \sigma_1, \quad x_1 < \frac{A}{2\sigma_1}; \quad \delta_1 = \frac{A}{2\sigma_1}$$

$$\frac{K_1}{\sqrt{2\pi\delta_1}} = \sigma_1$$

$$\delta_1 = \frac{A^2}{2\sigma_1^2}$$

This is contrary to Kobayashi and Ohr's expt. (1980)

$$\frac{Ab}{2\delta_0} - \frac{Kb}{(2\pi\delta_0)^{\frac{1}{2}}} = \sigma_1 b$$

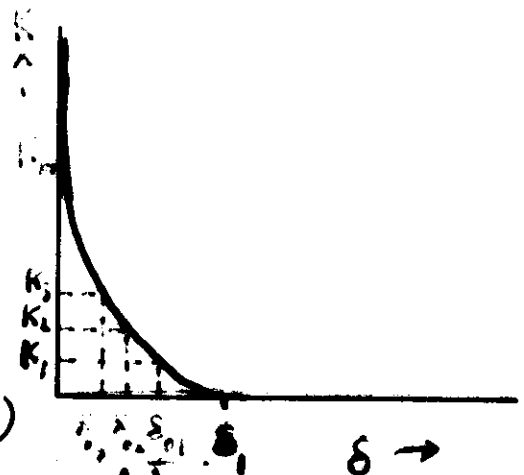
$$\delta_0 = \left(\frac{A^2 b^2}{2} \right) \left[\left(Ab^2 \sigma_1 + \frac{K^2 b^2}{2\pi} \right) + \sqrt{\left(Ab^2 \sigma_1 + \frac{b^2 K^2}{2\pi} \right)^2 - A^2 b^4 \sigma_1^2} \right]^{-1}$$

$$(1) \quad \frac{\partial \delta_0}{\partial K} < 0$$

$$(2) \quad K=0, \quad \delta_0 = \frac{A}{2\sigma_1} = \delta_1$$

$$(3) \quad \delta_0(K) \neq b, \quad K = K_m$$

$$(4) \quad K > K_m, \quad \text{no DFZ} \quad !! (\delta < b)$$



The crack inhibits the formation

of DFZ !!!

$$\text{Negative DZ size } x_0 \approx 0.64 \alpha^2 c / (\pi^2 (1-\alpha^2)) \quad (\alpha = \sigma^A / \sigma_1)$$

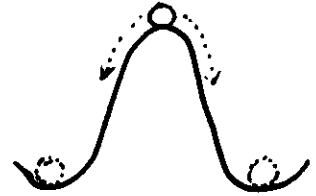
$$(1) \quad x_0 = 0 \rightarrow \infty \quad (\alpha: 0 \rightarrow \sigma_1); \quad (2) \quad \frac{\partial x_0}{\partial \alpha} > 0, \quad x_0(\alpha) \uparrow \quad (22)$$

Dislocations annihilation mechanism

The dislocation distribution calculated as above is not at the lowest energy state though forces are in equilibrium.

energy for new surface creation

$$\delta_s b \approx \frac{Eb^2}{20}$$



Selfenergy of a dislocation
 $\sim \mu b^2$

Positive and negative dislocations at the crack tip

prefer to annihilate to lower the energy of the system.

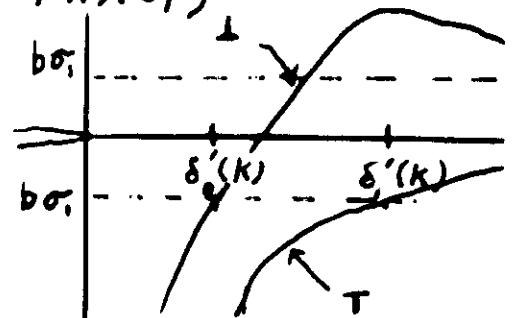
$$\bar{f}_d = \frac{-Kb}{(2\pi x_1)^{1/2}} - \frac{Ab}{2x_1}$$

Let $\bar{f}_\alpha = -\sigma, b$, we have $s'_i(K) = z_i$

$$\delta_1'(K) = \left(\frac{A^2}{2}\right) \left\{ \left(\frac{K^2}{2\pi}\right) - \frac{A\sigma_1}{2} - \sqrt{\left(A\sigma_1 - \frac{K^2}{2\pi}\right)^2 - A^2\sigma_1^2} \right\}^{-1}$$

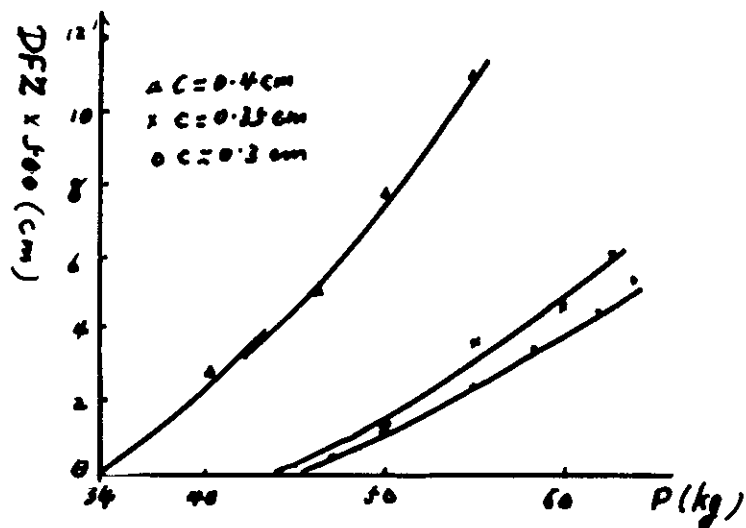
1. $\delta_i'(k) > \delta_0'(k)$
2. If $\delta_i'(k) > \text{negative dist. zone size}$

$$\delta_1'(k) \simeq \frac{1}{2} \alpha^2 a \propto \sigma^2 a$$



- $$3. \frac{\partial \delta'_i(k)}{\partial k} > 0$$

C. W. Lung, in *Progress in Physics, Chemistry & Mechanics*, (1988), 38. 23



Dependence of the size of DFZ on crack length
and applied load in Fe-3%Si single crystal.

$$1, \frac{\partial (DFZ)}{\partial K} > 0$$

$$2, DFZ \sim P^2$$

$$3, K_{DFC} \text{ exists } (P_{DFC} \sqrt{a} \sim 78; 82; 68 \text{ kg} \cdot \text{mm}^{\frac{3}{2}})$$

(mean value: 76 kg·mm^{3/2})

K. F. Ha et al. SMR/390-4

Fig. 13.

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