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SMR/390 - 12

WORKING PARTY ON "FRACTURE PHYSICS"
(29 May - 16 June 1989)

DISLOCATION SHIELDING
(Part II)

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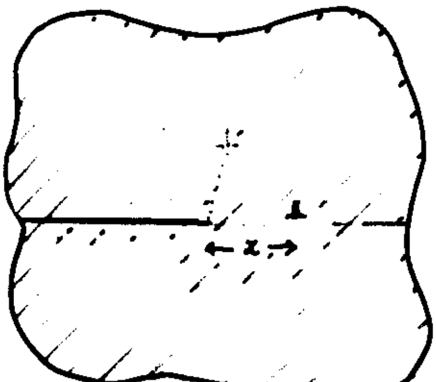
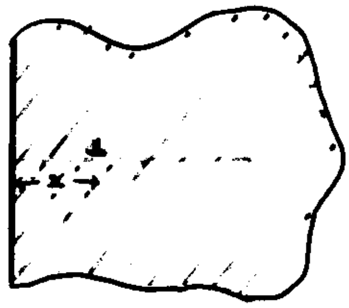
Image Force on the Dislocation near a Crack

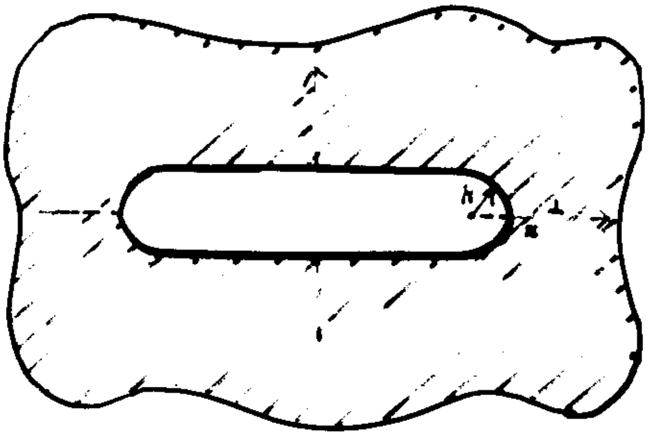


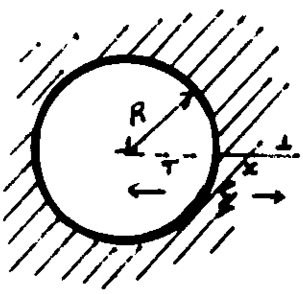
$$\boxed{\frac{Ab}{2x}}$$



? How about finite length crack

(Rice and Thomson (1973), Asaro (1975))



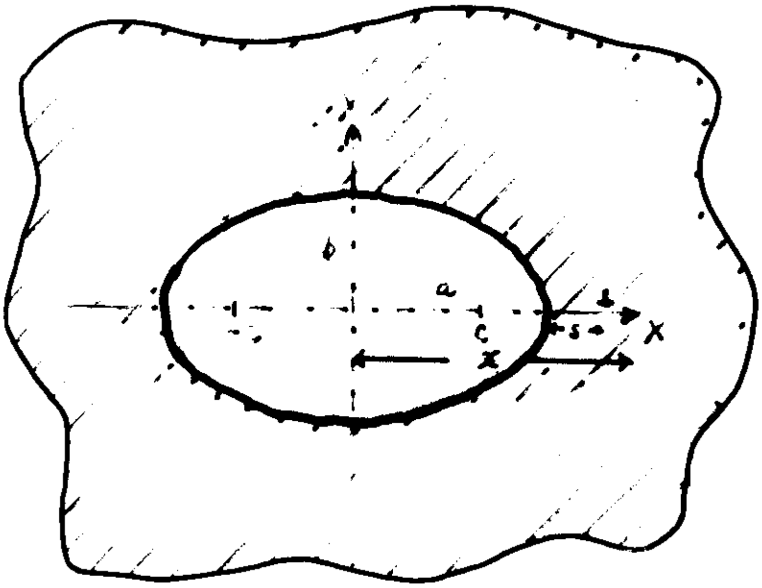


$$F^I = -\frac{\mu b^2}{2\pi}\frac{1}{\xi\left(\frac{\xi^2}{R^2}-1\right)}$$

(Eshelby)

$$\boxed{F^I \to 0 \quad (\text{as } R \to 0)}$$

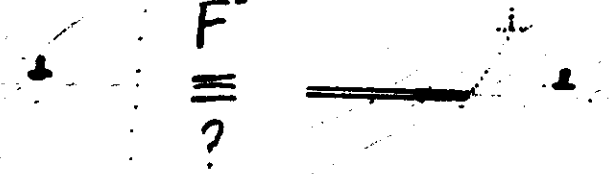
$$F^I_\perp = \frac{Ab}{\xi}$$



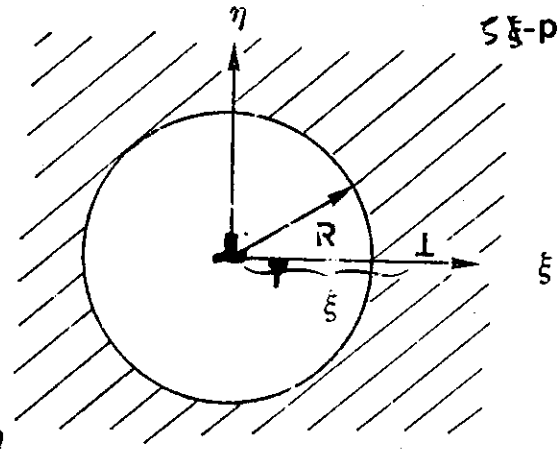
$$\boxed{F^I = F(x, a, b)}$$

1. geometrical shape
2. approximate answer
3. $\sum_i b_i = 0$ in a cavity

50, 5 (1984) L19
(L. and W.)
phy. stat. sol. (a)
85. (1984) K113
(L.)



$F^{I} \equiv ?$



$$F_{\xi}^{I} = -\frac{\mu b^2}{2\pi}\left\{\frac{1}{\xi - \frac{R^2}{\xi}} - \frac{1}{\xi}\right\}$$

$$= -\frac{\mu b^2}{2\pi}\,\frac{R^2}{\xi(\xi^2 - R^2)}$$

Fig. 1

$Z = Z(\zeta) = \frac{c}{2}(\zeta + \zeta^{-1})$

$\zeta = \zeta(Z) = (Z + \sqrt{Z^2 - c^2})/c$

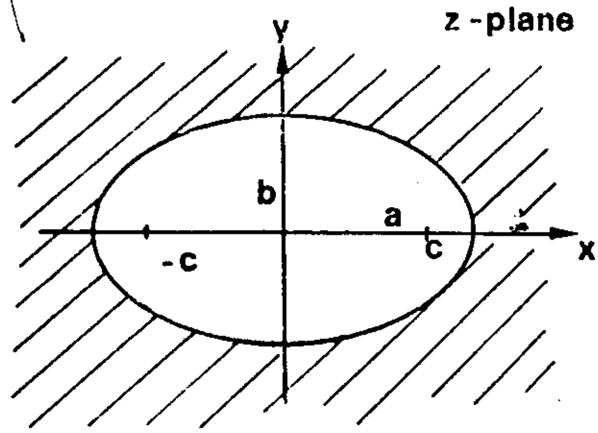
ICTP Preprint, IC/84/63,
(Lung, Wang)

$E(z)$ (z-plane)

$= E[Z(\zeta)]$

$= E(\zeta)$ (ζ-plane)

$$\mathrm{Re}\,E(\zeta) = \left(\frac{\mu b_s^2}{4\pi}\right)\ln\left[\frac{(\xi^2 - a^2)}{\xi^2}\right] + \text{const}$$



$$F_{z}^{I} = -\frac{dE(z)}{dZ}$$

$$= F_{\zeta}\left|\frac{d\zeta}{dZ}\right|$$

Fig. 2

$$\boxed{F_x = -(\mu b_s/2\pi)(a+b)^2\left\{\sqrt{x^2 - a^2 + b^2}\left[\left(x + \sqrt{x^2 - a^2 + b^2}\right)^2 - (a+b)^2\right]\right\}^{-1}}$$

(b = 0, the crack limit case.)

2
10

1. $b/a = k$ (const.), $(a+b) = a(1+k)$

F_x approaches zero as $a \to 0$

2. $a/b = 1$. a circular hole

$$F_x = \left(\frac{\mu b_s^2}{2\pi}\right) \frac{a^2}{x(x^2-a^2)}$$ Eshelby's expression $x = a+s$, $s \ll a$, $F_x \approx \frac{Ab_s}{2s}$

3. Let $x = a+s$ and $s \ll \rho$ $\left(\rho = \frac{b^2}{a}\right)$

$$F_x \approx \left(\frac{\mu b_s^2}{2\pi}\right) \frac{1}{2s} = \frac{Ab_s}{2s} \qquad \left(A = \frac{\mu b}{2\pi}\right)$$

Rice and Thomson (1973), Asaro (1975)

If the dislocation is much closer to the tip of the crack than the minimum radius of curvature, the dislocation sees a plane.

4. Let $b = 0$. This is the crack-limit case.

$$F_x = -\left(\frac{\mu b_s^2}{4\pi a}\right)\left\{\sqrt{\left(\frac{x}{a}\right)^2-1}\left[\left(\frac{x}{a}+\sqrt{\left(\frac{x}{a}\right)^2-1}\right)^2-1\right]^{-1}\right.$$

let $s \ll a$; then $F_s \approx \frac{Ab_s^2}{4s} = \frac{1}{2}F_s \left(s \ll \frac{b^2}{a}\right)$

3

Let $x = a + s$:

1. $F_{pl.} > F_{cir.} > F_{el.} > F_{cr.}$

2. $F_{pl.} \approx F_{cir.} \approx F_{el.} \approx F_{cr.}$ (s, $s/a < 0.005$)

3. $F_{el.} \approx F_{pl.}$ ($s << \rho\ (= b^2/a)$) (Rice and Thomson's case)

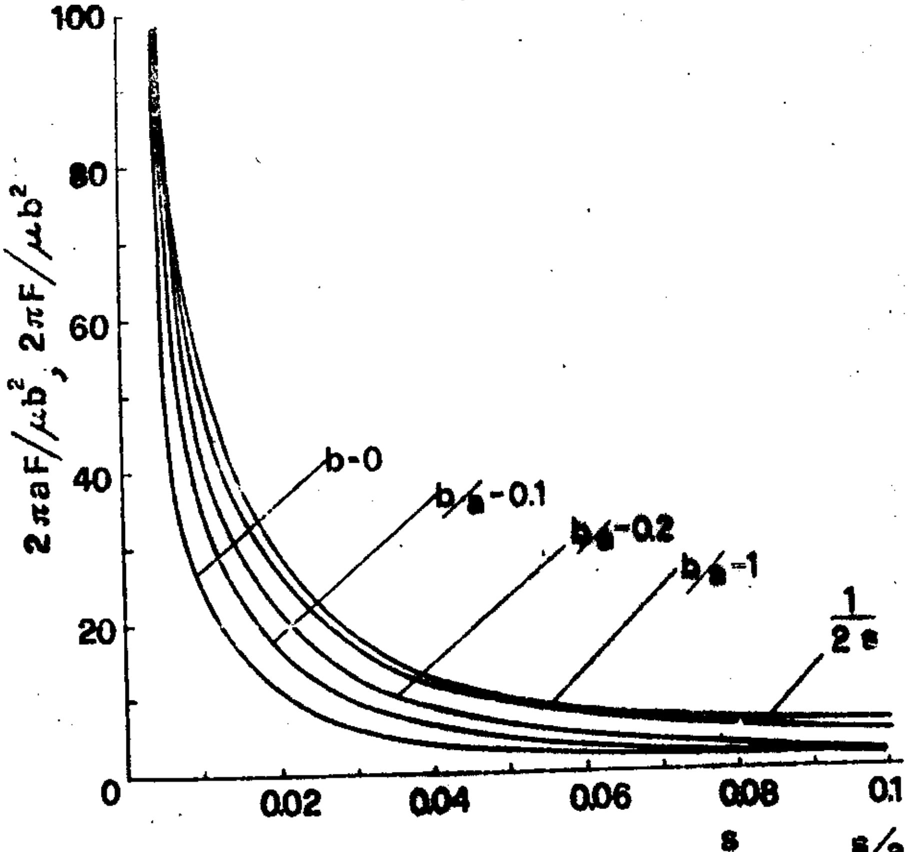


Fig.3

$$F_x = -(\mu b_s^2/2\pi a)\left\{\sqrt{(x/a)^2-1}\left[\left(x/a+\sqrt{(x/a)^2-1}\right)^2-1\right]^{-1}\right.$$

($b=0$) The crack limit case

4, $F_{cr.} \approx \frac{1}{2} F_{pl.}$ ($s << a$) (in general, for finite length crack)

Dislocation Emission from Cracks

The force component in the slip plane

$$Re(\bar{f}_d^{sp}) = \frac{k_{III} b_s}{\sqrt{2\pi r}} \cos(\theta/2) + \frac{b_e}{2\sqrt{2\pi r}}[k_I \sin\theta \cos(\theta/2)$$

$$+ k_{II}(2\cos(3\theta/2) + \sin\theta \sin(\theta/2))] - \frac{\mu}{4\pi r}\left(b_s^2 + \frac{b_e^2}{1-\nu}\right)$$

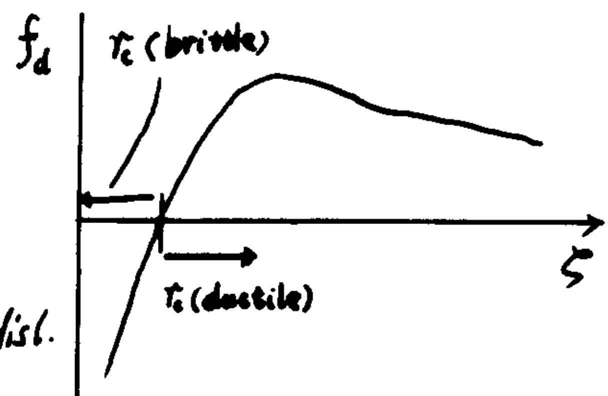
R. Thomson: ICTP SMR/390 — 1 (28.1)

let $\bar{f}_d^{sp} = 0$ solving for r_o

1, If $r_o > r_c\ (\sim b)$ Dislocations never traverse an attractive regime

2, If $r_o < r_c\ (\sim b)$ the net force is always repulsive

Let $r = r_c$, $f_d(K) = 0$ for the value of k which must be exceeded in order to emit a disl.



For simplicity, $k_{II} = k_{III} = 0$. Then

$$k_{IE} = \frac{\mu}{\sqrt{2\pi r_c}} \frac{b_s^2 + b_e^2(1-\nu)^{-1}}{b_e \sin\theta \cos(\theta/2)}$$

The Griffith criterion for cleavage $k_{Ic} = 2\sqrt{\mu\gamma/(1-\nu)}$

The combined criterion for cleavage/emission in pure Mode I

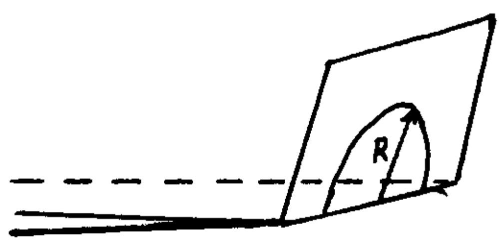
$$K_{IE} < K_{IC} \qquad \text{emission}$$

$$K_{IE} > K_{IC} \qquad \text{cleavage}$$

Thermally activated emission

In 3-D, a saddle-point configuration of a dislocation loop of finite size exists; the loaded crack can emit a dislocation by thermal fluctuation.

E_{act} *is an important quantity.*



Mode-I emission in 3D the shape of the critical nucleus is assumed to be a half-circle.

	K_{Ic}	K_{Ie}	K_{me} (cal.)	K_{me} (exp.)	E_{act}
Cu	6.4	3.21	1.5	0.4	
Al	3.4	2.17	1.0	0.7	
Ni	8.4	5.47	2.6	1.4	
Fe	8.8	9.34	4.2		2.2
Mo			8.2	9.2	

K in $10^5 N m^{3/2}$, E in eV

phys. stat. sol. [illegible] 198[illegible]
Acta met. Sinica, 19, A[illegible] (1983)
(in Chinese)

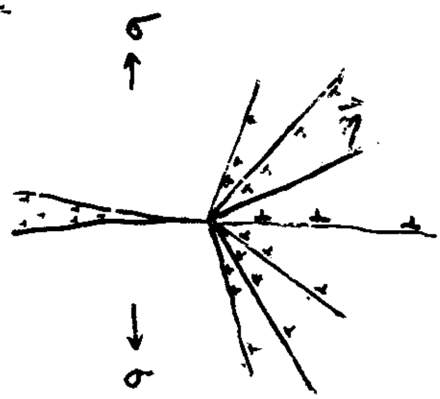
$\underline{\sigma_y - T}$ [3]:

$$\underline{\sigma_y(T) = \sigma^*(0)\, e^{-BT}}$$

$\underline{G_{ic} - T}$ [1]:

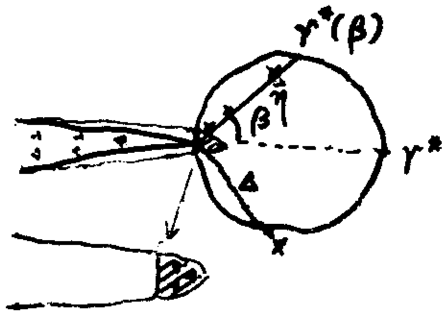
$$\underline{G_c(T) = G_0\, e^{T/T_0}}$$

$$L \cdot D^c(\eta) \int_0^{\eta} \tau(\vec{r})\, b\, dr$$



$$W(\vec{\eta}) = L \int_0^{r^*(\beta)} D^p(\vec{\eta})\, d\vec{\eta} \int_0^{\eta} \tau(\vec{r})\, b\, dr$$

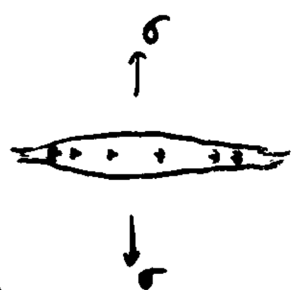
$$W_T = 2L \int_0^{\pi/2} d\beta \int_0^{r^*(\beta)} D^p(\vec{\eta})\, d\vec{\eta} \int_0^{\eta} \tau(\vec{r})\, b\, dr$$



$$\boxed{G_{ic}^P = 2\gamma_p = \frac{W_T}{L\Delta}}$$

$$\underline{\sigma_{yy}^A(x) + A \int_{-a}^{a} \frac{D(x')\, dx'}{x - x'} = 0}$$

$$\left(A = \frac{Gb}{2\pi(1-\nu)} \right)$$



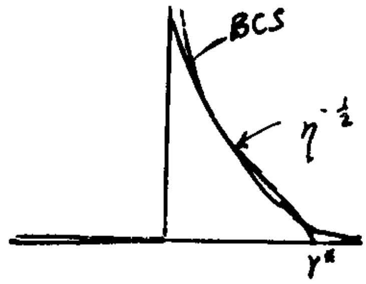
$$\sigma_{ij}^D(\vec{r}) = b \int_{-a}^{a} D_y(x')\, \sigma_{ij}^y(x - x', y, z)\, dx'$$

$$\underline{D(-s) \cong \frac{K}{\pi A (2\pi)^{1/2}} (-s)^{1/2}}, \qquad \underline{\sigma(s) \approx \frac{K}{(2\pi)^{1/2}}\, s^{-\frac{1}{2}}}$$

7/2

BCS model,

$D^p(s) \propto \cosh^{-1}\left|\frac{m}{s}\right|$, $\quad (m \cong 2ar_y^*/(r_y^*+a) \approx 2r_y^*)$



as $s/m < 0.1$, $D\sigma s \to \infty$ due to $s \to 0$;

as $s/m > 0.5$, $D\sigma s \to 0$ due to $D \to 0$.

$0.1 < s/m < 0.5$ $\quad (s/m)^{1/2}\cosh^{-1}(m/s) \approx 1$

In calculating W_i, we take

$$\underline{D^p(\eta) \cong C_0 B_0 \eta^{-\frac{1}{2}}}$$

$$C_0 = \frac{3}{4}\left(\frac{a}{r^*}\right)^{\frac{1}{2}}, \qquad B_0 = \frac{K_{ic}^\circ}{A\pi(2\pi)^{1/2}}$$

$$\tau_{r\theta} = \frac{K_I^\circ}{\sqrt{2\pi r}} F_1(\theta) \qquad (\text{Mode I})$$

$$\tau_{r\theta} = \frac{K_{II}^\circ}{\sqrt{2\pi r}} F_2(\theta) \qquad (\text{Mode II})$$

$$F_I(\theta) = \frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}$$

$$F_{II}(\theta) = \frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}$$

$$\frac{r_1^*}{r_2^*} \approx \left(\frac{A_1}{A_2}\right)^{1/2}$$

$$W_T = 2b\, B_0 C_0 D_0 L \pi r^* \propto r_1^{*\frac{1}{2}} \quad (\because C_0 \propto r^{*-\frac{1}{2}})$$

$$G_{Ic}^p = 2\gamma_p = \frac{W_T}{L\Delta} \propto r^{*\frac{1}{2}}$$

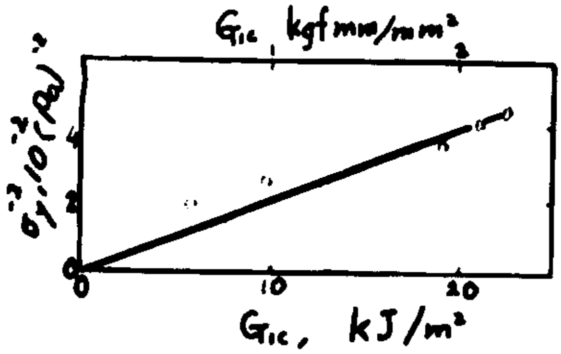
③

$$\overset{\text{work done}}{G_{Ic}(T) \propto r_c^*(T)^{1/2}} \overset{\text{stress equil.}}{\propto} \left(\frac{G_{Ic}(T)}{\sigma_y(T)}\right)^{1/2}$$

$$\boxed{G_{Ic}(T) \propto \sigma_y(T)^{-2}}$$

$$\frac{1}{T_0} = 2B$$

$G_{Ic}\cdot\sigma_y^{2}$ = 常 和Kraft[illegible]所得
(Const.) (consistent with)
结果[illegible]一致。



0.03 wt-%C的Fe-C钢的G_{Ic}（淬火后650°C回火2h）[7]
和σ_y^{-2}（800°C真空退火）[10]。

UNDER HOMOGENEOUS STRESS (K. Sieradzki, 1985)

For 2-D fracture problem the surface energy may be expressed as:

$$\gamma_s = \gamma_s^{\circ} \left(L/L_o \right)^{\bar{d}-1} \qquad (1)$$

where L, L_o ($\approx L_{av}$) the upper and lower limits of the self similar cracks respectively. L_{av} ~ average crack length. According to Griffith's criterion of fracture

$$\frac{\delta}{\delta L_{av}} \left[\frac{\pi \sigma^2 L_{av}^2}{E} + 2 L_{av} \gamma_s^{\circ} \left(L/L_o \right)^{\bar{d}-1} \right] = 0 \qquad (2)$$

is the applied stress. E is the elastic modulus of the system. Near the percolation threshold coherence length and the modulus may be expressed by

$$L_{av} \approx L_o \left(p - p_c \right)^{-\nu} \qquad (3)$$

$$E \approx E_o \left(p - p_c \right)^{\tau} \qquad (4)$$

Evaluating eq.(2) and employing (3) and (4)

$$\sigma_f \approx \left(\frac{\bar{d}}{2\pi L_o} \right)^{1/2} k_c^{\circ} \left(p - p_c \right)^{[\tau + \nu(2-\bar{d})]/2} \qquad (5)$$

where $k_c^{\circ} = (2E^{\circ}\gamma_s^{\circ})^{1/2}$. Near the percolation threshold $\bar{d} \approx 2$, so that

$$\sigma_f \sim \left(p - p_c \right)^{\tau/2} \qquad (6)$$

This implies that the rigidity exponent in the simulation performed by Ray and Chakrabarte might have been close to 2.

Ray and Chakrabarte : *(private communication, 1987)*

$$\sigma_c \sim E_F^{1/2} \, |p - p_c|^{\tau'} \sim |p - p_c|^{\tau'},$$

where $\tau' = [\tau + (d - d_B)\nu]/2$, E_F : limiting average strain energy of bonds.

10

UNDER INHOMOGENEOUS STRESS FIELD

In many actual fracture processes, microcracks combine to be a main long crack and then. the final fracture process is the propagation of the main crack. It is well known that the macrocrack propagates step by step. The local material just ahead of the crack tip consists of many microcracks, voids etc. It seems reasonable to assume that crack propagation is due to the local fracture of the porous structure just ahead of the crack tip.

$$\sigma(r) = \frac{k_{ef}}{\sqrt{2\pi r}}$$

$$r_f = \frac{1}{2\pi}\left(\frac{k_{ef}}{\sigma_f}\right)^2$$

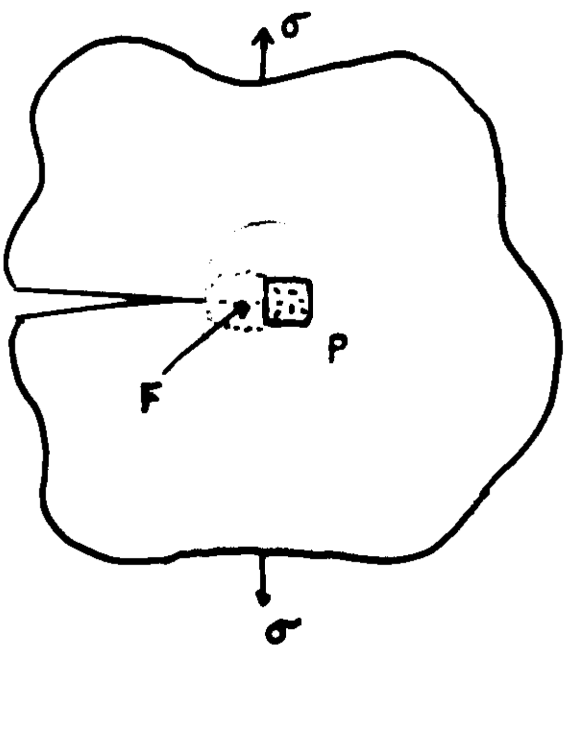
r_f is the distance of the crack propagation step.

Expressing k_{ef} as a function of dislocation parameters and σ_f as above, we may obtain the r_f in more detail form.

We remember that (Lung and Gao, 1985)

$$G_c^p \approx 2\gamma_p = \frac{W_i}{L r_f}$$

The critical crack extension force may be obtained.



THE EFFECT OF EXTERNAL DISLOCATION ON CRACK TIP PROCESS

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BACKGROUND

experiments

1) dislocation emission from crack and external sources (Ohr 1986, adequately large applied stress);

2) existence of negative dislocations at immediate crack tip (Ohr 1987, in stainless steel);

3) many dislocation dipoles around a crack tip (Narita et al 1987, with etch pits technique in Nacl).

previous theories

Dai and Li 1982, Majumdar and Burns 1983, Ohr 1985, Pande et al 1988, etc only discussed influence of crack-tip -generated dislocations on crack tip process

existence of negative dislocations at crack tip was pointed out by Lung and Xiong (1983). Thomson and Lin have done some work on it.

$\boxed{\text{what effect of external disloctions on crack tip process?}}$

ANALYSIS

mainly using a superdislocation pair model.

In terms of Li's analysis, force on negative dislocation

$$f = \frac{K_{III}}{\sqrt{(2\pi X)}}(-b) + \frac{\mu(-b)}{2\pi}\left[\frac{-(-b)}{2X} + \frac{\sqrt{d}}{\sqrt{X}}\left(\frac{-b}{d-X}\right)\right] \quad (1)$$

Critical stress intensity factor for an external screw dislocation pair emission is

$$K_{IIIp} = \sqrt{[2\pi(S - \frac{L}{2})]}\,(-\sigma_f + \frac{\mu b}{2\pi L}) \quad (5)$$

Condition for the negative one of an external dislocation pair annihilation at crack is

$$X_1 > r_c \quad \text{or} \quad K_{III} > K^-_{IIIa} \quad (6)$$

where

$$K^-_{IIIa} = \sqrt{(2\pi r_c)}\{\sigma_f - \frac{\mu}{2\pi}\left[\frac{b}{2r_c} - \frac{\sqrt{d}}{\sqrt{r_c}}\left(\frac{b}{d-r_c}\right)\right]\} \quad (7)$$

According to Majumdar and Burns (1981), stress intensity factor due to an emitted external dislocation pair is

$$K_{IIId} = -\frac{\mu}{\sqrt{(2\pi)}}\left(\frac{-b}{\sqrt{X}} + \frac{b}{\sqrt{d}}\right) \quad (8)$$

Stress exerted on external dislocation source is

$$\sigma_S = \frac{K_{III}}{\sqrt{(2\pi S)}} + \underbrace{\frac{\mu\sqrt{X}}{2\pi\sqrt{S}}\left(\frac{b}{X-S}\right) - \frac{\mu\sqrt{d}}{2\pi\sqrt{S}}\left(\frac{b}{d-S}\right)}_{Q} \quad (9)$$

Stress intensity factor due to negative dislocations

$$K^{-}_{IIId} \approx \int_{-\pi/2}^{\pi/2} \int_{0}^{2R\cos\theta} \left[-\frac{\mu(-b)}{\sqrt{(2\pi r)}}\right] \cos\left(\frac{\theta}{2}\right) g \frac{\sqrt{(2R)}}{\sqrt{r}} \, r\,dr\,d\theta \qquad (13)$$

Critical stress intensity factor to operate external source

$$K_{IIIs} = \sigma_{op} \sqrt{(2\pi S)} \qquad (14)$$

Critical stress intensity factor for dislocation emission from a crack defined by Ohr (1985), that is,

$$K_{IIIe} = \frac{\mu b}{\sqrt{(8\pi r_c)}} + \sigma_f \sqrt{(2\pi r_c)} \qquad (15)$$

DISCUSSION AND CONCLUSIONS

let $r_c = b/2$, $\sigma_f = \sigma_{op} = \mu/100$

(i) From (7) and (15), $K^{-}_{IIIa} < K_{IIIe}$ if $d > 1.3$ b.

An external negative disolcation can annihilate at a crack before a dislocation can emit from the crack tip.

$$\text{energy} \approx \begin{cases} (\mu b^2 + \gamma b) > 0 \\ (-\mu b^2 + \gamma b) < 0 \quad \text{(surface energy, } \gamma \approx \mu b/20) \end{cases}$$

Why negative dislocation existence at crack tip was observed rarely?

(ii) Since X < d, $K_{IIId} > 0$ from (8) i.e. net effct of an external dislocation pair on crack tip is antishielding.

Let $d = 200b$ and $K_{III} = 0$, then $X_1 \approx 9b$ from (8). In this case,

$K_{IIId} \approx 48\ \%\ K_{IIIe}$.

K_{IIIe} $\left\{\begin{array}{l}\text{measured values, about 50 kPa } \sqrt{m} \text{ in LiF (Burns 1986)} \\ \text{theortical values, about 230 kPa } \sqrt{m}\text{, using theories} \\ \text{(Rice and Thomson 1974, Ohr 1985)}\end{array}\right.$

(iii) For Frank-Read Source, if $L > 16\ b$, $K_{IIIp} < 0$ from (5).

(iv) From (14) and (15), if $S < 143\ b$, then $K_{IIIs} < K_{IIIe}$.

if sufficiently close to a crack, an external source can emit dislocations more easily than the crack,

Hockey's MgO observations: many cases where sources of dislocations operate near the crack position (or near a previous crack position), but only a few where the crack itself may have been the source of the dislocations)

(v) Let $2R = 143b$, $K^-_{IIId} > K_{IIIe}$ if $N^- > 5.5$ ($\rho > 2.6 \times 10^{15}/m^2$)

dislocation densities for a recrystallized and a heavily worked metal are about $10^{11}\ /m^2$ and $10^{17}/m^2$ respectively a physical picture:

when k_p, k_a, k_s < K < k_c, k_e, net effect of external dislocations on crack should be shielding so that k < k_a if positive dislocations and remaining portions of negative dislocations reach an equilibrium distribution.

If negative dislocations pile up against barriers in front of crack tip, antishielding effect is also likely to appear.

DFZ formation ?

external negative dislocations
1. may be closest to crack tip
2. annihilate or form locked dipoles with them
3. influence their emissiom from crack tip.

external dislocations play a very significant role in crack tip process and should be taken into account adequately.

ACKNOWLEDGMENTS

The authors would like to thank Professor L Y Xiong and Dr R Fu for helpful discussion.

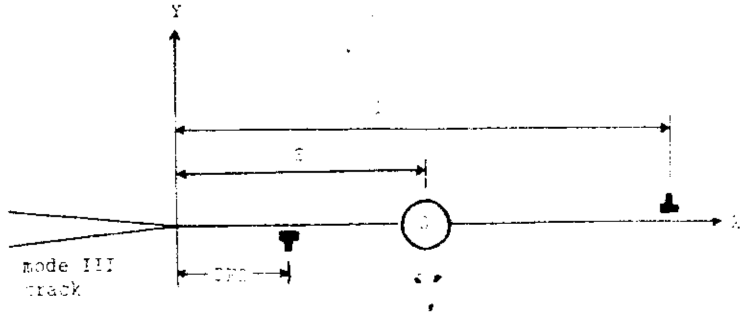


Figure 1. A pair of screw dislocations emitted from an external source labelled S in front of a sharp model III crack.

Force on the negative dislocation at x

$$f = \frac{K_{III}}{\sqrt{2\pi x}}(-b) + \frac{\mu(-b)}{2\pi}\left[\frac{-(-b)}{2x} + \sqrt{\frac{d}{x}}\left(\frac{-b}{d-x}\right)\right]$$

$$f = \begin{cases} \sigma_f & x_1 \\ -\sigma_f & x_2 \end{cases}$$

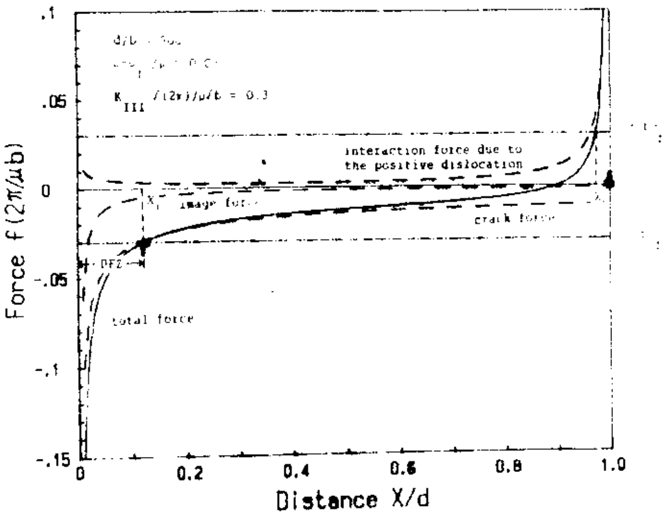


Figure 2. The force on the negative external dislocation near a crack tip of a mode III crack.

$$\frac{dx_1}{dK_{III}} > 0 \qquad \frac{dx_2}{dK_{III}} > 0$$

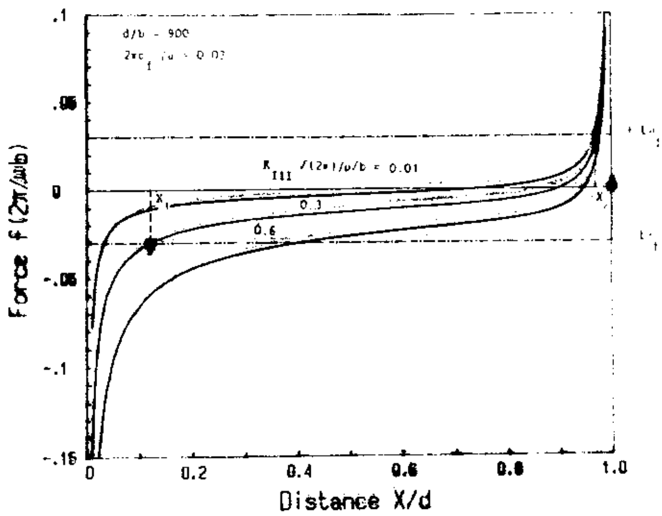


Figure 3. The effect of the crack tip stress intensity on the force exerted on the negative external dislocation.

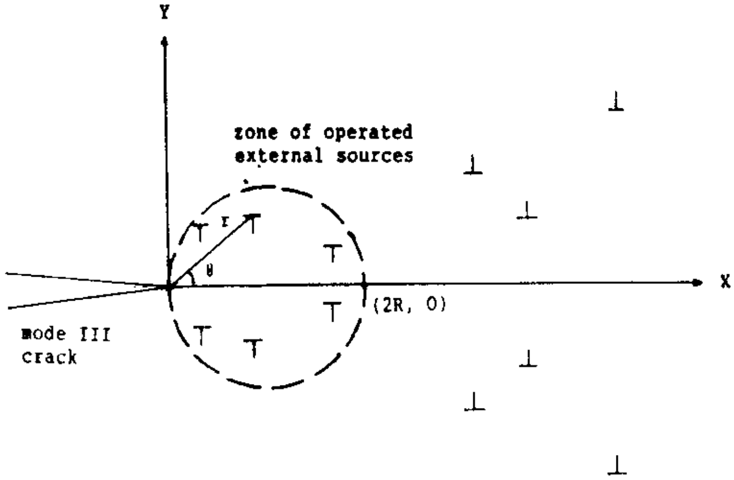


Figure 4. Schematic showing two dimensional distribution of negative and positive external dislocations near a sharp mode III crack tip

$$K_{III} = \left(\sigma_f + \frac{A}{L}\right)\sqrt{4\pi\left(s - \frac{L}{2}\right)} \approx \sigma_f\sqrt{4\pi s}$$

$$(L \to 0)$$

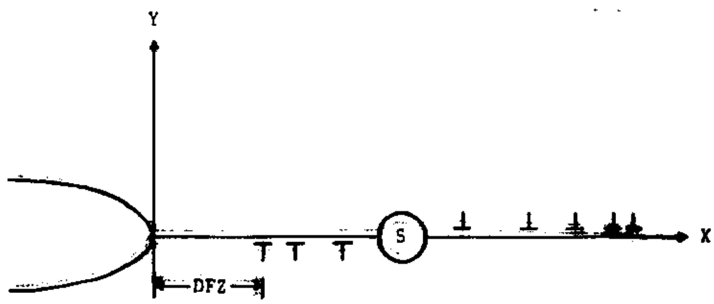


Figure 5. The positive external dislocations and the remaining negative external dislocations are in equilibrium in front of the bluated crack.

$$\boxed{\begin{matrix} K_i & & K_{ic} \\ & > & \\ G_i & & G_{ic} \end{matrix}}\quad 2\gamma_s$$

1. Dislocation theory

$$k_{eff} = K_i - \sum_j \frac{\mu b_j}{(2\pi \xi_j)^{\frac{1}{2}}}$$

2. Gauge theory

$$F_K = \int_\pi \left(L \delta_\ell^K - \frac{\partial L}{\partial X_{f,\ell}} X_{f,k} - \frac{\partial L}{\partial A^M_{\nu,\ell}} A^M_{\nu,k} \right) n_\ell \, ds$$

$$f_d^* = \sigma_i + Bv/b$$

B, due to phonon drag, electron drag and reradiation drag owing to emission of elastic waves by an accelerating dislocations

$2\gamma_p$

(1) $\sigma_y \delta_{cr.}$

(2) $2\gamma_s f[b, \rho^d, T, v^d \cdots]$
(A.S. Tetelman)

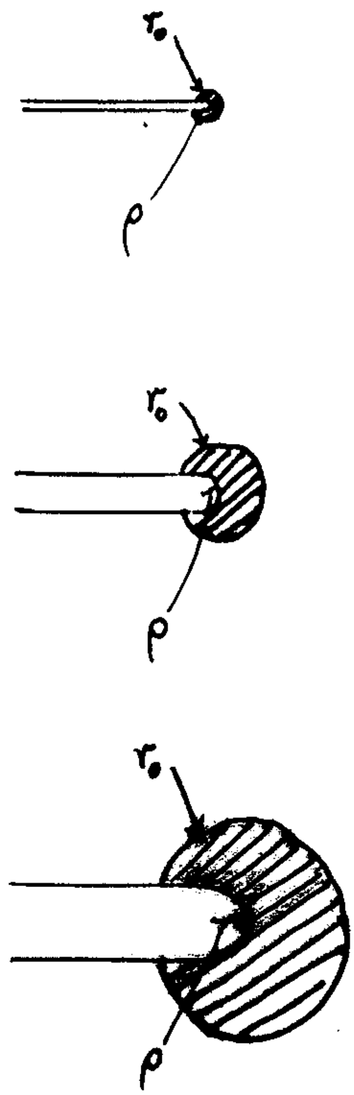
(3) $\propto r_c^{*\frac{1}{2}}(T)$
(phy. stat. sol. (a) 87, (1985) 565.

(4) $2\mu (2/\pi)^{\frac{1}{2}} K_c^* B x_1^{1/2}$
(superdislocation model IC/86/109)

(5) $\int_0^{r_1^*} f_d \, dr$

$$f_d = \sigma^K - \sigma^{1c} + \sigma^{2I} + \sigma^{3I} - \sigma_i - Bv/b$$

σ_i — frictional stress
(dissipated energy, irreversible)



ρ : the mean radius of curvature

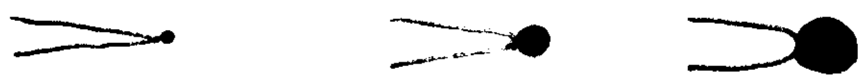
r_0 the core region radius

Is it still right for considering the stress field near $z=0$ only, as the ρ becomes larger ?

Gauge theory of defects

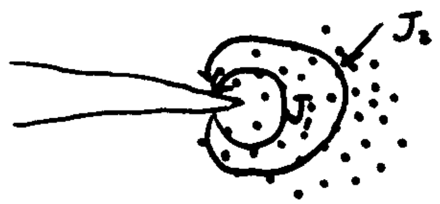
— Limitation of the local k approach

$\sigma_{z\to 0} \cong k/\sqrt{2\pi z}$ only true for the region near the crack tip. It needs a parameter to reflect



the intensity of stress field in a larger region.

— J Integral is not path independent in medium with defects in it.



$J_1 \neq J_2$

In J, there are no singularities representing the dislocations of the plastic zone, only a nonlinear stress-strain "constitutive law".

Is it possible to find another path indepeddent integral in a defect medium?

Edelen's Gauge Theory:

Noting that the group $SO(3) \triangleright T(3)$ may be viewed as a 6-parameter gauge group that leaves the Lagrangian of elastcity theory invariant, the Yang-Mills universal gauge theory construction is used to erect a complete continuum theory of material bodies with dislocation and dislination fields. Breaking of the homogeneity of the action of $SO(3)$ is shown to give rise to disclinations and rotational dislocations while homogeneity breaking of $T(3)$ gives rise to translational dislocations.

« A Gauge Theory of Dislocations and Dislocations » 1983.

Cracks can be treated as a distribution of virtual "crack" dislocations. Dislocations and disclinations are taken as gauge field of classical elastic field.

A covariant derivative is taken as

$$\partial_\beta X^i \longrightarrow D_\beta X^i = \partial_\beta x^i + g\, T_{kj}^{\,i} A_\alpha^\mu$$

where $T_{kj}^{\,i}$ are the structure coefficients, and A_α^μ are gauge potentials.

After proving E. Noether's theorem and the conservation laws, field equations of the system with both of two kinds of defects are obtained.

The total driving force (crack extension force) acting on a crack can be expressed as a path independent integral under the translation group of space.

Following Noether's theorem and standard steps in gauge field theory, we get field equations of a continuum with defects (general) braking symmetry of $SO(3) \triangleright T(3)$.

$$\begin{cases} \dfrac{\partial L}{\partial X_i} - \left(\dfrac{\partial L}{\partial X_{i,\nu}}\right)_{,\nu} = 0 \\ \dfrac{\partial L}{\partial A^M_\mu} - \left(\dfrac{\partial L}{\partial A^M_{\mu,\nu}}\right)_{,\nu} = 0 \end{cases}$$

For an elastic medium with defects (elastic) (in plane statics).

$$\begin{cases} A^N_{1,22} - A^N_{2,12} = 0 \qquad (N = 1, 2, \cdots 6) \\ A^N_{2,11} - A^N_{1,21} = 0 \\ ig(\hat{\sigma}_{11} a_2 - \hat{\sigma}_{12} a_1) + A^3_{1,22} - A^3_{2,12} = 0 \\ ig(\hat{\sigma}_{12} a_2 - \hat{\sigma}_{22} a_1) + A^3_{2,11} - A^3_{1,21} = 0 \\ -ig\hat{\sigma}_{11} + A^4_{1,22} - A^4_{2,12} = 0 \\ -ig\hat{\sigma}_{12} + A^4_{2,11} - A^4_{1,21} = 0 \\ -ig\hat{\sigma}_{21} + A^5_{1,22} - A^5_{2,12} = 0 \\ -ig\hat{\sigma}_{22} + A^5_{2,11} - A^5_{1,21} = 0 \end{cases}$$

For one single straight edge dislocation, (special defect)

$$b = -i\frac{2\pi(1-\nu)}{\mu g} \lim_{a_1 \to 0} a_1 (A^4_{2,11} - A^4_{1,21})\Big|_{a_2=0}$$

For a crack as dislocation pile ups, (special defect)

$$K = -i\frac{\pi}{gb}\left(\frac{\pi}{a}\right)^{1/2} \frac{1}{\frac{\pi}{2} - \arcsin\frac{c}{a}} \lim_{a_1 \to a} (a_1 - a)(A^4_{2,11} - A^4_{1,21})\Big|_{a_2=0}$$

where $c \in (-a, a)$, $2a$ is the length of a crack.

The criterion for a singularity motion (or a crack propagation) in a generalized continuum with defects. We can prove

$$\oint_{\Sigma}\left(L\delta_{\ell}^{k}-\frac{\partial L}{\partial X_{f,\ell}}X_{f,k}-\frac{\partial L}{\partial A_{\nu,\ell}^{M}}A_{\nu,k}^{M}\right)n_{\ell}\,ds=\frac{d}{dt}\int\left(\frac{\partial L}{\partial \dot{X}_{\nu}}X_{\nu,k}+\frac{\partial L}{\partial \dot{A}_{\nu}^{M}}A_{\nu,k}^{M}\right)dV$$

The l.h.s is a path independent integral, the r.h.s is the energy release rate or the force acting on the singularity (or crack extension force).

$$F_{k}=\int_{\pi}\left(L\delta_{\ell}^{k}-\frac{\partial L}{\partial X_{f,\ell}}X_{f,k}-\frac{\partial L}{\partial A_{\nu,\ell}^{M}}A_{\nu,k}^{M}\right)n_{\ell}\,ds$$

$$\boxed{F_{K} > F_{Kc}}$$ break off !

where F_{Kc} is the resistance to crack propagation, (a material constant).

In the case of defect free materials,

$$F_{k}=\int_{\pi}\left(L_{f}\,\delta_{i}^{k}-\frac{\partial L_{f}}{\partial X_{j,i}}X_{j,k}\right)n_{i}\,ds$$

J. D. Eshelby's energy-momentum tensor

J. R. Rice's J-integral.

$$F_k = \int_{\pi} \{ [L_0(\hat{C}_{\mu\nu}) - \frac{1}{4}(A^M_{[\nu,\mu]} + g T^M_{N\Gamma} A^N_\mu A^\Gamma_\nu)\cdot(A^M_{[\nu,\mu]} + g T^M_{NI} A^N_\mu A^I_\nu) \delta^k_i - \frac{\partial L_0}{\partial \hat{C}_{\alpha\beta}} \chi_{j,k} (\chi_{j,(\alpha}\delta^i_{\beta)} - g T^M_{j\lambda} \chi_\lambda A^M_{(\alpha}\delta^i_{\beta)}) + (A^M_{[\nu,i]} + \frac{1}{2} g T^M_{N\Gamma} A^N_{[i} A^\Gamma_{\nu]}) A^M_{\nu,k} \} n_i \, ds$$

where T^M_{NK} are structure constants of Lie algebra of local group $G = SO(3) \triangleright T(3)$, and

$$A^M_{[\mu,\nu]} = A^M_{\mu,\nu} - A^N_{\nu,\mu}, \quad \chi_{j(\alpha}\delta^i_{\beta)} = \chi_{j\cdot\alpha}\delta^i_\beta + \chi_{j\cdot\beta}\delta^i_\alpha$$

The integral F_k is the generalization of J. R. Rice's J-integral.

(The details please see: Dong, Zhang and Lung, ICTP Preprint, IC/86/144, or Int. J. Solid Structures, 1988, in press.)

We are on the way of exploring physical understanding of fracture, an every day problem in materials science and engineering.

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