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WORKING PARTY ON "FRACTURE PHYSICS"
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CRACK DISLOCATIONS

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DISLOCATIONS AND CRACKS

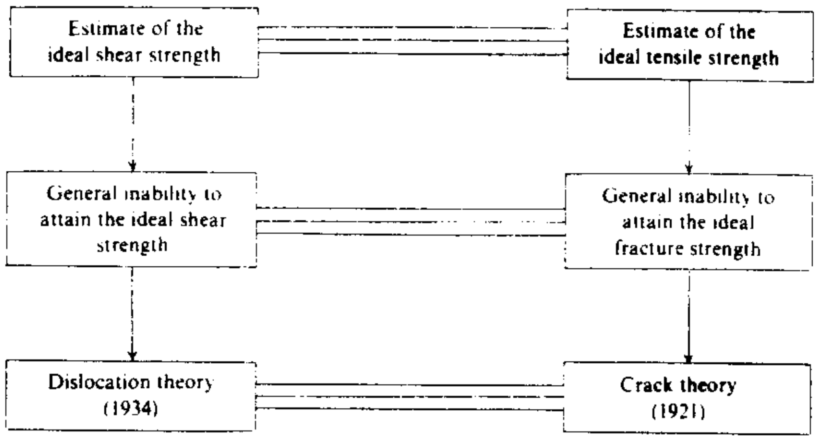
Deformation (non-elastically)

Fracture

Table I
The foundations of dislocation and crack theories

$\sim \mu/10$ $\sim E/10$

$(\sim 10^{-4}$ ~ 10[illegible]$)\mu$ $\sim 10^{-3}$ E



A particularly important example of a material with a low fracture strength is ordinary window glass which breaks at a stress of about 10^4 psi as compared with an ideal tensile strength of about 10^6 psi. The reason is that its surface contains cracks, typically about 10^{-4} cm deep, which arise for example during abrasion: these cracks can be detected [12] by decorating the surface with sodium vapor. Near-ideal tensile strengths have been obtained for some bulk solids such as glass, silica and sapphire, by taking exceptional care to avoid surface defects: thus, for example, a tensile strength of $E/20$ has been measured for 0.25 in (6 mm) diameter glass rods [13]. In this context it should be noted that fibres and whiskers have tensile strengths near the ideal value because their mode of preparation and handling is conducive to a good surface finish [14]. The importance of surface stress concentrations is recognized in many practical situations: thus the surface of glass in motor car windscreens is

Crack:
1. as a distribution of virtual "crack" dislocations
2. Dislocation shielding and anti-shielding.

(3. Dislocation role on crack nucleation and growth.)

29 Nov 1989

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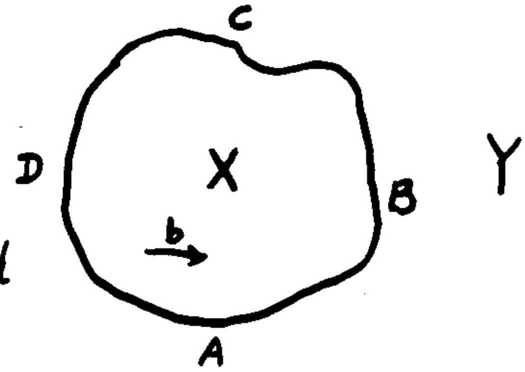
DISLOCATIONS

Dislocation:

The boundary line between a slipped and an unslipped area is called a dislocation line (ABCD).

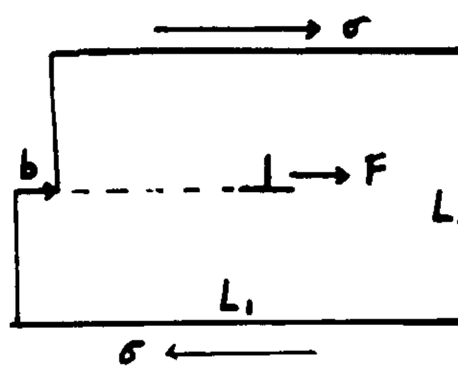


A dislocation line can never end within a crystal; it must form a closed ring, or end at a free surface, or be joined to other dislocation lines.

Imperfections in real crystal — The observed yield strengths of the softest crystals are in the range $10^{-5}\mu$ to $10^{-4}\mu$, i.e. several orders of magnitude below the shear strength of a perfect lattice.

Force on a dislocation:

$$F = (\sigma L_1 b)/L_1$$
$$= \sigma b$$

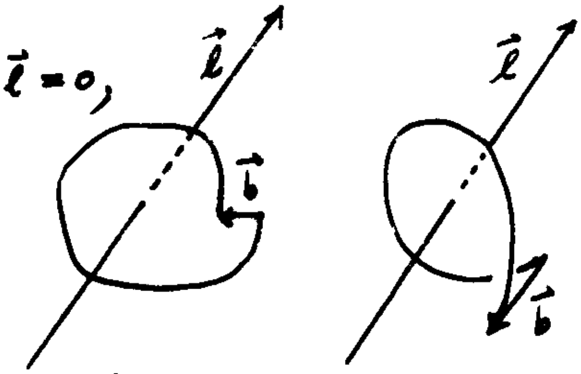


Screw and Edge Dislocations

Edge: $\vec{b}_e \times \vec{l} \neq 0$, $\vec{b}_e \cdot \vec{l} = 0$,

Screw: $\vec{b}_s \times \vec{l} = 0$

$\vec{b}_s \cdot \vec{l} \neq 0$



mixed type: $\vec{b} = \vec{b}_s + \vec{b}_e$

Elastic field of dislocations

an edge:

$$\sigma_{rr} = \sigma_{\theta\theta} = -D\sin\theta/r\,, \quad \sigma_{r\theta} = D\cos\theta/r$$

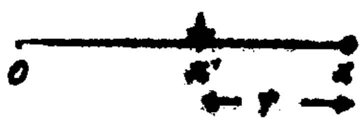
where $D = \mu b/2\pi(1-\nu)$ $(= A)$

an screw

$$\sigma_{\theta z}\ (= \sigma_{z\theta}) = \frac{\mu b}{2\pi r}$$

1-D:

$$\sigma^D = \frac{A}{x-x'} \quad \left(= \frac{A}{r}\right)$$



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An infinite isotropic continuum:

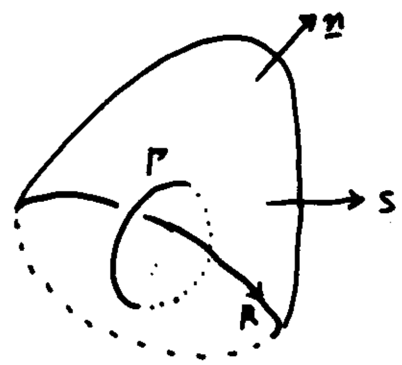
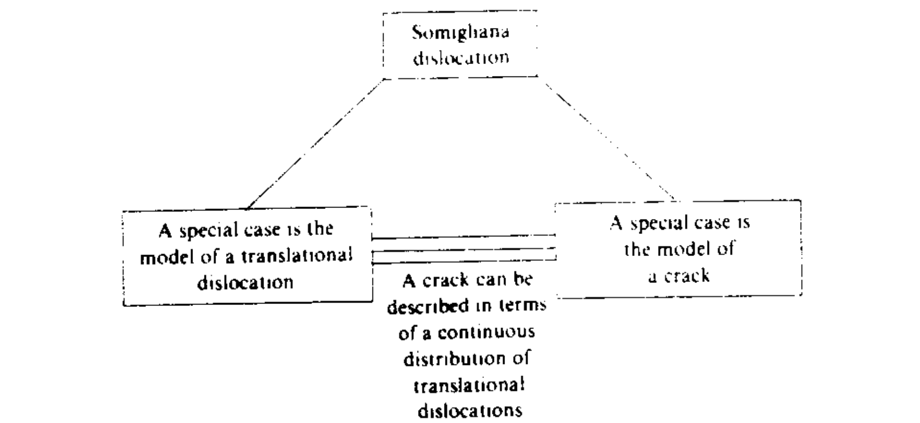


Table 2
Schematic representation of the relationship between the elastic continuum models of a translational dislocation and a crack



A cut is made over an open surface S with unit outward normal n; the positive side of the cut to which n points is displaced with respect to the negative side by a small constant vector d, which represents the magnitude and direction of the slip displacement; materials is removed, gaps are filled, the solid the being rewelded.

The translational dislocation ring R is the boundary of S.

$-d = b$ Burgers vector (RH/FS convention)

Crack — not rewelded.

Crack dislocation:

1. Inserting material with $D_1(x_1) > 0 \ (x_1 > 0)$; $D_1(x_1) < 0 \ (x_1 < 0)$

2. Generating stress distribution $p_{22}^{D}(x_1)$ along $x_2 = 0$,

3. An additional stress p_{22}^{A}; $D_1(x_1)$ is adjusted so that $p_{22}^{A} + p_{22}^{D} = 0$

4. The inserted material being removed without disturbing stress field.

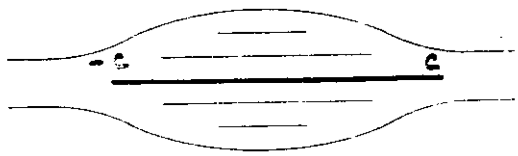


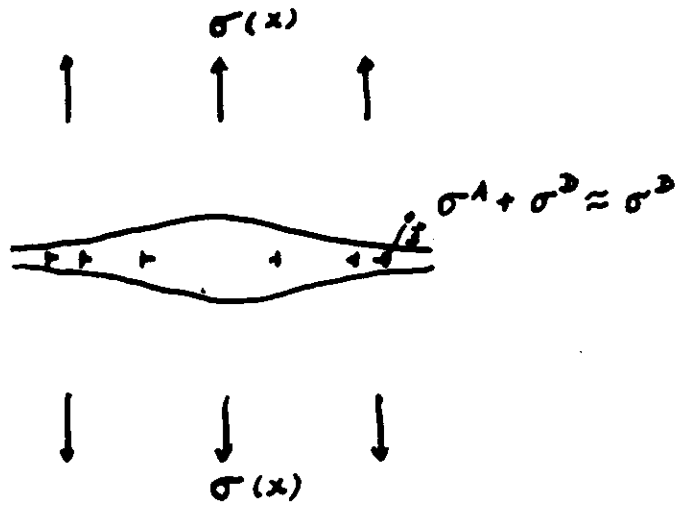
Fig. 10. The model used [28] for the interpretation of crack dislocations: material initially inserted

Then, in the region: traction-free — the behaviour of a crac[k]

These are crack dislocations

Crack dislocation	Ordinary dislocation
virtual	real
cuts unwelded	cuts rewelded
$p_{ij} n_j = 0$ on S	$p_{ij} n_j$ are continuous across S
$p^A \neq 0$; $D_1(x_1) \neq 0$	$p^A = 0$; $D_1(x_1) \neq 0$

Crack as a continuous distribution of dislocations



Traction-free surface condition (in crack region)

$$\sigma^A + \sigma^D = 0$$

$$\sigma_{yy}^A(x) + A\int_{-a}^{a} \frac{\mathcal{D}(x')\,dx'}{x - x'} = 0 , \qquad A = \frac{Gb}{2\pi(1-\nu)}$$

$$\sigma_{ij}^D(x) = b\int_{-a}^{a} \mathcal{D}_y(x')\,\sigma_{ij}^y(x-x', y, z)\,dx'$$

$$\sigma^D(s) \approx \frac{K^D}{(2\pi)^{1/2}}\, s^{-\frac{1}{2}} \qquad (x = s + a)$$

$$\mathcal{D}(s) \approx \frac{K^D}{\pi A (2\pi)^{1/2}}\,(-s)^{-\frac{1}{2}}$$

$$K^D = (2\pi)^{\frac{1}{2}} \lim_{s\to 0} s^{1/2}\,\sigma^D(s)$$

$$(K^D)^2 = 2\pi^3 A^2 \lim_{x\to a} (a-x)\,\mathcal{D}^2(x)$$

Bilby and Eshelby: 1968.
in « Fracture »

The solution of the Integral Equation

$$\mathcal{D}(x) = \frac{1}{A\pi^2}\int_{-a}^{a}\left(\frac{a^2-x'^2}{a^2-x^2}\right)^{1/2}\frac{\sigma^A(x')\,dx'}{x-x'} + \frac{n}{\pi(a^2-x^2)^{1/2}}$$

where, $\quad n = \int_{-a}^{a}\mathcal{D}(x)\,dx$

= the total number of dislocations

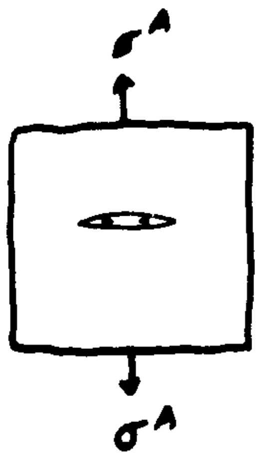
It may be determined by boundary condition.

Central Crack under Uniform Tension

$\sigma^A = P$ (const.),

$n = 0$

$$\mathcal{D}(x) = \frac{Px}{\pi A}(a^2-x^2)^{1/2}$$

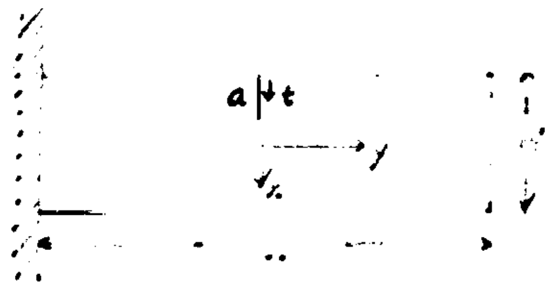


$K = P\sqrt{\pi a}$

<u>*The same as obtained in fracture mechanics.*</u>

[illegible]

$$\sigma_y(x)\Big|_{y=0} = -\alpha x$$



where,

$$\alpha = \frac{M}{2I}, \quad I = \frac{1}{12}B(W-a)^3 \qquad \left(\frac{W+a}{2}, 0\right)$$

and $\quad x = t - \frac{W-a}{2} - a = t - \frac{W+a}{2}$

The integral equation becomes

$$-\alpha\left(t - \frac{W+a}{2}\right) + \int_0^a \frac{D(t')\,dt'}{t-t'} = 0$$

The solution is (under the boundary condition

[illegible])

$$D(t) = \frac{\alpha}{A\pi}\left[(a^2-t^2)^{\frac{1}{2}} - \frac{a^2}{(a^2-t^2)^{1/2}}\right] + \frac{\alpha(W+a)t}{2\pi A(a^2-t^2)^{1/2}}$$

Then, $\quad K = \dfrac{M\,Y(a/W)}{B(W-a)^{3/2}}$

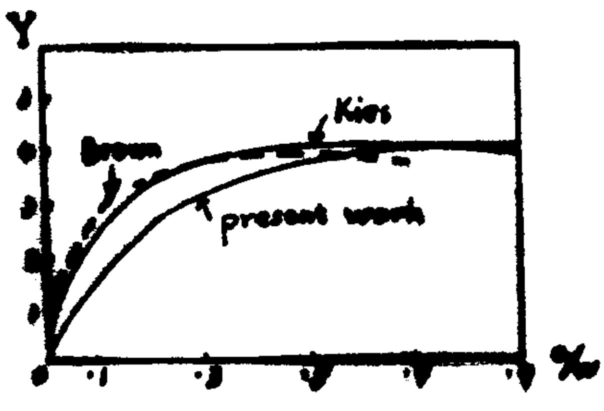
$$Y\left(\frac{a}{W}\right) = 3\sqrt{\pi}\left(\frac{a}{W}\right)^{1/2}\left(1-\frac{a}{W}\right)^{-1/2}$$

This function may be improved by adding a modified term:

$$Y\left(\frac{a}{W}\right) = 3\sqrt{\pi}\left(\frac{a}{W}\right)^{1/2}\left[1 - \frac{a}{W} + \beta\left(\frac{a}{W}\right)^2\right]^{-1/2}$$

β may be determined by Comparing to Wilson's analysis $Y_2\left(\frac{a}{W}=1\right) = 4,\qquad \beta_2 = 1\cdot 74$

Y \ a/W	0	0·1	0·3	0·5	0·7	1·0
Kies	0	2·82	3·87	4.09	4·12	4·12
Brown	0	3·00	3·83	3·98		
the above	0	1·74	3·14	3·88	4·14	4·0
(× 1·12)		(1·95)	(3·51)			



The differences between the present work and Kies' result is quite small as $\frac{a}{W} \geq 0\cdot 5$.

Lung : Proc. Beijing Conf. Fract. Mech. (1978).

[Acta Metallurgica Sinica, vol 14 (1978) 118 (Lung).]

Chebyshev polynomials: (for complicated applied stress)

$T_0(x) = 1, \quad T_1(x) = x, \quad T_2(x) = 2x^2 - 1, \quad T_3(x) = 4x^3 - 3x \cdots$

$U_0(x) = 0, \quad U_1(x) = 1, \quad U_2(x) = 2x, \quad U_3(x) = 4x^2 - 1 \cdots$

$$\boxed{\frac{1}{\pi}\int_{-1}^{1} \frac{1}{y-x}\left[\frac{T_n(y)}{(1-y^2)^{1/2}}\right] dy = U_n(x)}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

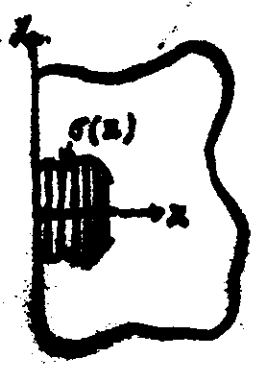
$$D(y) \qquad\qquad \sigma_{yy}^A(x)$$

$$\hookrightarrow K^D$$

$$\boxed{A\int_{-1}^{1} \frac{D(\frac{y}{a})\, d(\frac{y}{a})}{(\frac{y}{a}) - \frac{x}{a}} = \sigma_{yy}^A\left(\frac{x}{a}\right)}$$

$$\sigma_{yy}^A(x) \longrightarrow \sum_n C_n(a)\, U_n\left(\frac{x}{a}\right)$$

$$\boxed{D\left(\frac{x}{a}\right) = \frac{1}{A\pi} \sum_n \frac{C_n(a)\, T_n(\frac{x}{a})}{\left[1 - (\frac{x}{a})^2\right]^{1/2}}}$$



for edge crack in an infinitely large slab.

If, $\underset{\text{(arbitrary distribution)}}{\sigma(x) = \sigma \sum_{n=0}^{3} C_n \left(\frac{x}{a}\right)^n}$, $\quad K^D = \sigma C_0 \underline{F(a)} \sqrt{\pi a}$;

$$\underline{F(a) = 1.12 + 0.56\left(\frac{C_1}{C_0}\right) + 0.56\left(\frac{C_2}{C_0}\right) + 0.42\left(\frac{C_3}{C_0}\right)}$$

$$\underline{F(a) = 1.12 + 0.677\left(\frac{C_1}{C_0}\right) + 0.532\left(\frac{C_2}{C_0}\right) + 0.438\left(\frac{C_3}{C_0}\right)} \left[\begin{array}{l}\text{Handbook} \\ \text{Sih G.C., (1973)}\end{array}\right]$$

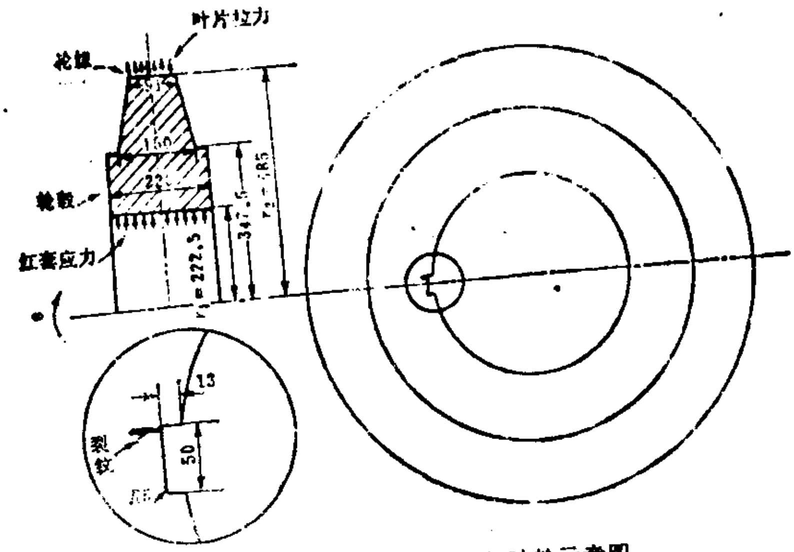


图6-1　Aπ-25-2型第14级叶轮示意图

2.5～3。1#机组自1952年投入运行至1975年发现裂纹，累积运行15.4万小时，工作情况正常。该叶轮材料金相组织为回火索氏体。化学成分和常规力性见表1和表2。

表1　材料的化学成分Wt%

C	Si	MN	Cr	Ni	Mo	P	S
0.33	0.20	0.47	[illegible].82	2.82	0.31	0.014	0.018

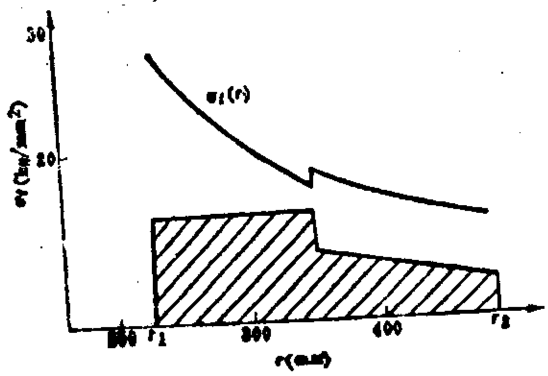


图6-2　Aπ-25-2型第14级叶轮正常运转时切应力沿径向分布图

表2　材料的常规力性

部　位	$\sigma_{0.2}$ (kg/mm²)	σ_b (kg/mm²)	δ_5 (%)	ψ (%)	a_k (Kg-M/cm²)	FATT (℃)
轮　缘	72.8	90.2	18.9	54.8	7.8	15
轮　毂	68.8	87.[illegible]	20.3	59.1	8.8	36

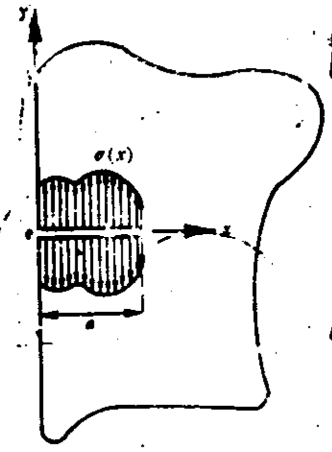
中国科学, Vol.XXIII, No.4 (1980) 441.

2

$$\boxed{\begin{gathered}\sigma_{yy}^{A}(x) + A\int_{-a}^{a}\frac{D(x')dx'}{x-x'} = 0 \\ K^2 = 2\pi^3 A^2 \lim_{x\to a}(a-x)D^2(x)\end{gathered}}$$

$$\sigma(x) = \sigma\sum_{n=0}^{n} C_n\left(\frac{x}{a}\right)^n$$

$$K = a_0 F(a)\sqrt{\pi a}$$



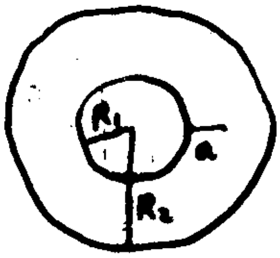
$$F(a) = 1.12 + 0.56\left(\frac{c_1}{c_0}\right) + 0.16\left(\frac{c_2}{c_0}\right) + 0.02\left(\frac{c_3}{c_0}\right) + \cdots$$

蒋易明:

$$F(a) = 1.12 + 0.677\left(\frac{c_1}{c_0}\right) + 0.52\left(\frac{c_2}{c_0}\right) + 0.438\left(\frac{c_3}{c_0}\right)\cdots$$

Fig. 2. A single edge crack under arbitrary distribution of applied stresses in a semi-infinitely large plate.

percentage are about 2—15%. The calculation based on the dislocation theory is simpler (Fig. 3).



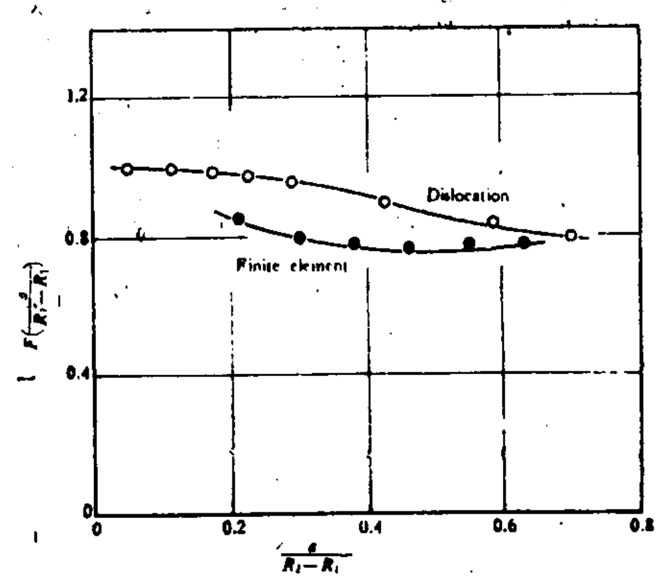


Fig. 3 The stress intensity factors of a rotary plate with an edge crack at the central hole.

The Chebyshev polynomials may be applied to more general cases. Any function $\sigma(x)$ may be considered as a "vector" in Hilbert space. The orthogonal fundamental function sets (such as $U_n(x)$ or $T_n(x)$) are the base vectors in Hilbert space. If the orthogonality and weight functions are known, the components of every base vector (the coefficient of certain $U_n(x)$ terms in the expansion of the function) can be calculated by the expansion of $\sigma(x)$. So any stress distribution function $\sigma(x)$ can be expressed in terms of $U_n(x)$ in principle.

汽轮机叶轮: 转速 3000 转/分, $K_{Ic} = 380$ kg/mm$^{3/2}$

$a_c = 60 \sim 70$ mm

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For a rotary plate with an edge crack at the central hole.

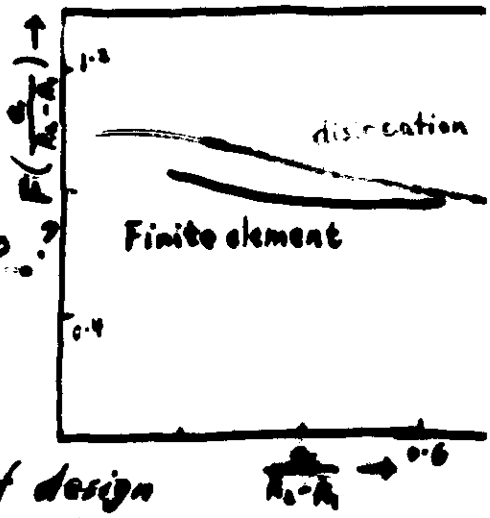
$\frac{\Delta K}{K}$ [illegible] % (in comparison with finite element method)

For an actual steam turbine wheel. ($f = 3000$ rot/min.)

$K_{Ic} = 380$ kg/mm$^{3/2}$ (experimental results)

a (cm)	1	2	3	4	5	6	7
K (kg/mm$^{3/2}$)	16[illegible]	230	277	314	344	371	394

a_c [illegible] ~ 70 mm



Is there any advantage of doing so?

1, Wide application possible

$\sigma_{yy}^{A}(x)$ (crack-free material) → data of design; → experimental measurements (Lin D[illegible] Act. Mech. Sinica, 2(1979)190.)

2, It would save us much time and labour of calculation.

Many engineering problems require prompt solution rather than accuracy.

6