INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE CENTRATOM TRIESTE

H4-SMR 393/2

SPRING COLLEGE ON PLASMA PHYSICS

15 May - 9 June 1989

SOLAR MAGNETOHYDRODYNAMICS

B. Roberts

University of St. Andrew's
Dept. of Applied Mathematics
Fife. Ky 169Ss
Scotland
U. K.

Lecture 1

B. Roberts
St. Andrews University

SOLAR MAGNETOHYDRODYNAMICS

The Sun is the Largest plasma that we will eve study in any detail.

But why study the Sun?

- because its there and has an intrinsic beauty and fascination
- because it offers a 'Rosetta Stone' for the understanding of stellar physics
- because it offers us the chance of studying a plasma in extremis, to test our understanding/faith in laboratory plasmas, numerical simula codes, other plasmas (magnetosphere, ...), etc.

What are the basic properties of the Sun? It is

HOT, COOL, WINDY and NOISY!

It is PLACID and DYNAMIC!

"—— it is reasonable to hope that in a not too distant future we shall be competent to understand so simple a thing as a star."

Sir Arthur Stanley Eddington,

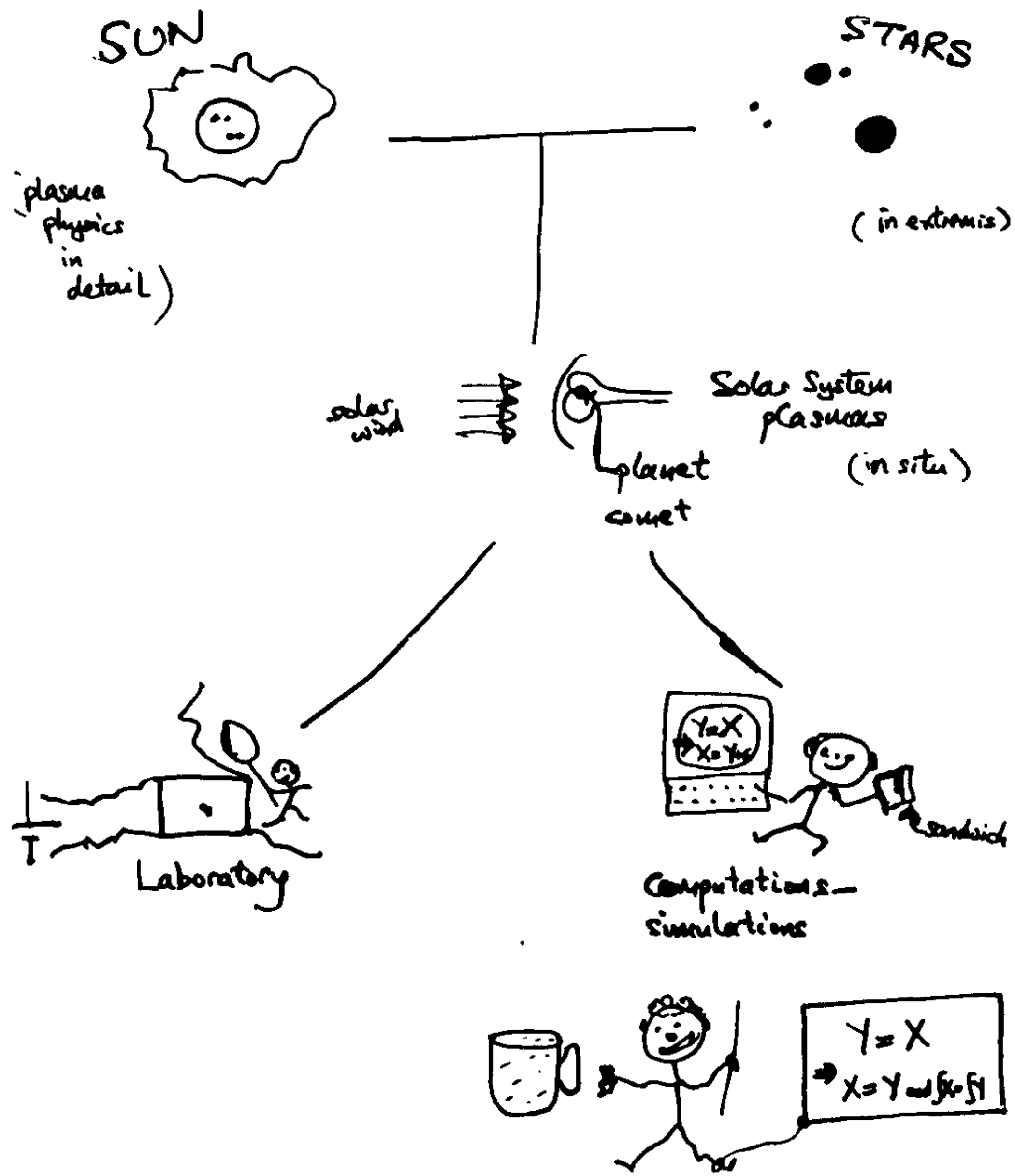
'The Internal Constitution of the Stars'

(1926)

"The fundamental equations of physics may contain all knowledge, but they are close-mouthed and do not volunteer that knowledge".

(E.N. Parker, 1979, 'Cosmical Magnetic Fields')

Solar — Stellar — Laboratory Connection



Understanding the Sun

2 key ingredients :—

$\exists$ of $\underset{\sim}{B}$

$\exists$ of small-scales in $\underset{\sim}{B}$ or ρ or T [cf. ...

These two ingredients lie at the heart of nearly all observed solar phenomena

Sun without $\underset{\sim}{B}$?

$\underset{\sim}{B}$ is a parasite!

Field strengths

	field strength (in gauss)
galaxy	10^{-6} G
Earth	0.4
household bar magnet	$10-10^{2}$
solar intense tubes	10^{3}
Sunspots	few $\times 10^{3}$
NMR (medicine)	$>10^{4}$
Industrial magnet	10^{6}
plasma (in laboratory)	10^{6-8}

Magnetic pressure $\propto B^2$, so ranges between $10^{-12} - 10^{2\ldots}$ (36 orders of magnitude

No $\underset{\sim}{B}$ means there would be

No

flares (reconnection of magnetic field)

corona (heated by magnetic field; chromosphere is partly accounted for by the dissipation of sound waves)

solar wind (since this is simply the expansion of the hot corona)

prominences (since no magnetic support and thermal instab. processes wouldn't operate if no corona!)

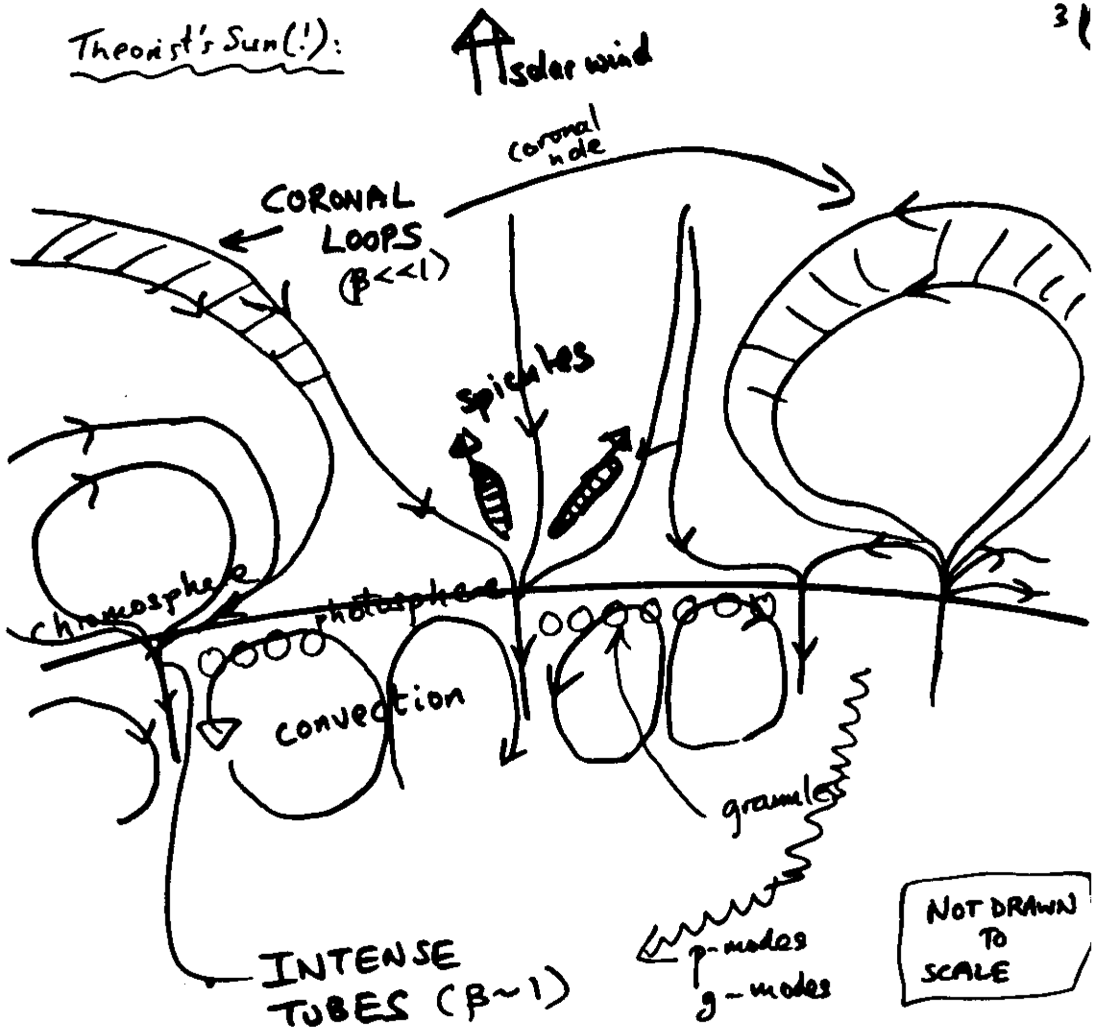
sunspots

etc.

– we would have a placid, almost perfectly spherical Sun – beloved of the Medieval Philosophers

– and a duller subject for sure!

Theorist's Sun(!):



Atmosphere is highly structured by magnetic field and strongly stratified by gravity

Magnetic structuring – may consider different regions
chromosphere/corona, $\beta << 1$
convection zone, $\beta >> 1$

$\beta = \frac{\text{gas press.}}{\ldots} = P/\ldots$

Solar MHD

Questions

(1) Why is the Sun's surface field so fragmented and concentrated into flux tubes? How universal is this?

(2) How does the surface field merge into the chromosphere and corona?
Why is the corona inhomogeneous?
· What is the temp. structure in loops? How do prominences form?

(3) How are sunspots (large flux tubes) formed; what is their internal structure like? (A monolithic tube or a 'spaghetti' clustering of small tubes?)

(4) How are spicules generated?

(5) How is the corona heated?

(6) What info. from p- & g-modes: helioseismology

(7) Where is the solar dynamo located and what is its nature? What causes solar (+stellar) activity?

(8) Implications for stars.

Magnetically,

May divide solar atmosphere into 3 regions:—

$$\beta \equiv \frac{\text{gas press.}}{\text{magn. press.}} = \frac{1}{B}$$

Corona / chromosphere

$\beta \ll 1$: magnetic forces dominate

photosphere – convection zone

$\beta \gg 1$: magnetic forces small

except in

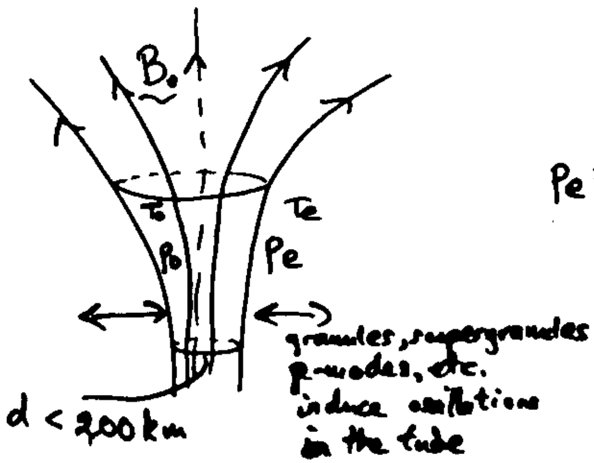
photospheric flux tubes

$\beta \sim 1$: magnetic and pressure forces comparable.

Three fundamental structures :-

- photospheric flux tubes
- coronal loops
- current sheets

Photospheric flux tubes (intense tubes)



$$p_e = p_0 + \frac{B_0^2}{2\mu_0}$$

$T_0 \simeq T_e$

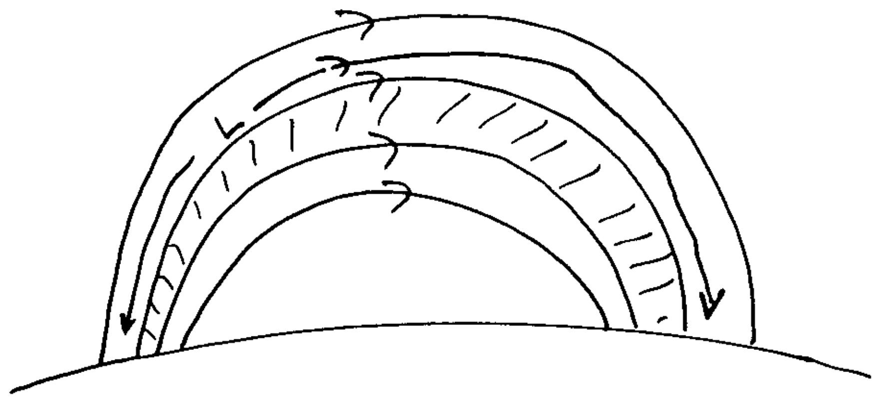
$p_0 < p_e$

$\rho_0 < \rho_e$

B outside of tube very small (a few gauss)

B inside tube, 1–2 kG.

Coronal loops

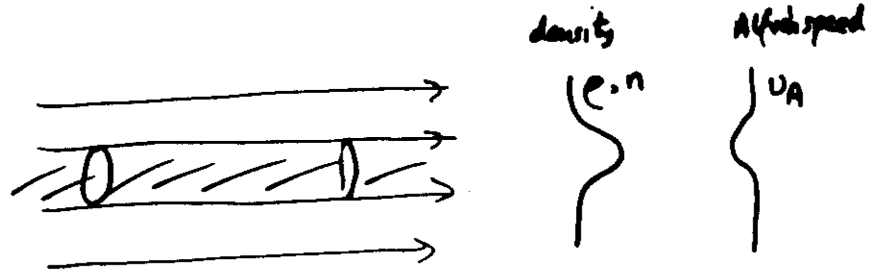


B probably almost uniform

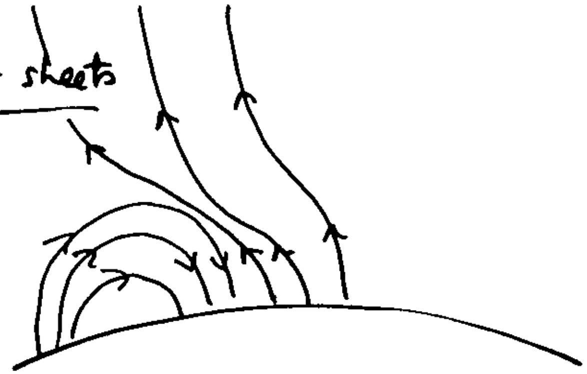
loops show up because of density /and/or temp. differences : emission $\propto n^2 T^\alpha$

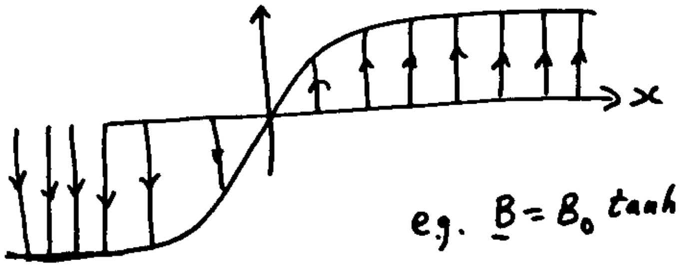
$T \sim 2 \times 10^6$ K , $L \sim 10^4 - 10^6$ km

$n \sim 10^9$ cm^{-3}



Current sheets





e.g. $\underline{B} = B_0 \tanh\left(\frac{x}{a}\right) \cdot \hat{\underline{z}}$

a = scale of current sheet
$a \sim 10^3$ km down to ? km

Important for flares, reconnection, etc.

- also important in magnetosphere, magnetotail

Equations of MHD

Continuity:

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \underline{v} = 0$$

Momentum:

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \nabla) \underline{v} \right) = -\nabla p + \rho \underline{g} + \underline{j} \times \underline{B}$$

pressure force; gravity (buoyancy); magnetic force

Isentropic (adiabatic):

adiabatic index ($\gamma = 5/3$)

$$\frac{\partial p}{\partial t} + \underline{v} \cdot \nabla p = \frac{\gamma p}{\rho} \left(\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho \right)$$

No heat losses

Ideal gas:

$$p = \frac{k_B}{\bar{m}} \rho T$$

k_B: Boltzmann's constant; $\bar{m}$: mean particle mass

Magnetic:

Faraday + Ohm

$$\frac{\partial \underline{B}}{\partial t} = \operatorname{curl}(\underline{v} \times \underline{B}) + \eta \nabla^2 \underline{B}, \quad \operatorname{div} \underline{B} = 0$$

η: diffusivity; $\operatorname{div} \underline{B} = 0$ (No monopoles)

$$\mu_0 \underline{j} = \operatorname{curl} \underline{B} \quad \text{(Ampere's law)}$$

Mechanics

$$\underline{j} \times \underline{B} = -\,\text{grad}\left(B^2/2\mu_0\right) + \frac{1}{\mu_0}(\underline{B}\cdot\text{grad})\underline{B}$$

pressure force; tension force

magnetic pressure $B^2/2\mu_0$ (mks units, B in tesla)

$\rightarrow B^2/8\pi$ (cgs units, B in gauss)

total pressure $p_T = p + B^2/2\mu_0$

Relative magnitude:

plasma beta $\boxed{\beta = p/(B^2/2\mu_0)}$

e.g. $B = 1500$ G in an intense tube

$B^2/8\pi = 9\times10^4$ dynes cm^{-2}

(cf. gas pressure at surface of the Sun, 2×10^5 dynes cm^{-2})

$\beta \sim 1$ in an intense tube

$\beta \sim \frac{1}{10}$ (or smaller) in corona

β large in convection zone

Sound and Alfvén speeds

sound: $c_s = (\gamma p/\rho)^{1/2} = \left(\frac{\gamma k_B T}{m}\right)^{1/2}$

Alfvén: $v_A = B/(\mu_0\rho)^{1/2}$ (Note: $\beta = \frac{2}{\gamma}\, c_s^2/v_A^2$)

e.g. in an intense tube, $c_s \sim 10$ km s^{-1}, $v_A \sim 10$ km s^{-1}

in corona, $c_s \sim 200$ km s^{-1}, $v_A \sim 10^3$ km s^{-1}

mercury under lab. condns.

$c_s \sim 1$ km s^{-1}, $v_A \sim 76$ cm s^{-1} for $B = 1000$ G

In Earth's core, $v_A \sim 1$ cm s^{-1}

The Induction Equation

Faraday's law + Ohm's law ⇒

$$\frac{\partial \underline{B}}{\partial t} = \text{curl}\,(\underline{v} \times \underline{B}) + \eta \nabla^2 \underline{B},$$

Induction Equation

with div $\underline{B} = 0$ (as an initial condn).

Decay of $\underline{B}$

Induction eqn with $\underline{v} = 0$

$$\frac{\partial \underline{B}}{\partial t} = \eta \nabla^2 \underline{B}, \quad \nabla \cdot \underline{B} = 0$$

– same as heat conduction eqn.

Diffusion timescale

$$\tau_{diff} \sim L^2/\eta$$

E.g. 1 cm radius sphere of mercury, $\tau_{diff} \sim 1$ sec

sunspot field, $\tau_{diff} \sim 300$ years (Cowling)

field of the Sun as a whole, $\tau_{diff} \sim 10^9$ yrs (cf comparable to age of solar system) [⇒ only [illegible]]

" " the galaxy, $\tau_{diff} \gg 10^9$ yrs

Earth $\sim 10^5$ yrs (cf. age of Earth $\sim 4.5 \times 10^9$ yrs)