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H4-SMR 393/11

SPRING COLLEGE ON PLASMA PHYSICS

15 May - 9 June 1989

QUASILINEAR THEORY

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QUASILINEAR THEORY

Ronchi, Similon, Sudan (1989)

$$n = n_L + n_s$$

$$\varphi = \varphi_L + \varphi_s$$

$$\langle \varphi_s \rangle = \langle n_s \rangle = 0$$

$$\frac{\partial n_L}{\partial t} + \frac{c}{B_0} \hat{y} \times \nabla\phi_L \cdot \nabla n_L = \nabla \cdot \left[\mu_{\perp e} n_L \nabla\varphi_L + D_e \nabla n_L \right] - \nabla \cdot \underset{\sim}{\Gamma} + Q - \alpha n_L^2$$

$$\underset{\sim}{\Gamma} = -\mu_{\perp e} \langle n_s \nabla\varphi_s \rangle - \mu_{He} \langle n_s \nabla\varphi_s \times \hat{y} \rangle$$

$$\frac{\partial n_s}{\partial t} + \frac{c}{B_0} \hat{y} \times \nabla\varphi_L \cdot \nabla n_s + \frac{c}{B} \nabla n_L \times \hat{y} \cdot \nabla\varphi_s = \nabla \cdot \left[\mu_{\perp e} n_L \nabla\varphi_s + D_e \nabla n_s \right]$$

Neglect non-linear terms in Eqn. for n_s

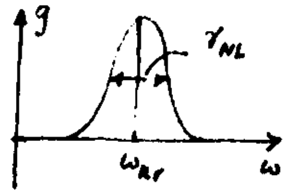
$$\varphi_{s,\underset{\sim}{k}\omega} = \frac{\omega + \mu_{He} \nabla\varphi_L \times \hat{y} \cdot \underset{\sim}{k} + i k^2 D_e}{\mu_{He} (\nabla n_L \times \hat{y}) \cdot \underset{\sim}{k} - i k^2 \mu_{\perp e} n_L} \, n_{s,\underset{\sim}{k},\omega}$$

$$\mu_{He} = \frac{c}{B_0}$$

$$\langle n_s \nabla\varphi_s \rangle$$

$$= \int d^2k \, d\omega \, \underset{\sim}{k} \langle |n_{s\underset{\sim}{k}\omega}|^2 \rangle \frac{k^2 D_e \frac{c}{B_0} \underset{\sim}{k} \times \hat{y} \cdot \nabla n_L - k^2 \mu_{\perp e} n_L \left(\omega + \frac{c}{B_0} \nabla\varphi_L \times \hat{y} \cdot \underset{\sim}{k} \right)}{\left[\frac{c}{B_0} \nabla n_L \times \hat{y} \cdot \underset{\sim}{k} \right]^2 + \left[\mu_{\perp e} n_L k^2 \right]^2}$$

$$\langle |n_{s\underset{\sim}{k}\omega}|^2 \rangle = \langle |n_{s\underset{\sim}{k}}|^2 \rangle \, g(\omega - \omega_{\underset{\sim}{k}r})$$



$$\langle n_s \nabla\varphi_s \rangle$$

$$= \int \frac{d^2k}{k^2} \underset{\sim}{k} \underset{\sim}{k} \times \hat{y} \left\langle \left| \frac{n_{s\underset{\sim}{k}}}{n_L} \right|^2 \right\rangle \left[\frac{\psi}{1+\psi} \frac{\Omega_e}{\nu_e} n_L \nabla\varphi_L - \frac{(2+\psi+\psi^2)}{1+\psi} \frac{T}{e} \frac{\Omega_e}{\nu_e} \nabla n_L \right]$$

$$= -\frac{1}{2} \frac{\psi}{1+\psi} \frac{\Omega_e}{\nu_e} \left\langle \left| \frac{\delta n(z)}{n_L} \right|^2 \right\rangle n_L \nabla\varphi_L \times \hat{y} + \frac{1}{2} \frac{(2+\psi+\psi^2)}{(1+\psi)^2} \frac{T}{e} \frac{\Omega_e}{\nu_e} \left\langle \left| \frac{\delta n(z)}{n_L} \right|^2 \right\rangle_s \nabla n_L \times \hat{y}$$

$$\because \int \frac{d^2k}{k^2} \left\langle \left| \frac{n_{s\underset{\sim}{k}}}{n_L} \right|^2 \right\rangle \underset{\sim}{k} \underset{\sim}{k} \times \hat{y} = -\frac{1}{2} \underset{=}{I} \times \hat{y} \left\langle \left| \frac{\delta n(z)}{n_L} \right|^2 \right\rangle_s$$

$$\underline{\Gamma} = -\mu_H^* n_L \nabla\varphi_L \times \hat{y} - D_H^* \nabla n_L \times \hat{y} - \mu_\perp^* n_L \nabla\varphi_L - D_\perp^* \nabla n_L$$

$$\mu_H^* = \frac{1}{2}\frac{\psi}{1+\psi}\left\langle \left|\frac{\delta n}{n_L}\right|^2\right\rangle_s \mu_{He}$$

$$D_H^* = \frac{1}{2}\frac{(2+\psi+\psi^2)}{(1+\psi)^2}\left\langle \left|\frac{\delta n}{n_L}\right|^2\right\rangle_s \frac{T}{e}\mu_{He}$$

$$\mu_\perp^* = \frac{1}{2}\frac{\psi}{1+\psi}\left(\frac{\mu_{He}}{\mu_{\perp e}}\right)^2\left\langle \left|\frac{\delta n}{n_L}\right|^2\right\rangle_s \mu_{\perp e}$$

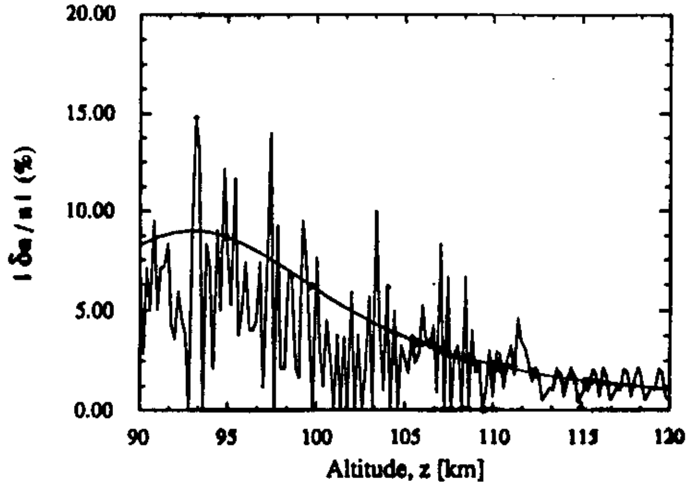
$$D_\perp^* = \frac{1}{2}\frac{(2+\psi+\psi^2)}{(1+\psi)^2}\left(\frac{\mu_{He}}{\mu_{\perp e}}\right)^2\left\langle \left|\frac{\delta n}{n_L}\right|^2\right\rangle D_e$$

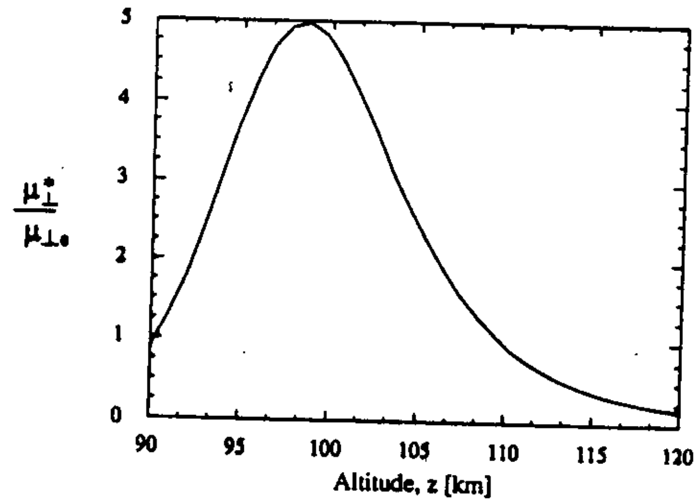
$$\left(\frac{\mu_{He}}{\mu_{\perp e}}\right)^2 = \frac{\Omega_e^2}{\nu_e^2}$$

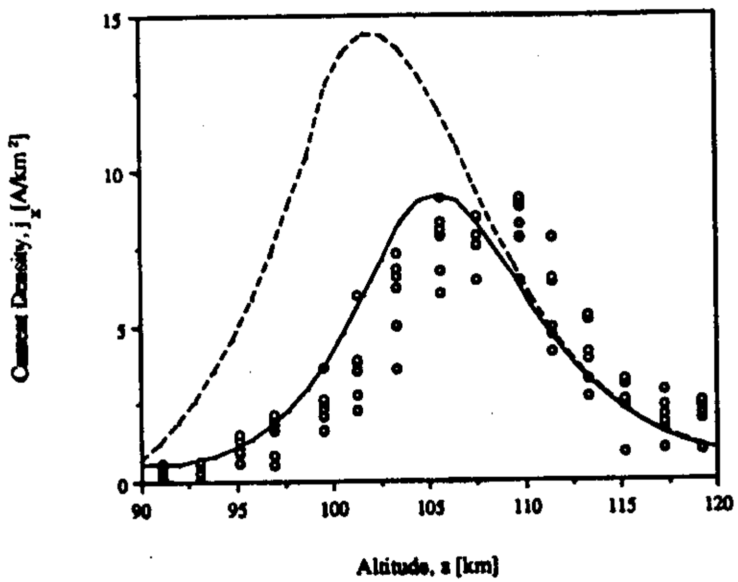
Finally :

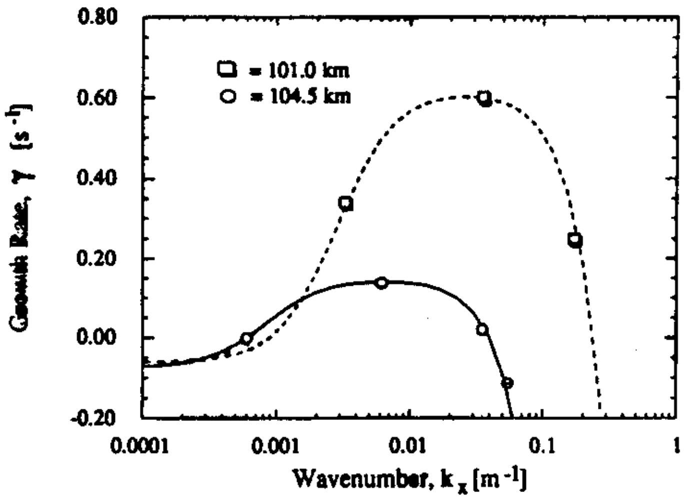
$$\frac{\partial n_L}{\partial t} + \frac{c}{B_0}\hat{y}\times\nabla\varphi_L\cdot\nabla n_L = \nabla\cdot\left[(\mu_{\perp e} + \mu_\perp^*)n_L\nabla\varphi_L\right] + \nabla\cdot\left[(D_e + D_\perp^*)\nabla n_L\right] + Q - \alpha n_L^2$$

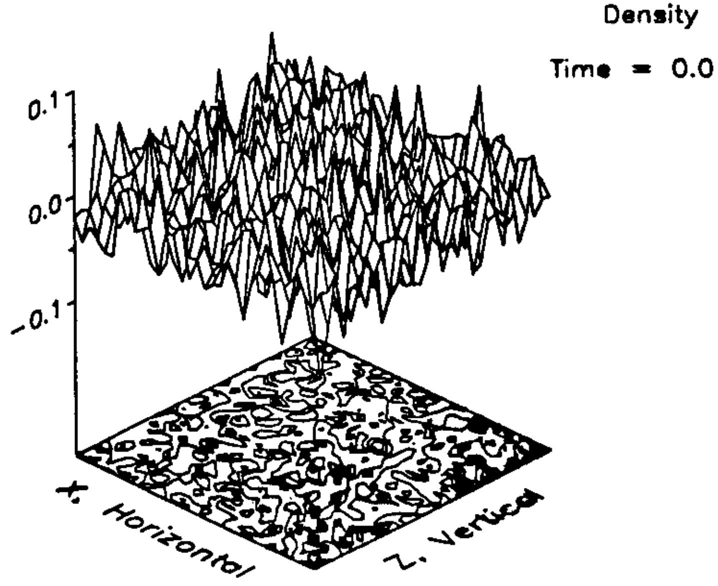
$$\mu_H^- \nabla n_L \cdot \nabla\varphi_L \times \hat{y} + \nabla\cdot\left[(\mu_\perp^- + \mu_\perp^*) n_L \nabla\varphi_L\right] + \nabla\cdot\left(D_i - D_e - D_\perp^*\right)\cdot\nabla n_L = 0$$

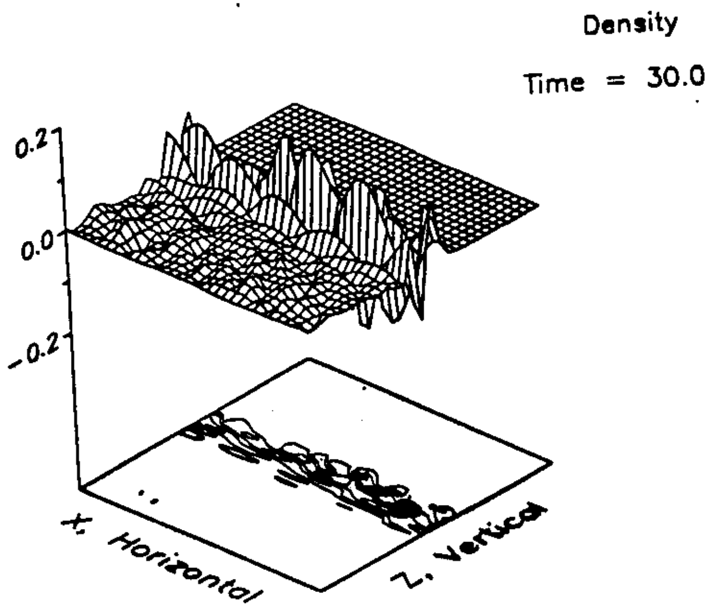


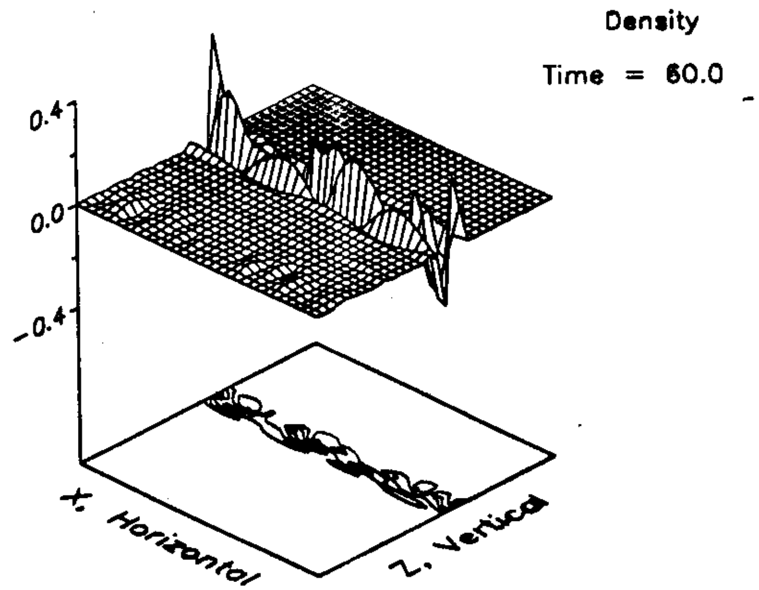

5
4
3
2
1
0
90
95
100
105
110
115
120
Altitude, z [km]


15
10
5
0
90
95
100
105
110
115
120
Current Density, j_x [A/km^2]
Altitude, z [km]


0.80
0.60
0.40
0.20
0.00
-0.20
□ = 101.0 km
○ = 104.5 km
Growth Rate, γ [s-1]
0.0001
0.001
0.01
0.1
1
Wavenumber, k x [m -1]


Density
Time = 0.0
0.1
0.0
-0.1
X, Horizontal
Z, Vertical


Density
Time = 30.0
0.2
0.0
-0.2
X, Horizontal
Z, Vertical


Density
Time = 60.0
0.4
0.0
-0.4
X, Horizontal
Z, Vertical

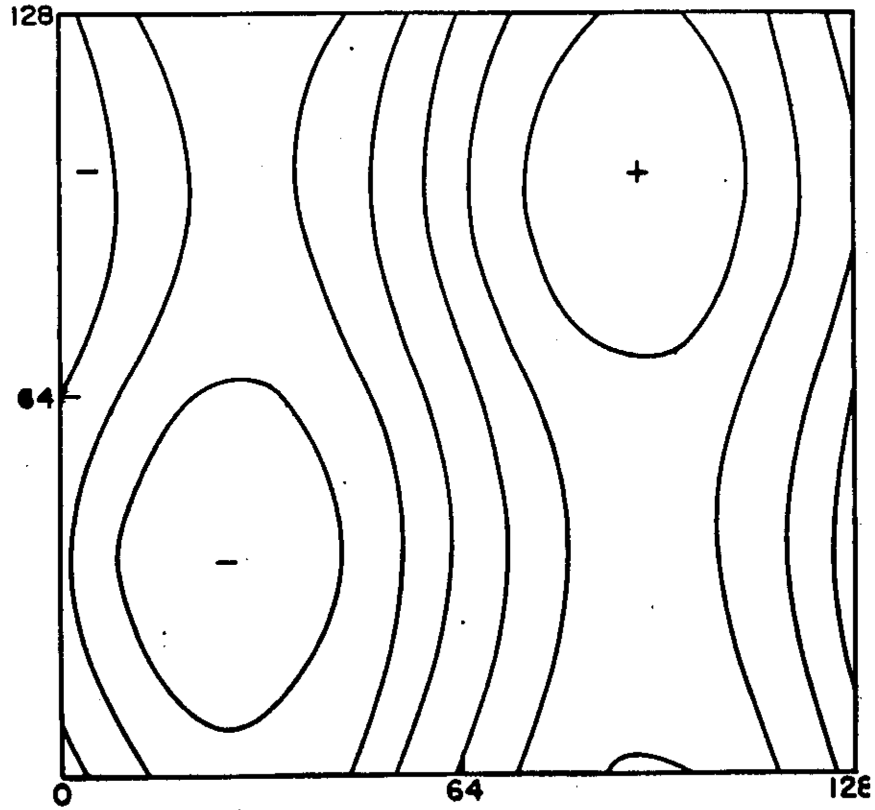


Fig. 5.2a

Keskinen, Sudan, Ferch J.G.R. 84, 1419 (1979)

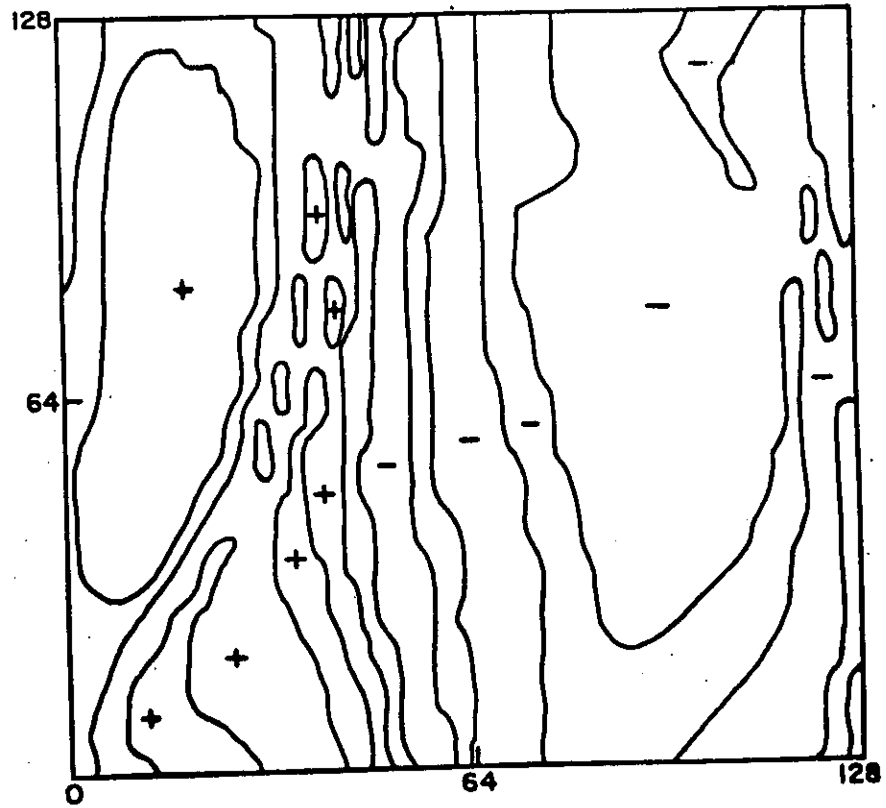


Fig. 5.2b

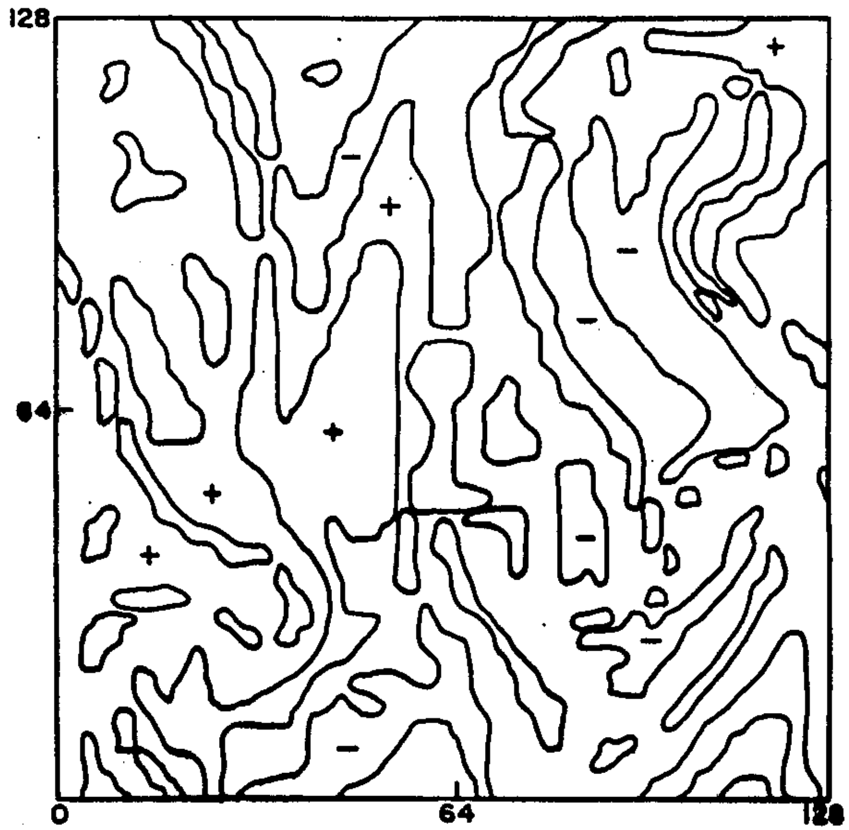
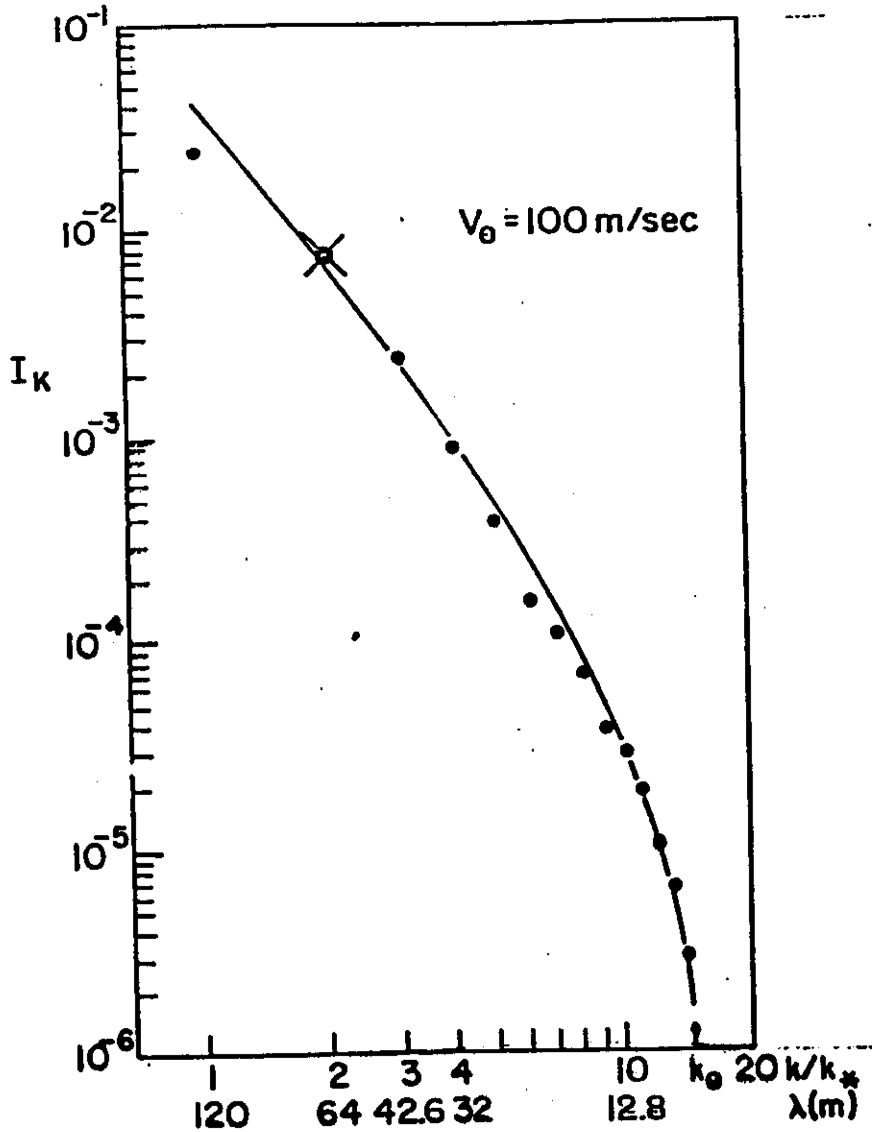
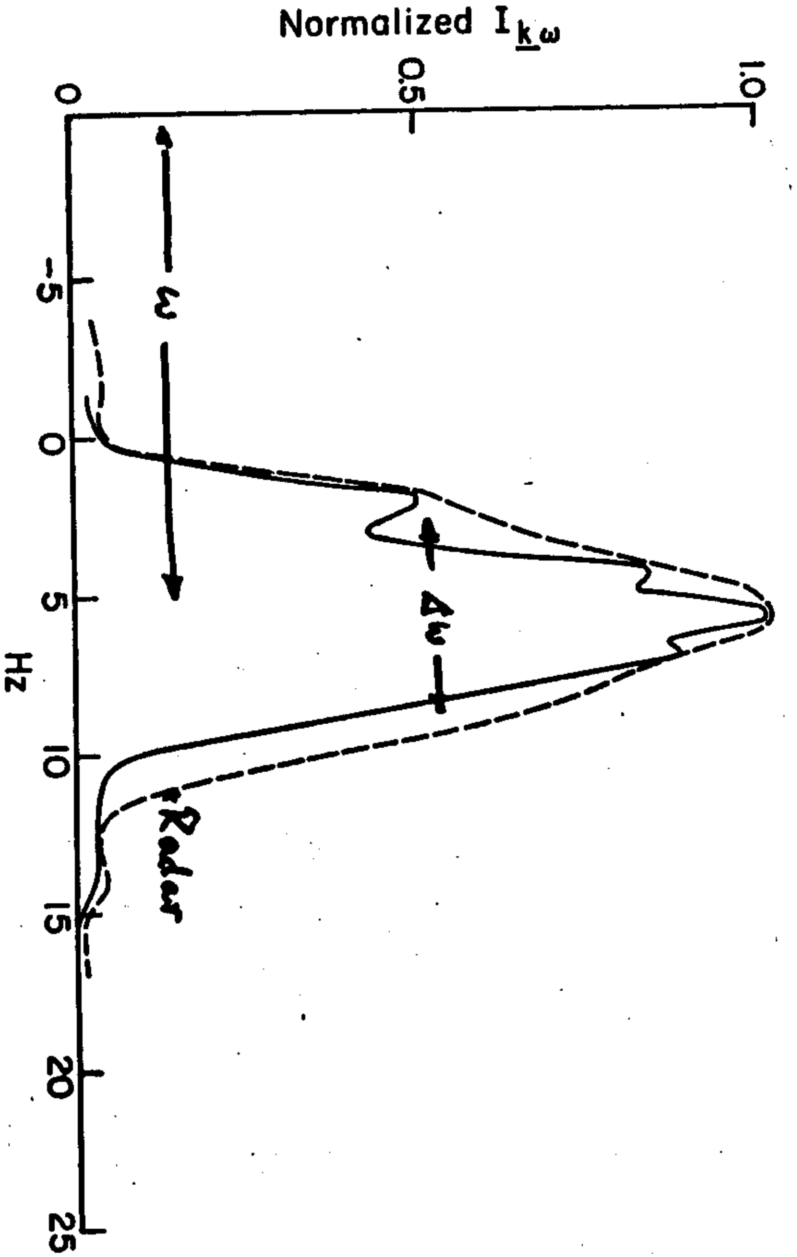

128
64
0
64
128
+
−

Fig. 5.2f


V_0 = 100 m/sec
I_K
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
1 2 3 4 10 k_0 20 k/k_*
120 64 42.6 32 12.8 λ(m)


Normalized $I_{\underline{k}\omega}$
0
0.5
1.0
-5
0
5
10
15
20
25
Hz
ω
Δω
Radar

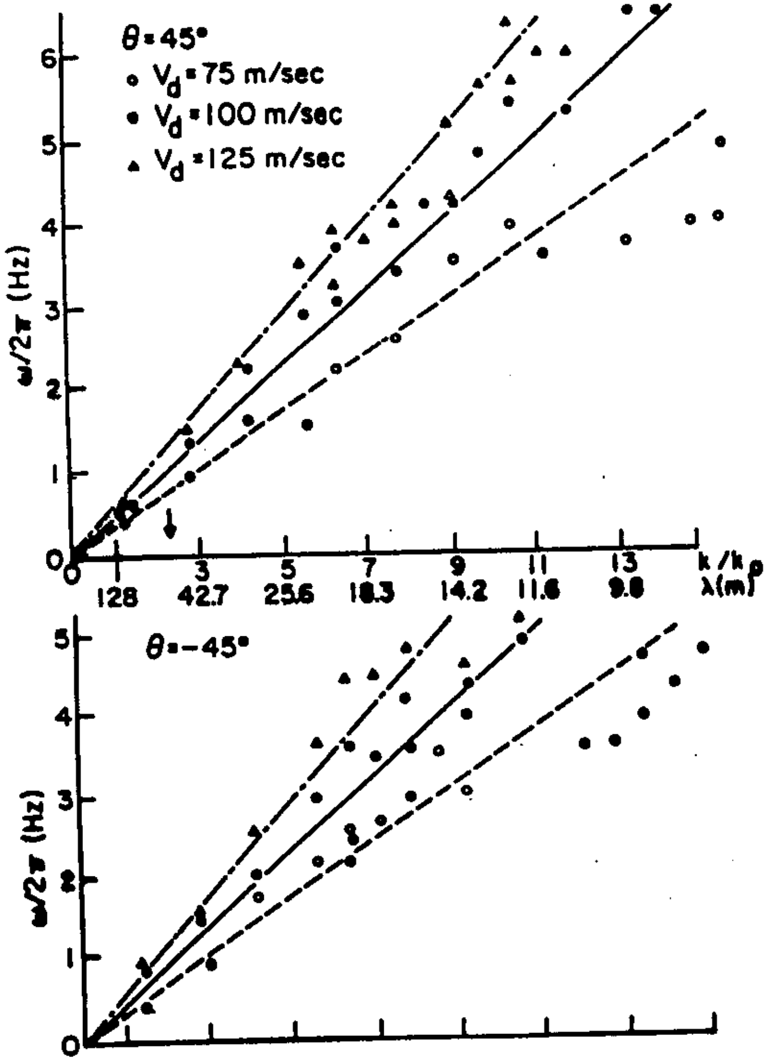

θ=45°
○ V_d=75 m/sec
● V_d=100 m/sec
▲ V_d=125 m/sec
ω/2π (Hz)
k/k_p
λ(m)
128
42.7
25.6
18.3
14.2
11.6
9.8
θ=-45°
ω/2π (Hz)

Fig. 5.7b

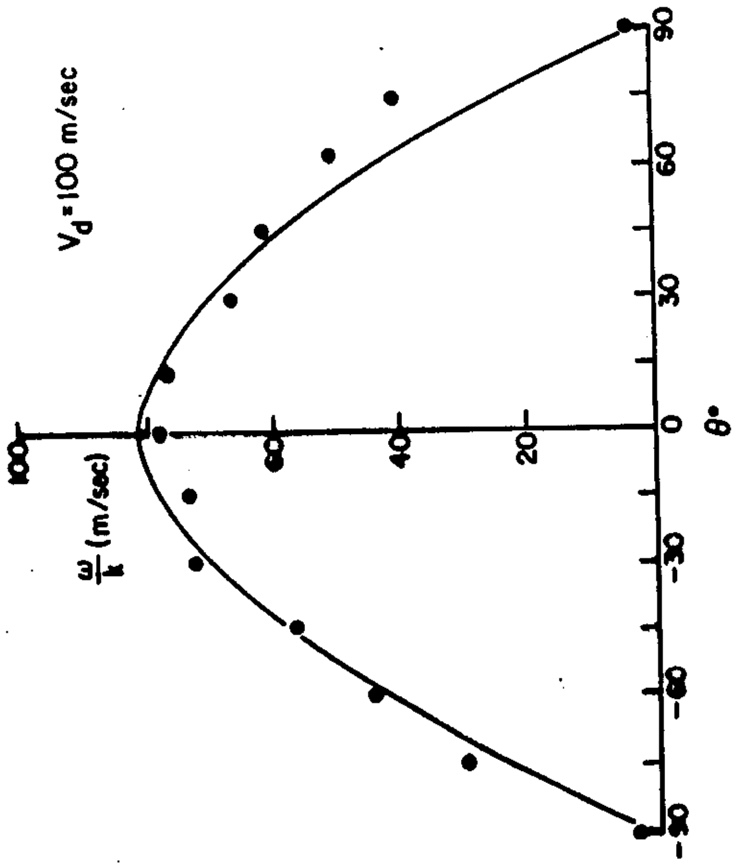


Fig. 5.8

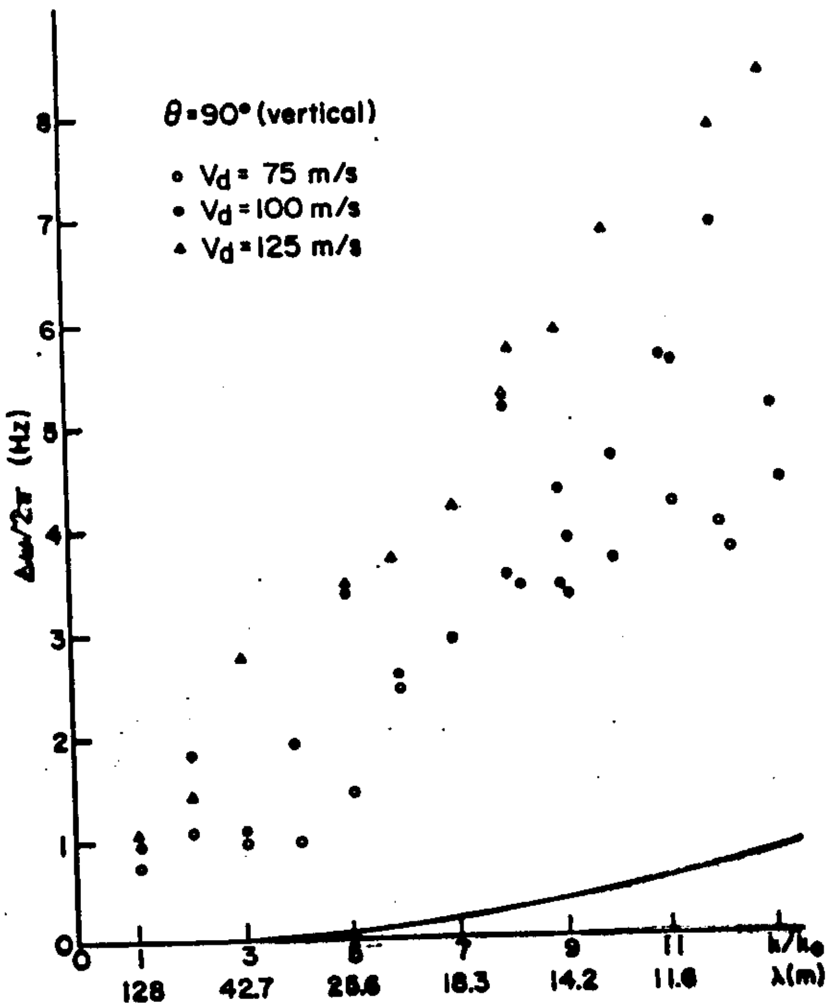


Fig. 5.9a

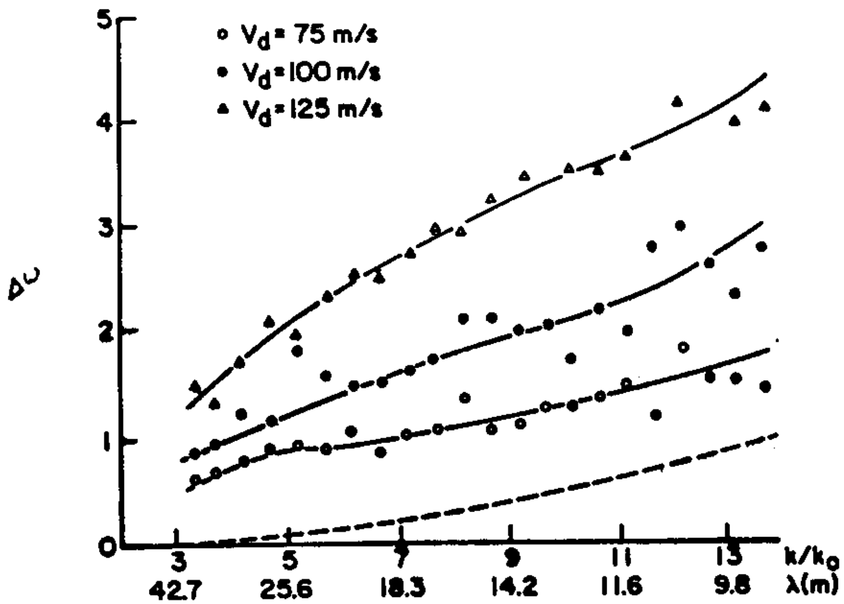


Fig. 6.3

* Almost 2 D ($k_\parallel \ll k_\perp$)

* Low frequency ($\omega \ll \Omega_i$)

* Low beta ($\beta < m_e/m_i$)

I Drift Wave turbulence

$$\boxed{\frac{\partial}{\partial t}\varphi_{\underset{\sim}{k}} + i(\omega_{\underset{\sim}{k}_\perp} + i\gamma_{\underset{\sim}{k}})\varphi_{\underset{\sim}{k}} = \sum_{\underset{\sim}{k}'} W_{\underset{\sim}{k}_\perp,\underset{\sim}{k}'_\perp}\varphi_{\underset{\sim}{k}'}\varphi_{\underset{\sim}{k}''}}$$

$$W_{\underset{\sim}{k}_\perp,\underset{\sim}{k}'_\perp} = \frac{1}{2}\hat{z}\cdot\underset{\sim}{k}'_\perp \times \underset{\sim}{k}''_\perp\,(k''^2_\perp - k'^2_\perp)/(1+k^2_\perp)$$

$$\omega_{\underset{\sim}{k}_\perp} = k_y/(1+k^2_\perp)$$ Hasegawa-Mima PRL 39,205(1977)

$$\underset{\sim}{k}'' = \underset{\sim}{k} - \underset{\sim}{k}'$$

II E X B Turbulence

$$\boxed{\frac{\partial}{\partial t}n_{\underset{\sim}{k}} + i(\omega_{\underset{\sim}{k}_\perp} + i\gamma_{k})n_{\underset{\sim}{k}} = \sum_{\underset{\sim}{k}'} W_{\underset{\sim}{k}_\perp,\underset{\sim}{k}'_\perp} n_{\underset{\sim}{k}'} n_{\underset{\sim}{k}''}}$$

$$W_{\underset{\sim}{k}_\perp,\underset{\sim}{k}'_\perp} = \alpha\frac{1}{2}\hat{z}\cdot\underset{\sim}{k}'_\perp \times \underset{\sim}{k}''_\perp \left(\frac{\underset{\sim}{k}''_\perp\cdot V_D}{k''^2_\perp} - \frac{\underset{\sim}{k}'_\perp\cdot \underset{\sim}{V}_D}{k'^2_\perp}\right)$$

$$\omega_{\underset{\sim}{k}_\perp} = \frac{\underset{\sim}{k}_\perp\cdot \underset{\sim}{V}_D}{1+\gamma}$$ Sudan, Keskinen PRL 39, 966 (1977)

$$\nu - \Delta V_D - k^2_\perp D$$

Kolmogorov Theory (1941)

← Energy injection ε per unit mass/sec.

Cascade of Energy

↓

→ Energy dissipation

k_F^{-1}

* Eddy velocity of scale size $\ell \sim k^{-1}$ $\quad \tilde{v}_k$

* Eddy life time $\tau_k \sim \ell / v_k \sim \frac{1}{k v_k}$

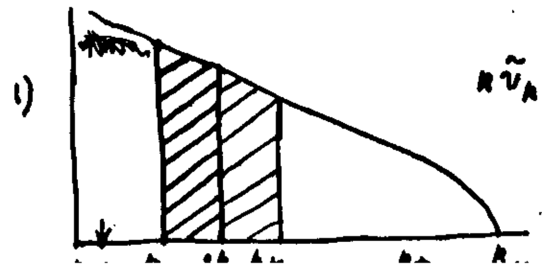
* Rate of energy transfer $= \dfrac{\tilde{v}_k^2}{\tau_k} = k \tilde{v}_k^3 = \varepsilon$ (constant)

$$\therefore \quad v_k \sim \varepsilon^{1/3} k^{-1/3}$$

Reynolds no: $\dfrac{v_L L}{\nu}$

$$\langle v^2 \rangle = \int dk\, E(k)$$

$$\tilde{v}_k^2 = k E(k) \quad \Rightarrow \quad E(k) \sim \varepsilon^{2/3} k^{-5/3}$$



$$k \tilde{v}_k = \frac{1}{\tau_k} \sim \nu k^2$$

$$k_K \sim \left(\varepsilon / \nu^3\right)^{1/4}$$

$$R^{3/4}$$

n_k is a stochastic variable

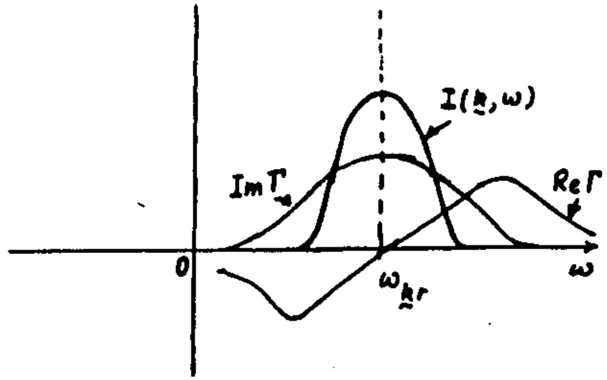
$$\frac{\langle n_{k\omega}\, n^*_{k',\omega'} \rangle}{n_0^2} = I_{k,\omega}\, \delta(k - k')\, \delta(\omega - \omega')$$

$$\frac{\langle n(x,t)\, n(x + X, t + T) \rangle}{n_0^2} = \langle n(0)\, n(X,T) \rangle$$

$$I_{k,\omega} = \int d^3X\, dT\, e^{i(k \cdot X - \omega T)} \langle n(0)\, n(X,T) \rangle$$

$$\left| \omega - (\omega_k + i\gamma_k + i\Gamma_{k\omega}) \right|^2 I_{k\omega} = \frac{1}{2} \sum_{k'\omega'} \left| W_{k,k'} \right|^2 I_{k'\omega'}\, I_{k''\omega''}$$

$$\Gamma_{k\omega} = -i \sum_{k'\omega'} \frac{W_{kk'}\, W_{k'',k}\, I_{k'\omega'}}{(\omega'' - \omega_{k''} - i\gamma_{k''}) - i\Gamma_{k''\omega''}}$$



Let

$$I(\underset{\sim}{k},\omega) = I(\underset{\sim}{k})\, g\left(\frac{\omega-\omega_{kr}}{\tilde{\Gamma}}\right)$$

* Solve for $\Gamma(\underset{\sim}{k},\omega_k)$; $\tilde{\Gamma} = i\int\frac{dq}{2\pi}\Gamma(\underset{\sim}{k},\omega_k)$

* $\int d\underset{\sim}{k} \rightarrow \int_{\alpha k} d\underset{\sim}{k}$

Result — Sudan & Keskinen P.F. 22, 2305 (1979)

$$\boxed{\tilde{\Gamma}(k_\perp) = D(\alpha)\,\beta v_o\, k_\perp^2\, I^{1/2}(k_\perp)}$$; $\beta = \frac{\nu_i}{\Omega_i(1+\psi)}$

Interpretation

from linearized eqns.

electron velocity $\underset{\sim}{v}_k \sim \beta v_o\left(\frac{\hat{b}\times \underset{\sim}{k}}{k^2}\right)\frac{n_k}{n_0}$

$$\tilde{\Gamma} \sim \tau^{-1} \sim \frac{\delta v}{\ell} \sim k\,\delta v \sim k\left[k^2\langle|v_k|^2\rangle\right]^{1/2} \sim \beta v_o k^2 I^{1/2}$$

τ : Eddy turnover time

Kolmogorov Cascade

* Assume isotropy perpendicular to $\underset{\sim}{B}$

$$\text{Energy Density} \propto \int_0^\infty dk_\perp \underbrace{k_\perp I(k_\perp)}_{E(k_\perp)} = \int_0^\infty \frac{dk_\perp}{k_\perp}\underbrace{k_\perp^2 I(k_\perp)}_{\mathcal{E}(k_\perp)}$$

Energy Conservation

$$\boxed{\frac{d}{d\ln k_\perp}\Big[\tilde{\Gamma}\; \underbrace{k_\perp^2 I(k_\perp)}\Big] = 2\underbrace{\gamma(k_\perp)}_{\gamma_o - k_\perp^2 G}\;\underbrace{k_\perp^2 I(k_\perp)} \qquad \tilde{\Gamma} \propto k_\perp^2 I^{1/2}}$$

Kulsrud and Sudan $\boxed{\Gamma \gg \gamma}$

For $\gamma = 0$

$$I(k_\perp) \propto k_\perp^{-8/3} \quad ; \quad \tilde{\Gamma}(k_\perp) \propto k_\perp^{2/3}$$

$$E(k_\perp) \propto k_\perp^{-5/3}$$

$$\boxed{\hat{I}\left(\frac{k_\perp}{k_o}\right) = \left(\frac{k_o}{k_\perp}\right)^{+\frac{8}{3}}\left\{1 - \left(\frac{k_\perp}{k_o}\right)^{-\frac{2}{3}} - \frac{1}{2R}\left[\left(\frac{k_\perp}{k_o}\right)^{4/3} - 1\right]\right\}^2}$$

Sudan and Pfirsch P.F. 28, 1702 (1985)

$\frac{Lu}{\nu}$ $\frac{1}{k_o}\frac{\gamma_o}{k_o G}$

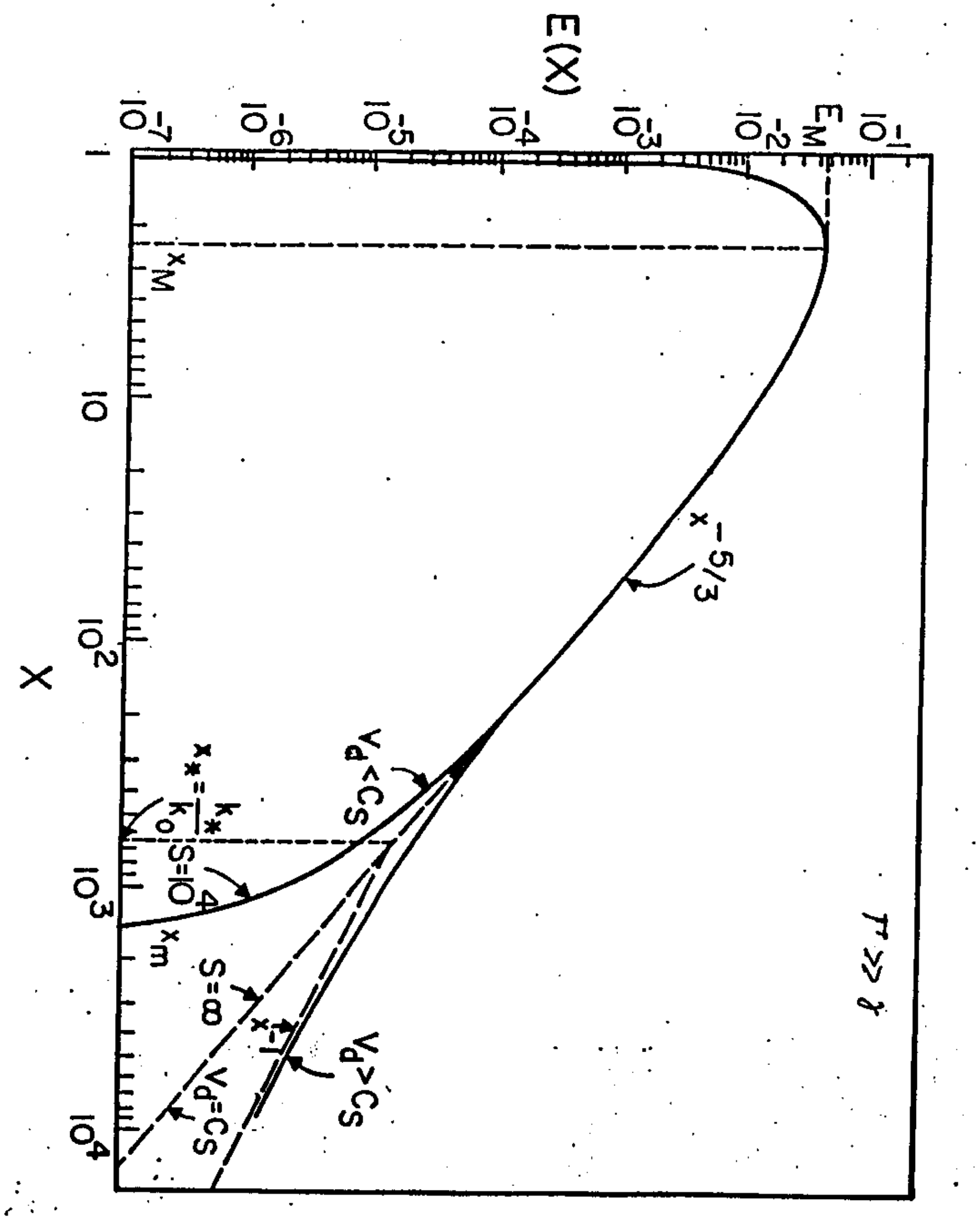
$R = \gamma_o / k_o^2 G$ equivalent Reynolds number

$$\hat{I} = \left(\frac{\gamma_o}{\beta v_o}\right)^{-2} k_o^4\, I(k_\perp)$$

Marginally stable $k = k_c = \sqrt{\gamma_o/G}$; $\frac{k_c}{k_o} = R^{1/2}$

Spectrum drops off at $\frac{k}{k_o} = \frac{k_K}{k_o} = (2R)^{3/4}$ Kolmogorov cut-off

$$\frac{k_K}{k_c} \sim R^{1/4} \quad ; R \sim 10^4$$



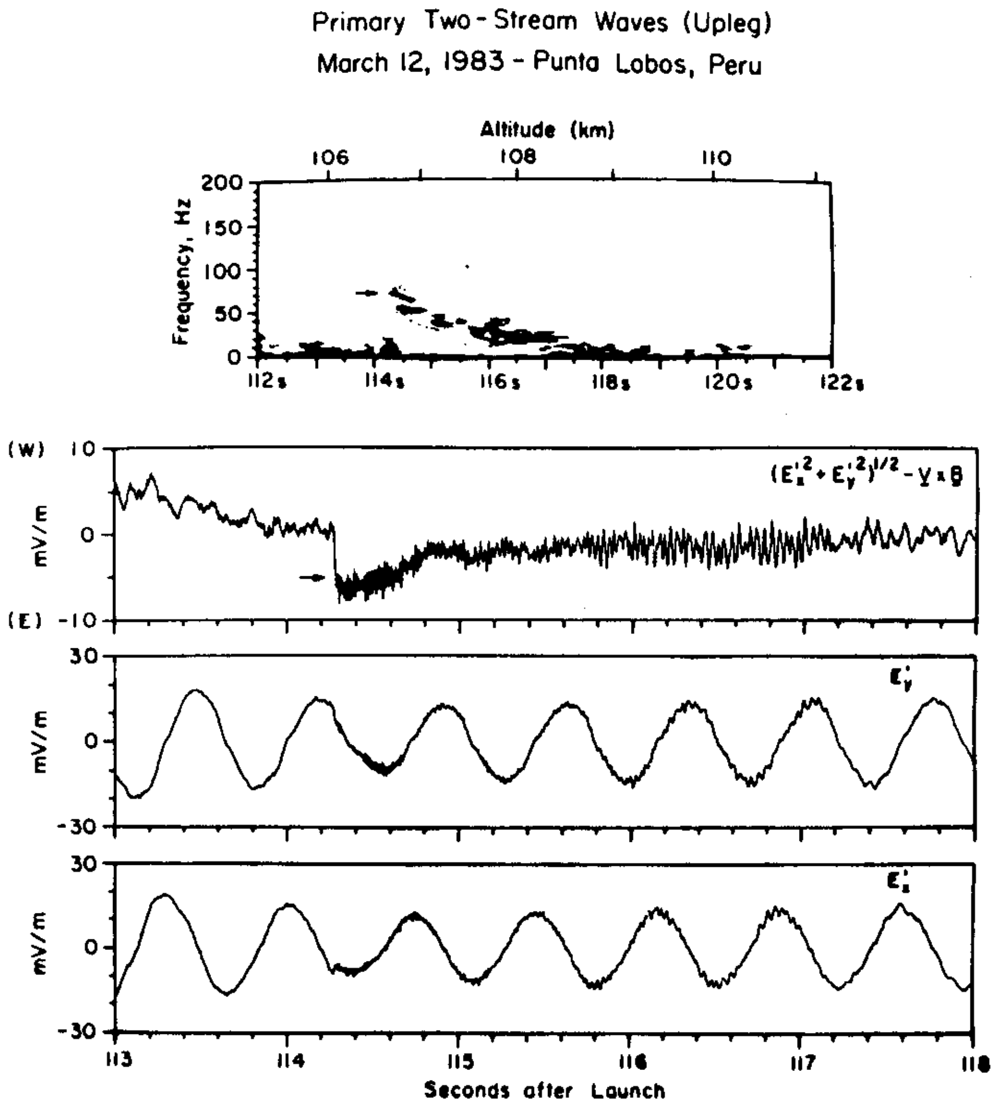


Fig. 4.29. Electric field observations of the two-stream waves for the upleg. The lower panels show the raw DC-coupled data, above which is plotted the square root of the sum of the squares of these waveforms. The upper panel shows a sonogram of these waves. (Note the change in the scale of the time axis.) An arrow indicates the onset of the strong burst of primary two-stream waves. (After Pfaff et al., 1987b. Reproduced with permission of the American Geophysical Union.)

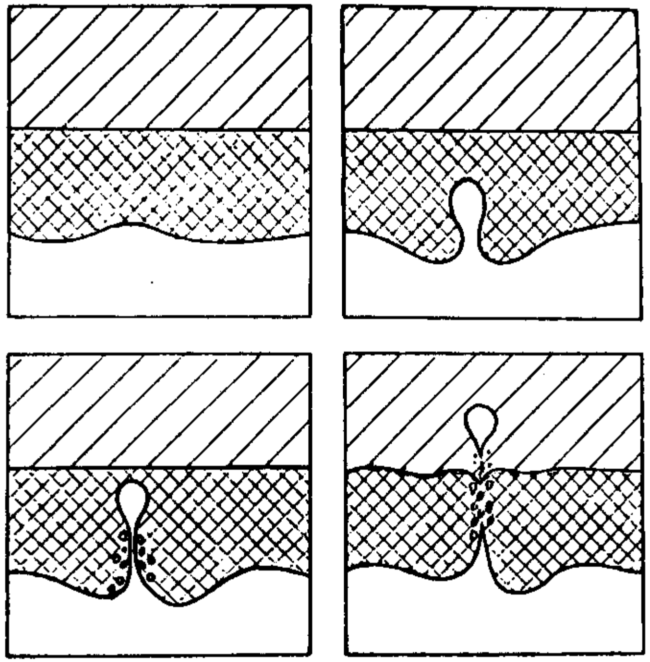
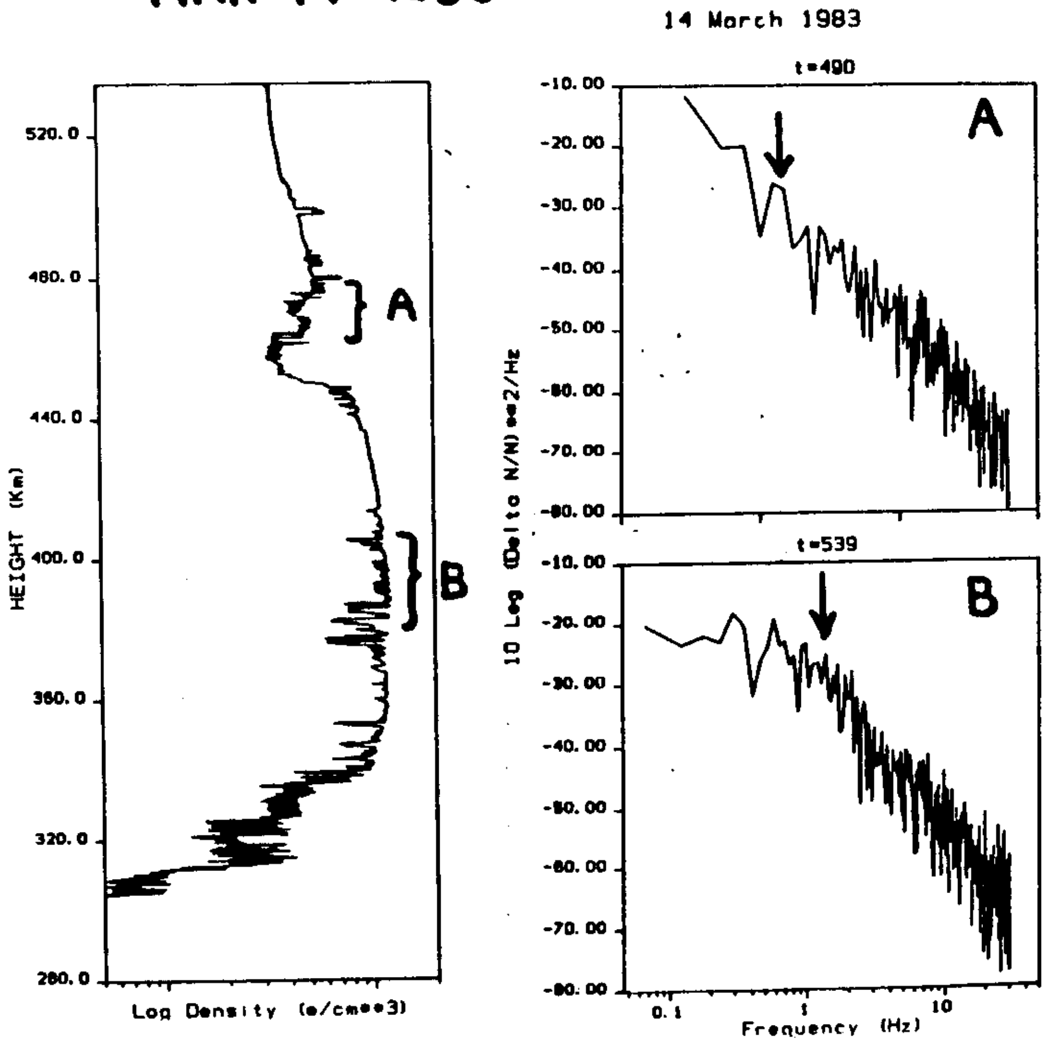


Figure 2.4: The model proposed by Woodman and LaHoz [1976] to explain topside spread F: the bottomside region, unstable to the gravitational Rayleigh-Tayler process, distorts into a pinched-off "bubble" which rises into the topside.

CONDOR
MAR 14 1983



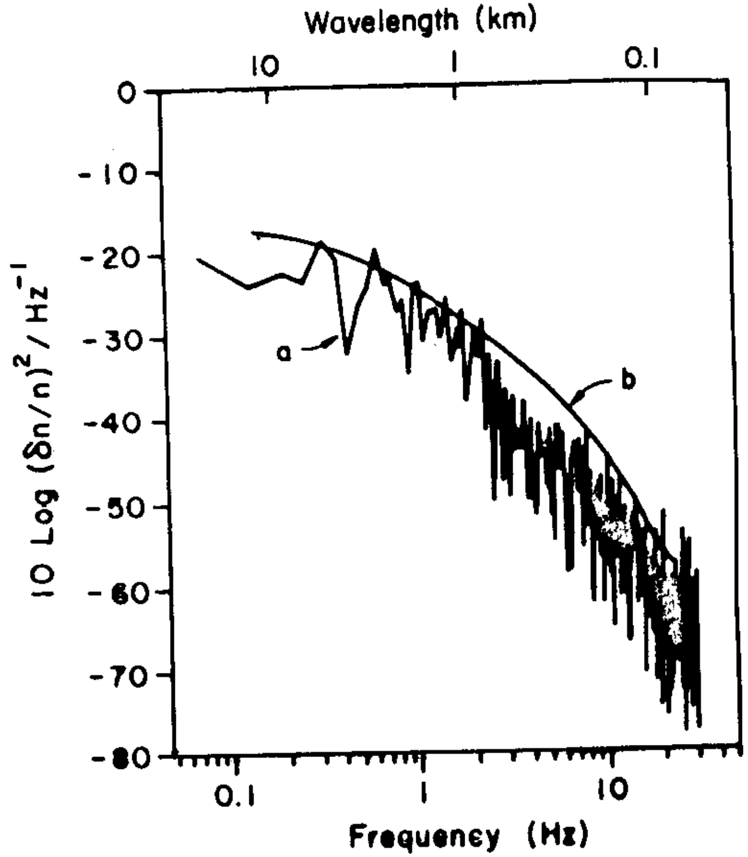


Fig. 4.15. (a) The spectrum of density irregularities measured at 390 km on the upleg of the March 14 rocket flight.

(b) The spectrum of density irregularities predicted by Sudan and Keskinen (1984). The wave number to frequency transformation has been approximated using the rocket velocity, and the theoretically predicted spectrum has been integrated over wave number to obtain the same units as the measured spectrum. (After LaBelle and Kelley, 1986. Reproduced with permission of the American Geophysical Union.) $D_\perp^* \simeq 400\, D_c$

E - Region

1. No turbulence where $\nabla n = 0$. However if $v_d > C_s/(1+\Psi)$ observe Farley-Buneman instability

2. $\nabla n > 0$, $v_d < \frac{C_s}{1+\Psi}$ } Observe turbulent fluctuations spectral index fair agreement.

3. Line width and Doppler shift from radar observations ; good agreement with simulations

F - Region

4. Above F-peak observed spectral index ~ 2.5 is greater than Kolmogorov index ; region of decaying turbulence

5. When turbulent activity is strong spectral index → 1.5

6. Breaks in spectrum occur at ~ 1 km and 100 m, flatter (k^{-1}) for λ < 1 km and steeper ($k^{-4.5}$) for λ > 100 m.

Moderate Turbulence

$$\gamma_0 = \Gamma \propto k^2 I_k^{1/2}$$

$$\therefore k I_k \propto k^{-3}$$