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I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

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Planetary Magnetospheres

A. Eviatar

Tela Aviv University
Geophysics & Planetary Science
Raymond & Beverly Sackler Faculty of Exact Sciences
Ramat Aviv 69978
Israel

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Planetary Plasma Physics

Professor Aharon Eviatar

Tel Aviv University

Ramat Aviv, Israel

Introduction

Planetary plasma physics, as all branches of plasma physics, deals with the motion and thermal state of ionized gas. The basic equations governing the behavior of plasma in a planetary environment, be it a magnetosphere or an extended ionosphere, are Maxwell's equations for the electromagnetic field, Vlasov's equation for the velocity space distribution function and the equations of motion of a charged particle in a nonuniform magnetic field. It is assumed in these lectures that the participants are familiar with these equations in general. I shall, therefore, dispense with any introductory review and address directly those aspects of plasma physics that are of particular interest in a planetary context. If a detailed review is needed, the following books are recommended:
Rossi & Olbert, Introduction to the Physics of Space
Alfvén and Fälthammar, Cosmic Electrodynamics

The solar system, outside of the solid bodies and inner atmospheres of planets is comprised mainly of plasma. The interplanetary gas is a supersonic hydrogen plasma flowing out from the sun which interacts with the various planetary objects orbiting the sun. In addition, the outer atmospheres of planetary objects and of comets are ionized by the solar ultraviolet radiation, creating an *ionosphere*. The plasma in an ionosphere is dominated by pressure gradient or hydromagnetic forces created by collisional coupling to a neutral atmosphere. Much of the interesting plasma and charged particle physics in the solar system takes place, however, neither in the solar wind nor in the collision dominated ionosphere, but rather in the *magnetosphere* of a planetary body.

The magnetosphere of an object is the region of space surrounding the object which is permeated by a magnetic field and in which the motion of plasma is dominated by electromagnetic forces, rather than by hydrodynamic or pressure gradient forces. The magnetic field may be intrinsic to the object, such as a magnetic planet [Earth, Jupiter etc.] or a star or pulsar, or it may be an induced field such as that which is draped around Venus or is trapped in the coma and tail of a fully developed comet [*e.g.* P/Halley or P/Giacobini-Zinner both of which were visited by spacecraft during the current decade].

The distant magnetic field of a pulsar or other object existing in the interstellar medium can be described quite satisfactorily by a vacuum field. A planet in the solar system, on the other hand, has its magnetic field grossly distorted by the solar wind, which flows past it at a supersonic velocity. Thus, instead of extending to infinity, the magnetic field of a planet is confined within a boundary called the *magnetopause* the shape of which is determined by the interaction with the solar wind. Since the wind is supersonic, there will be a bow shock standing off some distance from the magnetopause. Between the bow shock and the magnetopause, there is a region of shocked plasma known as the *magnetosheath* in which the flow is subsonic and the temperature and entropy are higher than they are in the solar wind.

Most of the planets in the solar system have been found to have intrinsic magnetic fields. The main exceptions are Venus and Mars and the latter may have some degree of magnetization. Comets also do not have intrinsic fields as was found by the Giotto and ICE spacecraft. Both Venus and comets have inducted magnetic fields caused by the draping of the interplanetary field around the object.

Magnetosphere plasmas are in general effectively collisionless and therefore can be regarded as ideal conductors. In this case, the frozen field theorem holds, *i.e.* that there can be no relative motion between the plasma and the magnetic lines of force. Thus the solar wind will be deflected around the magnetopause of a magnetic planet or the ionopause of a nonmagnetic object. In reality, there must be a force which deflects the solar wind, in other words there must be a surface current the Lorentz force of which is responsible for the deflection.

In order to understand this better, recall that the frozen flux theorem refers to the plasma as a continuum. On the scale of a particle gyroradius, the individual particles can penetrate the field for a distance of a gyrodiameter. Since the protons and electrons have vastly different gyroradii and are curved by the field in opposite directions, a surface current is indeed set up which

performs the deflection.

The problem of finding the field at the boundary was solved as a magnetostatics problem by Chapman and Ferraro in 1931. Consider a plane surface located at a distance R from a magnetic dipole. On the other side of the dipole a flowing plasma is impinging and exerting a pressure. This can be represented by an image dipole located at a distance 2R from the original dipole. The total field will be the sum of the two dipole fields:

$$B_t(r) = \frac{B_0}{r^3} + \frac{B_0}{(2R - r)^3} \qquad (1$$

where B_0 is the field at unit distance [the unit commonly used in planetary magnetosphere research is the radius of the planet under consideration]. Note that at r = R, the field is exactly twice the field caused by the original dipole. This extra field is that generated by the surface currents. In the absence of displacement fields the relationship is [the symbols have their usual meanings]:

$$\nabla \times B = \frac{4\pi}{c} j \qquad (2$$

which in terms of characteristic values can be expressed:

$$\frac{B}{\rho} = \frac{4\pi q N v_\perp}{c} \qquad (3$$

where N is the density, $v_\perp$ is the perpendicular velocity and ρ is the gyroradius. This leads to the pressure balance across the surface:

$$\frac{B^2}{8\pi} = \frac{Nmv_\perp^2}{2} \qquad (4$$

We can now find the distance to the magnetopause for any given planet in units of the planetary radius a:

$$\frac{r_m}{a} = \left(\frac{B_0^2}{4\pi Nmv_\perp^2}\right)^{\frac{1}{6}} \qquad (5$$

We see that the magnetopause distance is relatively insensitive to the external solar wind parameters. For Earth parameters, B_0 = .3G, N = 10 cm^{-3} and a solar wind speed of 400 km/s, we find about $8R_E$ while for Jupiter where B_0 = 4G, N = .05 cm^{-3} and the solar wind speed is roughly the same, we find a distance of about $40R_J$. These are merely rough estimates and the distances can vary considerably.

In general, the field of a planet can be represented by the sum of three component currents:

B_d : dipole field

B_p : plane surface currents

B_c : curvature surface currents

Thus the internal field B_i and the external field which is assumed to vanish in these simple models can be expressed in terms of the above components.

$$B_i = B_g + B_p + B_c$$

$$0 = B_g - B_p + B_c \qquad (6$$

$$\Rightarrow B_i = 2(B_g + B_c)$$

There exists a problem in that in that B_c is unknown unless the shape of the magnetosphere is known and the shape itself depends on the total field including B_c. The way around this is to iterate. The first approximation is $B_c = 0$ and the iterations are continued until a self consistent surface is found. The zero order field is taken to be that of a vacuum dipole, *i.e.* a field containing no currents which can be derived from a scalar potential. The potential is written as an expansion in spherical harmonics, expanded to second order and used in the iteration. Of course, once the curvature is included, the field is no longer potential, but now we have a basis from which to proceed iterating back and forth between the shape and the field until a self consistent picture is attained. The distortion of the field is found to have azimuthal asymmetry and the drift shells of the particles [to be discussed below] are different on the night and

day sides of the planet.

Magnetospheric plasma particle populations

A planetary magnetosphere can contain various genera of charged particles, differing from one another in composition, source, energy and spatial and temporal distribution. We shall deal separately with the high energy or radiation belt particles and the thermal particles at the various planets. At Earth, the energetic and auroral particles are in general of solar origin, while at Jupiter and Saturn, the many satellites whose orbits are contained within the magnetosphere provide the thermal plasma and at Jupiter the energetic and auroral plasma as well.

Early in the history of magnetosphere studies of the Earth, the question arose as to the role of the interplanetary magnetic field [IMF] in the injection of solar particles into the magnetosphere. In figures 1 the open field model of Voight and Fuchs [1979] is shown. If the IMF has a northbound component, a null surface is formed and magnetic merging or reconnection can take place. A second neutral point or surface must exist in the tail, which is caused by the sweeping of flux tubes by the incident solar wind. Plasma can enter through the tail and thus reach the polar regions. Because of plasma flow into the neutral zones with finite conductivity [in the ionosphere], there must be an electric field corresponding to the potential drop across the polar cap caused by the work done by the solar wind there.

$$\Psi = \oint \frac{vB}{c}\, ds \qquad (7$$

Typical terrestrial ionosphere wind speeds are about .1 km/s so the potential over a polar cap dimension of 6000km with B ≈ .5G is about 30 kV. Since σ_0 is very large, the field lines should be equipotentials and the same potential drop should map out to the neutral zone in the tail. Thus, this electric field will be directed across the tail from dawn to dusk and its effect will be to drive plasma towards the planet. The importance of this effect will vary from planet to planet.

Originally the question of the open or closed nature of the magnetosphere relative to the IMF was controversial. The closed model depends on viscous effects to provide the energy input to the magnetosphere from the solar wind. It is certainly true that there are viscous effects deriving from friction associated with wave-particle interactions in the magnetopause region, but it is now generally accepted with respect to the terrestrial magnetosphere that connectivity to the solar wind field is the major source of both energy and solar matter. The means by which the field lines connect is called magnetic merging or reconnection and is still a major area of plasma physics research. An understanding of at least the basic elements of the merging process is essential to understanding the basic facts of magnetospheric physics. We must also understand the division of the magnetospheres into particular regions characterized by the domination of particular physical processes.

There are two main sources of bulk plasma motion in a planetary magnetosphere. One is the rotation of the planet which causes an electric field to come into existence. The picture is most easily understood in the equatorial plane. If the planet is rotating in the clockwise direction and the field is directed from from north to south, as it is at Jupiter and Saturn, then the plasma will drift in the direction of the rotation if there is an electric field in the outward radial direction. This field will be dominant near the planet. The other source of convective motion is the cross-tail electric field described above which is induced by the flow of the solar wind past the planet. It will drive plasma towards the poles by its interaction with the component of the magnetic field parallel to the neutral sheet, the so called plasma sheet, and towards the Earth by its interaction with the normal component of the magnetic field. It will be the dominant mechanism in the far tail.

There will be an intermediate region where the two electric fields will oppose one another, *i.e.* their vector sum will tend to the null vector. The line or surface upon which it vanishes, the separatrix is known as the *plasmapause*. Inside the plasmapause the plasma density is high and outside of it the plasma density is low.

In the polar regions, there will be a region of space near the ionosphere which is connected to the far tail by open field lines. This region will have much less plasma in it than the regions immediately equatorward of it and is known as the plasma trough. Ideally, a particle which was created in the plasmasphere, the region inside the plasmapause, can never cross the separatix and similarly the plasmasphere is a forbidden region to particles that come from the tail. In the magnetosphere of the Earth, the plasmapause is located at about 3 R_E while at Jupiter and Saturn where the planet is huge and rotates rapidly and the solar wind is weak, the plasmapause and magnetopause more or less coincide. The magnetosphere of Uranus seems to be convection dominated at the present epoch while nothing is known about the situation at Neptune.

Radiation Belts in Magnetospheres

Very early in the history of spacecraft exploration of the environment of the Earth, Van Allen and others discovered that the magnetosphere contains a population of very high energy particles. The study of the morphology, composition and energy budget of these belts of particle radiation, known as the Van Allen belts, was the main topic of research during the first decades of the world wide space science program.

The motion and trapping of these particles in the in the geomagnetic field depends on the theory of adiabatic motion of charged particles in a magnetic field, in this case, the dipole or near-dipole field of a magnetized planet. It was found by Alfvén the the motion of such a particle can be described with full relativistic generality by three invariants or first integrals of the motion. The term adiabatic is used in the sense that the invariance of the quantity under discussion is valid if some parameter varies very slowly in either time or space. We shall see that as a result of these invariants, the motion of a particle in the field of a planet will have three modes, widely separated from one another in time scale.

The first mode of motion that we consider is the gyration of the particle around a magnetic field line. Its time scale is the gyroperiod, which for example in the field of the Earth for a 1 Mev electron is about 10 millisecond. If the magnetic field varies very little during that time or little spatially over the corresponding gyroradius which is about 7 km, it can be easily shown that the magnetic moment of the particle is preserved. Thus

$$\frac{P_\perp^2}{B} = \text{const.} = \frac{P^2\sin^2\alpha}{B} \qquad (8$$

where P is the momentum and α the pitch angle. This is a well known result that is applied to laboratory mirror machines as well as to planetary dipole magnetospheres. Since the energy is conserved, we may define the *mirror ratio*:

$$\frac{B}{B_m} = \sin^2\alpha \qquad (9$$

The maximum value of the magnetic field magnitude is attained at the turning point where $\sin\alpha = 1$. It is well known that a static magnetic field does no work and therefore the total energy, *i.e.* the square of the total momentum is conserved. We can therefore write the parallel component of the momentum in the form:

$$P_\parallel = P\cos\alpha = P\left(1 - \frac{B}{B_m}\right)^{\frac{1}{2}} \qquad (10$$

Thus we see that the particle can move along the field line until the ambient magnetic field becomes equal to the maximum field. It will then turn and move back towards weaker field. On a closed field line, the particle will perform a bounce motion between its two mirror points which we denote as m_1 and m_2. The period of this motion will be given by

$$\tau_b = 2\int_{m_1}^{m_2}\frac{ds}{v_\parallel} = \frac{2}{v}\int_{m_1}^{m_2}\frac{ds}{\sqrt{\left(1 - \frac{B(s)}{B_m}\right)}} \qquad (11$$

It can easily be shown by means of a Taylor expansion that the motion

of a charged particle whose amplitude of motion about the equator is small is very well described by the equation of motion of a harmonic oscillator.

As a particle bounces back and forth along a closed field line, it will experience a force associated with the fact that the magnetic field is not uniform.. This gradient drift is caused by a Lorentz force perpendicular to B and in the direction of $-\nabla_\perp B$. It is known that the drift caused by a force with a component $\perp$ to B is given by:

$$V_d = \frac{m}{q}\frac{F\times B}{B^2}\ . \qquad (12$$

In the case of a slowly varying magnetic field, then the cyclotron averaged component of the Lorentz force normal to the field will be given by:

$$\langle F \rangle = \frac{q}{2\pi c}\int_0^{2\pi} v_\perp(B + \nabla_\perp b\ \rho_g\cos\phi)\cos\phi d\phi \qquad (13$$

This leads to the magnetic gradient drift velocity:

$$v_g = \frac{mv_\perp^2\nabla_\perp B}{qB^2} \qquad (14$$

Note that this drift differs significantly from the electric field drift. It depend on both energy and charge, so that electrons and protons in a magnetosphere will drift in opposite directions, which gives rise to a current. This current, known as the "ring current" plays an important role in planetary magnetospheres.

The periodic motion along the field line between mirror points is associated with the so-called second invariant. In terms of canonical momentum and conjugate coordinates, this second invariant or first integral of the motion can be written:

$$J_2 = \oint P_\parallel ds = 2pI = \int_{m_1}^{m_2} ds\ \left(1 - \frac{B(s)}{B_m}\right)^{\frac{1}{2}} \qquad (15$$

in which I is an integral that depends on the field geometry. As the particle drifts around the planet and bounces back and forth between the mirror points, it describes a surface. If the particle is acted upon by forces that remain virtually constant over a bounce period, then J_2 is an adiabatic invariant of the motion. If the first two adiabatic invariants are conserved, then, we may write the general expression for an action- angle first integral with the aid of the momentum and the vector potential of the magnetic field, A:

$$J_1 = \oint ds \cdot \left(P + \frac{q}{c} A \right) \qquad (16$$

where the integrand is the canonical momentum of the particle, in a simpler form for a particle drifting around the planet. The first term will be smaller than the second term by the ratio between the gyroperiod and the drift period, so that we can write the third invariant:

$$J_3 = \frac{q}{c} \oint ds \cdot A = \frac{q}{c} \Phi \qquad (17$$

in which Φ is the flux through the drift surface. The line integral is along all the field lines on which the particle orbits as it drifts around the planet. The surface thus created has a magnetic flux through it which is conserved in the absence of forces or if the magnetic field does not vary significantly during a bounce period.

In figure 2, we see that there are regions of stable trapping of such particles and other regions in which particles cannot remain trapped. This is a result of the asymmetry imposed on a magnetosphere by the solar wind. For purposes of understanding the motion of a particle in a magnetosphere, a special coordinate system has been devised. It was found that the position of a particle can be defined by the magnitude of the magnetic field and a parameter L which identifies the drift shell on which the particle is drifting. In a dipole field, the intersection of the drift shell and the equatorial plane is a circle of radius LR_p where R_p is the radius of the planet. Since the equation of a dipole field line is

$$r = LR_p \cos^2\lambda \qquad (18$$

where λ is the magnetic latitude, we can also define L in terms of the invariant latitude Λ of a flux shell, the latitude at which the shell intersects the surface of the planet:

$$\Lambda = \arccos\left(L^{-\frac{1}{2}} \right) \qquad (19$$

Since the magnetic gradient drift is perpendicular to both B and $\nabla_\perp B$, it follows immediately that for a particle trapped in the equatorial plane, the drift trajectory will be along a surface of constant B. The two constant parameters, B and L are sufficient to identify the position of a particle in a field having dipole symmetry. The parameter L is, of course, intimately related to the second adiabatic invariant.

If the field were indeed symmetrical, particles would drift around in circles and nothing else would happen to them. In fact, it turns out that in order that a particle be trapped, its mirror points must lie above the dense atmosphere and its drift surface must contain only closed field lines. Since the solar wind compresses the magnetosphere, a given value of the magnetic field will lie closer to the planet on the night side than on the day side.

If the mirror point, i.e. the point at which the particle experiences the maximum field is at a latitude such that the field line is closed at all longitudes, then the particle is said to be trapped. If the particular field line is closed on the day side, but is one of the field lines that is swept out into the tail on the night side, then the particle cannot complete a drift trajectory of 2π radians and it is said to be quasi-trapped. If the field line is open to the solar wind at all latitudes, then it is a non trapping region and there will be no particles bouncing and drifting on it. The three regions are shown in figure 2.

The above considerations hold primarily for the energetic particles of the Van Allen belts. There is another set of considerations that delineates permitted and allowed regions for thermal plasma particles as was discussed above in the section dealing with the distortion of the magnetosphere by the solar wind flow interaction..

If the planet is a giant rapid rotator, such as Jupiter, then there exists a means by which the energy of the rotation of the planet can find its way into the energy budget of charged particles. This is a process known by the term *pickup*. If a neutral particle is ionized in the presence of a magnetic field it will begin to gyrate around the lines of the field at a perpendicular velocity equal to that required to transform its rest frame into that in which there will be no electric field associated with the motion.

The Earth

As mentioned above, the magnetosphere of the Earth is solar dominated in the sense that the energetic particles which it contains derive their energy from the sun and not from any intrinsic characteristic of the Earth itself. In this section, we shall deal with transport phenomena which have been observed over the last three decades in the magnetosphere of the Earth and are therefore best known there. They are, however, by no means unique to the Earth and we shall encounter them in our discussion of other magnetic planets.

Violation of Adiabatic Invariants

If indeed the particles of the radiation belts were able to gyrate, bounce and drift while conserving all the invariants, they would remain forever trapped in the magnetosphere and our discussion could end now. If fact, the condition for the conservation of the adiabatic invariants are often violated and the particles have very finite lifetimes in the magnetosphere.

A major sink for radiation belt particles is the atmosphere of the planet. Obviously a particle having a zero pitch angle at the equator will have a zero pitch angle everywhere so that $v = v_{\parallel}$ and the particle will collide witha the planet. Since planets have

atmospheres, the particle will be thermalized by collisions with atmospheric molecules and atoms long before it reaches the surface. Thus there will be a range of equatorial pitch angle space on every field line in which the associated mirror point will be deep enough in the dense atmosphere so that the particle will undergo collisions with atmospheric gas particles and will not return on its orbit. This range of pitch angles defines a cone known as the loss cone. If B_a denotes the field magnitude at the top of the collision dominated region of the polar ionosphere, B_e the field magnitude at the equator and α_a the equatorial pitch angle of a particle that mirrors at B_a, then conservation of the first adiabatic invariant implies that the loss cone is defined for all equatorial pitch angles, α_e which satisfy:

$$0 < \alpha_e < \sin^{-1}\left[\left(\frac{B_e}{B_a}\right)^{\frac{1}{2}}\right] \qquad (20$$

Any particle that has its equatorial pitch angle in this range will not survive more than one half bounce and will be precipitated into the atmosphere. Thus, if the adiabatic invariants are conserved, these trajectories will always be empty of particles. The rest of the sphere of pitch angle space will, on the other hand, contain energetic particles. For the particular case of the terrestrial magnetosphere it may be noted that the radiation belts consist of two components, an inner belt in which the main energetic component is proton and an outer belt made up of energetic electrons. We need not worry here about charge neutrality since the radiation belt particles are very few in number compared to the ambient thermal plasma and their compensating particles can be at lower energies.

An inner zone proton whose L parameter, *i.e.* its maximum distance from the planet is a few planetary radii will in general encounter residual atmospheric density along its entire orbit. Mostly it will collide with free and bound electrons in the upper ionosphere. Since it is massive compared to these electrons, it will undergo little or no scattering in pitch angle, but it will be decelerated. In general, the lifetime of a typical proton against deceleration by

Coulomb collisions is of the order of 1000 years. This is comparable to the time scale for secular variation of the magnetic field (≈ -.016 G/century), so that contraction of drift shells can make up the energy loss. This follows from the fact that if the magnetic field increases and the magnetic moment is conserved, then the energy must increase as well.

For protons at the lower end of the radiation belt energy scale, there is another sink which is more effective than Coulomb deceleration. This sink is charge exchange. The energetic proton becomes a fast neutral hydrogen atom which is no longer confined by the magnetic field, while the newly created proton will have a much lower energy and will not be a radiation belt particle. For protons of magnetic moment 1 Mev/G, the lifetime against charge exchange will ambient hydrogen atoms is as low as a fraction of a day at $2R_E$ in the magnetosphere of the Earth. At Jupiter and Saturn, different considerations hold, which will be discussed when we deal with these planets specifically.

Inner zone electrons undergo pitch angle scattering and energy diffusion [range straggling], since their mass is the same as that of the electrons with which they are colliding. Thus an electron that is not on a trajectory that reaches the atmosphere, can, in principle, be scattered onto a trajectory that will reach the atmosphere. In actuality, the scattering takes place in general in the upper atmosphere and an electron that mirrors above the atmosphere would, in a symmetric dipole, continue to do so almost indefinitely. In truth, there is a major asymmetry in the near field of the Earth, known as the South Atlantic anomaly. This is a region [discovered by Halley in the early 18th century] in which the field is much weaker than it is at other longitudes. Thus an orbiting particle must plunge deeper into the atmosphere to reach its particular B_m determined by its equatorial pitch angle. The net result is an increase in the magnitude of the loss cone as the particle drifts across the longitudes that correspond to the South Atlantic anomaly. This is the ultimate sink for inner zone radiation belt particles.

If the energetic electron flux beyond 1.25 R_E in the terrestrial magnetosphere is increased [by a magnetic storm or a high altitude nuclear explosion], it is found to decay more rapidly than would be expected as a result of atmospheric interactions only. The situation is most extreme at the larger distances, beyond $4R_E$ where storm associated enhancements of the flux at E ≈ .5 Mev will decrease by exp(-1) on a time scale of about 5 days. Such a decay at those distances would require a time of thousands of years to be created by collisions with the local residual atmosphere. It is, therefore, natural to invoke other types of interactions to explain these results. The interactions are collective interactions in the plasma involving waves that are eigenmodes of the plasma and they are generally known as *Wave-Particle Interactions*. The main point of these interactions is that they constitute forces that can violate any or all of the three adiabatic invariants. For example a wave whose frequency is comparable to the gyrofrequency or whose wavelength is comparable to the gyroradius of a particle can cancel the condition that the magnetic field should not vary significantly in time over a gyroperiod or in space over a gyroradius. If a particle encounters such a wave, its magnetic moment can be changed. If the interaction is elastic, i.e. the energy is conserved, then the only thing that can be changed is the pitch angle. Similarly, if the particle encounters a wave whose period is near that of the bounce orbit, the second invariant can be violated.

The first type of wave driven diffusion that we shall consider will be diffusion in pitch angle space. We must bear in mind that there are two different kinds of diffusion in this space and the absorbing wall is represented by the dense atmosphere. We define the mirror ratio, M, as the ratio between the maximum field B_m and the minimum field, B_e which in the geomagnetic case is the equatorial field on a given field line. A particle is said to be in the loss cone if its maximum field is found at an altitude that is inside the dense atmosphere. We define a trapped particle as a particle whose trajectory at the equator is outside the loss cone:

$$\alpha_e > \alpha_c = \sin^{-1}\left(M^{-\frac{1}{2}}\right) \quad (21$$

For a laboratory machine, $\alpha_c \approx 45^0$ while for the Earth's magnetosphere it is about 3^0 at $L = 6$. Thus, in a laboratory mirror machine, a particle will be diffused readily into the loss cone in a time of the order of the inverse of the diffusion coefficient and will not diffuse back out of it. This is known as Strong Diffusion and it will cause the distribution in pitch angle space to be isotropic. In the magnetosphere, on the other hand, a particle will random walk in pitch angle space to the edge of the loss cone and will be lost in the next half bounce. Thus the loss cone will be empty while the trajectories up to the edge of the loss cone will be full. This provides a gradient in pitch angle space which can serve as a source of free energy to drive the wave modes that provide the pitch angle scattering of the particles. If $D_{\alpha\alpha}$ is the diffusion coefficient, defined by the diffusion equation:

$$\frac{\partial f}{\partial t} = \frac{1}{\sin\alpha}\frac{\partial}{\partial a}\left[D_{\alpha\alpha}\sin\alpha\frac{\partial f}{\partial\alpha}\right] \quad (22$$

Liouville's theorem tells us that in the absence of collisions we can track the general pitch angle distribution function back to the equatorial pitch angle distribution function along a particle trajectory. Thus:

$$f(\alpha) = f_e(\alpha_e) \quad (23$$

and the variation of α along the trajectory is given by:

$$\sin^2\alpha = \frac{B}{B_e}\sin^2\alpha_e \quad (24$$

Let us rewrite equation 11 for the bounce period for a dipole field thus:

$$\tau_b = \frac{4R_E L}{v}\, s(\alpha_e) = \tau_0 s(\alpha_e) \quad (25$$

where the geometrical function $s(\alpha_e)$ is given by the latitude integral along the field line:

$$s(\alpha_e) = \int_0^{\lambda_M}\frac{\cos\lambda\sqrt{(1+3\sin^2\lambda)}\,d\lambda}{\sqrt{\left(1-\sin^2\alpha_e\sqrt{(1+3\sin^2\lambda)}\cos^{-6}\lambda\right)}} \quad (26$$

where λ_M is the mirror point latitude. We note that this function has a rather weak dependence of the equatorial pitch angle.

We wish to consider the time-averaged form of the diffusion equation, so we average equation 22 over a bounce period:

$$\tau_0 s(\alpha_e)\frac{\partial f_e}{\partial t} = \int_0^{\tau_b}\frac{1}{\sin\alpha}\frac{\partial}{\partial a}\left[D_{\alpha\alpha}\sin\alpha\frac{\partial f}{\partial\alpha}\right]dt \quad (27$$

We next transform the integral over time to an integral over latitude by means of:

$$dt = \frac{1}{v_\parallel}\frac{ds}{d\lambda}\,d\lambda$$

where for a dipole field $\frac{ds}{d\lambda} = R_E L\cos\lambda\left[1+3\sin^2\lambda\right]^{\frac{1}{2}}$. We transform all the expressions from pitch angle space to equatorial pitch angle space. The diffusion coefficient transforms according to the relation:

$$D_{\alpha\alpha} \propto \frac{(v\Delta\alpha)^2}{2\Delta t}$$

and

$$\frac{(v\Delta\alpha_e)^2}{2\Delta t} = \frac{(v\Delta\alpha)^2}{2\Delta t}\left[\frac{\partial\alpha_e}{\partial\alpha}\right]^2 \quad (92$$

which leads to the bounce averaged pitch equatorial pitch angle diffusion equation to which we have added an atmospheric loss term:

$$\frac{\partial f_e}{\partial t} = \frac{1}{s(\alpha_e)\sin\alpha_e\cos\alpha_e}\frac{\partial}{\partial\alpha_e}\left[s(\alpha_e)\sin\alpha_e\cos\alpha_e\langle D_{\alpha\alpha}\rangle\frac{\partial f_e}{\partial\alpha_e}\right] - \frac{f_e}{\tau_a} \quad (28$$

in which the bounce averaged diffusion coefficient is given by:

$$\langle D_{\alpha\alpha} \rangle = \frac{1}{\tau_b} \int_0^{\tau_b} D_{\alpha\alpha} \left(\frac{\partial \alpha_e}{\partial \alpha} \right)^2 dt$$
$$= \frac{1}{2s(\alpha_e)} \int_{\lambda_1}^{\lambda_1} D_{\alpha\alpha} \left(\frac{\partial \alpha_e}{\partial \alpha} \right)^2 \frac{\cos^2\alpha}{\cos^2\alpha_e} \cos^7\lambda \, d\lambda \qquad (29$$

In principle, the problem could be solved by integration of equation 28 for the time evolution of the pitch angle distribution. It is, however, more illustrative for our purposes, to attempt to evaluate the content of this equation in terms of the physical processes that take place.

A classical paper on this subject is that of Kennel and Petschek [1966]. They dealt with the steady state limit on the flux of outer zone energetic particles which have a resonance with whistler waves at the Doppler shifted electron gyrofrequency. We need not go into the mathematical details of their study in order to understand the physics behind it.

The source of free energy that drives the wave spectrum is in the anisotropy of the particles pitch angle distribution. A loss cone distribution is inherently unstable and the waves will grow to a large amplitude. On the other hand, the pitch angle diffusion coefficient which is inversely proportional to the scattering lifetime of the particles is proportional to the square of the amplitude of the waves, i.e. to their energy density. Thus if the waves grow in amplitude, the particles will tend to be scattered, thus reducing the source of free energy, i.e. the growth rate of the waves. Eventually, the distribution will become isotropic and the growth rate will vanish, which leaves a certain finite intensity of waves. Thus the tendency is to reach marginal stability at which the waves neither grow nor decay.

In order to understand this interaction a bit better, consider a single electron interacting with a whistler wave at resonance, i.e. under the condition:

$$kV_r = \omega + \Omega_e = kv_\parallel \qquad (30$$

Whenever the electron absorbs or emits a quantum, the wave energy will change by an amount $\hbar\omega$ and the momentum will change by $\hbar k$. Thus if the wave grows, the parallel energy of the particle will change by $-\hbar k V_r$ and the total particle energy will change by $-\hbar\omega$ so that the ratio becomes:

$$\frac{dE}{dE_\parallel} = \frac{\omega}{kV_r} = \frac{1}{1 + \frac{\Omega_e}{\omega}} < 0 \qquad (31$$

since the whistler wave frequency is less than the electron gyrofrequency. Thus if the waves are growing, then the majority of the resonant particles are losing energy, i.e. $dE < 0$, which then implies that $dE_\parallel > 0$ for this same majority. The result is that the anisotropy that is the source of the wave growth is diminishing and eventually the waves will stabilize at a finite amplitude. Note that in accordance with the correspondence principle, Planck's constant h does not appear in the final result.

In the magnetosphere, if there is a magnetic storm resulting from a solar event that injects a large number of energetic particles into the magnetosphere, the waves will grow accordingly and thus enhance the rate of scattering of the electrons until the electron flux is reduced to the limiting flux. The electrons provide amplification for the waves which in the steady state must balance the loss of wave energy caused by imperfect reflection at the ionosphere. Thus the limiting flux can be calculated by equating the growth caused by the electrons to the losses at the ionospheric mirrors.

The steady state is maintained if there is a source of accelerated electrons which can replace the electrons that are precipitated into the atmosphere by the whistler waves. It is interesting and initially counter-intuitive that the flux of electrons that can be stably trapped is independent of the strength of this source. Resonant electrons will accumulate until their flux is sufficient to generate whistler waves that will pitch angle scatter them. Any further electrons that are supplied will be scattered into the loss cone and will not contribute to the resident population.

This theory, which has been expanded and extended by various people has turned out to be correct. Kennel and Petschek considered only parallel propagating electromagnetic waves for electrons and ions. It has been shown that if off-angle waves are allowed, the allowed resonances will include higher harmonics of the gyrofrequency and the scattering will be correspondingly enhanced.

These considerations hold only for those particles that have sufficient energy to attain resonance with the waves. It has been shown by Kennel and Petschek that the condition to achieve this is:

$$E \geq E_r = \frac{B^2}{8\pi n} \qquad (32$$

The result will be obtained below for a particular pitch angle distribution.

This is the magnetic energy per particle and as may be noted is related to the square of the Alfvén velocity. We also note that inside the plasmasphere, the resonant energy is lower because the particle density is much higher. Thus particles of lower energy can now interact with the wave and the result is a general loss of particles. The slot between the two radiation zones is related to this phenomenon. During magnetic storm injection events, the slot fills up with electrons and is emptied by the waves that are created by the existence of the electrons on a time scale of hours to days.

The general form of the dispersion relation for waves propagating along a magnetic field is given by:

$$\omega^2 - c^2k^2 + \pi \sum_j \int_{-\infty}^{+\infty} dp_\parallel \int_0^{+\infty} p_\perp^2 \, dp_\perp \frac{1}{\gamma_j(\omega - k_\parallel v_\parallel) \pm \Omega_j} \left[(\omega - k_\parallel v_\parallel) \frac{\partial f_j}{\partial p_\perp} + k_\parallel v_\parallel \frac{\partial f_j}{\partial p_\parallel} \right] = 0 \qquad (33$$

This equation can be solved for both the real and imaginary parts of ω. It is customary to use the Cauchy principle value theorem to evaluate the integral and what is found is that the Im ω value, *i.e.* the growth rate of the waves is proportional to the flux of energetic particles. The distribution function can be represented as the sum

of a cold plasma represented by a Maxwellian distribution and a "tail" of hot or radiation belt particles described usually by a power law distribution in energy and $\sin^{2s}\alpha_e$ distribution in pitch angle space. If the distribution function of the energetic particles is indeed of the form:

$$F_j(p,x) = \left(\frac{p_\perp}{p}\right)^{21} \left(\frac{p_\perp}{p}\right) p^{-2} J_\perp\left(\frac{p_\perp^2}{2m_j}\right)$$

where $J_\perp$ is the flux of particles normal to the field line and we assume that the particles are at most mildly relativistic, $\gamma \approx 1$. In such a case, the integration can be performed and the growth rate written explicitly in terms of known functions:

$$\mathrm{Im}\,\omega \approx -\frac{2\pi^3 v_g v_p}{\omega c^2} \frac{q_j^2}{|k_\parallel|} \left(\frac{p_\perp}{mv_r}\right)^{21} B(s+1,1) \left\{\omega + s(\omega - |\Omega_j|)\right\} J_\perp\left(\frac{p_\perp^2}{4m_j}\right) \qquad (34$$

where B is the beta function which is related to the Euler gamma function thus:

$$B(s+1,1) = \frac{\Gamma(s+1)\Gamma(1)}{\Gamma(s+1+1)} .$$

This shows that for interaction of electrons with whistler waves or for the interaction of protons with ion cyclotron waves, the growth rate will be positive for frequencies that satisfy:

$$0 < \omega < \left| \frac{s\Omega_j}{s+1} \right| \qquad (35$$

It should be understood that this condition of instability does not guarantee that there will be a significant energy density of waves in the magnetosphere, since it was derived on the assumption that the plasma is both infinite and homogeneous. In fact, in order that there be such a density available to drive the diffusion, a certain amount of internal wave reflection is required. This is provided by the fact that the waves tend to be ducted along the field lines and since their frequency is less than the plasma or upper hybrid frequency of the ionosphere, there will be a certain finite reflectivity, R. This analogous to what causes the internal amplification in a maser. Then the condition that spontaneous wave generation has a positive energy budget is given in terms of the reflectivity R, the group velocity v_g and the path length between wave reflections which is approximately $2R_EL$. The condition is that in a time given by

$$\frac{2R_EL}{v_g}$$

the exponential growth of the wave amplitude will balance the loss because of the imperfect reflection, R < 1. The condition can be written in the form:

$$\text{Im}\ \omega \approx \left| \frac{v_g \ln R}{2R_EL} \right| \qquad (36$$

Since Im ω is proportional to the flux of particles, wave energy will be generated and trapped until the diffusion coefficient becames so large that that excess particles are scattered into the loss cone. Then equation 35 will no longer be satisfied and the wave energy density will now longer increase. In fact, if as shown above, the result of a total decrease in energy will be an increase in $p_\parallel$ so that the pitch angle which is given by

$$\alpha = \tan^{-1} \frac{v_\perp}{v_\parallel}$$

will decrease as a result of the wave particle interaction. The result of this is that there will be a limiting flux, of particles above the resonant energy given above. Schulz and Lanzerotti [1974] have plotted the theoretical limiting integral flux of particles having energies above 40 keV and have compared it to observed integral fluxes (see figure 3). The integrated flux is calculated from an energy equal to the resonance energy, which itself follows from the resonance condition:

$$p^* = \frac{m_j}{k_\parallel} \left(\omega + \Omega_j \right) \qquad (37$$

where $\omega = \frac{s}{s+1}\Omega$. This can be coupled with the cold plasma relation from the real part of the dispersion relation:

$$\left(\frac{\omega}{k_\parallel c} \right)^2 \approx - \frac{\omega(\Omega_e + \omega)}{\omega_{pe}^2} \qquad (38$$

for electrons and

$$\left(\frac{\omega}{k_\parallel} \right)^2 \approx c_A^2 \left(\frac{\Omega_i - \omega}{\Omega_i} \right)]]] \qquad (39$$

for ions. Put these back into equation 36, square and express in terms of energy to get the required values of the energy for the two cases:

$$E_e^* = \frac{B^2}{8\pi n_e s(s+1)^2}$$
$$E_i^* = \frac{B^2}{8\pi n_e (s+1)s^2} \qquad (40$$

Violation of the Second Invariant

A force field can violate the second invariant while conserving the first through a resonant interaction with the bounce motion of the particle. Such a force can be associated with magnetospheric micropulsations, which are compressional waves, or with electrostatic waves. In either case, the force must have a component along the magnetic field in order to effect the bounce motion. We can write the equation of motion in the form:

$$\frac{dp_\parallel}{dt} + \frac{M}{\gamma}\frac{\partial B}{\partial s} = f_\parallel(s,t) \qquad (41$$

where $p_\parallel = \gamma m_0 v_\parallel$. We take the unperturbed field to be static, multiply the equation by $p_\parallel$ and obtain:

$$\frac{dw}{dt} = \frac{p_\parallel}{m_0} f_\parallel \qquad (42$$

where $w = \frac{p_\parallel^2}{m_0}$. This tells us that the force will primarily affect the energy of the particle motion in the parallel direction without influencing the first and third components, i.e. M and Φ will remain constant. For a dipole field is is found that the ensuing diffusion coefficient varies as $\sin^{-2}\alpha_e$. The result is that this mechanism works primarily on oscillatory particles that mirror near the equator. Thus it can provide a seed population for a pitch angle scattering mechanism that is most effective far off the equator. The result will be a two stage loss mechanism for radiation belt particles.

In both of the above cases, violation of M or violation of J, it should be kept in mind that diffusion can be either "weak" or "strong" in the sense defined above. If the diffusion is fast enough so that the particle can diffuse back out of the loss cone in a time shorter than a half bounce period, the loss cone will be full of particles and the distribution of the particles in pitch angle space will tend to isotropy. On the other hand, if the particle cannot diffuse out of the loss cone in the time available before it reaches its mirror point, it will be scattered by atmospheric particles and will not mirror. In that case, the loss cone will be empty of particles and the distribution will remain anisotropic. Since anisotropy is a source of free energy for the support of wave growth, the scattering will continue as long as there are particles left to support the waves.

Radial Diffusion: Violation of the Third Invariant

While pitch angle scattering associated with the violation of either or both of the first two invariants is primarily a loss mechanism for radiation belt particles, diffusion connected with the violation of Φ is usually associated with the creation of the belts. In particular, this is true of radial diffusion, in which M and J are

conserved, since the particles which come from a source on the outside gain energy as they approach the planet. Thus diffusion in Φ space which conserves M and J plays the double role of accelerating particles to higher energies and injecting them into the inner part of the magnetosphere.

In addition to the particles that are transported into the magnetosphere from interplanetary space or from the far tail, the magnetosphere contains particles that are born locally. They come from the decay of neutrons $n \rightarrow p + e + \bar{\nu}$, so called β decay. When a solar proton (E > 100 MeV) or a galactic cosmic ray particle collides with an atmospheric particle, it causes the target to emit a neutron. Those neutrons which fly out of the atmosphere can enter the magnetosphere freely for the first ten minutes of their flight for they are uncharged. The charged particles that are created by either SPAND [solar proton albedo neutron decay] or by CRAND [cosmic ray albedo neutron decay] are the main source of particles in the inner zone. CRAND is about an order of magnitude more intense than SPRAND. If an oxygen nucleus, for example, is excited to an energy of above 8.6 MeV, which is the first binding energy of a neutron in that nucleus, its most probable mode of decay is to emit a neutron. For these inner zone particles, the role of radial diffusion is to provide the observed spatial distribution.

During the period 1958-1963 high altitude nuclear explosions were set off by various countries and contributed significantly to the population of the inner radiation belt. By 1968, these artificial radiation belts had decayed to below the intensity of the natural radiation belts. In the process, they provided major information on the radial diffusion coefficient for radiation belt particles.

In the outer zone, the role of radial diffusion is to provide the source of the trapped radiation. In figure 4 we show direct observational evidence for the occurrence of third invariant violation. The data were obtained by a satellite on board a spacecraft in synchronous orbit at the longitude of Hawaii. A magnetic disturbance arrives at the spacecraft several minutes before showing up on the ground at Honolulu. All channels are affected simultaneously, but they recover on a time scale equal to half the local drift period and then drop back down a full drift period later. The interpretation is that the recovery is the arrival of particles from the night side where the storm had less influence and the reappearance of the particles that had been at the day side at the time of the event. These drift period echoes persist well after the end of the disturbance that caused them. The fact that each energy channel flux oscillates at its own characteristic drift frequency is convincing evidence for the drift phase organization of the particles which are now dispersed with respect to Φ. The finite bandwidth in energy corresponds to a bandwidth in frequency and eventually phase mixes the the observations after a few drift periods. The particles had initially different energies and flux invariant values, but the detector can no longer distinguish between them although the separate identities are in reality maintained.

In order that the transport be indeed diffusive, it is necessary that a degree of stochasticity be introduced into the motion of the particles. This is derived from the above mentioned phase mixing and the fact that the sudden impulses that drive the diffusion are statistically uncorrelated on the time scale of a drift period ≈ 100 s. Thus it is meaningful to speak of third invariant violation in terms of diffusion in

$$\Phi = \frac{2\pi B_0 R_E^2}{L}$$

space. We thus obtain a diffusion equation which by use of a simple Jacobian be expressed in L space:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \Phi}\left(D_{\Phi\Phi}\frac{\partial f}{\partial \Phi}\right)$$

$$= L^2\frac{\partial}{\partial L}\left[\frac{1}{L^2}D_{LL}\frac{\partial f}{\partial L}\right] \qquad (43$$

where we have used the relation: $D_{LL} = \left(\frac{dL}{d\Phi}\right)^2$ and the particle energy distribution function is related to the observed quantity, the normal flux by: $f = \frac{J_\perp}{p^2}$ evaluated on a surface generated by the mirror

points of particles having the same values of M and J.

Radial diffusion differs from pitch angle diffusion in that it is parasitic, *i.e.* the electric fields that drive the random motion are not created by the free energy associated with the particles themselves. There are two major sources of such electric fields, motions of the magnetopause caused by fluctuations in solar wind dynamic pressure and the potential field of the magnetosphere. A third source, the electric field associated with tidal motions of the atmosphere is more important at the giant planets than it is a the Earth. The diffusion coefficients derived from these two sources are very different from one another.

The magnetic impulses are caused by rapid contractions and expansions of the magnetosphere, *i.e.* by changes in the stand off distance of the sub solar point of the magnetopause. The perturbation in the magnetic field will have two components, a symmetric perturbation and an asymmetric component. The symmetric component will affect all particles equally at all drift phases and is therefore reversible. It will not violate the third invariant. The asymmetric component will on the other hand affect the trapped particles by means of an induced electric field, which can be derived from the Maxwell induction equation. As a result, the expansion or contraction of the magnetosphere which determines the direction of the electric field can be used to derive the diffusion coefficient by means of the equations of magnetohydrodynamics. In truth, Maxwell's equations are not causal equations and the laws of causality are violated on times shorter than the travel time of an Alfven wave through the magnetosphere, $\approx \frac{b}{c_A}$. Our drift periods are, however, of the order of several minutes in duration and for all practical purposes, the arrival time of an impulse at a given L shell is independent of drift phase. After much painful algebraic manipulation given in all detail by Schulz and Lanzerotti [1974], an expression for the electric field created by fluctuations in the magnetopause distance can be derived. From this we can proceed to evaluate the response of the particles. The field will be in the direction normal to the magnetic field and the L value of the particle will be changed, thus changing $\Phi \propto L^{-1}$. The result of calculating the quantity $\langle(\Delta L)^2\rangle$ is a diffusion coefficient that varies as L^{10}. It is also nearly independent of energy and, of course, as an electric field drift, independent of species.

$$D^M_{LL} = L^{10} \sum_{m=1}^{\infty} A_m \Omega_d^2 \, P(m\Omega_d) \tag{46}$$

where Ω_d is the drift frequency, the A_m are weighting factors and P^M is the power spectral density of the magnetic fluctuations evaluated at harmonics of the drift frequency. These are the so-called drift "echoes" that have been observed in the trapped fluxes.

Electric field fluctuations may be of various types. The azimuthal electric field caused by the creation of the tail will have a spectral power density caused by its fluctuations, which ultimately are also connected with the fluctuations in the solar wind. A general expression for it can be written in the form:

$$D^E_{LL} = L^6 \sum_{m=1}^{\infty} A_e \, P^E(m\Omega_d) \tag{47}$$

This type of diffusion differs from the magnetic type in that the coefficients of the series are energy dependent. The L dependence is such that the electric fluctuations tend to dominate at the low L values and the lower particle energies, whereas at larger distances and at higher energies, the magnetic fluctuations are more important in determining the distribution of radiation belt particles. For the case of electric field fluctuations driven by atmospheric tides, $D_{LL} \propto L^3$. In this case, the insignificance of these modes in the outer magnetosphere is even more evident.

Obviously, radial diffusion and pitch angle diffusion, while deriving from different sources, must certainly interact in the magnetosphere itself. For example, a particle diffusing inwards from an external source will conserve its first two adiabatic invariants. In a dipole field, this means that:

$$\frac{E_\perp}{B} = \frac{E_\perp L^3}{B_0} = \text{const.} \tag{48}$$

i.e. the energy will increase as L^{-3}. It should be noted that the perpendicular energy is what increases and the parallel energy is not affected. Thus the particles will become more anisotropic and this will lead to an enhancement of the growth rate of whistler waves. As the particles diffuse inwards, they approach the plasmapause, inside of which the resonant energy $\frac{B^2}{4\pi n}$ decreases as a result of the increase in particle density. Ray tracing calculations have shown that a local instability can fill the entire plasmasphere with noise by means of multiple reflections. As a result of these multiple reflections, the whistler waves will no longer propagate along primarily parallel to the magnetic field, but will have oblique angles. This causes them to have higher order $n > 1$ cyclotron resonances ($kv_r = \omega - n\Omega$) with electrons. These higher order resonances will couple to electrons whose parallel energy is higher than the fundamental by a factor of order n^2. Since the magnetic energy per particle increases with latitude, high energy electrons will have $n = 1$ resonance at high latitude, $n = 2$ resonance at intermediate latitude and higher order resonances at the equator. In addition there will be a so called Landau resonance for which $n = 0$. All electrons will have a Landau resonance near their mirror points where the Doppler shift is negligible and low frequency electrostatic waves can scatter the particles. At low energy, the Landau resonances are effective for large pitch angles and at smaller pitch angles, the higher order cyclotron resonances take over. It is expected that there will be very strong gradients in pitch angle space in the "hole" between the Landau and cyclotron resonances.

It is possible now to take the precipitation lifetime term and to add it to the diffusion equation to obtain:

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} \left[\frac{1}{L^2} D_{LL} \frac{\partial f}{\partial L} \right] - \frac{f}{\tau_p} \tag{49}$$

Solution of this equation with the calculated L-dependent pitch angle scattering lifetimes and the Coulomb lifetimes gives rise to the radial profiles shown in figure 5. We see that the slot in low energy electron fluxes is a natural result of the physics involved.

This concludes our very superficial tour of the magnetosphere of the Earth. In the remainder of this course, we shall apply what we have learned to the magnetospheres of the giant planets and shall study the similarities and differences between the various planetary magnetospheres in the solar system.

MAGNETOSPHERES OF THE GIANT PLANETS

We shall embark on a comparative study of the magnetospheres of the solar system. We shall see that Earth, which we have discussed in some detail can be considered a prototype of one type of solar system magnetosphere, one that is dominated by the Sun. The next example we shall consider is vastly different and represents a type of magnetosphere in which the planet itself is far more important than the Sun in determining what goes on in the magnetosphere and how the energy budget is balanced. Before considering this Jovian case in detail, we shall, however, discuss in some broad aspects the characteristics of the outer planet magnetospheres, which can be summarized in the table. There are several comments to be made with respect to the entries in the table. We note first that all these planets are much larger than the Earth and also much more massive. They are gas giants with rocky cores and their mean density is considerably lower than that of the terrestrial planets. The Earth, for example, has a mean density of about 5.5 gr cm^{-3}. Another important point to note is that for all except Uranus, which is a very special case, the main source of plasma in these magnetospheres is the satellites, of which there are very large numbers. These giant planets are also very rapid rotators, which coupled with their huge size, makes the rotation of the planet the main source of energy in the magnetosphere. This is in sharp contrast with the Earth, where the main source of energy is solar input.

Jupiter

Jupiter is a unique object in the solar system. In addition to its huge size and mass, it has a magnetic moment second only to that of the Sun. The magnetic moment of Jupiter was first detected

accidentally by radio astronomers in 1955 and then studied intensively in both decimetric and decametric wavelengths to determine it remotely. Various gross characteristics of the field were estimated with the aid of radiation models and the observed cutoff of the decametric emission at 40 MHz. The latter phenomenon if interpreted as an electron gyrofrequency corresponds to a magnetic field intensity of about 14 G. These early radio observations were able to determine the southward polarity of the field and its tilt angle of 9.6^{0}. One of the amazing results of the radio astronomy observations was the discovery in 1962 by an Australian radio astronomer that the decametric emissions of Jupiter are controlled to a large extent by the satellite Io.

In addition to the radio astronomy, observations were made in the visible spectrum after the surprising discovery of sodium emission from Io in 1973 at the Mt. Hopkins Observatory in Arizona, USA. In 1974, observations made at the Wise Observatory showed that the sodium cloud was spread around the orbit of Io and that a fast component of it reflected the corotation velocity. Since the sodium observed is neutral, this implies that the observed fast atoms were once corotating ions that had undergone charge exchange. In 1975, again at the Wise Observatory, the first observation of the forbidden lines of ionized sulfur in the magnetosphere of Jupiter was made, near the orbit of Io, thus proving that Io is a source of plasma for the magnetosphere.

During the 1970's, four different spacecraft flew through the magnetosphere of Jupiter, Pioneer 10 (1973), Pioneer 11 (1974) and Voyagers 1 and 2, both in 1979. These spacecraft confirmed in general the remote observations and added a vast amount of new information about the magnetic field, the thermal plasma and the radiation belts of Jupiter. Intense auroral emissions were also found. Since 1979, observations have been continued from the ground and from Earth orbit in the ultraviolet part of the spectrum and an immense effort has been devoted to interpretation of the data provided by the various instruments on board the spacecraft. Many questions have been answered, but many new questions have arisen, whose resolution awaits the Galileo mission which will place a spacecraft in orbit around Jupiter.

The Magnetic Field

As may be seen from the table, Jupiter has an intense magnetic dipole moment, which corresponds to a surface field of 4 G at the equator. In addition it was found to have significant quadropole and octupole moments, which mainly contribute to the intensity close to the planet. Isointensity curves shown in figure 6 indicate the complex nature of the magnetic field at the surface of the planet.

As in the case of the Earth, the magnetic field stands off the solar wind, which is 27 times less intense there, at a large distance. If it were visible, the Jovian magnetosphere would be the largest object in the sky as seen from Earth, larger than either the Sun or the Moon. The structure is the same as at the Earth, a bow shock caused by the super- Alfvenic flow of the solar wind, a magnetosheath in which the flow is turbulent as a result of the effect of the shock wave and a magnetopause surface where the pressure balance between the field and the solar wind dynamic pressure holds.

The magnetosphere itself can be divided into three main regions, an *inner magnetosphere* extending out to the region between 6 to $10R_J$, a *middle magnetosphere* which extends from outside the orbit of Io at 6 R_J where currents in the magnetosphere begin to distort the symmetry significantly out to to the poorly defined region 30-50 R_J where magnetopause asymmetry and tail effects become important and an *outer magnetosphere* extending from there out to the magnetopause and including the very long zenomagnetic tail, which can reach out to the orbit of Saturn.

The most spectacular aspect of the Jovian system is the extreme volcanism of the satellite Io. This was predicted shortly before its discovery by Pease et al. [1979] who noted that the next satellite out Europa had a 2:1 orbital period resonance with Io. As a result there is an extremely strong tidal interaction with Jupiter which keeps Io in its locked orbit against the perturbation of Europa at the cost of a tidal energy dissipation of about a 100 TW. The result

of this is the melting of most of the interior of Io, which is presumed to have a solid crust of just a few kilometers in thickness The main gaseous output from the volcanoes is sulfur dioxide, most of which condenses onto the surface of the planet. Some fraction of it, however, is injected into the magnetosphere where it can become ionized and begin to mass load the magnetic field. It is this plasma derived from a small satellite in the inner magnetosphere that dominates the entire magnetospheric volume.

Another interesting difference between Jupiter and Earth is the absence of a plasmapause in the Jovian magnetosphere. Essentially, the plasmasphere extends out to the magnetopause, at least on the day side and the magnetosphere is completely corotation dominated.

The Inner Magnetosphere

The dominant feature in the inner magnetosphere is the body of plasma associated with the satellite Io, commonly known as the Io torus, because of its essentially toroidal shape. It was discovered in 1975 by observations made at the Wise Observatory of Tel Aviv University and was investigated in detail *in situ* by the instruments on board the two Voyager spacecraft that flew through the magnetosphere of Jupiter in 1979. The torus was found to contain dissociation products of sulfur dioxide, in various states of ionization. The Plasma Science detector on board Voyager which measures the ratio of energy to charge, between 10 eV and 6 keV, was able to distinguish between the various species on the assumption of a known corotation velocity wherever the Mach number was significantly greater than unity.

The plasma torus was found to have two components, the S^+ torus or cold torus that was observed initially from the ground and the hot torus which contains sulfur and oxygen ions in higher ionization states and at a much higher temperature. The hot torus was observed remotely by the Voyager ultraviolet spectrometer which observed emissions from S^{++},O^{++} and S^{+++}. Measurements by the Plasma Wave and Planetary Radio Astronomy experiments provided information on the electron density along the spacecraft trajectory by means of the detection of density dependent critical frequencies in the plasma and the Low Energy Charged Particle (LECP) detector extended the radiation belt measurements down to 30 keV, an energy much lower than that observed by the Pioneer detectors.

A two dimensional model of the Io plasma torus density distribution is shown in figure 7. The isodensity lines were obtained by assuming a balance between electrostatic, centrifugal and thermal pressure gradient forces, which can determine the plasma density distribution along a field line. It is fairly simple to show that for such a balance the distribution of density along a field line near the equator is Gaussian with a scale height given by:

$$H = \left(\frac{4T}{3m\Omega^2}\right)^{\frac{1}{2}} \quad (50$$

It is apparent from figure 8 that there are indeed two different plasma tori. The inner or cold torus has a scale height of ≈ .2 R_J while the outer or hot torus has a scale height of ≈ 1 R_J. The difference between the two plasma bodies is shown dramatically in the profiles of density and temperature obtained by the Voyager 1 flyby displayed in the figure.

In the cold torus it is possible, with the aid of the high Mach number to separate out the various peaks in the spectra and to associate them with particular species. The main difficulty is in the lack of uniqueness of some of the mass to charge ratios, in particular the common ratio of O^+ and S^{++} both of which have the value 16. In the warm torus, the Mach numbers are smaller and there is serious overlap between the contributions of the various ions. During the years since the encounter, vast efforts have been devoted to determining the ionic composition, temperature and density of the plasma in the Io torus. Much progress has been made, but many issues are still unresolved.

The neutral matter injected in the torus eventually undergoes ionization by either solar UV or electon impact. The newly created ion will sense an electric field created by the corotation of the magnetized plasma with the planet. As a result, the guiding center will begin to participate in the bulk motion of the corotation and

the particle itself will begin to gyrate around the field line in random phase. This creates a temperature of 270 eV for oxygen and 540 eV for sulfur in the torus. In fact, what is observed is a common temperature of about 100 eV for all the ions outside of about 5.9 R_J. This discrepancy was one of the first difficult problems to be raised by the Voyager encounter.

The UV experiment on Voyager observed ionic emissions of the order to 10^{12}W from the torus. Most of this power comes from features centered at 685 Å and 833 Å. The former is composed primarily of overlapping multiplets of S^{++} and a contribution from O^{++}, while the latter is caused by nearly coincident multiplets of O^+ and O^{++} along with smaller contributions from S^{++} and S^{+++}. There are also many other less intense features, most of which are not resolved since the UVS instrument has a resolution of at best 30 Å. In addition, spectral features from various states of sulfur and oxygen have been observed from Earth orbit at higher spectral resolution by means of IUE (International Ultraviolet Explorer) including the neutral atoms.

The emissions are the result of allowed transitions caused by electron- ion collisions and prompt radiative decay. At the time of the pickup process described above, the electron acquires very little energy because of its miniscule mass. The ions on the other hand become fairly energetic as described above and begin to collide with the electrons and to share their energy with them. The electrons, as soon as they acquire a few volts of energy, begin to lose by collisions with the bound electrons in the ions. This energy is radiated away as ultraviolet photons and the process continues. As a result of this rapid sink of energy, the final steady state is that in which the ion temperature is reduced to about 100 eV and the electron temperature is pegged at about 5 eV. In actuality, the electron distribution cannot be well described by a single thermal Maxwellian, but is really a core distribution at about 5-10 eV and a suprathermal tail or "halo" of much lower density and a mean energy of about 100-120 eV. This suprathermal component is very important for the understanding of the ionization states observed in the Io plasma torus. It is also the only way in which the observed ultraviolet emission of the torus can be maintained since 5 eV electrons cannot provide the needed excitation. One source for these suprathermal electrons that has been proposed is secondary electron production by heavy energetic ions which precipitate into the atmosphere (more on this below). The average energy of such a collision is in the range of 50 eV or so, but there will be a tail of higher energy electrons of energies up to a keV. These electrons, if created above the exobase, will spiral out the field line to the torus region, while bringing with them by ambipolarity an equal flux of cold ionospheric protons, whose energy will be below the 10 eV threshold of the PLS detector.

The existence of such a strong source of plasma in the inner magnetosphere can be expected to give rise to strong diffusive transport. The source of the electric field will, in this case, be the plasma itself. The conserved quantity, *i.e.* that which diffuses is the so-called flux tube content. This is the total density of particles integrated over a flux times the radius squared, *i.e.* for a Gaussian scale height distribution:

$$Y = NL^2 = 2\pi R_J^2 \, H \, L^3 n(0) \qquad (51$$

If the variation of this quantity is such that an inner flux tube has a much higher mass content than an outer flux tube, the situtation is described as being unstable to the centrifugal interchange mode. This is a magnetohydrodynamic wave mode in which the loaded inner flux tube which has more centrifugal energy than the empty outer flux tube, changes places with it and as a result free energy is released. We see in figure 9 that in fact, there is a fairly flat region, a "ramp" and then again a flattening out in the so-called disc region of the magnetosphere. In fact, there are severe doubts about the degree of steepness of the ramp and the question is not fully resolved. What is clear is that the torus should have dissipated itself by centrifugally driven transport, while in fact it is very much present. It is clear that something is impounding the torus and keeping it from flying off.

The initial candidate for the role of impounder of the torus was the radiation belt or ring current particle population, especially those observed by the LECP experiment which detected ions in the range 30 keV to 150 MeV and electrons between 14 keV and 20 MeV. These particles, while fewer in number than the thermal (PLS) particles, are probably the major source of plasma pressure in the magnetosphere, where their temperature is tens of keV. Their composition is sulfur and oxygen from Io plus protons from the solar wind, along with other solar heavy ions such as helium and carbon in high states of ionization. These particles are in the process of radial diffusion inwards since their source is in the outer magnetosphere. The Iogenic particles reach the outer magnetosphere in a manner to be described below and are energized there. Thus both they and the protons diffuse inwards under the influence of the same random electric fields that diffuse the thermal particles outward. The gradients of both distributions are maximal, *i.e.* the diffusion coefficient is minimal at about 9 R_J which corresponds to the outer fringe of the torus controlled region. It was found, however, that the observed ring current population with energies above 30 keV lacks sufficient energy to provide the pressure needed to contain the torus. Thus it was proposed that there must exist a "phantom plasma" with energy in the range between 4 keV and 30 keV *i.e.* in the gap between the PLS and LECP instruments. This problem has not yet been resolved and research is continuing in this area.

The question of the aurora of Jupiter which has been observed in the ultraviolet to require an energy input to the upper polar atmosphere of the order of over 10^{13}W is also of great interest. The energy must eventually come from the rotation of Jupiter for the auroral emissions are observed in regions which are connected magnetically to the outer part of the Io torus and definitely not to the tail and through it to the solar wind. At Earth, the aurorae are produced by local acceleration of electrons which are supplied from the Sun. At Jupiter, we must seek an entirely different source.

As mentioned above, the energy extracted from the rotation of

Jupiter by the act of ionization will be that corresponding to the velocity of the corotating plasma in the rest frame of the neutral atoms which will have the local Keplerian orbital velocity. It has been shown [Eviatar and Siscoe, 1980] that only half of this energy will be available for dissipative purposes, since half of it is taken up by the bulk velocity of the plasma. The channelling of this available energy into the ultimate production of ultraviolet photons of Lyman-α from atomic hydrogen and the Werner bands from H_2 in the upper polar atmosphere of Jupiter is not a straightforward or simple process and considerable effort has been devoted to understanding it.

The first proposal was precipitation of electrons by means of enhanced whistler waves in the outer part of the torus. It is a simple calculation, however, to show that even if all the electrons created were precipitated, they would all have to have an energy of 100 keV to provide the required power. It thus appears obvious that the aurora must be generated by precipitation of energetic ions.

It has been observed that the phase space density for given magnetic moment of energetic ions shows a sharp decrease at about L = 8. This cannot be the result of Coulomb scattering or charge exchange since the rates for these processes are less than the rate of inward radial diffusion at which these energetic ions are supplied. Thus the logical candidate for pitch angle scattering at the rate of strong diffusion is a wave particle interaction that involves ion cyclotron waves and the ions. If there is no local acceleration source, then the steady state configuration is described by a balance between radial diffusive transport and precipitation loss:

$$L^2 \frac{\partial}{\partial L} \left(\frac{1}{L^2} D_{LL} \frac{\partial F}{\partial L} \right) = \frac{F}{\tau_p} \qquad (52$$

where τ_p is the lifetime agains strong pitch angle diffusion and the rest of the3 notation is as used in the discussion of the terrestrial magnetosphere. At Jupiter, the radial diffusion will be driven either by centrifugal interchange instability or possibly by neutral winds in the planetary ionosphere. This is in sharp contrast with

the Earth where neither of these processes is important and diffusion is driven by electric fields associated with the interaction of the magnetosphere with the Sun. In figure 10, we show minimum strong diffusion scattering lifetimes and radial diffusion time scales, $\tau_D = D_{LL}^{-1}$ as functions of radial distance. It should be noted that the two time scales become comparable near 8 R_J while in the outer atmosphere the supply by radial diffusion is much more rapid than the pitch angle scattering ($D_{LL}\tau_p \gg 1$) and the distribution function F(L) becomes independent of L. Thus the differential flux, for a fixed magnetic moment, $j(E) = p^2F$ will scale in direct proportion to the ambient magnetic field. For a dipole field this gives, $j(E) \propto L^{-3}$ and the integral flux $J(>E) \propto L^{-4.5}$ or L^{-6} in accordance with whether or not the particles are relativistic. If, however, the pitch angle scattering becomes more rapid than the radial diffusion supply, a sharp radial gradient will develop in order to maintain the balance between supply and loss. If no other factors (such as large scale electric field convection) intervene, we expect to find a maximum in energetic particle flux at the point where $D_{LL}\tau_p \approx 1$. Such precipitation ledges are indeed observed in the LECP data set. The sharp drop in 1 MeV ions requires strong pitch angle scattering while the small decrease in 1.5 MeV electrons requires only weak diffusion. It should be noted that these fluxes do not display the azimuthal asymmetry that would be associated with large scale electric field driven convection.

It is observed that in most of the torus region, the electron fluxes are above the Kennel-Petschek critical limiting flux. We expect, therefore, that there will be much energy density in unstable cyclotron waves available for particle scattering. The fluxes are not driven down to their critical values by the scattering because of the continuous resupply by the rapid radial diffusion. It should be kept in mind that the source of this diffusion is the centrifugal interchange instability of the thermal plasma of the Io torus. Inside the torus, however, the situation is radically different.

It is of interest to compare the observed phase space density profiles with solutions of the steady state diffusion equation to test the assumption of balance between pitch angle scattering and radial diffusion. For protons of magnetic moment 70 MeV G^{-1} for which the minimum lifetime is

$$\tau_p = \tau_0 L^{\frac{11}{2}} \quad (53$$

where $\tau_0 \approx 1$ s for energetic protons and the diffusion coefficient is modeled as:

$$D_{LL} = D_0 L^m \quad (54$$

then the diffusion equation has an analytical solution:

$$F = L_*^{\frac{(m-3)}{2}} K_\nu(z) \quad (55$$

where $K_\nu(z)$ is a modified Bessel function of the second kind of order ν and argument z where:

$$\nu = \frac{2m - 6}{2m + 7}$$

$$L_* = \left(\left(\frac{2m + 7}{4} \right)^2 D_0 \tau_0 \right)^{\frac{2}{2m + 4}} L \quad (56$$

$$z = L_*^{-\frac{2m + 7}{4}}$$

It is known that m = 4 corresponds to radial diffusion driven by centrifugal interchange diffusion. It is seen in figure 11 that this fits the data very well for L > 7, but that inside it does not work. Here there is indeed a very steep gradient in the phase space density of the energetic particles. This is also the region of a steep outward gradient of the thermal torus particles. A solution of the diffusion equation with m = 12 is seen to fit the observed distribution very well in the inner region.

There are still many unresolved issues in this area. For example, it is not yet clear what the observed slope of the flux tube content ramp is. It is also not clear how the energetic particles

are precipitated into the upper atmosphere of Jupiter, although it is clear that the reduction in resonant energy caused by the high density in the torus makes a much larger flux available for scattering by the waves. The identify of the waves is not clear since the wave detectors on Voyager did not observe them unambiguously. It is to be noted however that the absence of loss inside the torus indicates that a density dependent mode is most likely to be the best candidate, such as the cyclotron modes that are most easily excited in the high density torus.

Various estimates have been made of the power required to supply the energy of the Jovian aurora. It is generally accepted that for 10 keV electrons, something like $6 \cdot 10^{13}$W would be required which would entail a total precipitation of $4 \cdot 10^{28}$ electrons per second. Alternativel, ions could be precipitated which in turn would create secondary electrons which would excite the aurora. This is a much less efficient process and there would be a need for about 10^{14}W at energies of about 100 keV. This would entail a smaller number flux of ions. Various mechanisms have been suggested for the creation of these ions and their acceleration to auroral energies. One of them is a recycling of charge exchanged matter and its transport inwards by radial diffusion as discussed in the attached reprint.

It is of interest to ask how much power is available for deposition into the Jovian atmosphere. The energetic particle detectors on Voyager could not provide an answer to this question, because they were operated in a mode designed to protect them from damage by the intense particle fluxes and therefore could not resolve the loss cone. It is possible, however, to obtain an estimate of the loss cone flux:

$$j_p(E) = \frac{2}{\alpha_c^2} \int_0^{\alpha_c} j(E,\alpha) \sin\alpha \, d\alpha \quad (57$$

by means of the knowledge of the trapped flux:

$$j_T(E) = 2 \int_{\alpha_c}^{\frac{\pi}{2}} j(E,\alpha) \sin\alpha \, d\alpha \quad (58$$

and the strong diffusion lifetime and the diffusion time, so that:

$$\frac{j_p(E)}{j_T(E)} = \frac{\tau_p(E)}{\tau_D(E)} \quad (59$$

Thus, unless the pitch angle diffusion is fast enough to match the radial diffusion, the precipitated flux will be less than the trapped flux. If strong diffusion conditions hold, then the trapped flux can be taken as an estimate of the precipitated flux and the total power precipitated into the atmosphere can be estimated. It has been estimated that the best candidates for the role of excitors of the aurora are ions of about 10-100 keV energy precipitated into the atmosphere by unidentified wave modes.

Corotation of the magnetospheric plasma

The definition of corotation is that the plasma in the magnetosphere rotates with the same angular velocity as the neutral gas in the upper atmosphere which in turn rotates at the angular speed of the planet. In general, the height integrated current in the ionosphere will be given by:

$$j' = \int_{z_0}^{z_1} j dz = \Sigma\left(E + v_n x B \right) \quad (60$$

where Σ is the conductance or height integrated conductivity of the ionosphere and v_n is the neutral wind speed. We make use of the fact that the electric and magnetic fields vary over characteristic distances of the order of a Jupiter radius which is huge compared to the vertical dimension of the ionosphere. We also make use of the assumption that the conductivity is sufficiently high to that the magnetohydrodynamic approximation holds:

$$E + vxB = 0 \quad (61$$

so that equation 61 can be written:

$$\frac{j'}{\Sigma} = (v_n - v)xB \quad (62$$

Thus we see that if the ionospheric conductance is very large, then the magnetospheric plasma will indeed move with the speed of the upper atmosphere neutral gas. The is what is meant when the term corotation is used, since the assumption $v_n \approx \Omega x r$ usually holds quite well. In fact, we can lay down four conditions for the corotation of the magnetospheric plasma with the planet:

1. There is sufficient transfer of angular momentum in the atmosphere to maintain corotation of the neutral atmosphere at ionospheric altitudes *i.e.* planet-atmosphere coupling.
2. The ionospheric conductance is sufficiently high to maintain ionosphere-magnetosphere coupling.
3. The magnetohydrodynamic approximation should hold.
4. There are sufficient magnetic stresses to balance the centrifugal acceleration of the plasma. These condition hold in general throughout most of the Jovian magnetosphere, except at large distances from the planet. There the moment arm becomes too long and the mass involved becomes larger than the torque that the ionosphere currents can supply.

In the ionosphere, ion-neutral collisions exert a drag force that is balanced by a Lorentz force:

$$F_\phi = j_\theta B_r = \sigma B_r^2 \, r\delta\omega\sin\theta \quad (63$$

where $\delta\omega$ is the angular speed of the plasma relative to the neutral gas and θ is the colatitude. The torque per unit magnetic flux can be written:

$$\frac{dT}{d\Phi} = \int \frac{F_\phi r\sin\theta}{B_r} dz \quad (64$$

where the integration is over the relevant height in the ionosphere in which collisions are important, *i.e.* the Pedersen conducting layer. This layer is thin enough so that the magnetic field and other quantities which vary on the scale of the radius of the planet can be regarded as being constant. Then equation 64 can be written in the form:

$$\frac{dT}{d\Phi} = \Sigma\delta\omega R_J^2 B_r \sin^2\theta \qquad (65$$

which for a dipole field in which

$$\sin^2\theta_s = \frac{1}{L^2}$$

$$B(R_J,\theta_s) = 2B_0\cos\theta_s$$

$$\frac{d\Phi}{dL} = \frac{2\pi R_J^2 B_0}{L^2}$$

becomes with the aid of the chain rule:

$$\frac{dT}{dL} = \frac{4\pi\Sigma\delta\omega R_p^4 B_p^2}{L^3}\left(1 - \frac{1}{L}\right)^{\frac{1}{2}}. \qquad (66$$

In this derivation we have replaced B and θ by their values at the surface of the planet. In the case of Jupiter, of course, the term "surface" refers to a reference layer such as the cloud tops. We may now proceed to derive a differential equation for the angular momentum as a function of radial distance, which will show the physical source of the corotation lag.

If $\frac{dM}{dt}$ is the rate of plasma production which is also equal to the outward flow of mass through a flux shell and l is the angular momentum per unit mass of particles distributed on a flux shell, then the radial profile of angular momentum is described by the following equation:

$$\frac{d\mathcal{L}}{dL} = \frac{d}{dL}\left(l\frac{dM}{dt}\right) \approx \frac{dM}{dt}\frac{dl}{dL} \qquad (67$$

where we have ignored the mass flux divergence compared to that of the angular momentum. The significance of this assumption is that everywhere the diffusive flux of plasma is greater than the local production rate. This is manifestly untrue in the inner magnetosphere and indeed there is little corotation lag there.

The angular momentum per unit mass is simply:

$$l = (\Omega - \delta\omega)r^2\sin^2\theta \approx (\Omega - \delta\omega)R_J L \qquad (68$$

where we have assumed in accordance with observation that the plasma is moving outwards in a thin equatorial disk so that $\sin\theta \approx 1$. We may then differentiate l respect to L and obtain the differential equation:

$$\frac{d\mathcal{L}}{dL} = \frac{dM}{dt}\Omega R_J^2 \frac{d}{dL}\left[L^2\left(1 - \frac{\delta\omega}{\Omega}\right)\right] \qquad (69$$

which when coupled with the conservation of angular momentum, $T = \mathcal{L}$, leads to the equation for the normalized corotation lag"

$$f = \frac{\delta\omega}{\Omega}$$

$$L^5\frac{df}{dL} + \left[2L^4 + 4L_0^4\left(1 - \frac{1}{L}\right)^{\frac{1}{2}}\right]f = 2L^4. \qquad (70$$

where $L_0^4 = \pi\Sigma R_J^2 B_0^2 \left(\frac{dM}{dt}\right)^{-1}$. This differential equation can be solved wilth the boundary condition that $f(1) = 0$ and the appropriate Green's function to give a solution at large L,

$$f(L) = 1 - L^{-2}\exp\left[-L^4\left(1 - \frac{1}{L^4}\right)\right] + \sqrt{\pi}\left(\frac{L_0}{L}\right)^2\exp\left(\frac{L_0^4}{L^4}\right)\left[\mathrm{erf}\left(\frac{L_0^2}{L^2}\right) - \mathrm{erf}\left(L_0^2\right)\right] \qquad (71$$

This theory predicts that for values of $L > L_0 \approx 20$ the plasma will lag significantly behind rigid corotation, which is in general true.

SATURN

The magnetosphere of Saturn has been explored *in situ* by three flyby spacecraft, Pioneer 11 in 1979, Voyager 1 in 1980 and Voyager 2 in 1981. It is in some ways similar to that of Jupiter and in other ways similar to that of Earth. It, thus, provides an interesting application and revision of magnetosphere theories developed at the previously studied planets. It differs from both of the others in that the magnetic moment is almost perfectly aligned with the rotation axis and the dipole symmetry is almost total.

As Voyager 1 approached Saturn, the first noteworthy phenomenon observed in the magnetosphere was a large cloud of neutral hydrogen, extending inwards from the magnetopause to about the orbit of Rhea at 8.8 R_S. The source of this hydrogen is the atmosphere of the giant moon Titan, which has an atmosphere with a surface pressure of about 1500 mb. The spacecraft flew past Titan and detected dense plumes of plasma flowing out of Titan as well as marked distortions of the magnetic field caused by the draping of flux tubes around the object, in a manner similar to the draping of field lines at Venus and comets. The plumes, as shown in figure 12 were identifiable after three Saturn rotations and their positions correlated correctly with observed motions of the magnetopause generated by variations in solar wind dynamic pressure. These plumes are distinguishable by their low temperature which reflects the ionosphere of Titan which is in sharp contrast with the magnetosphere of Saturn which is dominated by pickup energy.

The magnetosphere of Saturn can be divided roughly into three regions, the inner plasma sheet, the outer plasma sheet or mantle and the Titan-magnetopause region as may be seen in figure 13. The Titan region is dominated by hot plasma created by ionization of neutral atoms escaped from Titan and the cold ionospheric plumes. A sharp increase in density is seen at about 17 R_S which apparently represents the maximal incursion of the magnetopause. Inside this region, plasma can remain in the magnetosphere for a reasonably long time and resolvable, *i.e.* cold spectra begin to appear. Most of these are not nitrogen ions as would be expected to come from Titan, but rather of the water group, which must be derived from the inner icy satellites. It it not clear how the oxygen survives the transport through the dense hydrogen cloud, but then neither is it clear how large the hydrogen cloud is in its radial dimension. It would appear that convection is more important in this context than diffusion, especially if we recall that there is nothing analogous to Io in the inner magnetosphere of Saturn that could create centrifugal interchange instability.

The inner magnetosphere, inwards of Rhea is characterized by a cooler oxygen-dominated plasma, derived from the various icy satellites. It is of interest to note that at Saturn the azimuthal velocity profile relative to rigid corotation is opposite to that at Jupiter. In fact, the greatest lag is near the satellite Dione, which is a source of material by means of sputtering as is shown in figure 14.

The force balance relationship for an annulus of magnetospheric plasma of width ΔL centered on a radial distance L, rotating with angular velocity Ω in the presence of an ionospheric Pedersen conductance Σ_p and a mass loading rate $\frac{dM}{dt}$ leads to an expression for the corotation fraction:

$$\frac{v}{R_S \Omega L} = \frac{\mu}{\mu + \frac{dM}{dt}} \tag{72}$$

where μ is given by:

$$\mu = \frac{8\pi \Sigma_p B_0^2 R_S^2 \Delta L}{c^2 L^5} .$$

The observed deviations from corotation near Dione would lead to a prediction of an extremely low Pedersen conductance in the ionosphere of Saturn. Alternatively, it is possible that one of the other conditions mentioned above for corotation is not satisfied, such as the coupling between the neutral atmosphere and the plasma and/or the coupling between the planet and the neutral atmosphere. Another possible explanation is an outflow, as discussed above for Jupiter, but it is not entirely clear that there is a continuous outflow from the inner magnetosphere. In fact, observation tends to show the opposite, especially if the strongest source is at Rhea and the plasma flows inwards. If the mass loading from Dione-Tethys is sufficiently strong and the conductivity and/or the transport of angular momentum in the atmosphere is sufficiently low, the strong corotation lag in the inner magnetosphere would be a natural consequence.

The corotational behaviour of the plasma in the magnetosphere of

Saturn differs in many essential features from that in the magnetosphere of Jupiter. In the outer magnetosphere, the plasma is nearly corotating which indicates that the four conditions are fairly well satisfied, while in the inner magnetosphere, the lag is as great as 70%.

Another interesting question is the steady state density and compostion of a plasma torus created by the sputtering of an icy satellite in a giant planet magnetosphere. A model has been developed and applied with varying degrees of success to the magnetospheres of Saturn, Uranus and Neptune and the satellites contained therein. The following rate equations have been set up for the various genera of particles that can come out of a source satellite or be created by atomic and molecular processes in the magnetosphere of a giant planet:

$$\frac{dn_j^{(m,n)}}{dt} = \frac{F_j^{(m,n)}}{V} - \left[\left(\alpha_j + \beta_j \right) n_e + \eta_j^{(i)} + \eta_j^{(d)} + v \sum_k \sigma_{jk} n_k^{(a+m,i)} \right] n_j^{(m,n)} \qquad (73$$

$$\frac{dn_j^{(m,i)}}{dt} = \left[\alpha_j n_e + \eta_j^{(i)} + v \sum_{j \neq k} \sigma_{jk} n_k^{(a+m,i)} \right] n_j^{(m,n)} - \left[\nu + \gamma^{(m)} n_e + v \sum_{k \neq j} \sigma_{kj} n_k^{(a+m,n} + C \right] n_j^{(m,i)} \qquad (74$$

$$\frac{dn_j^{(a,n)}}{dt} = \frac{F_j^{(a,n)}}{V} - \left[\left(a_j + \beta_j \right) n_e + \eta_j^{(i)} + v \sum_k \sigma_{jk} n_k^{(a+m,i)} \right] \qquad (75$$

$$\frac{dn_j^{(a,i)}}{dt} = \left[\alpha_j n_e + \eta_j^{(i)} + v \sum_{k \neq j} \sigma_{jk} n_k^{(a+m,i)} \right] n_j^{(a,n)} - \left[\nu + v \sum_{k \neq j} \sigma_{kj} n_k^{(a+m,n} + C \right] n_j^{(a,i)} \qquad (76$$

where the following notation has been used: The subscripts j and k denote individual species, the superscripts (m,n), (m,i), (a,n) and (a,i) signify neutral molecules, ionized molecules, neutral atoms and ionized atoms respectively. The superscripts (a+m,n) and (a+m,i) imply that k is to be summed over both atomic and molecular species, neutral and ionized. The source strengths of neutral molecules and atoms are denoted respectively by $F^{(m,n)}$ and $F^{(a,n)}$ respectively, α_j is the electron impact ionization rate coefficient, β_j is the electron impact dissociation rate coefficient, $\eta^{(i)}$ and $\eta^{(d)}$ are the photoionization and photodissociation rates, σ_{pq} is the charge exchange cross section between neutral p and ion q., v the relative velocity between the reactants, $n_j^{(p,q)}$ is the density of constituent j of structure p (m or a) and charge state q (n or i), n_e is the electron density which is, of course, equal to the total charge density of the positive ions, ν is the plasma transport rate and C is the frequency of collisions of ions with the source satellite.

This model has been compared to the observations made in the inner magnetosphere by the plasma detector on Voyager. In particular, it has been used to estimate how much neutral atomic hydrogen can exist in the inner magnetosphere. This is important because the presence of neutral atomic hydrogen constitutes a major sink for oxygen ions.

When the Voyager spacecraft flew into the inner magnetosphere, they found a warm plasma torus at Dione and Tethys (6.3 and 4.9 R_S) but no local warm plasma at Enceladus or Mimas. Voyager 2 crossed the ring plane at 2.8 R_S and found a dense, but very cold layer of heavy ion plasma with a scale height of a fraction of a Saturn radius. This plasma was also found to be anisotropic, *i.e.* $T_\perp > T_\parallel$, which would appear to be paradoxical, since the diffusion coefficient is smaller than the Coulomb scattering rate. The explanation proposed for this phenomenon is the fact that the rate of dissociative recombination of the molecule ions obtained from the icy moons is greater than the rate of isotropization and radiative cooling by means of Coulomb collisions. Wave-particle interactions are not relevant in this context because these are thermal particles whose energy is well below the magnetic energy per particle. The plasma consists, therefore, of a hot anisotropic molecular component superimposed on a cold atomic oxygen plasma, which has a long lifetime against recombination and can diffuse inwards towards the ring system if the lifetime against charge exchange with atomic hydrogen is long enough, *i.e.* if the atomic hydrogen density is sufficiently low. One way of testing this is to run the model given by equations 137-140 for varying background neutral hydrogen density in order to see what value will reproduce the observed variation of flux tube plasma content.

Saturn, as Jupiter, is a radio source. It radiates, primarily, in the kilometric range and despite the high degree of symmetry, their appear to be two sources that are localized in local time. It has been proposed that one is located in each hemisphere, but there are still difficulties with the interpretation of the data.

It is clear that much remains to be done to understand the magnetospheres of Jupiter and Saturn. One of the main difficulties is the lack of a long time series of observations which would make it possible to delineate time dependent phenomena and to carry out observations throughout a greater portion of the magnetosphere and at a greater number of satellites. The Galileo project for Jupiter and the proposed Cassini project for Saturn will, if carried out, provide this additional enlightenment.

Uranus

Voyager 2 encountered the planet Uranus on January 24, 1986. In contrast to the other planets, Uranus has its rotation axis tilted 97^{0} to the ecliptic normal and at the present time, it is pointing towards the Sun. All the theoretical work done before the encounter was based on the assumption that the magnetic axis would be more or less in the direction of the rotation axis, since at all the other magnetic planets it is within 10^{0} or so. As early as 1975, Siscoe and others discussed in the |iterature the possible modes of coupling of solar wind energy into the magnetosphere. In 1985, just before the encounter, the model described above was applied to an aligned Uranian magnetosphere and plasma tori associated with the various icy satellites were predicted. It was generally expected that radio noise from Uranus was disclose the existence of the magnetosphere well before the encounter.

All of the above expectations were confounded by an incredible

reality. No radio emissions were detected until January 23, barely a day before the encounter. Eventually it was found that all the radio emission was coming from the dark side of the planet instead of from the sunlit side as might have been expected. The plasma in the magnetosphere was found to consist totally of protons, apparently from ionization of a planetary hydrogen corona and no heavy ions that could be attributed to the icy satellites were found. The most amazing discovery, however, and the one that is responsible for all the other anomalies, was the fact that the magnetic axis is tilted approximately 60^{0} to the rotation axis. The result of this weird configuration is that in the frame of the corotating magnetosphere there is a constant electric field across the magnetosphere which convects all the plasma generated in the plane of the satellites sunward to the magnetopause and then out the tail on a time scale of about 30 hours. This is much more rapid than any of the production processes and the net result is a total absence of heavy ion plasma in the Uranian magnetosphere.

Another interesting consequence of this strange configuration is the fact that reconnection starts and stops on a time scale of a Uranian day which is 17.3 hours. A possible reflection of this effect may have been seen in the downstream flow velocities and densities.

It is interesting to consider what will be the situation in the magnetosphere of Uranus in the future. In 21 years the rotation axis will be normal to the Sun-planet direction and there will be a 17.3 hour day more or less equally divided between light and darkness. The magnetic axis will sample all angles between 0 and 360^{0} as the planet rotates, while the tail will remain in the antisolar direction. It is clear that a very complex magnetic configuration will evolve.

Uranus has proved to be a rich source of radio astronomy studies. Two smooth radio sources, a low frequency SLF (20-347 kHz) and a high frequency source SHF (up to 686 kHz), were observed, along with broadband and narrow band bursty components(b-bursty, 78-750 Khz and n-bursty 40-270 kHz), periodic events and very low frequency emissions (1.2 to 20 kHz). Detailed polarization studies have been carried out which have made it possible to draw inferences about the locations of the various source regions. The main conclusions are as follows:

1. The SLF source is located near the northern magnetic pole in the latitude range 13^0 to 38^0, in the west longitude range 26^0 to 70^0 and in the altitude range 1.23-1.39 R_U and the radiation is in the ordinary mode.

2. The SHF and the b-bursty component are both near the southern magnetic pole, with the SHF in the latitude range -44^0 - 52^0, in the west longitude range 228^0-256^0 and in the height range 1.77-1.85 R_U while the b-bursty is confined in a smaller region within the SHF source volume.

It is noteworthy that at present the south pole is in continuous darkness while the southern part of the northern magnetic pole region has alternating day and night conditions. It appears that the most likely free energy source for this nightside emission which is the main emission from Uranus lies in the radiation belts. The energy is tapped, according to one model, by a maser type instability caused by enhanced precipitation at the opposite pole which in turn is driven by variations in atmosphere scale height. The precipitating electrons create secondary backscattered electrons which propagate to the southern hemisphere where they mirror above the atmosphere and return, thus producing counterstreaming electron beams. It is well known that such a distribution is unstable to electromagnetic modes.

Neptune

In general, very little is known about this outermost planet. It can be expected to have a magnetosphere large enough to contain the orbit of its large moon Triton. The surface of Triton appears to be composed of solid and liquid nitrogen and methane. Gaseous methane has been observed in the atmosphere of Triton. It may be anticipated then that matter from Triton will reach the magnetosphere and may

become ionized to create a plasma torus. Neptune should also have radiation belts whose interaction with Triton will contribute to the removal of particles and the creation of the plasma torus. The Voyager 2 spacecraft will arrive at Neptune in August of 1989 and we shall then acquire initial information on the nature of the magnetosphere of Neptune.

Figure Captions

Figure 1. The magnetic field lines of the Earth in the noon-midnight meridian plane calculated from a theoretical model of Voigt and Fuchs [1979]. The interplanetary field was assumed to be 50 μG, directed southward in the middle panel and northward in the bottom panel. The value of C_0 denotes the fraction of the interplanetary field that penetrates the magnetopause. In the top panel in which $C_0 = 0$, the magnetospheric configuration is independent of the direction and strength of the interplanetary field.

Figure 2. A noon-midnight plane schematic of the various regions and components of the terrestrial magnetosphere.

Figure 3. Flux of electrons of energy greater than 40 keV measured at the equator. The dotted lines denote the Kennel-Petschek limiting flux value [from Schulz and Lanzerotti, 1974].

Figure 4. Drift periodic echoes of a magnetic perturbation seen in outer zone electron fluxes [Schulz and Lanzerotti, 1974].

Figure 5. Radial distribution of energetic electron fluxes [Schulz and Lanzerotti, 1974].

Figure 6. Jovian field intensities as a function of latitude measured by the Voyager magnetometer represented by means of isocontours [Acuña et al., 1983].

Figure 7. Isocontours of elementary charge concentration observed in the inner Jovian magnetosphere by the Voyager plasma experiment. The dotted line represents the spacecraft trajectory [Belcher, 1983].

Figure 8. Radial profiles of plasma density and temperature observed in the Jovian magnetosphere by the Voyager plasma experiment [Belcher, 1983].

Figure 9. Radial profile of Jovian magnetosphere flux tube content [Siscoe et al., 1981].

Figure 10. Strong diffusion lifetimes for energetic electrons and protons as a function of radial position in the Jovian magnetosphere. Two estimates of the radial diffusion time are plotted for comparison.

Figure 11. Phase space density of particles of magnetic moment 70 Mev/G as a function of radial position in the Jovian magnetosphere. Two theoretical curves are plotted for comparison [Thorne, 1983].

Figure 12. Density and temperature plots in the vicinity of Titan. The plasma plumes of Titan ionosphere plasma are identified.

Figure 13. Electron density and temperature plots, along with scale heights and spacecraft position through the Saturn encounter of Voyager 1 [Sittler et al., 1983].

Figure 14. Corotation lag curves in the magnetosphere of Saturn [Richardson, 1986].

References

Acuña, M.H., K.W. Behannon and J.E.P. Connerney, Jupiter's magnetic field and magnetosphere, 1-50 in *Physics of the Jovian Magnetosphere*, ed. A.J. Dessler, Cambridge University Press, Cambridge, 1983.

Belcher, J.W., The low-energy plasma in the Jovian magnetosphere, in *Physics of the Jovian Magnetosphere*, 68-105, ed. A.J. Dessler, Cambridge University Press, Cambridge, 1983.

Eviatar, A. and G.L. Siscoe, Limit on rotational energy available to excite Jovian aurora, *Geophys. Res. Letters*, *7*, 1085-1088, 1980.

Kennel, C.F. and H.E. Petschek, Limit on stably trapped particle fluxes, *J. Geophys. Res.*, *71*, 1-28, 1966.

Peale, S.J., P. Cassen and R.T. Reynolds, Melting of Io by tidal dissipation, *Science*, *203*, 892-894, 1979.

Richardson, J.D., Thermal ions at Saturn: Plasma parameters and implications, *J. Geophys. Res.*, *91*, 1381-1389, 1986.

Schulz, M. and L.J. Lanzerotti Jr., *Particle Diffusion in the Radiation Belts*, Springer, Berlin 1974.

Siscoe, G.L., A. Eviatar, R.M. Thorne and J.D. Richardson, Ring current impoundment of the Io plasma torus, *J. Geophys. Res.*, *86*, 8480-8484, 1981.

Sittler, E.C., Jr., K.W. Ogilvie and J.D. Scudder, Survey of low-energy plasma electrons in Saturn's magnetosphere, *J. Geophys. Res.*, *88*, 8848- 8870, 1983.

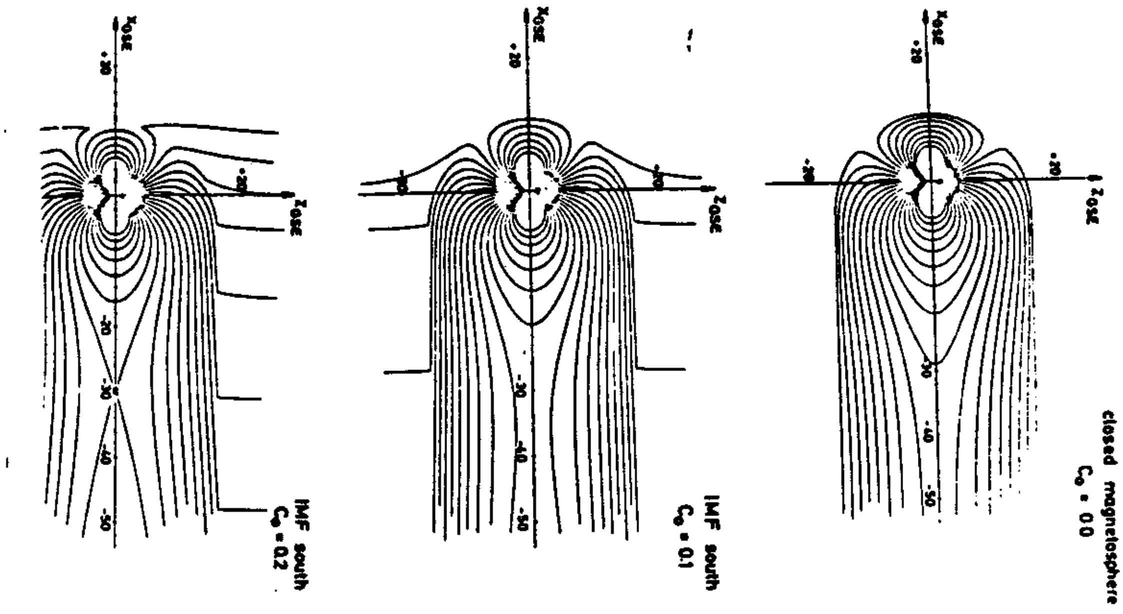
Thorne, R.M., Microscopic plasma processes in the Jovian magnetosphere, in *Physics of the Jovian Magnetosphere*, 454-488 ed.

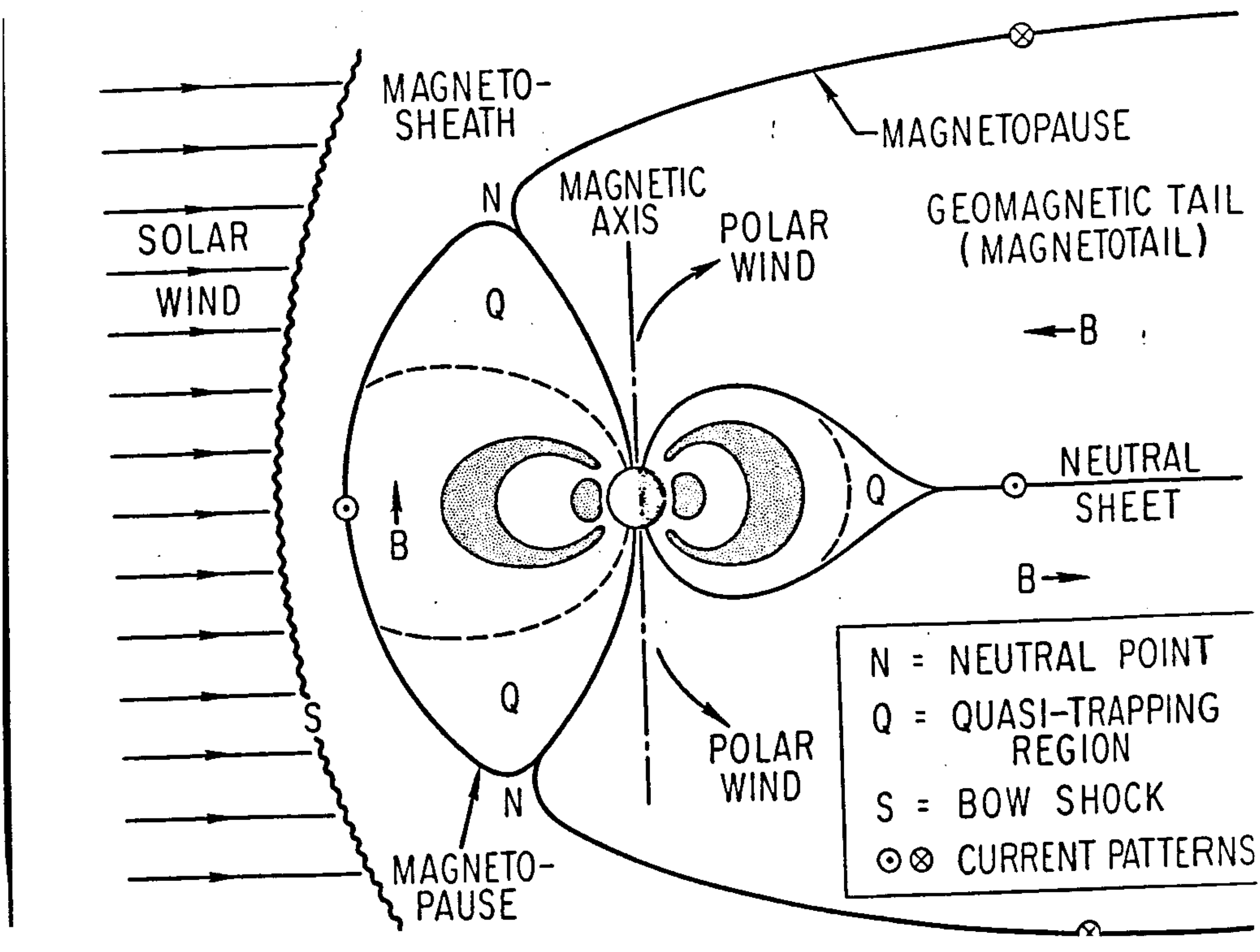
A.J. Dessler, Cambridge University Press, Cambridge, 1983.

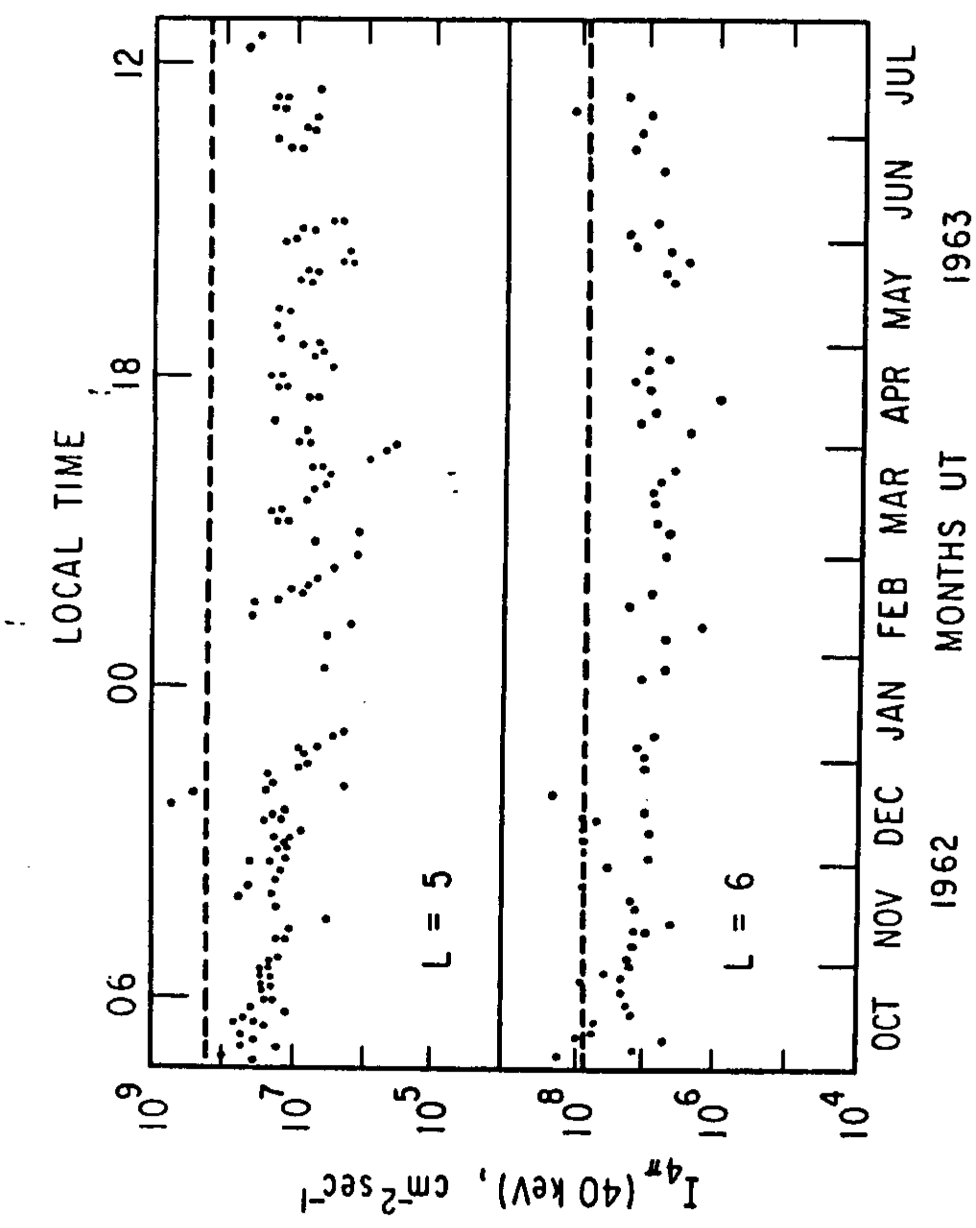
Voigt, G.H. and K. Fuchs, A macroscopic model for field line reconnection between the magnetosphere and the interplanetary space, in *Quantitative Modeling of Magnetospheric Processes*, 448-459, ed. W.P. Olson, American Geophysical Union Monograph 21, 1979.

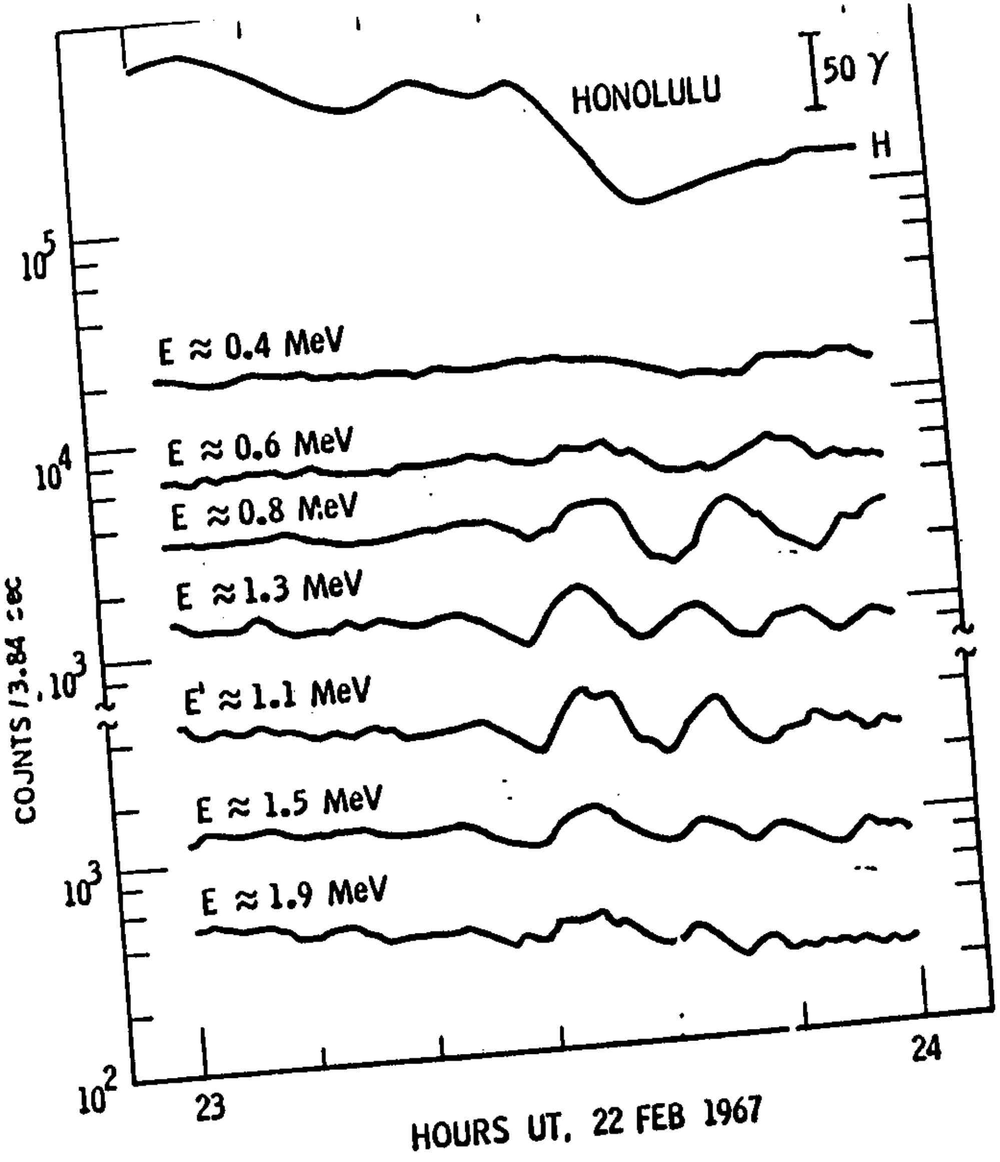
Giant Planet Characteristics

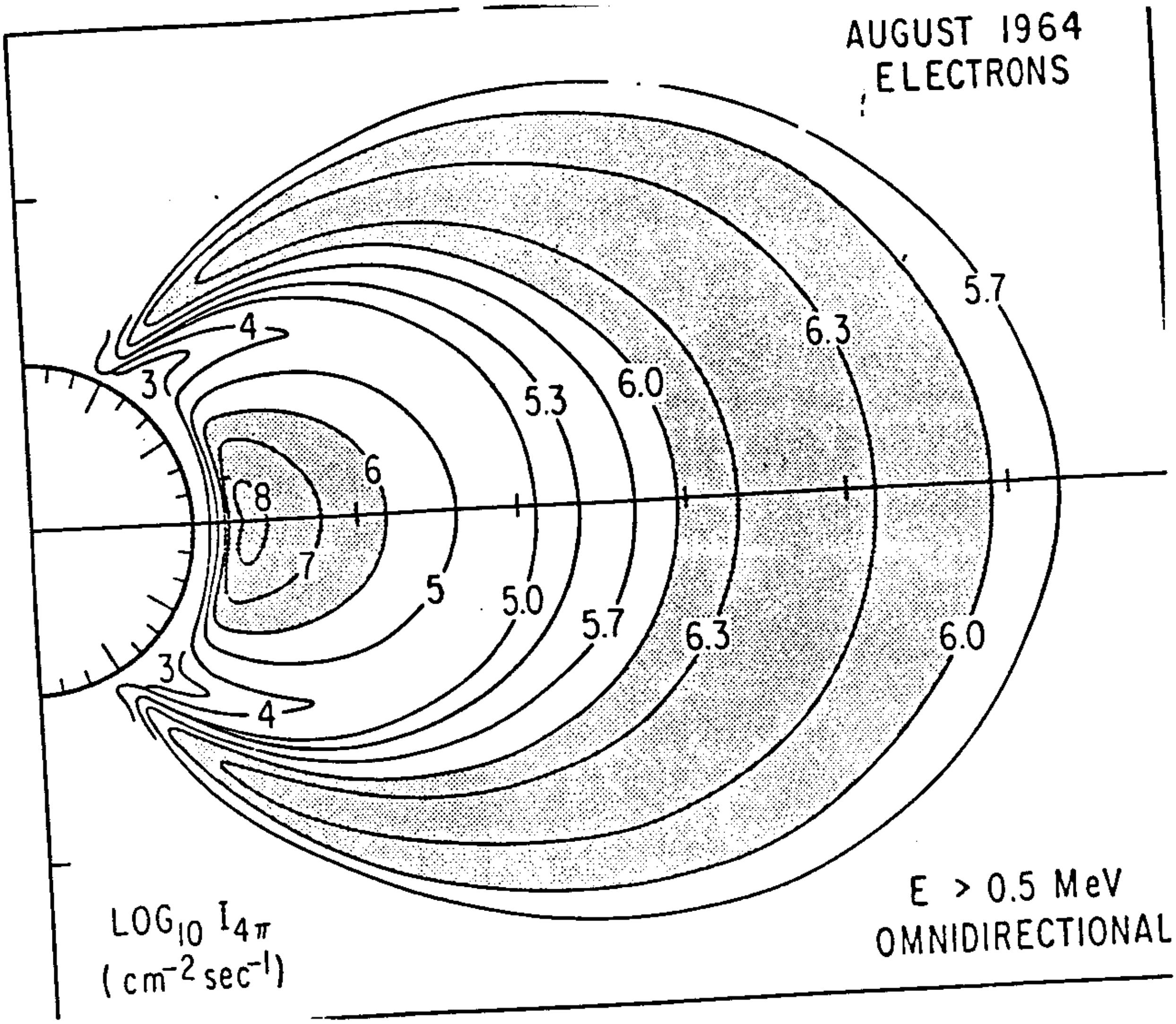
	Jupiter	Saturn	Uranus	Neptune
Radius km	71,000	60,300	25,600	≈ 25,000
Radius Earth = 1	11.2	9.5	3.8	3.9
Mass Earth = 1	318	95	4.0	17
Density gr cm^{-3}	1.3	0.6	1.4	1.6
Orbit Radius AU	5.20	9.54	19.18	30.07
Period years	11.9	29.47	84.00	164.90
Rotation rad s^{-1}	$1.74 \cdot 10^{-4}$	$1.64 \cdot 10^{-4}$	$1.01 \cdot 10^{-4}$	$\approx 1 \cdot 10^{-4}$?
Inclination °:'	3:05	26:44	97:55	28:48
Magnetic dec. °	9.6	0	60	?
Mag. Moment G cm^3	$1.4 \cdot 10^{30}$	$4.8 \cdot 10^{28}$	$3.6 \cdot 10^{27}$	$1.6 \cdot 10^{26-28}$?
Magnetopause R_p	50-100	18-25	18	10-50?
Plasma sources	Io	Titan, icy moons	Planet	Triton?
Plasma Composition	H,S,O,Na,K	H,H_2,H_2O group,N	H	H,N,N_2,CH_n
Source Strength s^{-1}	$>10^{28}$	10^{26}	10^{25}	10^{25}?
Lifetimes	months-years	months	days	months?
Total ion content	10^{34}	10^{32}	10^{30}	10^{31}?
Corotation Mach #	1-20	1-5	.2	≈ 1?


closed magnetosphere
C_0 = 0.0
IMF south
C_0 = 0.1
IMF south
C_0 = 0.2
X_{GSE}
Z_{GSE}

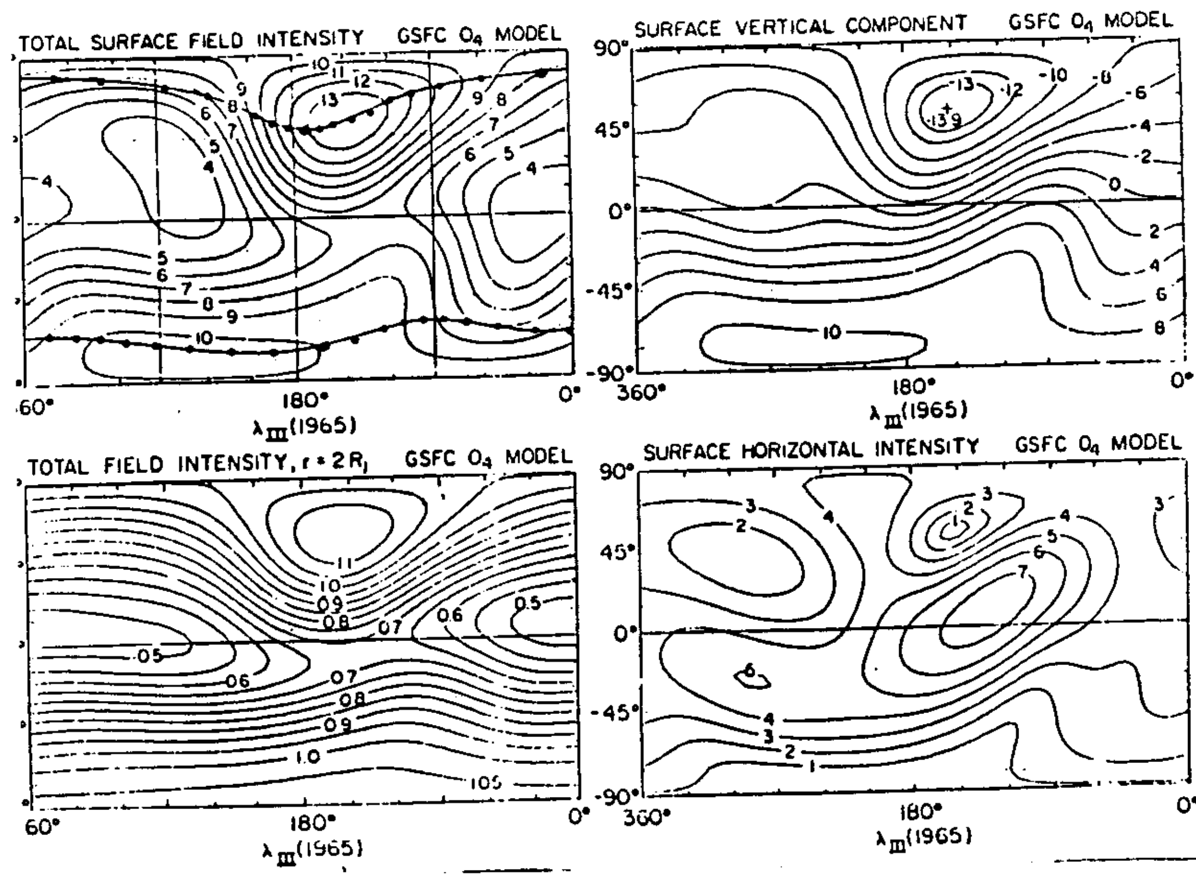

MAGNETO-
SHEATH
MAGNETOPAUSE
SOLAR
WIND
N
MAGNETIC
AXIS
POLAR
WIND
GEOMAGNETIC TAIL
(MAGNETOTAIL)
Q
B
NEUTRAL
SHEET
S
POLAR
WIND
N
MAGNETO-
PAUSE
N = NEUTRAL POINT
Q = QUASI-TRAPPING REGION
S = BOW SHOCK
⊙⊗ CURRENT PATTERNS


LOCAL TIME
12
18
00
06
L = 5
L = 6
OCT NOV DEC JAN FEB MAR APR MAY JUN JUL
1962
1963
MONTHS UT
10^9
10^7
10^5
10^8
10^6
10^4
$I_{4\pi}$ (40 keV), $cm^{-2} sec^{-1}$

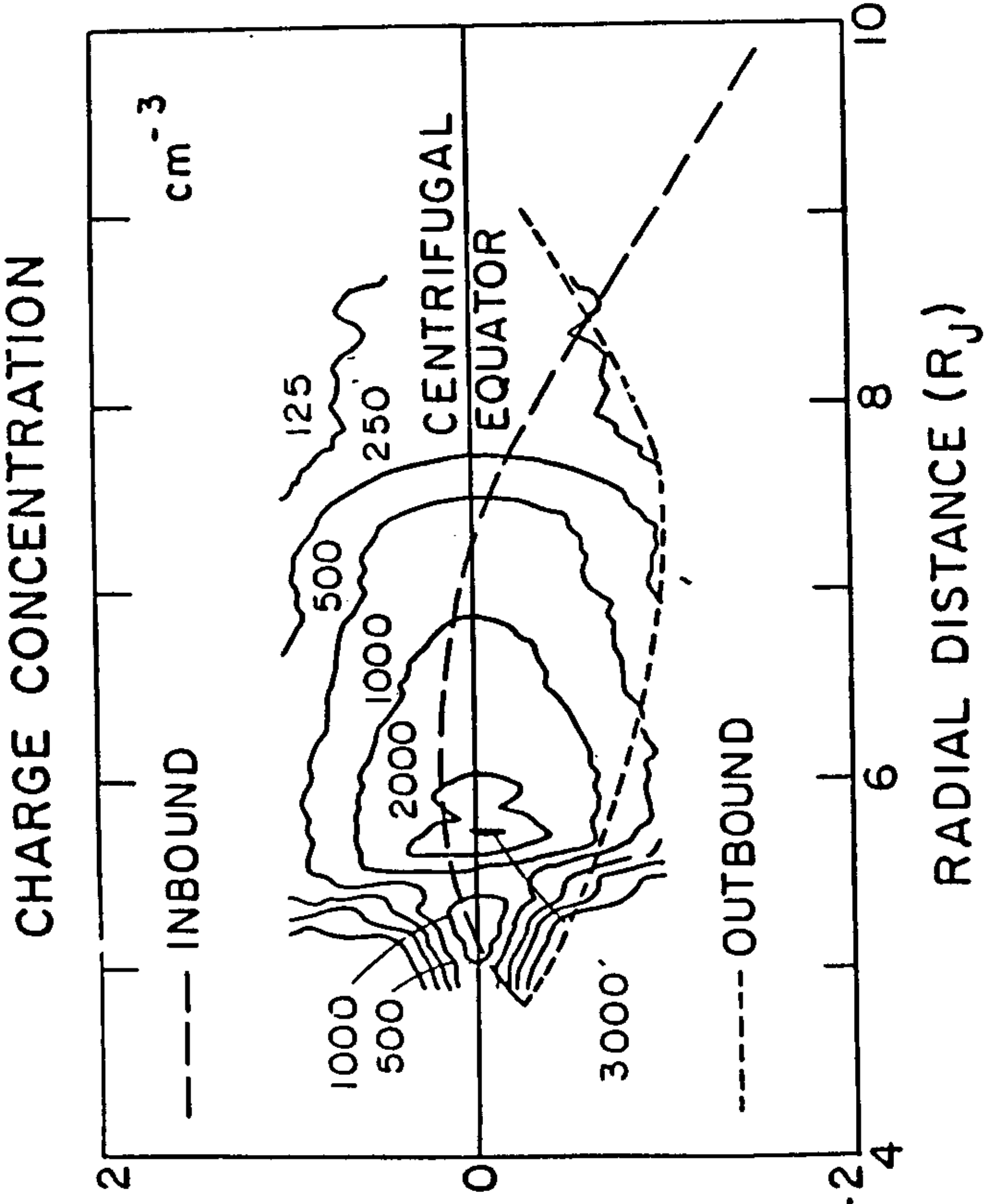

HONOLULU
50 γ
H
E ≈ 0.4 MeV
E ≈ 0.6 MeV
E ≈ 0.8 MeV
E ≈ 1.3 MeV
E ≈ 1.1 MeV
E ≈ 1.5 MeV
E ≈ 1.9 MeV
10^5
10^4
10^3
10^3
10^2
COUNTS / 3.84 sec
23
24
HOURS UT, 22 FEB 1967



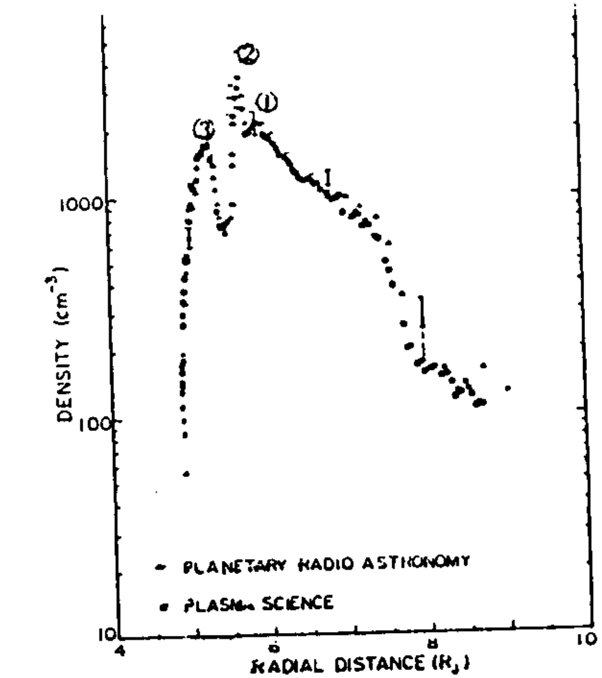
AUGUST 1964
ELECTRONS
5.7
6.3
6.0
5.3
6
8
7
5
5.0
5.7
6.3
6.0
4
3
3
4
E > 0.5 MeV
OMNIDIRECTIONAL
LOG10 I4π
(cm-2 sec-1)



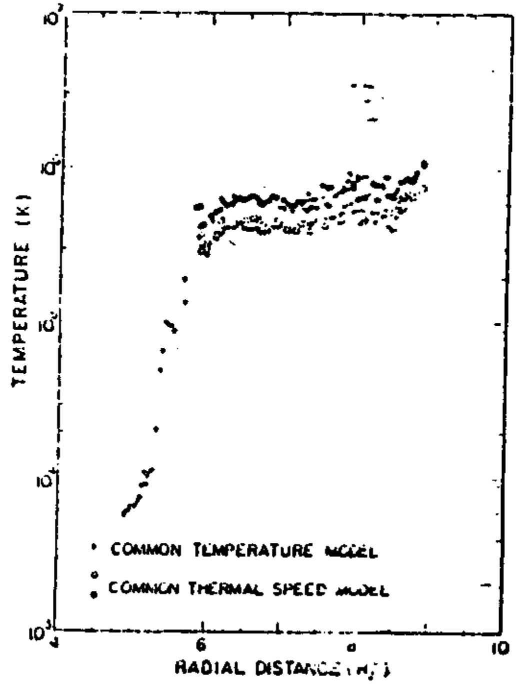
TOTAL SURFACE FIELD INTENSITY GSFC O4 MODEL
λIII(1965)
SURFACE VERTICAL COMPONENT GSFC O4 MODEL
λIII(1965)
TOTAL FIELD INTENSITY, r = 2 RJ GSFC O4 MODEL
λIII(1965)
SURFACE HORIZONTAL INTENSITY GSFC O4 MODEL
λIII(1965)



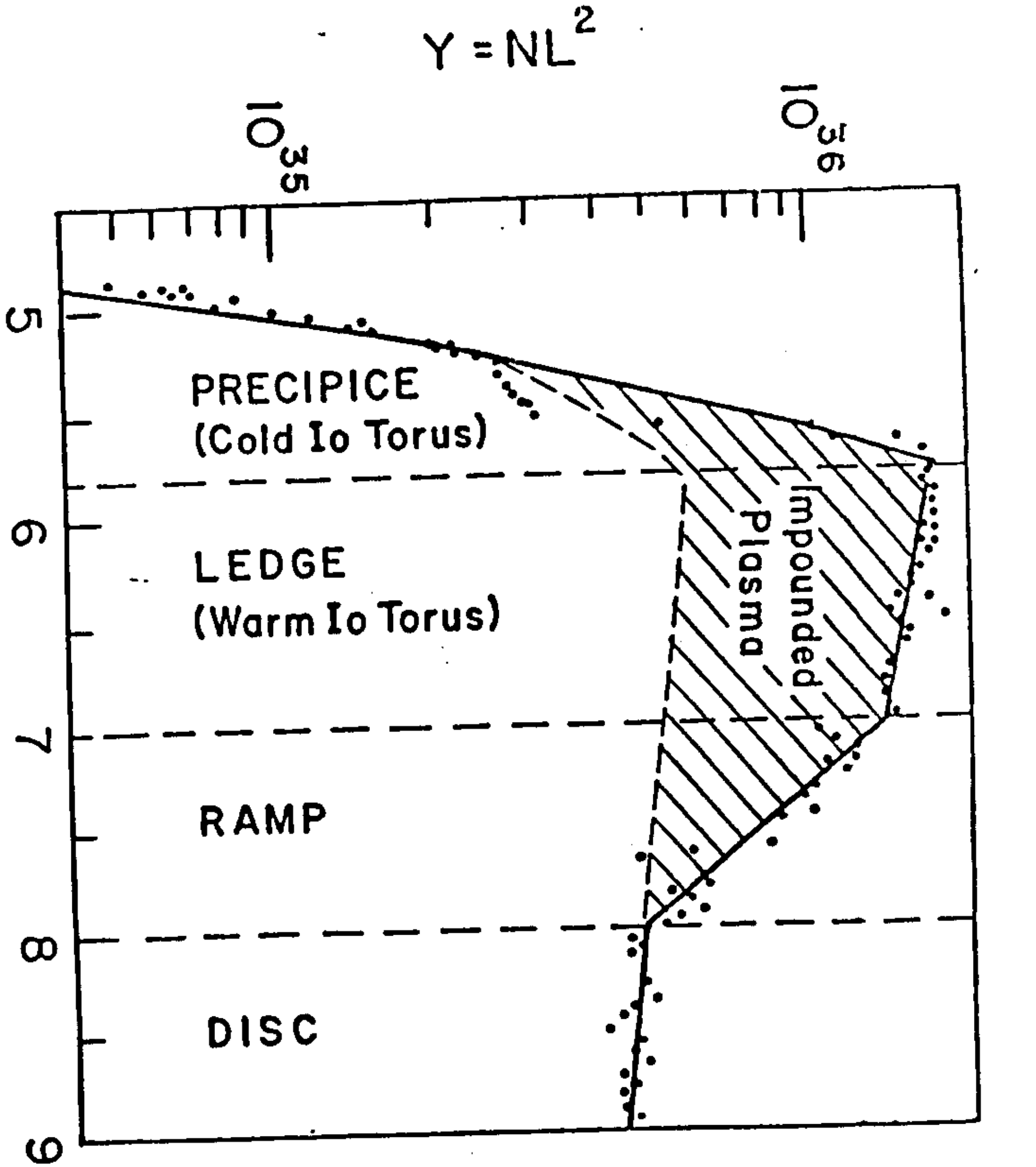
CHARGE CONCENTRATION
cm-3
INBOUND
OUTBOUND
CENTRIFUGAL EQUATOR
125
250
500
1000
2000
1000
500
3000
2
0
-2
4
6
8
10
RADIAL DISTANCE (RJ)



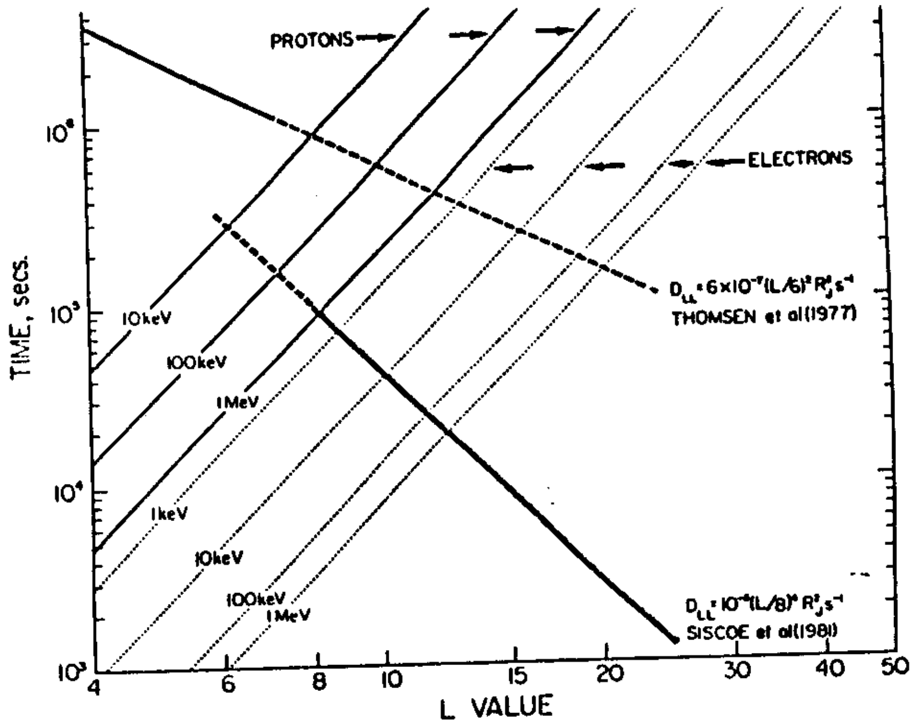
DENSITY (cm-3)
1000
100
10
4
10
RADIAL DISTANCE (RJ)
PLANETARY RADIO ASTRONOMY



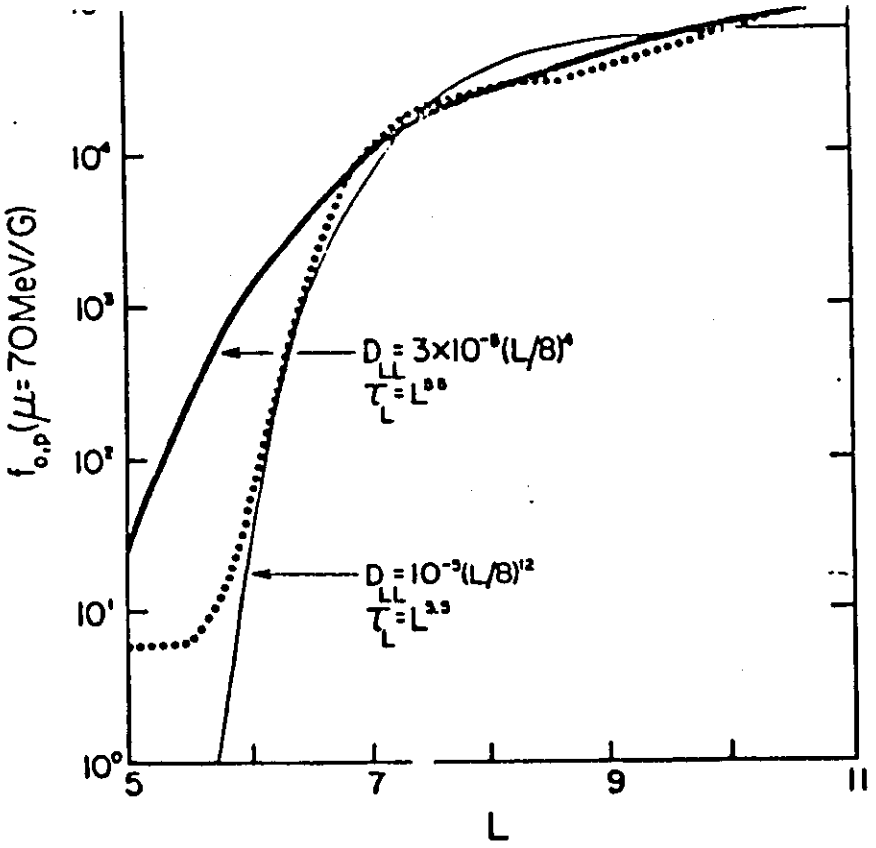
TEMPERATURE (K)
COMMON TEMPERATURE MODEL
COMMON THERMAL SPEED MODEL
6
10



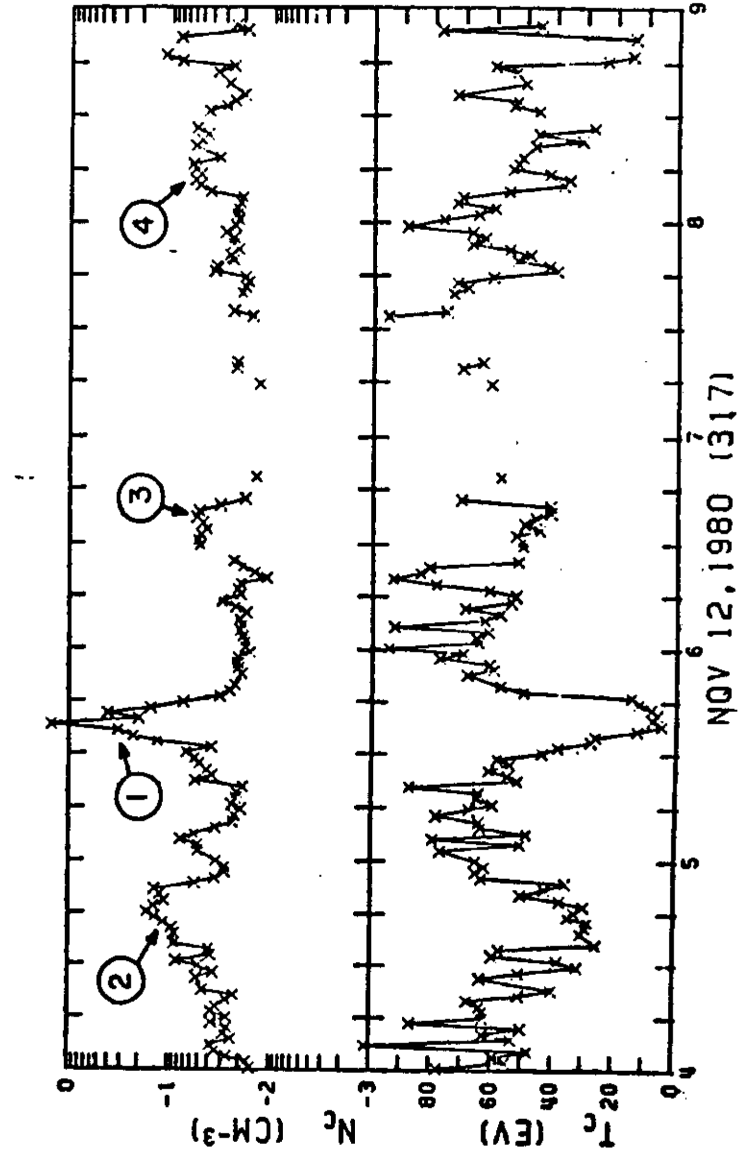
Y = NL^2
10^{35}
10^{36}
5
6
7
8
9
PRECIPICE
(Cold Io Torus)
LEDGE
(Warm Io Torus)
RAMP
DISC
Impounded Plasma



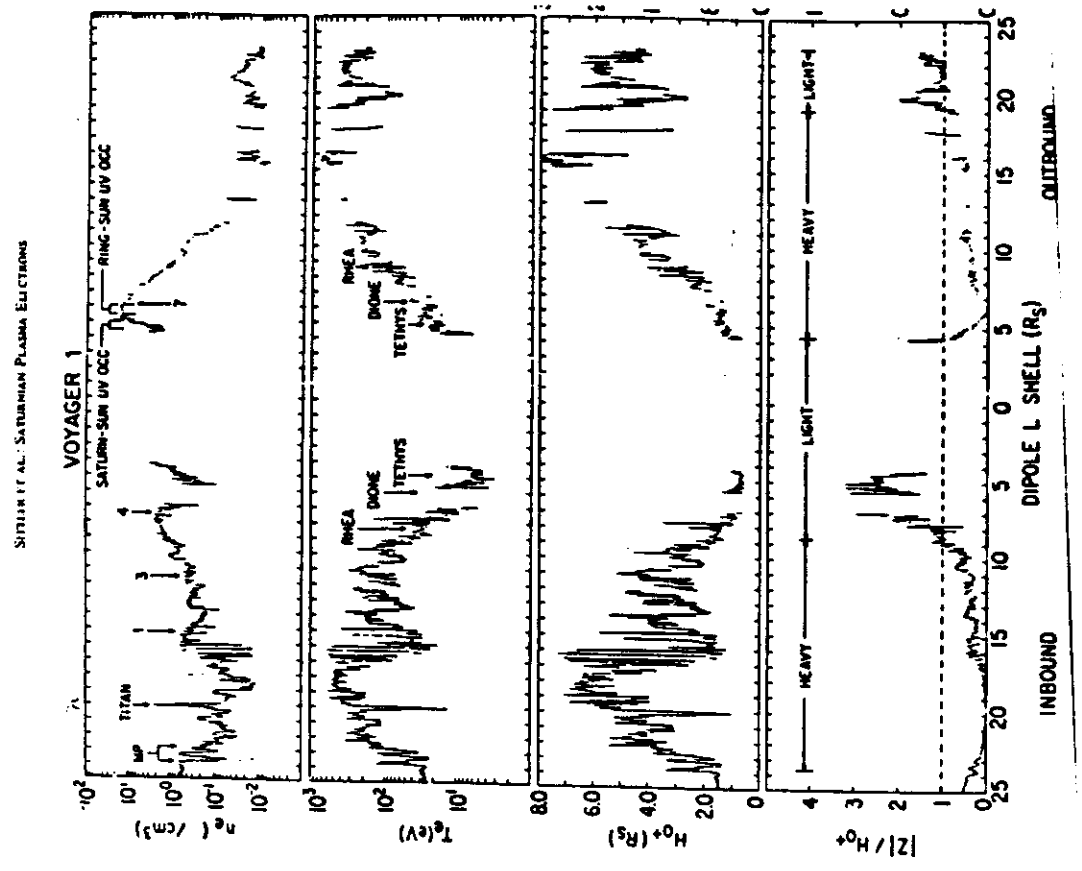
PROTONS
ELECTRONS
TIME, secs.
L VALUE
10keV
100keV
1MeV
1keV
10keV
100keV
1MeV
$D_{LL} = 6\times10^{-7}(L/6)^{3} R_J^2 s^{-1}$
THOMSEN et al (1977)
$D_{LL} = 10^{-6}(L/8)^{4} R_J^2 s^{-1}$
SISCOE et al (1981)
4
6
8
10
15
20
30
40
50

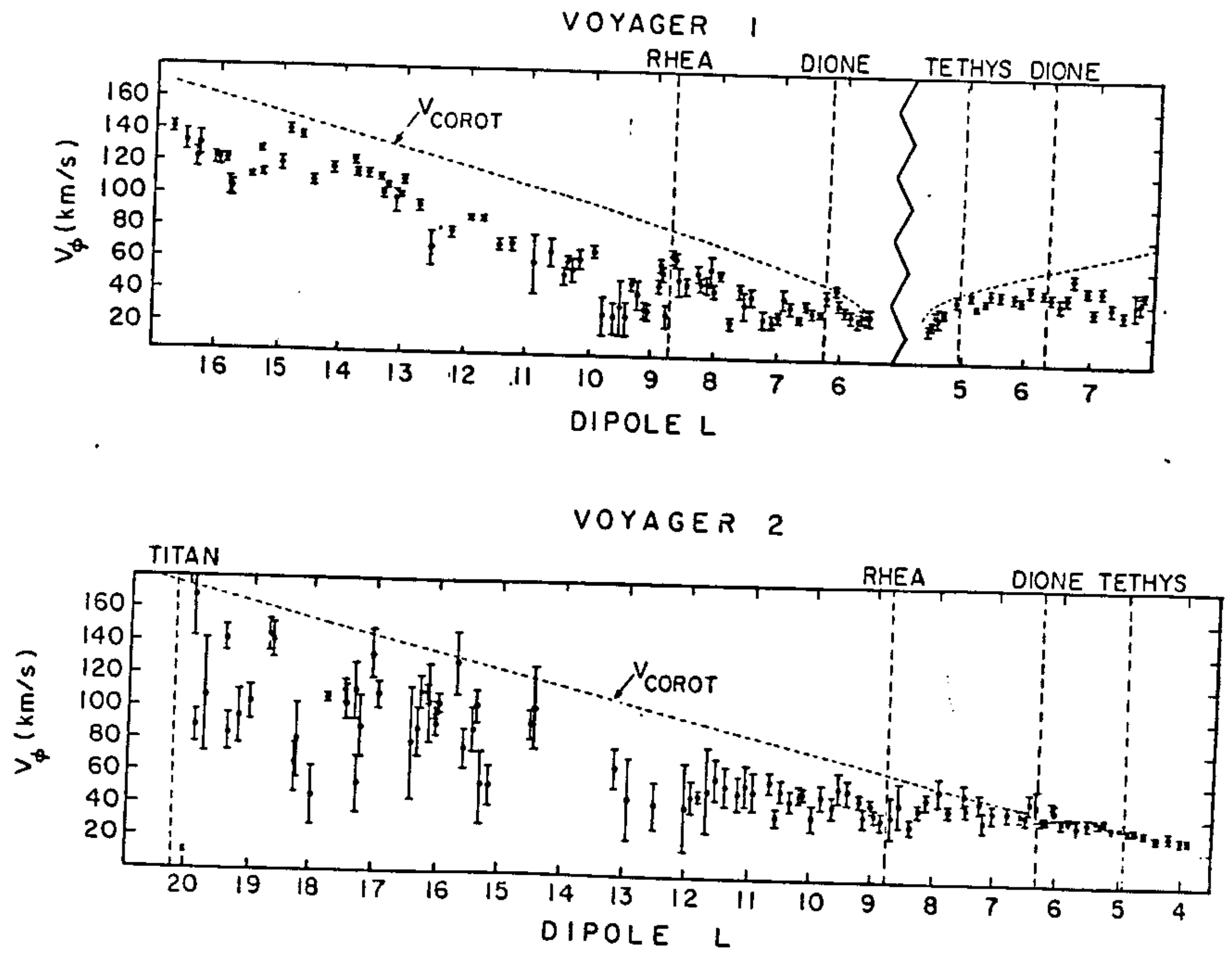


$f_{0,p}(\mu = 70MeV/G)$
10^4
10^3
10^2
10^1
10^0
$D_{LL} = 3\times10^{-9}(L/8)^4$
$\tau_L = L^{3.5}$
$D_{LL} = 10^{-3}(L/8)^{12}$
$\tau_L = L^{3.5}$
5
7
9
11
L



1
2
3
4
N_c (CM-3)
0
-1
-2
-3
T_c (EV)
80
60
40
20
0
4
5
6
7
8
9
NOV 12, 1980 (3317)




VOYAGER 1
RHEA
DIONE
TETHYS DIONE
V_{COROT}
V_ϕ (km/s)
DIPOLE L
VOYAGER 2
TITAN
RHEA
DIONE TETHYS
V_{COROT}
V_ϕ (km/s)
DIPOLE L