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Introduction to the Physics of Large Amplitude Plasma Waves

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Understanding the behavior of large amplitude plasma waves is important for many practical applications ranging from particle acceleration to plasma heating. A rich variety of nonlinear effects can limit the amplitude of a driven electron plasma wave, including nonlinear frequency shifts, wave-particle interactions, and coupling to other waves. A useful upper limit is provided by wavebreaking, which leads to a strong distortion of the electron velocity distribution. In general, energy coupling into other waves by ion density fluctuations is very important. A feedback mechanism allows this coupling to be very efficient, even starting from thermal level ion waves. The result is a collapse into shorter wavelength electron plasma waves which transfer their energy into a suprathermal electron tail.

INTRODUCTION

A key feature of a plasma is its ability to support various kinds of waves or collective modes of interaction. In the simplest case, these plasma waves correspond to charge density fluctuations along with their associated electric fields. These electric fields can accelerate particles to a high energy, a feature which is sometimes desirable and sometimes not. In laser fusion applications,[1] very energetic electrons can prematurely heat the fuel in a capsule and make efficient implosions difficult. Hence one tries to avoid exciting plasma waves. In accelerator applications, one aims to produce efficient acceleration by a plasma wave which is intentionally excited. In either case, it is important to understand the nonlinear effects which determine how large and how coherent a plasma wave can be excited.

In this introductory lecture, we will focus on the nonlinear behavior of large amplitude electron plasma waves. We will begin with a discussion of the linear characteristics of these waves. Then we will consider nonlinear frequency shifts and the amplitude limitation due to the onset of strong wave-particle interactions. Finally, we will show that the long-term nonlinear evolution is strongly influenced by the interaction with low frequency density fluctuations.

ELECTRON PLASMA WAVES IN A COLD PLASMA

Let us first treat high frequency charge density oscillations using a physically appealing Lagrangian description.[3] We assume a cold electron plasma with a uniform background density n_0, an immobile neutralizing background of ions, and no imposed magnetic fields. A one-dimensional treatment suffices, since the motion is along the wave vector for these electrostatic fluctuations. The position of each electron is

$$x = x_0 + \xi(x_0, t),$$

where $\xi(x_0,t)$ is the displacement of an electron from its initial position x_0. The displacement ξ leaves behind a positive charge per unit area $\sigma = n_0 e\xi$. The electric field due to this positive charge provides a restoring force. Since $E = 4\pi\sigma$, the electron equation of motion becomes

$$\ddot{\xi} = -\frac{4\pi n_0 e^2}{m}\xi. \tag{1}$$

Equation 1 clearly describes high frequency charge density oscillations at the electron plasma frequency ω_{pe}, where $\omega_{pe} = \sqrt{4\pi n_0 e^2/m}$.

It is important to note that equation 1 is exact under the assumption that electrons do not cross one another. Hence there is no nonlinear frequency shift in the cold plasma limit. However, there is a nonlinear frequency shift associated with relativistic corrections to the motion. Equation 1 then becomes

$$\frac{d}{dt}\frac{\dot{\xi}}{\sqrt{1-\frac{\dot{\xi}^2}{c^2}}} = -\omega_{pe}^2\xi, \tag{2}$$

where $\dot{\xi} = d\xi/dt$ and c is the velocity of light. If we assume $\dot{\xi}/c \ll 1$, equation 2 can be written as

$$\ddot{\xi} = -\omega_{pe}^2 \xi \left[1 - \frac{3}{2}\frac{\dot{\xi}^2}{c^2}\right]. \tag{3}$$

We look for a solution of the form

$$\xi = \bar{\xi} \sin[kx_0 - \omega_{pe} t + \phi(t)], \tag{4}$$

where $\phi(t)$ varies slowly in time and $\dot{\phi} = \delta\omega$ represents the small frequency shift proportional to $\bar{\xi}^2$. We substitute into equation 3, neglect $d^2\phi/dt^2$, and use the trigonometric identity

$$\sin x \cos^2 x = (\sin x + \sin 3x)/4.$$

Neglecting the response at the third harmonic and noting that $\bar{\xi} = eE/m\omega_{pe}^2$, we then obtain[4]

$$\Delta\omega = -\frac{3}{16}\left(\frac{eE}{m\omega_{pe}c}\right)^2 \omega_{pe}. \tag{5}$$

This negative frequency shift is due to the relativistic increase in the electron mass due to motion in the field of the electron plasma wave.

WAVEBREAKING AMPLITUDE

We are now ready to consider an important nonlinear limit, which is due to the onset of a very efficient interaction between the wave and the plasma electrons. Neglecting relativistic effects, we consider a simple wave-like solution to equation 1; i.e.,

$$\xi(x_0, t) = \bar{\xi} \sin(kx_0 - \omega_{pe} t). \tag{6}$$

As already mentioned, this solution is exact until electron crossing occurs. Since $x = x_0 + \xi(x_0, t)$, crossing is determined by $\partial\xi/\partial x_0 = -1$, which clearly occurs when $|k\bar{\xi}| = 1$. Since $E = 4\pi n_0 e\xi$, the crossing condition defines the so called cold wavebreaking amplitude:

$$\frac{eE_{max}}{m\omega_{pe}} = \frac{\omega_{pe}}{k}. \tag{7}$$

At the wavebreaking amplitude, electron motion in the wave becomes "disordered." Streamers are formed in electron phase space, as some electrons are accelerated to nearly

twice ω_{pe}/k. Of course, this strong wave-particle interaction corresponds to the onset of an extremely efficient damping of the wave. Hence the wavebreaking amplitude provides a useful estimate of the maximum amplitude of an electron plasma wave.

A physically-appealing interpretation of the wavebreaking amplitude in a cold plasma can be given. Slow electrons simply oscillate in the wave. However, resonant particles with velocity near ω_{pe}/k see a nearly constant field and can be efficiently accelerated. At the wavebreaking amplitude, the oscillation velocity of an electron in the wave $(eE/m\omega_p)$ becomes as large as the phase velocity (ω_{pe}/k). Hence numerous electrons are *nonlinearly* brought into resonance with the wave, leading to the onset of a very strong damping which limits the amplitude.

The calculation of the wavebreaking amplitude is readily extended to include relativistic dynamics.[5,6] Taking a time derivative of equation 2 and changing variables to $p = \frac{\dot{\xi}}{c}/\sqrt{1 - \dot{\xi}^2/c^2}$ gives

$$\frac{d^2}{dt^2} p = -\omega_{pe}^2 \frac{p}{\sqrt{1 + p^2}}. \tag{8}$$

After multiplying by $\dot{p} = dp/dt$, we obtain

$$\frac{d}{dt}\left[\frac{\dot{p}^2}{2} + \omega_{pe}^2 \sqrt{1 + p^2}\right] = 0. \tag{9}$$

Hence

$$\frac{\dot{p}^2}{2} + \omega_{pe}^2 \sqrt{1 + p^2} = \omega_{pe}^2 \sqrt{1 + p_\phi^2}, \tag{10}$$

where $\dot{p} = 0$ for $p = p_\phi = p(\dot{\xi} = v_p)$. The maximum value of $\dot{p}$ obtains when $p = 0$:

$$\dot{p}_{max} = \sqrt{2}\left\{\sqrt{1 + p_\phi^2} - 1\right\}^{1/2}. \tag{11}$$

Since $\dot{p} = -eE/mc$, the maximum value of the electric field is

$$\left|\frac{eE_{max}}{m\omega_{pe}c}\right| = \sqrt{2}\left\{\frac{1}{\sqrt{1 - \frac{v_p^2}{c^2}}} - 1\right\}^{1/2}. \tag{12}$$

For $v_p \ll c$, equation 7 is recovered. Note that the wavebreaking amplitude increases as the relativistic mass of an electron moving at the phase velocity increases.

EFFECTS OF PLASMA TEMPERATURE

Let us now consider electron plasma waves in a plasma with a finite temperature. In particular, we no longer assume that the electron thermal velocity v_e is negligible compared to ω_{pe}/k. The linear effects of electron temperature are well known. There is a thermal correction to the frequency as well as a damping (or growth) due to electrons with velocity in the neighborhood of ω_{pe}/k. For $k\,\lambda_{De} \ll 1$, the frequency of an electron plasma wave becomes

$$\omega = \omega_{pe}\left(1 + \frac{3}{2}k^2\lambda_{De}^2\right) + i\gamma_L, \tag{13}$$

where $\lambda_{De} = v_e/\omega_{pe}$ and γ_L is the Landau damping decrement. This decrement is conveniently expressed as

$$\frac{\gamma_L}{\omega} = \frac{\pi}{2}\frac{\omega_{pe}^2}{k^2}\frac{\partial \bar{f}_e}{\partial v}\left(\frac{\omega}{k}\right), \tag{14}$$

where $f_e = n_0\bar{f}_e$ is the electron velocity distribution.

Plasma temperature also significantly reduces the wavebreaking amplitude. First, electrons with a finite initial velocity are easier to nonlinearly bring into resonance with the wave. Secondly, the pressure fluctuations enhance the force accelerating electrons. To illustrate these effects, we adopt a water bag model,[7] which corresponds to replacing a Maxwellian distribution with a velocity distribution which is constant between $\pm\sqrt{3}v_e$. This idealized distribution is convenient, since it yields the same pressure force as a Maxwellian distribution with thermal velocity v_e, yet has a well-defined maximum initial velocity of $\sqrt{3}v_e$. Although there are particles with an arbitrarily high velocity in a Maxwellian distribution, the number of particles is not sizable until $v \lesssim 2v_e$. Hence a water bag distribution should roughly model the condition that a significant number of particles are nonlinearly brought into resonance.

In the water bag model, the average density n and velocity u of the electrons satisfy the same equations as those for a warm electron fluid:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0 \tag{15}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{-eE}{m} - \frac{1}{mn}\frac{\partial p}{\partial x}. \tag{16}$$

Since the heat flow vanishes for a water bag distribution, the pressure p is given by the adiabatic equation of state; i.e.,

$$\frac{p}{n^3} = c^{st}. \tag{17}$$

Equations 15-17 are readily derived by taking the first three moments of the Vlasov equation.

Representing $E = -\partial\phi/\partial x$ and transforming to the wave frame with velocity v_p gives

$$nu = n_0 v_p \tag{18}$$

$$u^2 - \frac{2e\phi}{m} + \frac{3v_e^2 n^2}{n_0^2} = v_p^2 + 3v_e^2, \tag{19}$$

where n_0 is the density of the uniform unperturbed plasma. Using equation 18 in equation 19, we obtain

$$\frac{2e\phi}{mv_p^2} = \frac{u^2}{v_p^2} - 1 - \beta + \beta\frac{v_p^2}{u^2}, \tag{20}$$

where $\beta = 3v_e^2/v_p^2$. We determine the extremum potential (ϕ_{cr}) by the condition $\partial\phi/\partial u = 0$, which gives

$$\frac{-2e\phi_{cr}}{mv_p^2} = (1 - \beta^{1/2})^2, \tag{21}$$

for $u/v_p = \beta^{1/4}$. This critical potential simply represents the condition that the energy of the fastest electron be zero in the wave frame.

The critical value of the electric field is found by using Poisson's equation

$$\frac{\partial^2\phi}{\partial x^2} = 4\pi e(n - n_0) \tag{22}$$

We multiply by $\dot{\phi} = \partial\phi/\partial x$ and use equations 18 and 19 to obtain

$$\frac{\dot{\phi}^2}{2} + 4\pi\left[n_0 e\phi - nmu^2 - n_0 m v_e^2\frac{n^3}{n_0^3}\right] = -2\pi n_0 m v_p^2\left[(1 - \beta^{1/2})^2 + \frac{8}{3}\beta^{1/4}\right]. \tag{23}$$

The constant has been evaluated by noting that $\dot{\phi} = 0$ where $\phi = \phi_{cr}$. The maximum electric field ($E_{max} = -\dot{\phi}_{max}$) occurs when $\phi = 0$:

$$\frac{e^2 E_{max}^2}{m^2\omega_{pe}^2\, v_p^2} = 1 + 2\beta^{1/2} - \frac{8}{3}\beta^{1/4} - \frac{\beta}{3}. \tag{24}$$

This maximum electric field is plotted in figure 1 as a function of $\sqrt{3}v_e/v_p$. For $v_e = 0$, the cold plasma wavebreaking amplitude is recovered. Note the substantial decrease of the maximum field as the plasma temperature increases. For example, for $v_p = 5v_e$, $eE_{max}/m\omega_{pe}v_p \simeq .29$.

The water bag model can also be used to determine the nonlinear frequency shift. For $E \ll E_{max}$ and $k\lambda_{De} \ll 1$, it can be shown that[7]

$$\Delta\omega \simeq \frac{15}{4}\omega_{pe}k^2\lambda_{De}^2\left(\frac{e^2E^2}{m^2\omega_{pe}^2v_p^2}\right). \tag{25}$$

Note that this nonlinear frequency shift vanishes in a cold plasma.

ROLE OF ION DENSITY FLUCTUATIONS

Thus far we have considered electron plasma oscillations in a plasma with a uniform background density. As we shall see, low frequency fluctuations in the background density can efficiently couple an electron plasma wave into other electron plasma waves. We start from the equation relating the electrostatic field fluctuation (E) to the fluctuation in the current density (J):

$$\frac{\partial E}{\partial t} + 4\pi J = 0. \tag{26}$$

Equation 26 is readily derived by combining Poisson's equation and the continuity equation for the charge density. We describe the electrons as a warm fluid with density n_e and mean velocity v_e. The high frequency component ($\omega \sim \omega_{pe}$) of equation 26 is

$$\frac{\partial E^h}{\partial t} = 4\pi e(n_{oe} + n_e^l)u_e^h. \tag{27}$$

Here n_{oe} is the uniform background density, n_e^l is the low frequency density fluctuation associated with an ion sound wave, and $u_e^h(E^h)$ is the high frequency component of the electron fluid velocity (the electric field) associated with the electron plasma wave. The ions are assumed to form a neutralizing background. Since the frequency of an ion wave is so much less than the electron plasma frequency, the electron inertia can be neglected for the low motion, and so $u_e^l \simeq 0$. Under the same assumption, a time derivative of equation 27 gives

$$\frac{\partial^2 E^h}{\partial t^2} = 4\pi e(n_{oe} + n_e^l)\frac{\partial u_e^h}{\partial t}. \tag{28}$$

The high frequency component of the force equation for the warm electron fluid becomes

$$\frac{\partial u_e^h}{\partial t} = -\frac{eE^h}{m} - \frac{3v_e^2}{n_{oe}}\frac{\partial n_e^h}{\partial x}. \tag{29}$$

Here n_e^h is the high frequency component of the electron density, and v_e is the electron thermal velocity. An adiabatic equation of state has been taken under the assumption that $\frac{\omega}{k} \gg v_e$, where ω is the frequency and k the wave number of the electron plasma wave. Using Poisson's equation to eliminate n_e^h from equation 29 and substituting into equation 28 finally yields

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - 3v_e^2\frac{\partial^2}{\partial x^2}\right)E^h = -\frac{4\pi e^2}{m}n_e^l E^h, \tag{30}$$

where $\omega_{pe}^2 = 4\pi n_{oe}e^2/m$.

For $n_e^l = 0$, equation 30 simply describes electron plasma oscillations at the Bohm-Gross frequency: $\omega = \sqrt{\omega_{pe}^2 + 3k^2v_e^2}$. It is clear that a low frequency density fluctuation couples an electron plasma wave with a given wavenumber into plasma waves with other wavenumbers. This coupling is simple to understand. Oscillation of electrons across a variation in density creates a high frequency density fluctuation; i.e.,

$$\delta n = n_e(\mathbf{x} + \mathbf{x}_\omega) - n_e(\mathbf{x}) \simeq \mathbf{x}_\omega \cdot \frac{\partial n_e(\mathbf{x})}{\partial \mathbf{x}}, \tag{31}$$

where $\mathbf{x}_\omega$ is the amplitude of the electron oscillation in the high frequency electric field ($\mathbf{x}_\omega = e\mathbf{E}^h/m\omega^2$). As shown in equation 30, this density fluctuation drives an electron plasma wave.

To illustrate the efficiency of this coupling, let us consider a plasma with a large amplitude, homogeneous pump field $E_0\sin\omega_{pe}t$ and a static density fluctuation $n_e^l = \Delta n\cos kx$. This pump field models, say, a large amplitude electron plasma wave with a wavenumber much less than k. Linearizing equation 30 then gives

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - 3v_e^2\frac{\partial^2}{\partial x^2}\right)E = -\omega_{pe}^2 E_0\sin\omega_{pe}t\cos kx, \tag{32}$$

where E is the amplitude of the driven plasma wave. The driven solution is then

$$E = -\frac{1}{3k^2\lambda_{De}^2}\frac{\Delta n}{n_{oe}}E_0\sin\omega_{pe}t\cos kx. \tag{33}$$

Hence a plasma wave with shorter wavelength is generated by the interaction of the pump field with the density fluctuation. Indeed, this interaction becomes so efficient[8] when $\Delta n/n_{oe} \gtrsim 3k^2\lambda_{De}^2$ that our linearized treatment breaks down. Multiple harmonics in wavenumber then need to be considered.

The coupling via low frequency density fluctuations is especially important since there is a feed-back mechanism due to the so-called ponderomotive force. Let us consider the low frequency component of the force equation for the electron fluid:

$$m\frac{\partial u_e^l}{\partial t} = -eE^l - \frac{1}{n_{oe}}\frac{\partial p_e^l}{\partial x} - \frac{m}{2}\frac{\partial}{\partial x}\langle (u_e^h)^2\rangle. \qquad (34)$$

Here the superscript l denotes the low frequency component and the brackets denote a time average of the oscillatory energy of the electrons in the high frequency electric field. Note that the gradient of this oscillatory energy gives rise to a low frequency force, just as the gradient of the pressure does. This ponderomotive force is

$$F_p = -\frac{\partial}{\partial x}\left(\frac{m}{2}\langle (u_e^h)^2\rangle\right). \qquad (35)$$

If we again neglect electron inertia, equation 34 determines the low frequency electric field which transmits the ponderomotive and the usual pressure force to the ions. In particular,

$$eE^l = -\frac{\theta_e}{n_{oe}}\frac{\partial n_e^l}{\partial x} - \frac{m}{2}\frac{\partial}{\partial x}\langle (u_e^h)^2\rangle, \qquad (36)$$

where θ_e is the electron temperature and an isothermal equation of state is used for the low frequency electron fluctuations.

To describe the low frequency fluctuations, we must also calculate the ion dynamics. The continuity and force equations for the density n_i and mean velocity u_i of the ion fluid are

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0 \qquad (37)$$

$$\frac{\partial}{\partial t}(n_i u_i) + \frac{\partial}{\partial x}(n_i u_i^2) = \frac{Ze}{M} n_i E^l, \qquad (38)$$

where M is the ion mass, Z is the charge state, and the ion pressure is neglected for simplicity. We take a time derivative of equation 37, a spatial derivative of equation 38, and combine to give

$$\frac{\partial^2}{\partial t^2} n_i - \frac{\partial^2}{\partial x^2}(n_i u_i^2) + \frac{Ze}{M}\frac{\partial}{\partial x}(n_i E^l) = 0. \qquad (39)$$

We next take $n_i = n_{io} + n_i^l$ and substitute for E^l from equation 36. Approximating $Zn_i^l = n_e^l$ and $u_e^h = eE^h/m\omega_{pe}$ and keeping only the lowest order response in the low frequency fluctuation amplitudes, we obtain

$$\left[\frac{\partial^2}{\partial t^2} - c_s^2\frac{\partial^2}{\partial x^2}\right] n_e^l = \frac{Z}{M}\frac{\partial^2}{\partial x^2}\left(\frac{\langle (E^h)^2\rangle}{8\pi}\right), \qquad (40)$$

where c_s is the ion sound speed ($c_s^2 = Z\theta_e/M$). Equation 40 describes the generation of low frequency ion waves by variations in the intensity of the high frequency electric field.

The feedback leading to instability can be illustrated quite simply. If we return to our example of a plasma with a large amplitude pump field and a density fluctuation,

$$F_p = -\frac{\partial}{\partial x}(m v_{os} u), \qquad (41)$$

where $u = eE/m\omega_{pe}$ and $v_{os} = eE_0/m\omega_{pe}$. Substitution from equation 33 gives

$$F_p = -\frac{1}{6k^2\lambda_{De}^2}\frac{\Delta n}{n_{oe}}\frac{e^2E_0^2}{m^2\omega_{pe}^2} k \sin kx. \qquad (42)$$

As shown in figure 2, this ponderomotive force acts to enhance the density fluctuation ($\Delta n \cos kx$); that is, to push electrons into regions in which there is already an excess.[9] Hence the ponderomotive force provides a feed-back mechanism by which an initial (even thermal level) density fluctuation can be amplified.

The instability in this ideal model is called the oscillating-two-stream [10-11] and gives rise to the growth of both shorter wavelength plasma waves and zero-frequency density modulations. If the pump frequency is greater than ω_{pe}, there is also a branch of instability called the ion-acoustic decay.[11-13] This represents the resonant decay of the pump field into an electron plasma wave plus an ion acoustic wave. Both branches of instability are readily derived from equations 30 and 40.

The coupling into shorter wavelength waves is crucial for understanding the long-term nonlinear behavior of large amplitude electron plasma waves. In a collisionless plasma, Landau damping provides the sink for wave energy. Wave energy is transferred into shorter wavelength, lower phase velocity waves which ultimately damp, creating a tail of suprathermal electrons. This characteristic feature of heating via excitation of plasma waves has long been emphasized by computer simulations. For example, figure 3 shows an electron distribution function from an early one-dimensional simulation[14] of a plasma driven by a pump field with frequency near ω_{pe}.

A general and powerful analysis of the physics described in equations 30 and 40 has been developed. In this theory of plasma wave collapse,[15-17] a local modulation in the wave intensity self-consistently creates a density depletion via the ponderomotive force. This density depletion in turn amplifies the intensity modulation, leading to a narrower and deeper cavity. The process terminates when the cavity becomes sufficiently narrow that the associated high frequency wave is damped by the electrons.

SUMMARY

In summary, the nonlinear behavior of large amplitude electron plasma waves is a very rich topic. The amplitude to which such a wave can be driven depends on nonlinear frequency shifts, wave-particle interactions, and the coupling with low frequency density fluctuations. This latter coupling can transfer wave energy to shorter wavelengths, which allows efficient transfer to a suprathermal tail of electrons.

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REFERENCES

1. J. H. Nuckolls, L. Wood, A. R. Thiessen, and G. Zimmerman, *Nature* **239**, 139 (1972).
2. T. Tajima and J. M. Dawson, *Phys. Rev. Lett.* **43**, 267 (1979); T. Katsouleas and J. M. Dawson, *Phys. Rev. Lett.* **51**, 392 (1983).
3. J. M. Dawson, *Phys. Rev.* **113**, 383 (1959).
4. M. N. Rosenbluth and C. S. Liu, *Phys. Rev. Lett* **29**, 701 (1972).
5. I am grateful to Dr. W. B. Mori for this derivation.
6. A. I. Akhiezer and R. V. Polovin, *Sov. Phys. JETP* **3**, 696 (1956); T. Katsouleas and W. B. Mori, *Phys. Rev. Lett.* **61**, 90 (1988).

7. T. P. Coffey, *Phys. Fluids* **14**, 1402 (1971).
8. C. Darrow, et. al., *Phys. Rev. Lett.* **56**, 2629 (1986); and references therein.
9. F. F. Chen, *Introduction to Plasma Physics*, Plenum Press, New York, (1974).
10. V. P. Silin, *Sov. Phys. JETP* **21**, 1127 (1965).
11. K. Nishikawa, *J. Phys. Soc. Japan* **24**, 1152 (1968).
12. D. F. DuBois and M. V. Goldman, *Phys. Rev. Lett.* **14**, 544 (1965).
13. V. N. Tsytovich, *Nonlinear Effects in Plasma*, Plenum Press, New York (1970).
14. W. L. Kruer and J. M. Dawson, *Phys. Fluids* **15**, 446 (1972); and references therein.
15. V. E. Zakharov, *Sov. Phys. JETP* **35**, 908 (1972).
16. V. E. Zakharov, *Handbook of Plasma Physics*, Vol. II (A Galeev and R. N. Sudan, eds.) p. 81-122, North Holland, Amsterdam, 1984.
17. M V. Goldman, *Rev. Mod. Phys.* **56**, 709 (1984).

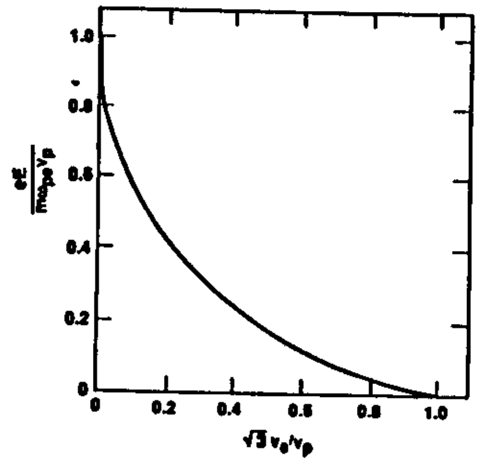

Figure 1. The wavebreaking amplitude as a function of the electron thermal velocity.

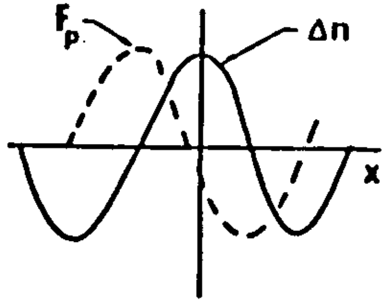

Figure 2. A schematic illustrating the feedback due to the ponderomotive force.

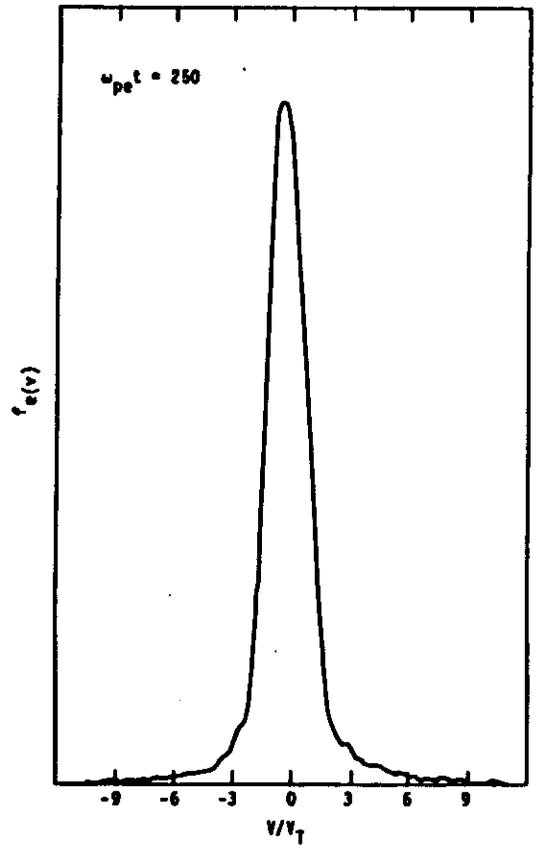

Figure 3. A heated electron distribution from a one-dimensional simulation of a plasma driven by a pump field with frequency near ω_{pe}.