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I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

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SPRING COLLEGE ON PLASMA PHYSICS

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EXCITATION OF LARGE AMPLITUDE PLASMA WAVES

Francis F. Chen

Department of Electrical Engineering
University of California Los Angeles
405 Hilgard Avenue
Los Angeles 90024
U.S.A.

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UCLA

What is "Large Amplitude"?

<u>Cold-plasma wavebreaking</u>

$$\nabla \cdot E = -4\pi e n_1 \qquad n_1 \simeq n_0$$

$$\nabla^2 \phi \simeq 4\pi e n_0 , \qquad |e\phi| \approx \frac{4\pi n_0 e^2}{m} \frac{m}{k^2} = \frac{\omega_p^2}{c^2 k^2} mc^2$$

Largest useful v_ϕ is $\frac{\omega_p}{k} \simeq c$

$$\therefore |e\phi_{max}| \approx mc^2 \approx 0.5 \text{ MeV}$$

Then $|E_{max}| \approx |k\phi_{max}| = \frac{\omega_p}{ce} mc^2 \approx 0.94\sqrt{n_0}$ V/cm

E.g.: for $n_0 = 10^{18}$ cm^{-3}, <u>$|E| \simeq 1$ GeV/cm</u>

This is $\simeq 1000$ times what is possible in a vacuum.

"Large amplitude" means $\epsilon > 10\%$ or so, where

$$\epsilon \equiv \frac{\phi}{\phi_{max}} = \frac{e\phi}{mc^2} .$$

Excitation Methods

1. Beat-wave
2. Wake-field
3. Laser wake-field
4. Relativistic electron beam
5. Stimulated Raman scattering

Properties of Large Amplitude Plasma Waves

1. Nonlinear frequency shift

The only non-linear $\delta\omega$ is due to the relativistic mass increase of the oscillating electrons: $m \to \gamma m$, $\omega_p^2 \to \omega_{p0}^2/\gamma$

... Mori, McKinstrie ... et al.

2. Wavebreaking limit

a) $T_e = 0$: $eE_{max} = m\omega_p v_\phi$

b) $T_e > 0$, $\gamma \simeq 1$: $\dfrac{eE_{max}}{m\omega_p v_\phi} = \left(1 - \frac{8}{3}\alpha_t^{1/4} + 2\alpha_t^{1/2} - \frac{1}{3}\alpha_t\right)^{1/2}$

where $\boxed{\alpha_t = 3KT_e/mv_\phi^2}$ (Coffey, 1971)

c) $T_e = 0$, $\gamma > 1$: $\dfrac{eE_{max}}{m\omega_p v_\phi} = \sqrt{2}\,(\gamma_\phi - 1)^{1/2}$

(Akhiezer & Polovin, 1956)

d) $T_e > 0$, $\gamma_\phi \gg 1$:

$$\frac{eE_{max}}{m\omega_p c} = \alpha_t^{-1/4}\left[\ln\left(2\gamma_\phi^{1/2}\alpha_t^{1/4}\right)\right]^{1/2}$$

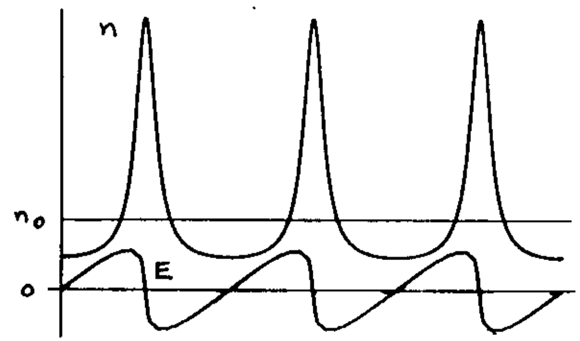
(Katsouleas & Mori, 1988)

PRL 61, 90

e.g.: $KT_e = 10\,\text{eV}$, $\gamma_\phi = 100$: $eE_{max}/m\omega_p c = 8.55$

3. Harmonic generation

$n_1/n_0 = .45$



Non-relativistic result:

m^{th} harmonic

$$\frac{n_m}{n_0} \approx \frac{m^m}{2^{m-1}\,m!}\left(\frac{n_1}{n_0}\right)^m$$

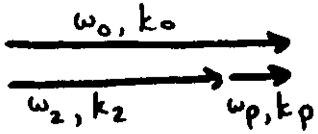
Beat-wave excitation

Matching conditions

$$\omega_p = \omega_0 - \omega_2 \qquad \underline{k}_p = \underline{k}_0 - \underline{k}_2$$

Optical mixing

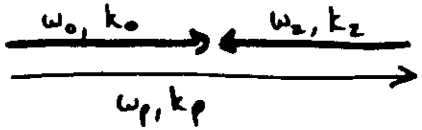
Parallel



"fast" wave

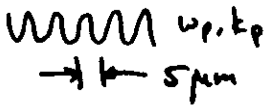
CO_2:

Anti-parallel



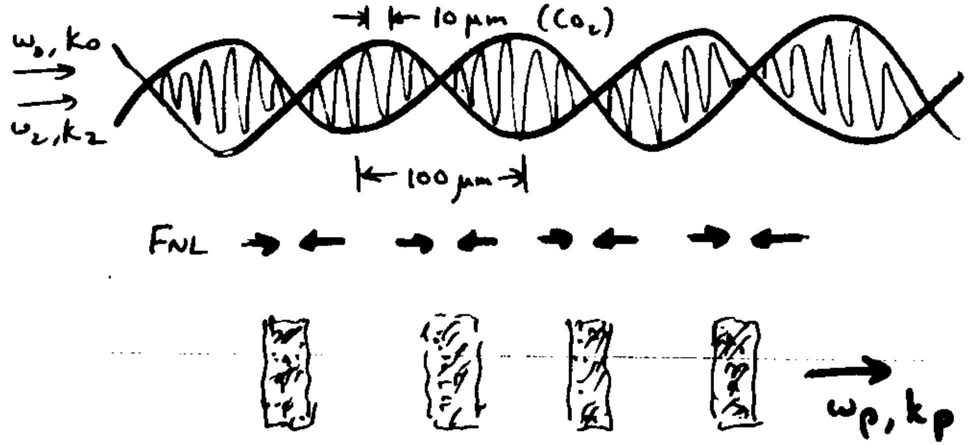
"slow" wave

CO_2:



Beat-wave excitation

Ponderomotive force



Electrons are bunched by the ponderomotive force

Group velocity matching

$$v_\phi = \frac{\omega_p}{k_p} \cong \frac{\Delta\omega}{\Delta k} \simeq v_g$$

$\therefore$ v_ϕ of plasma wave is synchronous with v_g of light wave

$$v_g = \left(1 - \frac{n}{n_c}\right)^{1/2} c, \quad \text{where} \quad \frac{n}{n_c} = \frac{\omega_p^2}{\omega_0^2}$$

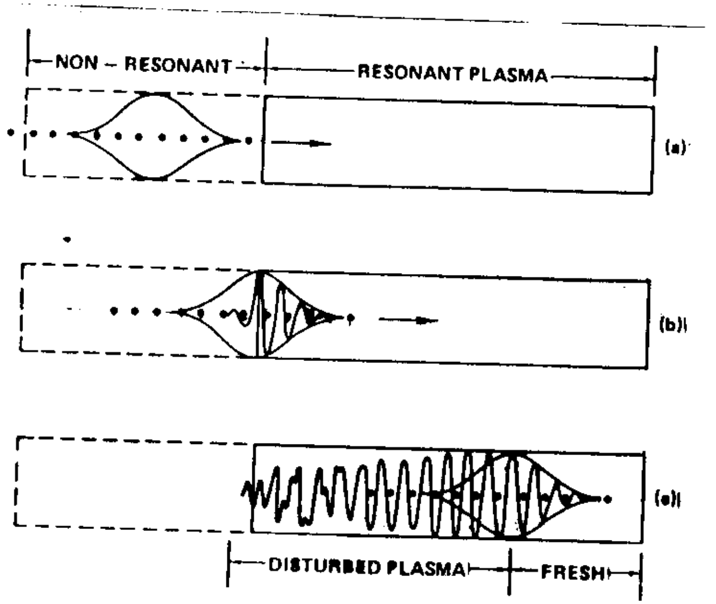
$$\gamma_\phi \equiv \left(1 - \frac{v_\phi^2}{c^2}\right)^{-\frac{1}{2}} = \left[1 - \left(1 - \frac{n}{n_c}\right)\right]^{-1/2}$$

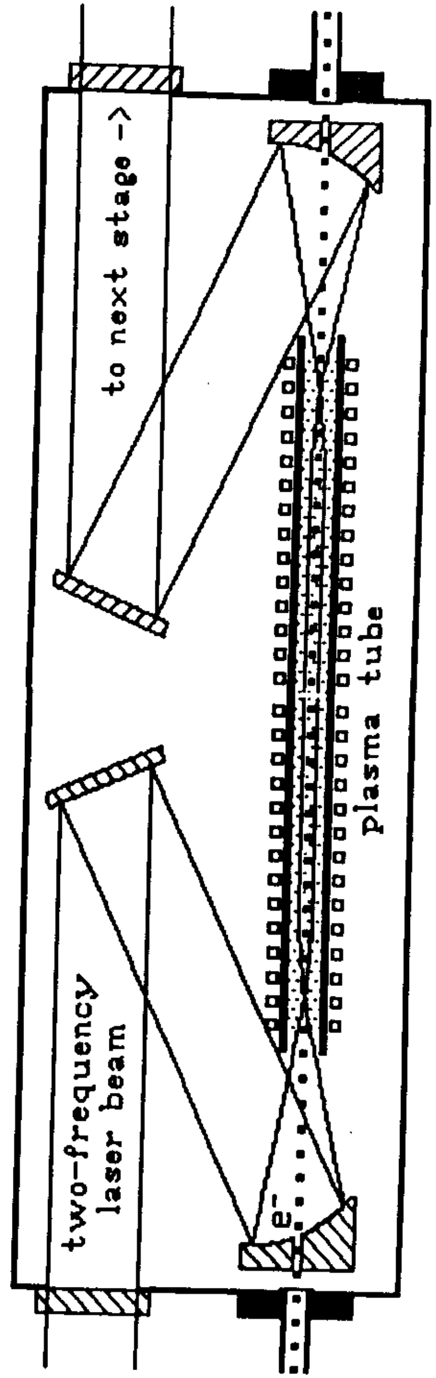
$$= \left(\frac{n_c}{n}\right)^{1/2} = \frac{\omega_0}{\omega_p}$$

$$\boxed{\gamma_\phi = \frac{\omega_0}{\omega_p}}$$

Beat-wave excitation

Laser Beat-wave Accelerator





Beat-wave excitation

Rosenbluth and Liu, Phys. Rev. Letters 29, 701 (1972)

Ponderomotive force

$$\underline{F}_{NL} = -\frac{\omega_p^2}{\omega_0\omega_2}\,\underline{\nabla}\,\frac{\langle|\underline{E}_0+\underline{E}_2|^2\rangle}{8\pi}$$

$$|F_{NL}| = \frac{\omega_p^2}{\omega_0\omega_2}k_p\frac{2E_0E_2}{8\pi}\cdot\frac{1}{2}\left[\cos(\omega_0-\omega_2)t+\cos(\omega_0+\omega_2)t\right]$$

$$= \frac{1}{2}k_p\frac{n_0e^2}{m}\frac{E_0E_2}{\omega_0\omega_2}. \qquad \boxed{\alpha \equiv \frac{v_{osc}}{c} = \frac{eE}{m\omega c}}$$

$$f_{NL} = \frac{1}{n_0}F_{NL} = \underline{\underline{\frac{1}{2}k_p mc^2\alpha_0\alpha_2}}$$

Electron motion $\quad m\dot{v} = -eE + f_{NL}$

Poisson's equation $\quad \nabla\cdot E = ik_pE = -4\pi e n_1$

Electron continuity $\quad \ddot{n}_1 = -n_0\nabla\cdot\dot{v} = -ik_p\frac{n_0}{m}(-eE+f_{NL})$

$$\ddot{n}_1 = -\underbrace{\frac{4\pi n_0e^2}{m}}_{\omega_p^2}n_1 - ik_p\frac{n_0}{m}\left(\frac{1}{2}k_pmc^2\alpha_0\alpha_2\right)$$

$$\ddot{n}_1 = -\omega_p^2n_1 - \frac{1}{2}ik_p^2c^2n_0\alpha_0\alpha_2$$

$$n_1 = n_1(t)\,e^{ik_px-i\omega_pt}:$$

$$\ddot{n}_1 \cong -2i\omega_p\dot{n}_1 - \omega_p^2n_1$$

$$\therefore\quad -2i\omega_p\dot{n}_1 = -\frac{1}{2}ik_p^2c^2n_0\alpha_0\alpha_2 \qquad k_p^2c^2\cong\omega_p^2$$

$$\dot{n}_1/n_0 = \frac{1}{4}\omega_p\alpha_0\alpha_2 \qquad \boxed{\epsilon = \frac{n_1}{n_0} = \frac{1}{4}\alpha_0\alpha_2\omega_pt}$$

Nonlinear saturation $\quad m\to\gamma m \quad\therefore\quad k_p\to\gamma k_p \neq \Delta k$

Frequency shift $\delta\omega$ due to mass increase:

$$(\omega_p+\delta\omega)^2 = \omega_p^2/\gamma, \qquad \gamma = \left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$\epsilon = \frac{n_1}{n_0} = \frac{k_p}{\omega_p}v = \frac{v}{v_\phi} \quad\therefore\quad \frac{v^2}{c^2} = \frac{v_\phi^2}{c^2}\epsilon^2,$$

$$\frac{1}{\gamma} = \left(1-\frac{v_\phi^2}{c^2}\epsilon^2\right)^{1/2}$$

$$\omega_p^2+2\omega_p\delta\omega \simeq \omega_p^2\left(1-\frac{1}{2}\frac{v_\phi^2}{c^2}\epsilon^2\right), \qquad \delta\omega \simeq -\frac{1}{4}\omega_p\frac{v_\phi^2}{c^2}\epsilon^2$$

$$\underline{\delta\omega \simeq -\frac{1}{4}\omega_p\epsilon^2(t)}$$

Growth stops when phase changes by $\frac{\pi}{2}$:

$$\omega_p\int_0^T\frac{1}{4}\epsilon^2(t)\,dt = \frac{\pi}{2}, \qquad \text{where}\quad \epsilon(t)\cong\frac{1}{4}\alpha_0\alpha_2\omega_pt$$

$$\therefore\quad \frac{1}{64}\omega_p^3\alpha_0^2\alpha_2^2\int_0^Tt^2\,dt = \frac{\pi}{2}, \qquad \frac{1}{3}\omega_p^3T^3 = \frac{32\pi}{\alpha_0^2\alpha_2^2}$$

$$\omega_pT = \left[\frac{96\pi}{\alpha_0^2\alpha_2^2}\right]^{1/3}, \qquad \epsilon_{sat}\cong\frac{1}{4}\alpha_0\alpha_2\omega_pT$$

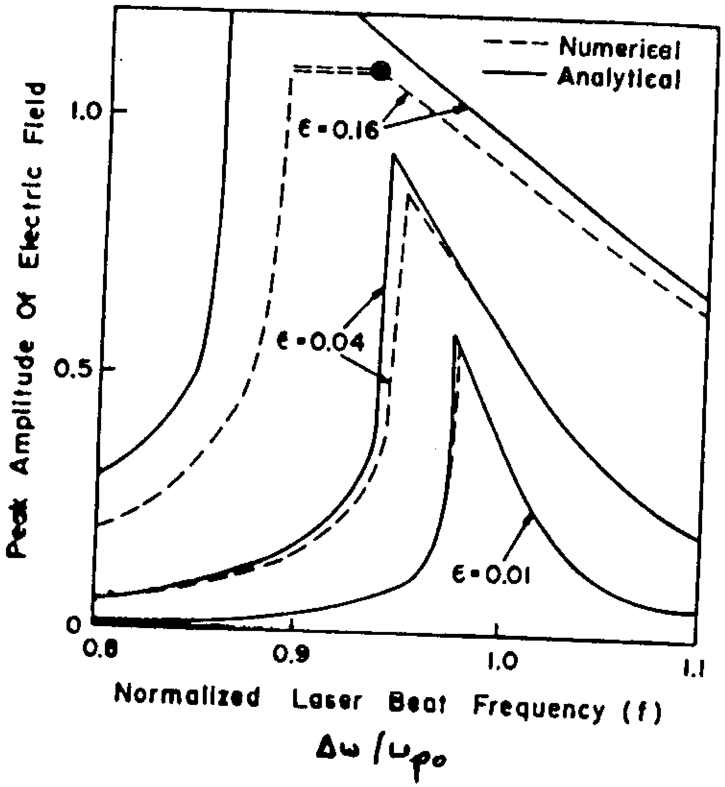
$$\therefore\quad \epsilon_{sat} = \left(\frac{3\pi}{2}\alpha_0\alpha_2\right)^{1/3} \qquad \propto I_0^{1/3}$$

More exactly: $\boxed{\epsilon_{sat} = \left(\frac{16}{3}\alpha_0\alpha_2\right)^{1/3}}$

Off-resonance driver

Tang, Sprangle, and Sudan, Phys. Fluids 28, 1974 (1985)

By tuning $\Delta\omega$ to a value below ω_{po}, it is possible to drive through the resonance at $\Delta\omega = \omega_{po}/\gamma$ and get a larger ϵ_{sat} by a factor ~ 1.6.



Beat-wave excitation

Other results relevant to accelerators

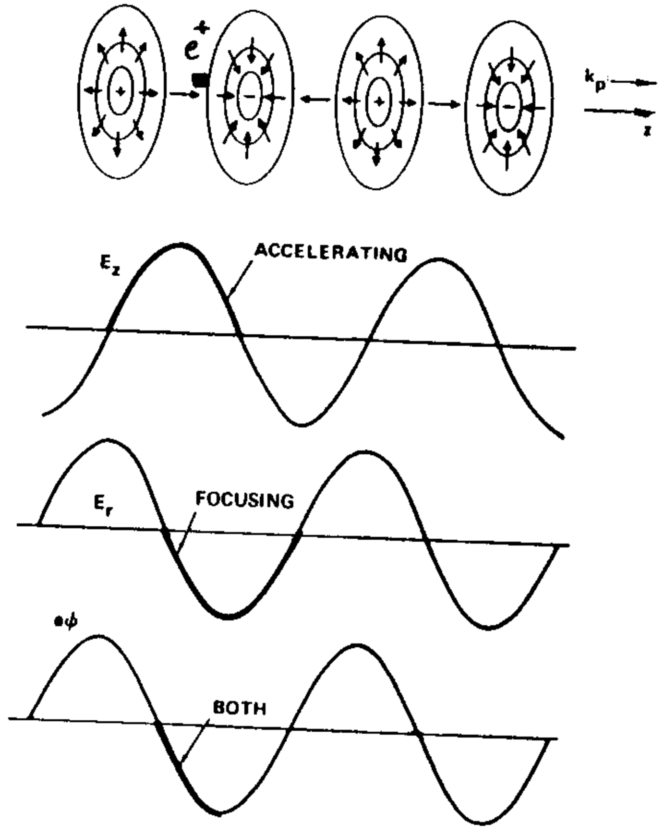
Energy gain

$$\Delta W \simeq 4\epsilon\gamma_\phi^2 mc^2$$

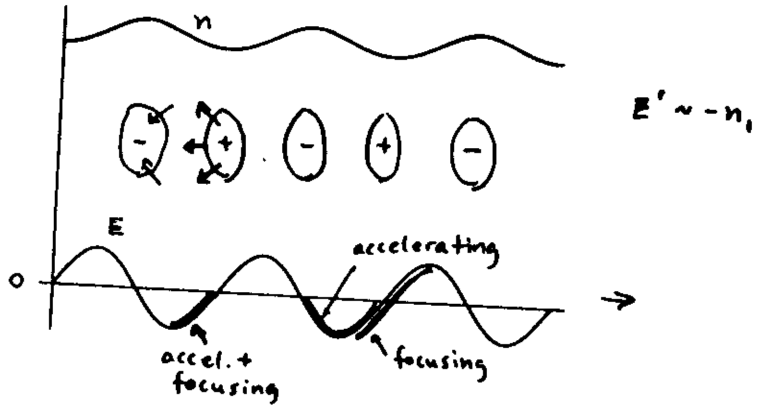
Required injection energy

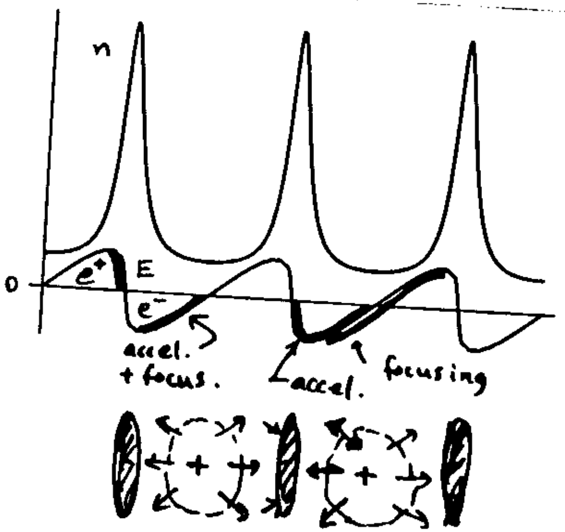
$$\gamma > \frac{1+\epsilon^2}{2\epsilon} \quad (1\text{-D})$$

Radial focusing



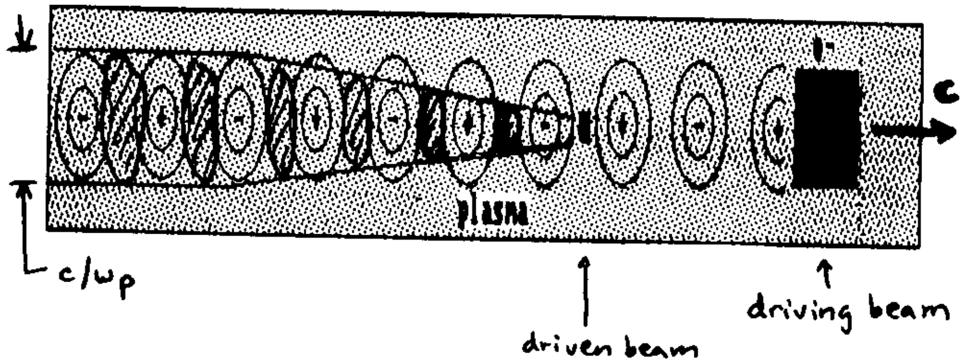
Injection phase space for nonlinear waves





But 2-dimensional plasma wave cannot stay in phase at all radii, since $v \sim \epsilon v_\phi \sim \epsilon c$, so γm varies with r.

Wake-field excitation

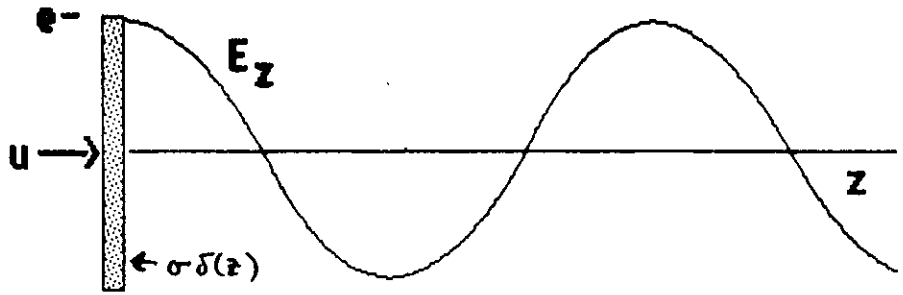


Fast driving beam displaces plasma electrons.
At end of beam, a large positive charge is left behind.
This causes a plasma oscillation at $\omega = \omega_p$.
The wake has $k \simeq \omega_p / c$.

Inject an electron bunch at proper phase to be accelerated.
The driven bunch also makes a wake.
This wake will cancel the original wake, thus taking most of the energy out of the plasma wave.
The driven bunch can be small, since a plasma wave has a minimum width of c/ω_p.
Thus, the accelerated electrons do not see the radial gradient in E.

Wake-field excitation

Green's function response



$\frac{\partial}{\partial t} = 0$ in wave frame. Plasma streams by at $u\hat{z}$.

$$m\,u\cdot\nabla v = -eE \qquad m u v' = -eE \quad (1) \qquad (\,)' = \frac{\partial}{\partial z}$$

$$n_0 \nabla\cdot v + u\cdot\nabla n_1 = 0 \qquad n_0 v' + u n_1' = 0 \quad (2)$$

$$\nabla\cdot E = E' = 4\pi\sigma\delta(z) - 4\pi e n_1 \qquad (3)$$

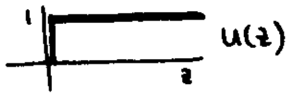
$$E'' = 4\pi\sigma\delta'(z) - 4\pi e n_1', \qquad n_1' = \frac{e n_0}{m u^2} E$$

$$E'' + \frac{\omega_p^2}{u^2} E = 4\pi\sigma\delta'(z) \qquad \boxed{\frac{\omega_p^2}{u^2} = k_p^2}$$

Solution:

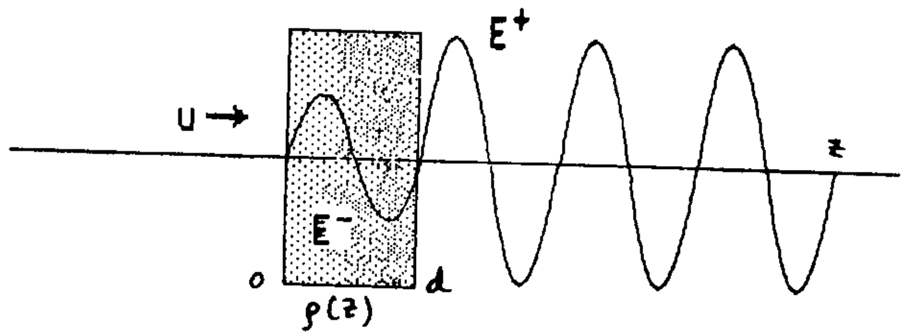
$$\boxed{E = 4\pi\sigma \cos k_p z \ast u(z)} = G(z)$$

where $u(z) = \int \delta(z)\,dz$:



Wake-field excitation

Transformer ratio $\qquad G(z) = 4\pi\sigma \cos k_p z$



$$E(z) = 4\pi \int_0^z \rho(z')\cos[k_p(z-z')]\,dz'$$

Case of $\rho(z) = \rho_0 = \text{const}$.

$$E(z) = \frac{4\pi\rho_0}{k_p}\left[\sin k_p(d-z) + \sin k_p z\right] \qquad z > d$$

$$= \frac{4\pi\rho_0}{k_p} \sin k_p z \qquad z < d$$

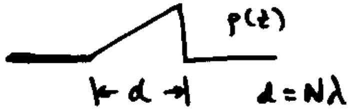
For $k_p d = 2\pi N$: $\quad E(z) = 2\sin k_p z \qquad z > d$

$$= \sin k_p z \qquad z < d$$

$$R \equiv |E^+/E^-|_{max}$$

$$\boxed{\text{Thus, for } \rho = \rho_0, \; R = 2}$$

Case of triangular bunch



Then $\rho = \rho_0 k_p z$

$$E_z = -\frac{4\pi\rho_0}{k_p} 2\pi N \sin k_p z \qquad z > d$$

$$= \frac{4\pi\rho_0}{k_p}(1 - \cos k_p z) \qquad z < d$$

$$E^+_{max} = 2\pi N\left(\frac{4\pi\rho_0}{k_p}\right), \quad E^-_{max} = 2\left(\frac{4\pi\rho_0}{k_p}\right)$$

$$\therefore \boxed{R = \pi N}$$

Add precursor: $G(z) = 4\pi\sigma \cos k_p z$

$$\rho(z) = \rho_0 k_p z + 4\pi\sigma\delta(z), \quad \sigma = \rho_0/k_p$$

Then

$$E_z = \frac{4\pi\rho_0}{k_p}(\cos k_p z - 2\pi N \sin k_p z) \qquad z > d$$

$$= \frac{4\pi\rho_0}{k_p} = \text{const.} \; (!) \qquad z < d$$

$$\boxed{R = \left[1 + (2\pi N)^2\right]^{1/2} \simeq 2\pi N}$$

1 GeV driver → 100 GeV beam if N = 16

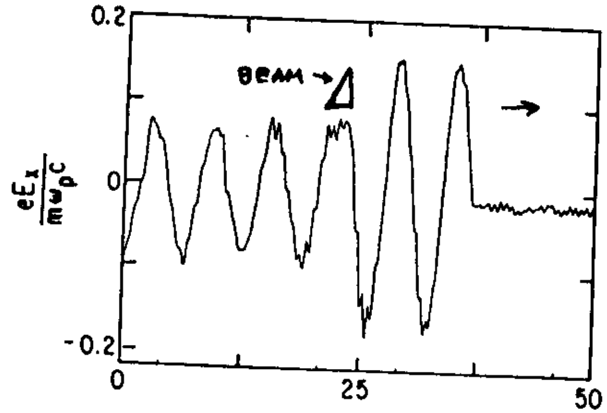
Reason: cold electrons are displaced over time, but beam ends suddenly.

Self-sharpening:

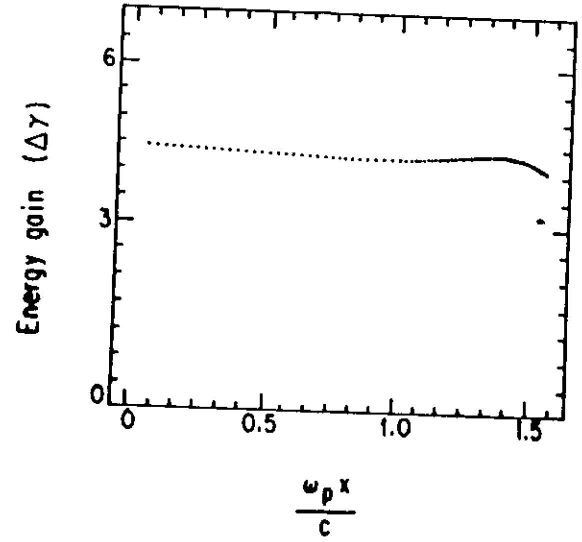
1. Rear of pulse catches up because E^- is smaller.
2. Relativistic mass increase makes $k_p = \omega_p/c$ decrease. Hence λ_p increases, and beam edge "seems" sharper to wave.

Beam loading by shaped beam

By shaping the driven beam as a reverse triangle, one makes E^+ = const. inside the beam; all electrons receive the same acceleration.



(a)



(b)

Here, $E = \frac{1}{2}E_{max}$ and efficiency $\eta = 75\%$

THE WAKEFIELD ACCELERATOR

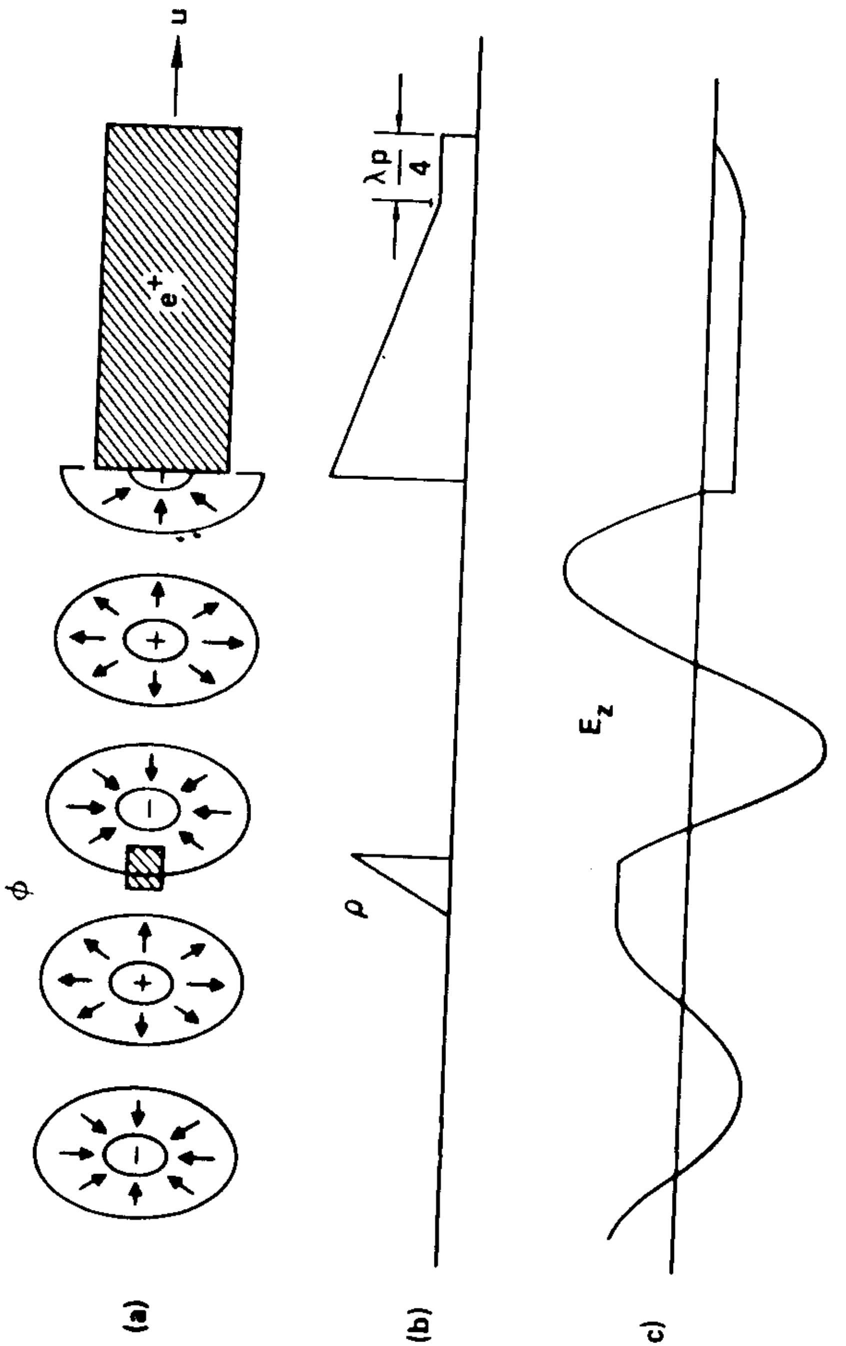
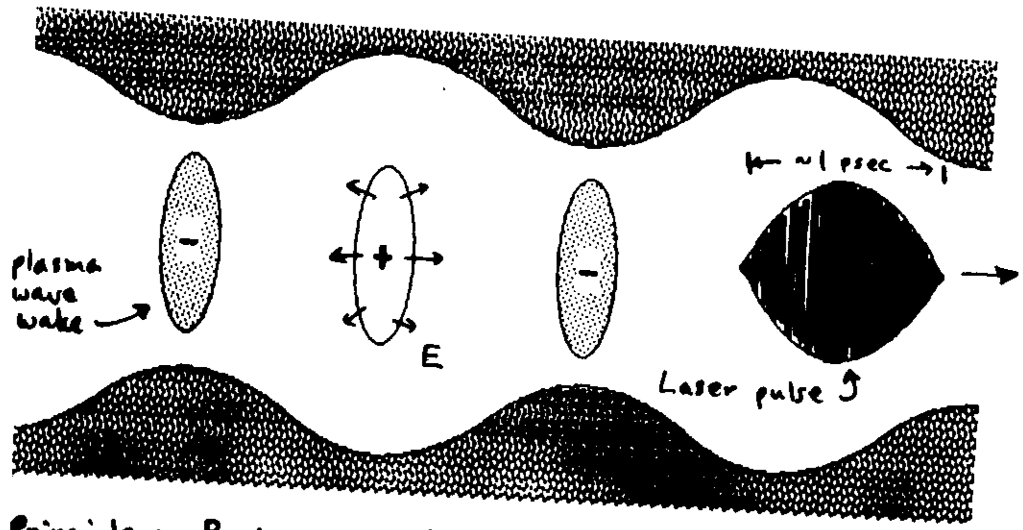


FIGURE 5

Laser Wake-field excitation

Sprangle, Esarey, Ting, and Joyce, Appl. Phys. Lett. 53, 2146, (1988)



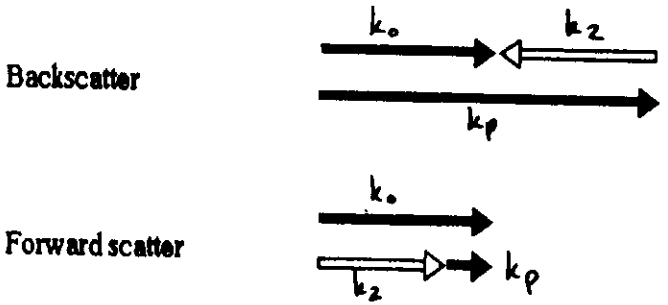
Principle: Ponderomotive force of short laser pulse ($L \sim \lambda_p = 2\pi c/\omega_p$) pushes cold electrons away. They oscillate at ω_p, thus forming a plasma wave wake.

Advantages:
1. Density need not be resonant, as in beat wave.
2. Relativistic self-focusing keeps laser pulse from diffracting if $P > 17(\omega/\omega_p)^2$ GW.
3. E_z larger by factor π than direct F_{NL} on accelerated particles, since they have $m = \gamma m_0$.

Examples:

Laser	λ	P	Accel. length	Final energy
CO_2	10.6 μ	2×10^{13} W	.54 m	1.4 GeV
Nd	1.06	2×10^{15}	54 m	140 GeV
KrF	0.26	3×10^{16}	860 m	2200 GeV

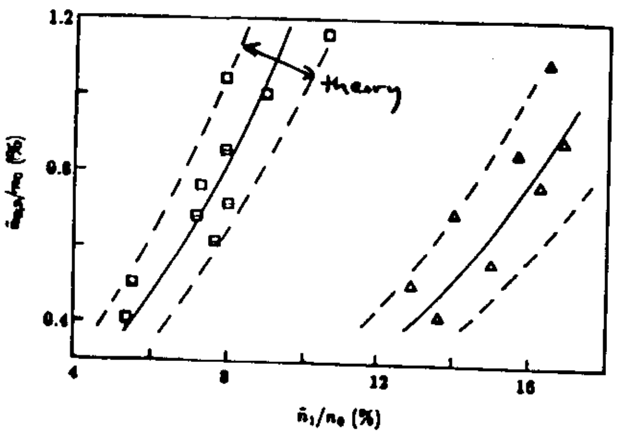
Stimulated Raman Scattering



In backscatter, short wavelength plasma waves (large k) can be excited to large amplitude; these are too slow to be of interest for acceleration.
In forward scatter, fast waves can be created, as in beat-wave excitation, but much more power is needed when there is only one pump, and the threshold is higher than for backscatter, so that backscatter will be dominant.

However, SRS has been used to study harmonic generation in large amplitude plasma waves:

Umstadter, Williams, Clayton, & Joshi, Phys. Rev. Lett. 59, 292 (1987)

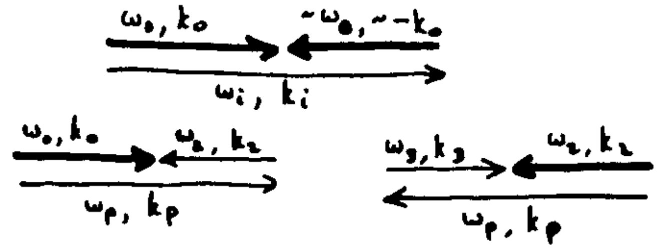


Stimulated Raman Scattering

Advantages: 1. Only one pump frequency.
2. Density resonance not needed.

Disadvantages: 1. High power needed.
2. Phase of plasma wave not controllable.
3. Brillouin scattering complicates result.

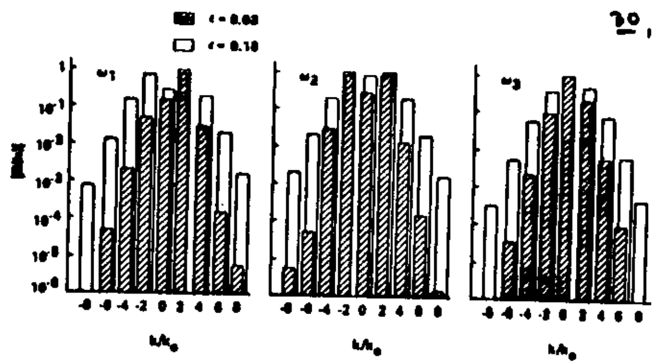
(3) unless $T_i \gtrsim T_e$, SBS will occur when SRS occurs.
This gives rise to: a) Two strong pumps, and SRS in both directions.



b) Mode coupling: $\omega_p = \omega_p(x)$, rippled by ion wave

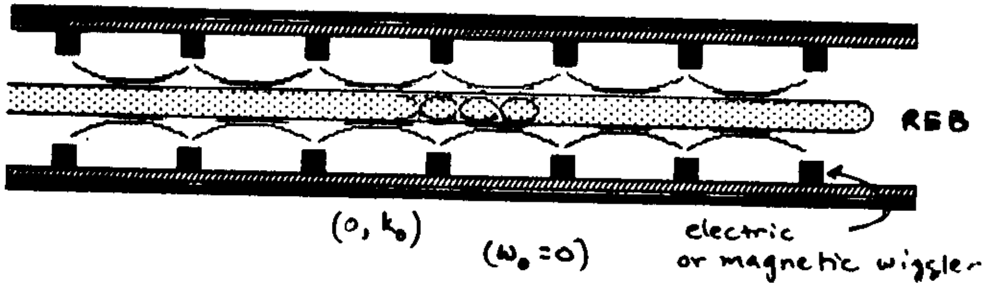
Get $k = k_p \pm k_i$, $k_p \pm 2k_i$, ... etc.

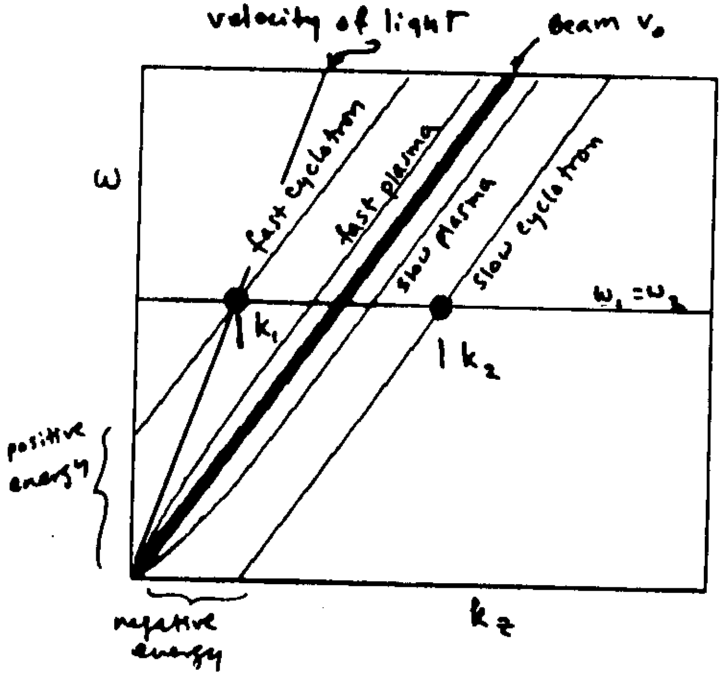
Barr & Chen, Phys. Fluids 30, 1180 (1987)



Relativistic Electron Beam Excitation

Nation, Anselmo, and Greenwald, (Orsay, 1987; CERN 87-11)





$$\omega_2 = \omega_1 + \omega_0 = \omega_1$$

$$k_2 = k_1 + k_0$$

Plasma Sources for Accelerators

Required characteristics

1. High density: 10^{14} -- 10^{18} cm^{-3}
2. Long: 10--100 cm, but can be only 4 mm in diameter
3. Quiescent
4. Density reproducible (if pulsed) or constant (if DC)
5. Density uniform longitudinally [with controllable gradient]
6. Density uniform radially [with controllable reverse gradient]
7. Fully ionised (low Z)
8. Heavy ions (high Z)
9. Zero or small magnetic field; or transverse B if surfatron
10. Beam access along axis
11. Simple, cheap, rugged, and dependable
12. Efficient, with simple power supplies

Uniformity is most important in the Beat-wave scheme. The nonlinear frequency shift is, at most

$$\delta\omega \simeq \frac{1}{4}\epsilon^2\omega_p .$$

The frequency shift due to density variations should be less than this. Thus

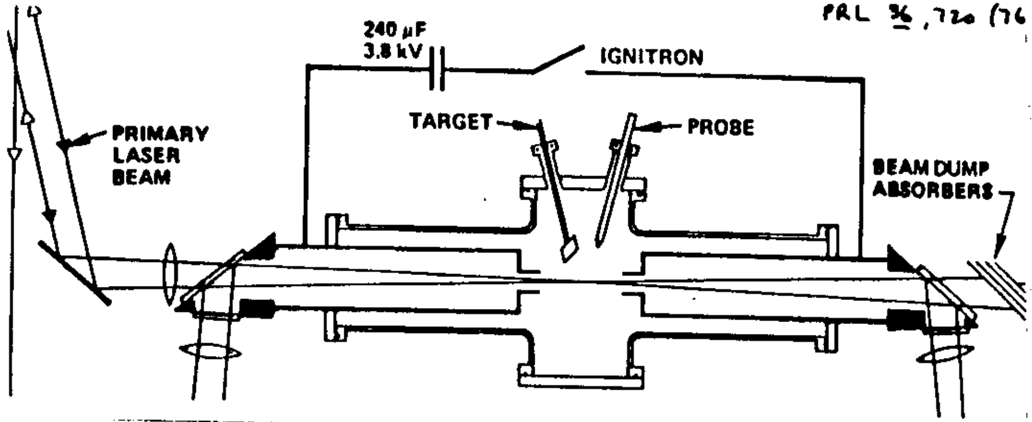
$$\frac{\delta\omega_p}{\omega_p} < \frac{1}{4}\epsilon^2 = \frac{1}{2}\frac{\delta n}{n}, \qquad \frac{\delta n}{n} < \frac{1}{2}\epsilon^2 \simeq 1\%$$

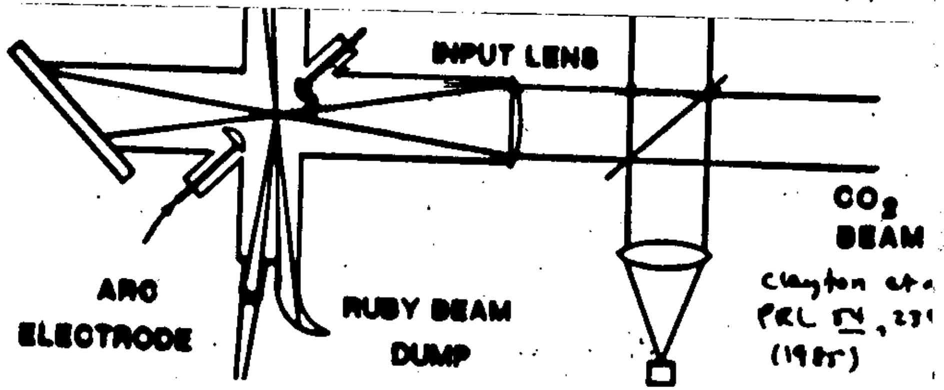
Plasma Sources
for high-density plasmas

1. High current arcs
2. Theta-pinch
3. Z-pinch
4. Multi-photon ionization
5. Laser (avalanche) ionization in gases
6. Laser ionization of foils or tubes
7. ECRH (electron cyclotron resonance)
8. RF (radiofrequency) ionization
9. Ultra-violet ionization
10. Capillary discharges

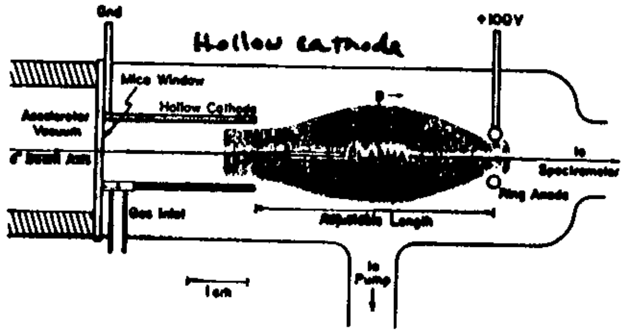
High current arcs

Turechek + Chen
PRL 36, 720 (76





Clayton et al.
PRL 54, 23?
(1985)

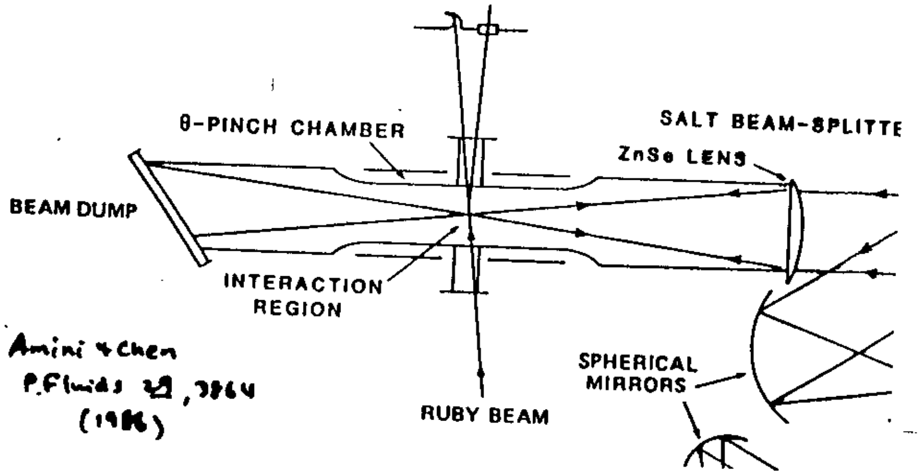


Rosenzweig
et al.
Malibu 191
p.226

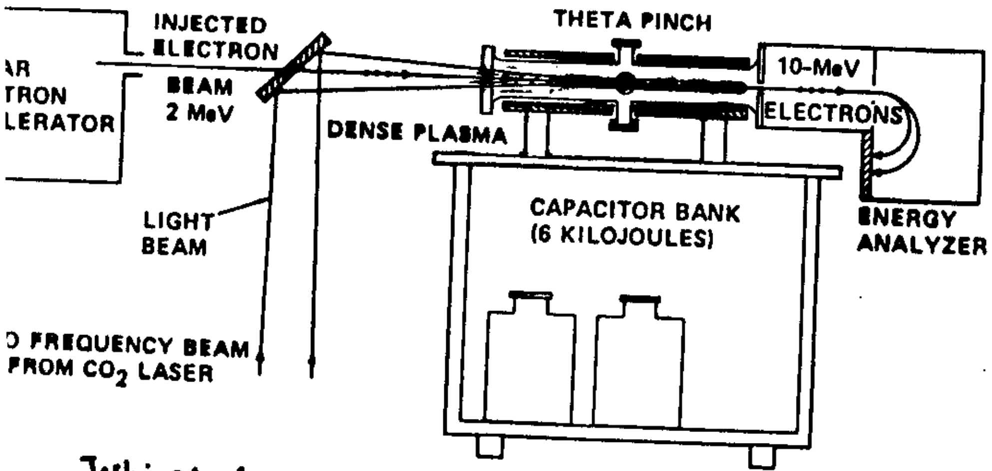
$n = 10^{13} - 10^{15}$
cm^{-3}

Fig. 2: Hollow cathode arc; experimental setup for plasma wake field accelerator test.

Theta pinch



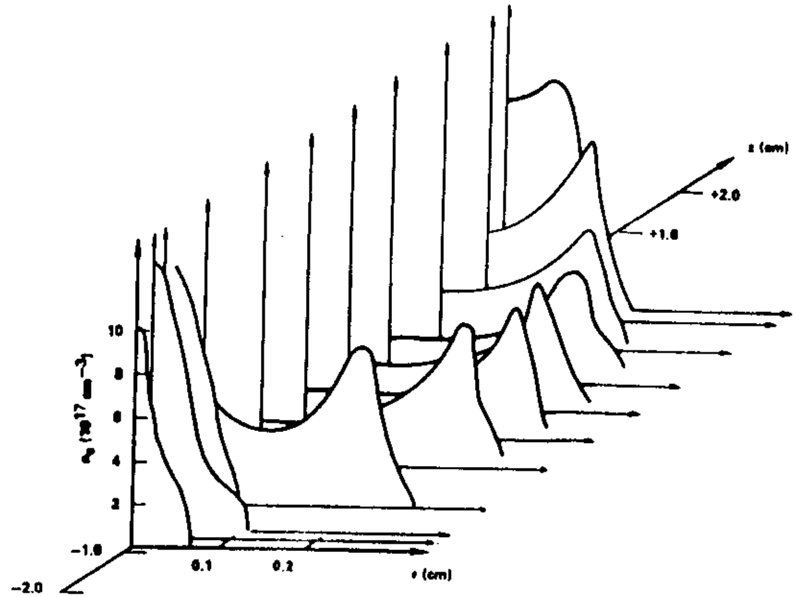
Amini + Chen
P.Fluids 29, 3864
(1986)



Joshi et al.
Orsay, 1987
p. 351

Laser-heated arc plasma (interferogram)

$n_{arc} \sim 2\times10^{16}$
$n_{final} \sim 10^{17}$ cm^{-3}
$T_{arc} \sim 3$ eV
$T_{final} \approx 10$-30 eV



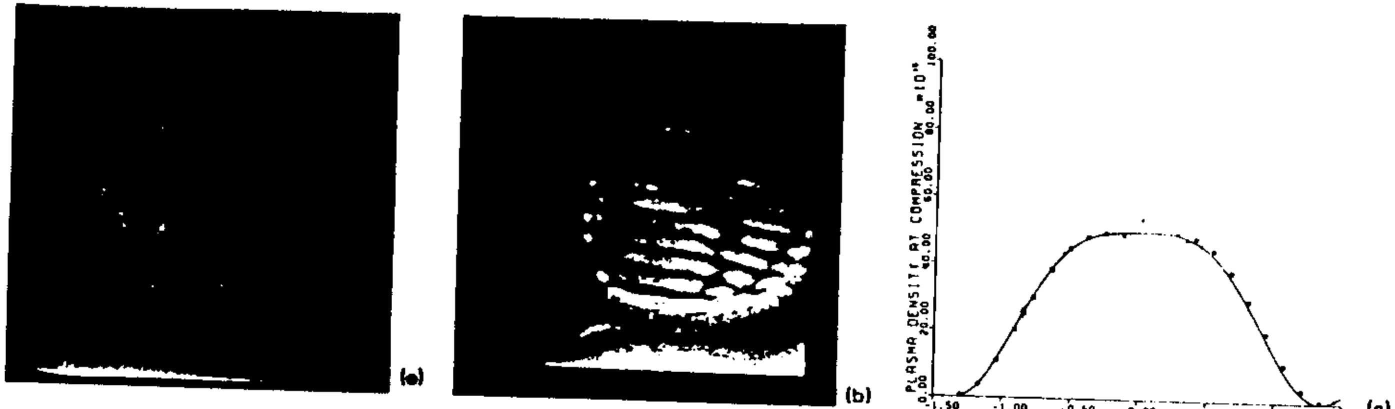


FIG. 2. Simultaneous end-on and side-on photographs and the Abel inversion. (a) End-on photograph of plasma taken at $t \sim 50$ nsec. Each fringe shows an electron density of 1.3×10^{16} cm^{-3}. (b) Side-on photograph taken simultaneously with (a). (c) Graph of the Abel inversion of the side-on fringe shift, photograph (b), the horizontal scale is in cm. Note both the density and the radial density profile in (a) and (c) are in reasonable agreement.

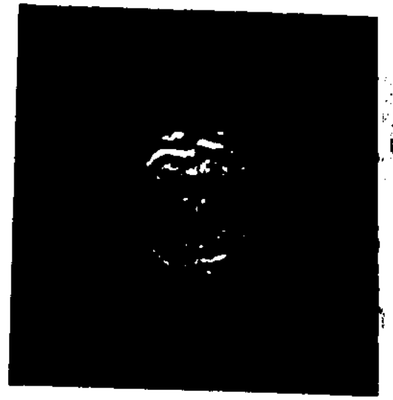


FIG. 3. Here $t\sim-50$ nsec: plasma ap proaches the axis. Existence of the instabi ity pattern caused by inward acceleration.

FIG. 4. Here $t\sim50$ nsec: inward motion has already come to a halt; termination of the first stage of the instability.

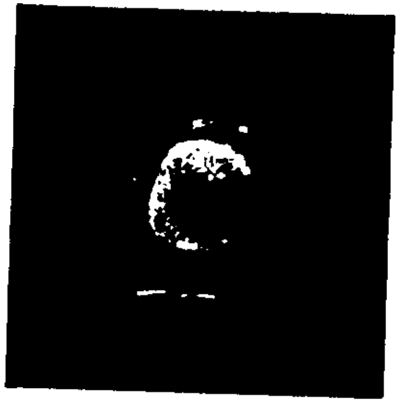


FIG. 5. Here $t\sim200$ nsec, plasma is in compressed stage, Rayleigh–Taylor instability has already been initiated.

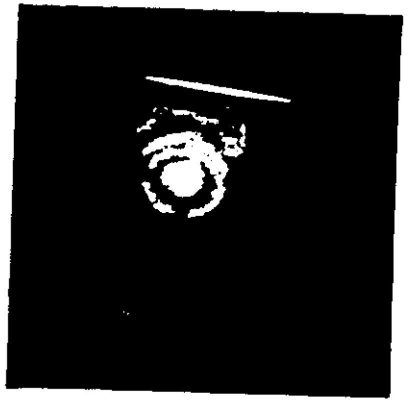


FIG. 6. Her $t\sim300$ nsec, th instability ha reached a highl nonlinear stage.

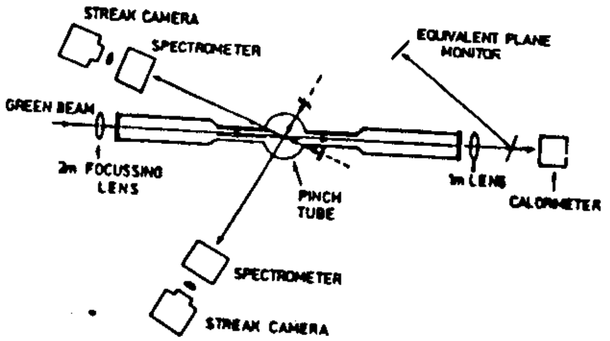
FIG. 7. Here $t\sim350$ nsec: initiation of wave breaking.

FIG. 8. Here $t\sim350$ nsec: plasma density is higher than in Fig. 7.

FIG. 9. Here $t\sim400$ nsec: the wave has already broken.

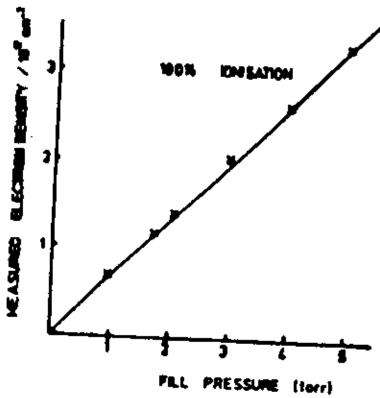
Multi-photon ionization (1.06 μm)

(also 0.53 μm)



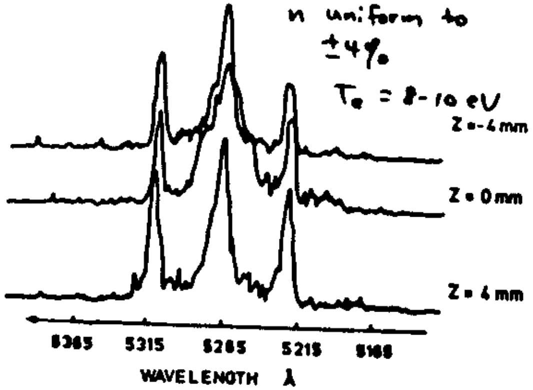
10 J, 200 psec
10^{14} W/cm^2
400 μm spot
H_2
0.5 - 4 torr

Fig. 3 Schematic arrangement of the multiphoton ionisation expt



Electron density as a function of fill pressure

n uniform to ±4%
T_e = 8-10 eV

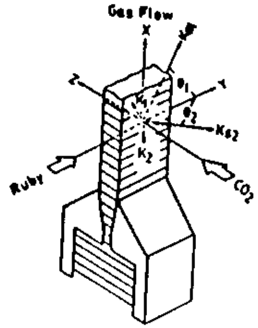


20 degree scattering signals from three axial positions

Advantages:
1) Fast, 100% ionization
2) Measured axial uniformity
3) Expected radial uniformity, from threshold effect

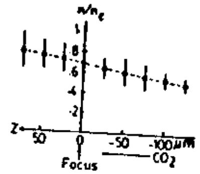
Disadvantages:
1) Limited diameter and length
2) Large laser needed
3) Timing is critical

Laser ionization of gas jets



Offenberger et al.

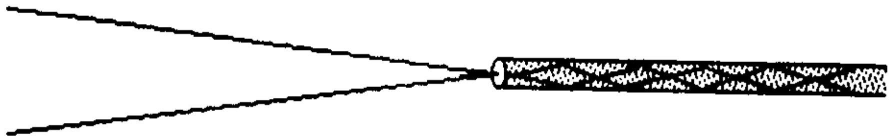
$h\nu\,(CO_2) = .12$ eV (~100 photons)
$h\nu\,(.5\mu m) = 2.34$ eV (~6 photons)
∴ collisional ionization



Laser ionization of foils

Figueroa + Joshi

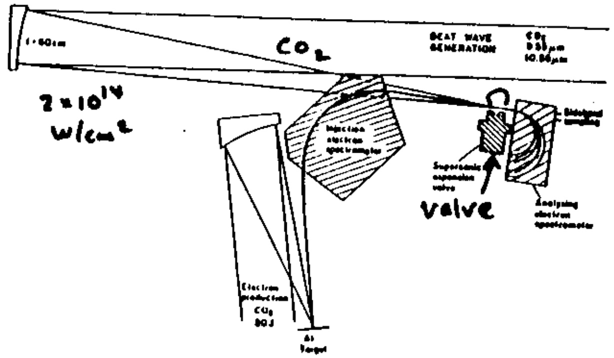
Laser ionization of tubes



Osaka (proposed)

Laser ionization of gas puffs

Martin et al. Madison, Wisc., 1986 (AIP Conf. Proc. 156, p.121



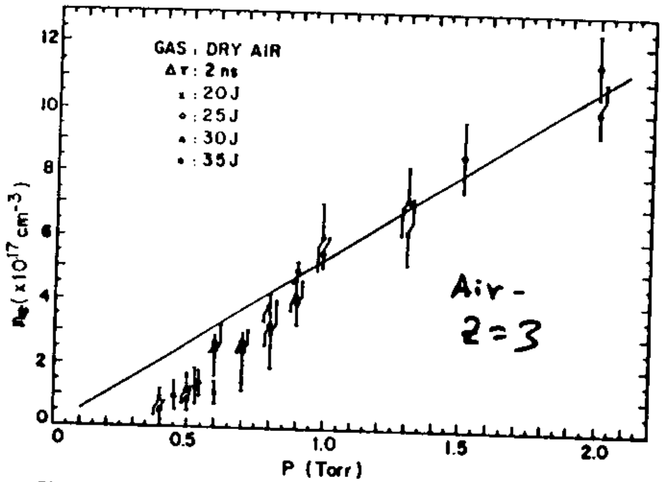
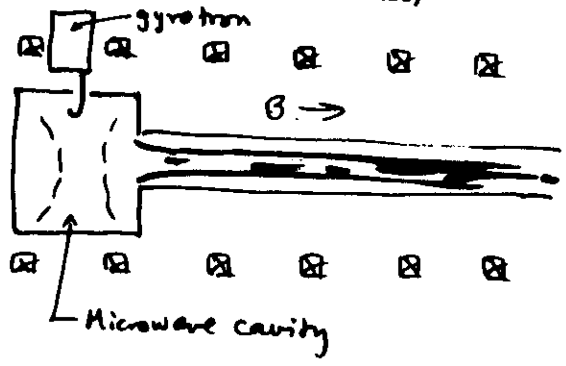


Figure 1: Electron density as a function of filling pressure for dry air

First 0.15 nsec: enhanced Keldysh (tunneling) ioniz.
Later: Collisional ionization

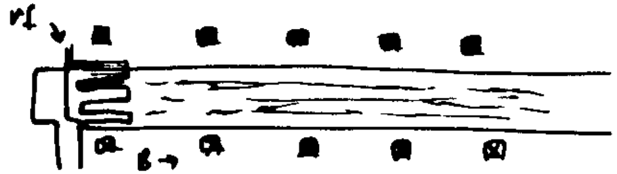
High-frequency and radiofrequency sources

ECRH (electron cyclotron resonance)

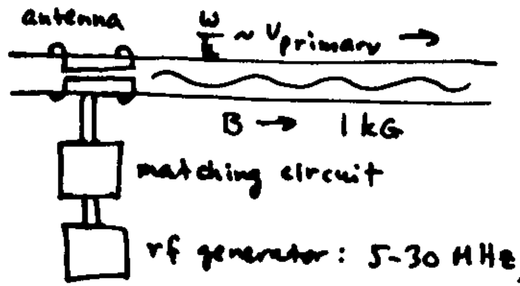


$n \simeq 10^{12}-10^{13}\ cm^{-3}$
$P \simeq 10-60$ kW
Partially ionized

Lisitano coil



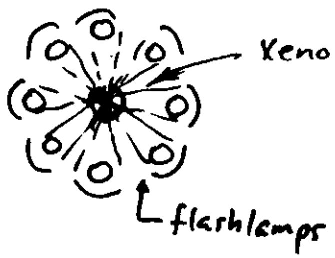
Helicon wave source



$10^{13}-10^{14}$
cm^{-3}
fully ionized

Proposed methods

Ultraviolet ionization



Capillary discharge