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RELATIVISTIC INSTABILITIES OF LARGE AMPLITUDE WAVES IN PLASMAS

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ABSTRACT

A critical review of relativistic instabilities of large amplitude electromagnetic waves in plasmas is presented. For illustrative purposes, we first summarize the small amplitude results. We then continue to take into account fully relativistic electron quiver velocity and large amplitude density fluctuations driven by a fully relativistic ponderomotive force and investigate modulational and filamentation instabilities of an intense circularly polarized electromagnetic wave. Thus, the nonlinearities arising from the relativistic electron mass variation and the relativistic ponderomotive force are treated exactly, so that our theory is valid for arbitrarily large values of the radiation intensity. Novel relativistic instabilities are shown to exist at large fluxes of the electromagnetic radiation. The relevance of our investigation to inertial confinement fusion and plasma based beat wave accelerators has been pointed out.

I. INTRODUCTION

There has been a great deal of interest in the study of interaction of strong electromagnetic waves with a plasma. This investigation is of fundamental importance with regard to the understanding of the various physical processes that occur in laser-plasma interactions and plasma-based beat-wave particle accelerators. Strong electromagnetic waves have also been observed in astrophysical plasmas.

The inertial confinement fusion concept requires a clear understanding of the detailed mechanisms by which the electromagnetic wave energy can be transformed into particle random energy leading to plasma heating. On the other hand, in the plasma based beat-wave accelerators, two collinear coherent electromagnetic waves whose frequencies differ by appoximately the electron plasma frequency, are employed for generating a large amplitude Langmuir wave. The longitudinal electric field of the latter can then be used to accelerate electrons to extremely high energies.

It is well known that a finite amplitude electromagnetic wave is subjected to a great variety of stimulated scattering, modulational, and filamentation instabilities [1-3] which belong to a class of parametric interactions. The parametric instabilities can affect the wave propagation, the wave absorption, and the electron energy transport in plasmas.

The nonlinear processes involve several distinct features. These are associated with the radiation pressure or the ponderomotive force effects [1-4], relativistic electron mass variations [5-22], harmonic generation [5,6,9], and the joule heating [23-25]. The radiation pressure effect drives the slow density fluctuations, the relativistic electron mass variation in the laser field causes the mass modulation, the harmonic generation produces the second order electron current density, whereas the joule heating gives rise to the temperature perturbation. The ponderomotive force and the joule heating effects may thus invlove the ion dymanics. On the other hand, the relativistic electron mass modulation and the harmonic generation nonlinearities arise on a time scale sufficinetly short so that the background slow plasma motion does not respond to the ponderomotive force.

The nonrelativistic ponderomotive force related stimulated scattering, modulational, and filamentation instabilities of a coherent electromagnetic wave have been investigated by many authors [1-4] A number of authors [6,9,10] have investigated the modulational and filamentational instabilities of an electromagnetic wave excluding the ponderomotive force effects but accounting for the relativistic electron mass variation and the second harmonic generation nonlinearities. The combined effects of ponderomotive force and relativistic mass variation nonlinearities have been incorporated in the study of the modulational and filamentation instabilities of circularly polarized electromagnetic waves in an unmagnetized plasma [14, 18-20]. New instability regiems have been found.

The purpose of the present review talk is to discuss relativistic instabilities of an electromagnetic wave in plasmas. In order to outline the essential physics, we have limited our efforts to an unmagnetized plasma. We shall start with the small amplitude theories for the modulational and filamentational instabilities of an electromagnetic wave. The role of relativistic electron mass variation shall be emphasized. We then continue developing finite amplitude theories for the parametric instabilities invloving arbitrarily large amplitude circularly polarized electromagnetic wave. Accounting for fully relativistic electron quiver velocity and large-amplitude density fluctuations driven by fully relativistic ponderomotive force of an intense circularly polarized electromagnetic wave, novel instabilities are shown to exist at high pump power. The relevance of our investigation to inertial confinement fusion and plasma based-beat wave accelerators has been pointed out.

2. GOVERNING EQUATIONS

The nonlinear interaction of a strong electromagnetic wave with the background plasma is governed by the continuity equation

$$\partial_t n_j + \nabla\cdot(n_j \vec{v}_j) = 0, \tag{1}$$

the relativistic momentum equation

$$(\partial_t + \vec{v}_j\cdot\nabla)\vec{p}_j = e_j\left(\vec{E} + \frac{\vec{v}_j}{c}\times\vec{B}\right) - \frac{1}{n_j}\nabla P_j, \tag{2}$$

the Maxwell equations

$$\nabla\times\vec{E} = -\frac{1}{c}\partial_t\vec{B} \tag{3}$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\partial_t\vec{E}, \tag{4}$$

and Poisson's equation

$$\nabla\cdot\vec{E} = 4\pi e(n_i - n_e), \tag{5}$$

where n_j , $\vec{v}_j$ and T_j are the number density, the fluid velocity, the temperature of the particle species j (equals e for the electrons and i for the ions), $\vec{p}_j = m_{jo}\vec{v}_j\gamma_j$, where $\gamma_j = (1 - v_j^2/c^2)^{-1/2}$, is the relativistic momentum, m_{jo} is the rest mass of the particle species j, e is the magnitude of the electron charge, P_j is the particle pressure, c is the speed of light, and $\vec{J} = n_i e\vec{v}_i - n_e e\vec{v}_e \equiv \vec{J}_i + \vec{J}_e$ is the total plasma current density. The usage of the nonrelativistic pressure gradient force in (2) is justified for the nonrelativistic plasma temperature.

The electromagnetic fields are usually represented as

$$\vec{E} = -\nabla\phi - c^{-1}\partial_t\vec{A}, \tag{6}$$

and

$$\vec{B} = \nabla\times\vec{A}, \tag{7}$$

where ϕ and $\vec{A}$ are, respectively, the scalar and vector potentials. We shall use throughout the Lorentz guage condition,

$$\nabla\cdot\vec{A} + c^{-1}\partial_t\phi = 0.$$

In order to close the above set of equations, one needs to specify an equation of state for the pressure. For isothermal processes, the equation of state is given by

$$P_j = n_j T_j, \tag{8}$$

whereas for adiabatic responses, one uses as equation of state:

$$d_t(P_j n_j^{-\alpha_j}) = 0, \tag{9}$$

where α_j is the usual adiabatic index which is the ratio of the specific heats. Equation (9) is valid for those thermodynamic processes where the net heat flux is zero. At relativistic temperatures the macroscopic equations of state as given by (8) and (9) are not valid.

A few comments must be made about the correctness of the continuity equation for processes where relativistic effects are taken into consideration. Since the density is not a Lorentz invarient quantity, it follows that one must account for changes in the particle number density due to relativistic effects. However, since all the calculations are carried out in one frame of reference, namely, the laboratory frame of reference or an inertial frame moving with respect to it, the

fact that the number density is not a Lorentz invarient quantity is not relevant to the present discussion. The effects due to the Lorentz invariance are important only when there is a change from one inertial frame to another. We shall be considering in this talk only those relativistic effects which arise due to the high-frequency motions of the electrons. For large field intensities, the latter becomes relativistic resulting in the nonlinearities due to the ponderomotive force and the particle-mass variation whereas the low-frequency motion and the plasma temperatures are still nonrelativistic.

3. DISPERSION RELATIONS OF WAVES

The dispersion relations for the Langmuir and electromagnetic waves including relativistic mass variations have been obtained from the fluid model presented in §2. In the weakly relativistic limit (viz., when the electron quiver velocity $v_0 = eE/m_0\omega$; where E is the wave electric field, ω is the wave frequency, and m_0 is the rest mass of the electron, is much smaller than c), one finds the electron plasma wave dispersion relation

$$\omega^2 = \omega_{pe}^2 \left(1 - \frac{3}{8} u_m^2\right), \tag{10}$$

where $\omega_{pe} = (4\pi n_e e^2/m_0)^{1/2}$ is the electron plasma frequency, and $u_m = v_0/c$. On the other hand, for $u_m \longrightarrow 1$, one obtains

$$\omega^2 \approx \frac{\pi^2}{8}\omega_{pe}^2 (1 - u_m^2)^{-1}, \tag{11}$$

where $eE = \sqrt{2}\, m_0\, c\, \omega_{pe}\, (1 - u_m^2)^{-1/4}$, so that

$$\omega \approx \pi m_0 c\, \omega_{pe}^2 / 2eE\,. \tag{12}$$

Equations (10) and (12) show that pure longitudinal waves with freqencies less than ω_{pe} can propagate in an electron plasma.

The relativistic electron mass variation as well as harmonic generation nonlinearities have been included by Sluijter and Montgomery[5] in their study of linearly polarized electromagnetic waves. In the weakly relativistic limit, their dispersion relation is

$$\omega^2 = k^2c^2 + \omega_{pe}^2\left[1 - \frac{v_0^2}{2c^2}\left(\frac{3}{4} - \frac{k^2c^2}{4\omega^2 - \omega_{pe}^2}\right)\right], \quad (13)$$

where k is the wave number. In the strongly relativistic limit $u_m^2 \gg 1$, one finds for quasi-transverse waves

$$\omega^2 \approx k^2c^2 + \pi\,\omega_{pe}^2/2\,u_m, \quad (14)$$

where $kc \gg \omega$ has been assumed. On the other hand, for circularly polarized waves, harmonic generation does not occur. Relativistic electron mass variation effects leads to [8]

$$\omega^2 = k^2c^2 + \omega_{pe}^2\left(1 + v_0^2/c^2\right)^{-1/2}. \quad (15)$$

Thus, waves in the frequency range

$$\omega_{pe}\left(1 + u_m^2\right)^{-1/4} < \omega < \omega_{pe} \quad (16)$$

can also propagate in the plasma. In the weakly relativistic limit, (15) becomes

$$\omega^2 \approx k^2c^2 + \omega_{pe}^2\left(1 - \frac{1}{2}\,\frac{v_0^2}{c^2}\right). \quad (17)$$

It is evident that near the cut-off (k=0) both the electron plasma waves and electromagnetic waves can propagate in the overdense region because of the electron mass variation nonlinearity which downshifts the local electron plasma frequency. Thus, relativistic nonlinear effects provides a novel mechanism for transporting the wave energy in the overdense region of plasmas.

4. MODULATIONAL INSTABILITIES

For illustrative purposes, we consider the modulational instabilities of a finite amplitude circularly polarized electromagnetic wave whose electric field is represented as

$$\vec{E} = E(\hat{x} + i\hat{y})\,\exp\,(ikz - i\omega t) + \text{c.c.}, \quad (18)$$

where c.c. stands for the complex conjugate. The relativistic electron mass modulation or the low-frequency modulations associated with slow plasma motion can give rise to an envelope of high-frequency electromagnetic wave packet. For the one-space dimension problem, the envelope of waves evolves according to the nonlinear Schödinger equation

$$i\left(\partial_t + v_g\partial_z\right)E + \frac{1}{2}v_g'\,\partial_z^2 E - \delta\omega\,E = 0, \quad (19)$$

where $v_g = \partial\omega/\partial k = kc^2/\omega$ is the group velocity of the wave packet, and $v_g' = \partial^2\omega/\partial k^2 = c^2/\omega$ is the group dispersion. The nonlinear frequency shift $\delta\omega$, including weak relativistic effect is given by

$$\delta\omega = \frac{1}{2}\,\frac{\omega_p^2}{\omega}\left[N - \frac{e^2|E|^2}{m_0^2\omega^2c^2}\right], \quad (20)$$

where $\omega_p = (4\pi n_0 e^2/m_0)^{1/2}$ is the unperturbed plasma frequency, $N = n_1/n_0$, n_1 is the electron number density perturbation in the average plasma density n_0 and is associated with slow plasma motion. We assume $n_1/n_0 \ll 1$. The second term in (20) comes from relativistic electron mass modulation.

We consider two types of plasma slow responses to the electromagnetic waves. First, the dynamics of non-resonant high-phase velocity (compared with the electron thermal velocity) electrostatic perturbations is governed by

$$\partial_t N + \partial_z v_{ez} = 0, \quad (21)$$

$$\partial_t v_{ez} = \frac{e}{m_o}\partial_z\phi - \frac{e^2}{m_o^2\omega_o^2}\partial_z|E|^2, \tag{22}$$

and

$$\partial_z^2\phi = 4\pi e n_1 , \tag{23}$$

where ϕ is the electrostatic ambipolar potential, and the second term on the right-hand side of (23) represents the ponderomotive effects. The time scales of the nonlinear interaction are assumed to be much greater than those of the ions, so that the latter do not participate in the motion, and form a neutralizing background. Combining (21)-(23), we find an equation for the driven electron plasma oscillations. We have

$$\left(\partial_t^2 + \omega_p^2\right) N = \frac{e^2}{m_o^2\omega^2}\partial_z^2|E|^2. \tag{24}$$

Next, we consider non-resonant low-phase velocity (comapred with the electron thermal velocity) electrostatic perturbations whose dynamics is governed by the inertialess electron fluid

$$\partial_z N = \frac{e}{T_e}\partial_z\phi - \frac{\omega_p^2}{\omega^2}\partial_z\frac{|E|^2}{4\pi n_o T_e} , \tag{25}$$

together with the continuity and momentum equations for the ion fluid. The low-frequency equations can be closed with the help of the quasi-neutrality condition $n_1=n_{1i}$. Thus, for $v_{ti}\partial_z \ll \partial_t \ll v_{te}\partial_z$, ω_{pi}, where ω_{pi} is the ion plasma frequency, and $v_{tj}=(T_j/m_{jo})^{1/2}$ is the thermal velocity, the density fluctuations associated with the driven ion-acoustic perturbations are governed by

$$\left(\partial_t^2 - c_a^2\partial_z^2\right) N = \frac{c_s^2\omega_p^2}{\omega^2}\partial_z^2\frac{|E|^2}{4\pi n_o T_e} , \tag{26}$$

where $c_a^2=c_s^2(1+\alpha_i T_i/T_e)$, $c_s^2=(T_e/m_o)$, $\lambda_{De}^2\partial_z^2 \ll 1$, and $\lambda_{De}=c_s/\omega_{pi}$ is the electron Debye length.

Let us now consider the modulational instability of a constant amplitude pump E_0 against the excitations of low-frequency, long wave length (compared with the electromagnetic wave) electrostatic perturbations. Physically, nonlinear interaction of the pump (ω ,k) with low-frequency electrostatic perturbations (Ω ,K) gives rise the upper ($\omega+\Omega$,k+K) and lower ($\omega-\Omega$,k-K) sidebands. The latter interact with the pump and produce a low-frequency ponderomotive force, which eventually reinforces the low-frequency oscillations. The modulational instability of an electromagnetic wave including the ponderomotive force driven density fluctuations and relativistic electron mass variations is governed by the nonlinear dispersion relation

$$\left[(\Omega - K v_g)^2 - K^2 v_g'\left(\tfrac{1}{4}K^2 v_g' - \gamma|E_o|^2\right)\right](\Omega^2 - \Omega_1^2) = K^2 v_g'\,\alpha(\Omega)\,|E_o|^2, \tag{27}$$

where

$$\gamma = e^2\omega_p^2 v_g/2 m_o^2 c^4 k\omega^2 ,$$

and

$$\alpha(\Omega) = e^2\omega_p^2 v_g \epsilon K^2/2 m_o^2 c^2 k\omega^2.$$

Note that $\Omega_1^2 = \omega_p^2$ and $\epsilon = 1$ for the nonlinear interaction involving driven electron plasma waves (24), and $\Omega_1^2 = K^2 c_a^2$ and $\epsilon = m_0/m_i$ for the nonlinear interaction involving driven ion sound fluctuations (26).

In the follwoing, we discuss several interesting analytical solutions of (27). First, in the absence of low-frequency density fluctuations (viz. $n_1=0$) there is an intrinsic instability of the electromagnetic wave because of relativistic electron mass modulation. Here (27) takes the form

$$(\Omega - K v_g)^2 = K^2 v_g' \left(\frac{1}{4} K^2 v_g' - \gamma |E_o|^2\right). \tag{28}$$

Letting $\Omega = kv_g + i\gamma_m$, we observe that (28) admits an instability for $\gamma |E_0|^2 > K^2 v_g'/4$. The growth rate of that instability is

$$\gamma_m = (\gamma v_g')^{1/2} K |E_o| . \tag{29}$$

The physical mechanism for the relativistic modulational instability is the relativistic correction to the mass of the electrons oscillating in the electric fields of the electromagnetic wave. As mentioned before, relativistic electron mass variation produces a nonlinear shift in the group velocity of the radiation. Accordingly, the energy accumulates around local maxima in the wave amplitudes.

Secondly, we consider the case in which the term $\gamma|E_o|^2$, which arises from relativistic electron mass increase, cancels the term resulting from the diffraction of the wave: i.e.,

$$\gamma |E_o|^2 \sim \frac{1}{4} K^2 v_g' . \tag{30}$$

For this case, (27) reduces to

$$(\Omega - K v_g)^2 (\Omega^2 - \Omega_1^2) = K^2 v_g' \alpha(\Omega) |E_o|^2. \tag{31}$$

We can analyze (31) in three limiting cases. Assuming that $\Omega = K v_g + i\gamma_m \ll \Omega_1$ ($\gamma_m \ll Kv_g$), we obtain from (31)

$$\Omega = K v_g + i K \left[v_g' \alpha(K v_g)\right]^{1/2} |E_o| / \Omega_1 . \tag{32}$$

Equation (32) exhibits an instability. Next, for $\Omega \gg \Omega_1$, the solution of (31) is

$$\Omega = \Omega_o \pm [\, \Omega_o^2 \pm K (v_g' \alpha_o)^{1/2} |E_o| \,]^{1/2}, \tag{33}$$

where $\Omega_o = Kv_g/2$ and $\alpha_o = e^2 \omega_p^2 v_g \epsilon / 2 m_o^2 c^2 k \omega^2$. Equation (33) admits an instability for $K^2 (v_g')^{1/2} |E_o| > \Omega_o^2$. Finally, letting $\Omega = Kv_g + \delta\Omega$ and assuming $\delta\Omega \ll Kv_g \approx \Omega_1$, we find from (31)

$$(\delta\Omega)^3 \approx (K' v_g' / 2 v_g)\, \alpha(K v_g) |E_o|^2. \tag{34}$$

Equation (34) yields an instability and the corresponding growth rate is given by

$$\mathrm{Im}\, \delta\Omega = (\sqrt{3}/2)\, [\, (K v_g' / 2 v_g)\, \alpha(K v_g) |E_o|^2 \,]^{1/3}. \tag{35}$$

The examples presented above clearly show that relativistic effects allow new regimes for the modulational instability of an electromagnetic wave.

We have also analyzed (27) by keeping the ponderomotive force driven density fluctuations and relativistic mass variations on an equal footing. For this case, letting $\Omega = Kv_g + i\gamma_m$ and $\gamma_m \ll Kv_g$, we obtain for $|E_o|^2 > K^2 v_g' / 4\alpha_o$

$$\gamma_m = |v_g'/4|^{1/2} K \,|\, 4 \alpha_o |E_o|^2 - K^2 v_g' \,|^{1/2}, \tag{36}$$

provided that Lighthill's criterion $v_g'/\alpha_o > 0$ is satisfied. Here, we have defined

$$\alpha_o \approx (e^2 v_g / 2 m_o^2 c^4 k \omega^2)(k^2 c^2 + \omega_p^2), \tag{37}$$

for the driven Langmuir perturbations,

$$\alpha_o \approx (e^2 v_g / 2 m_o^2 c^4 k \omega^2)\left(\frac{k^2 c^2}{Q} + \omega_p^2\right), \tag{38}$$

for the driven ion-sound perturbations, and

$$\alpha_0 = \frac{e^2 v_g \omega_p^2}{2 m_0^2 c^4 k \omega^2} \left(\frac{m_0 c^2}{T} + 1\right) \tag{39}$$

for the quasi-static modulations for which $\Omega \ll Kc_a$ and $\alpha_i = 1$ in Eq.(26). In (38), we have defined $Q = k^2 \lambda_{De}^2 (c_a^2 - v_g^2)/c_s^2$.

5. FILAMENTATION INSTABILITIES

For the convective amplification, Eq. (19) must be written in multi-space dimension. For the stationary filamentation of an electromagnetic wave propagating along the z axis, we have

$$i v_g \partial_z E + \frac{1}{2} v_g' \nabla_\perp^2 E + \alpha_0 |E|^2 E = 0, \tag{40}$$

where α_0 is given by the expression (39). In Eq. (40) we have assumed $\nabla_\perp^2 \gg \partial_z^2$. The filamentation instability of a constant amplitude pump can be investigated from the dispersion relation which is derived from (40). We have

$$K_z^2 = \frac{K_\perp^2 v_g'}{4 v_g^2} \left(K_\perp^2 v_g' - 4\alpha_0 |E_0|^2 \right) \tag{41}$$

If we set $K_z = -iK_m (K_m > 0)$ in (41), we see that the convective amplification occurs for

$$|E_0|^2 > K_\perp^2 v_g' / 4\alpha_0 . \tag{42}$$

Noting that the electromagnetic wave intensity is represented as $I = kc^2 |E_0|^2 / 8\pi\omega$, we can express (42) as

$$I > 10^{-7} kc^2 v_g' K^2 / 32\pi\omega\alpha_0 \quad \text{watt/cm}^2. \tag{43}$$

The mode number of the most unstable wave is

$$K_m = (2\alpha_0 E_0 / v_g')^{1/2}. \tag{44}$$

The corresponding maximum spatial amplification rate is

$$K_i = \alpha_0 |E_0|^2 / v_g. \tag{45}$$

The minimum scalelength over which the wave filamentation occurs is $L = 2\pi/K_i$. The critical power of the radiation for the filamentation process is

$$P = IA \approx (kc^2 |E_0|^2 / 8\pi\omega)\ \pi a^2, \tag{46}$$

where $A = \pi a^2$ is the cross-sectional area, and a is the radius of the filament. For the filamentation of a plane wave, we can roughly take $a \approx \lambda_\perp /2$, where $\lambda_\perp = 2\pi/K_m$ is the minimum perpendicular wavelength of the perturbation. Hence, (46) can be written as $P = 10^{-7} P_0$ watt, where

$$P_0 = \pi^2 k c^2 v_g' / 16\alpha_0$$

$$\equiv 2 \left(\frac{\pi m_0 c^3}{4 e\, \omega_p}\right)^2 \frac{\omega k}{[(m_0 c^2/T) + 1]} . \tag{47}$$

The filametation instability can break up a laser beam into small pipes, which can affect the propagation of the radiation and they might also cause localized plasma heating.

In the next section, we focus our attention on the modulational and filamentation instabilities of strong circularly polarized electromagnetic waves in an unmagnetized plasma.

6. FULLY RELATIVISTIC INSTABILITIES

Here we present an investigation of the modulational and filamentation instabilities of an intense circularly polarized electromagnetic wave taking into account an arbitrarily large amplitude relativistic electron quiver velocity as well as large amplitude electron density perturbations that are created by relativistic radiation pressure. Both of these effects become significant for laser intensity beyond 10^{16} watt/cm^2 because of the large quiver velocity v_0 ($=eE_0/m_0\omega$ = eA_0/m_0c, where A_0 is the vector potential) which obeys the well-known scaling

$$v_0/c = 8.5\times 10^{-10}\sqrt{I}\;\lambda \quad , \tag{48}$$

where the electromagnetic intensity I is expressed in terms of watts/cm^2 and λ is the laser wavelength in microns. For example, for a CO_2 laser with λ =10.6 μm and I=10^{16} W/cm^2, we find $v_0 \approx$ 0.9c. Clearly, in such a situation the results of §4 and §5 do not apply and a fully relativistic theory for the modulational and filamentation instabilities must be developed. This has been done by Shukla, Bharuthram and Tsintsadze [19,20] and their results are summarized below.

The wave equation for the circularly polarized electromagnetic wave is obtained from (4) and (7). We have

$$(\partial_t^2 - c^2\nabla^2)\vec{A} = -\frac{4\pi}{c} e n_e \vec{v}_e , \tag{49}$$

where the ion current is noted to be small, and has been neglected here. The right-hand side of (49) represents the nonlinear electron current density arising from the interaction of the relativistic quiver velocity and finite amplitude slow electron number density

variations. Furthermore, $\vec{A}$ is the perpendicular component of the vector potential and

$$\vec{v}_e = \vec{p}_e/m_0\gamma_e \equiv \vec{p}_e/m_0(1+p_e^2/m_0^2c^2)^{1/2} . \tag{50}$$

Inserting (6) and (7) into (2) it can be shown that the relativistic momentum equation is satisfied by a high-frequency part

$$\vec{p}_e = e\vec{A}/m_0c , \tag{51}$$

and a driven equation for the slow plasma motion

$$m_0c^2\nabla(1+|\vec{p}_e/m_0c|^2)^{1/2} = e\nabla\phi - T_e\nabla \ln N_e , \tag{52}$$

where $N_e=n_e/n_0$ and the inertia of the slow electron fluid for the temporal modulation has been neglected, whereas the electron inertial forces are unimportant for the spatial modulation. Equation (52) dictates that the fully relativistic ponderomotive force (the term on the left-hand side) can drive the finite amplitude slow ambipolar potential ϕ as well as the electron density variation n_e. The expression for the latter can be found for two classes of perturbations. First, for the forced Raman interaction, the ions form the neutralizing background and the ambipolar potential is directly created by the radiation pressure. Thus, from (52) we have

$$\phi = \frac{m_0c^2}{e}\left[(1+|\vec{\psi}|^2)^{1/2} - 1\right] , \tag{53}$$

where $\vec{\psi} = \vec{p}_e/m_0c \equiv e\vec{A}/m_0c^2$. On substituting (53) into the Poisson's equation, we can determine the slow electron number density

$$N_e^R = 1 + \beta \nabla^2 (1 + |\vec{\Psi}|^2)^{1/2}, \qquad (54)$$

where $\beta = c^2/v_{te}^2$. Secondly, for the quasistatic (QS) adiabatic response to the electromagnetic radiation, we find

$$N_e^Q = \exp[\Phi + \beta - \beta(1+|\vec{\Psi}|^2)^{1/2}], \qquad (55)$$

and

$$N_i^Q = \exp(-\sigma\Phi), \qquad (56)$$

where $\Phi = e\phi/T_e$ and $\sigma = T_e/T_i$. Eliminating Φ from (55) and (56) we obtain for $N_e^Q = N_i^Q$

$$N_e^Q = \exp[\beta_s - \beta_s(1+|\vec{\Psi}|^2)^{1/2}], \qquad (57)$$

where $\beta_s = \beta\sigma/(1+\sigma)$. Equations (49) and (54) or (57) are a pair of coupled equations for the study of the relativistic instabilities of intense electromagnetic waves in a uniform unmagnetized plasma.

A. Modulational Instability

As mentioned before, the interaction of either the slow plasma motion with the radiation field

$$\vec{\Psi} = \psi(\hat{x} + i\hat{y})\exp(i\vec{k}\cdot\vec{r} - i\omega t) + c.c., \qquad (58)$$

or relativistic electron mass modulation in the radiation field can give rise to a slowly varying envelope of waves which is governed by

$$i\varepsilon\partial_t\psi + i\vec{v}_g\cdot\nabla\psi + \Delta\psi + \beta\nabla^2\psi = \frac{N_e\psi}{(1+|\psi|^2)^{1/2}}, \qquad (59)$$

where $\varepsilon = 2\omega/\omega_p$, $\vec{v}_g = 2\vec{k}c^2/\omega_p^2\lambda_{De}$, $\Delta = (\omega^2 - k^2c^2)/\omega_p^2$, and the time and space variables are normalized by ω_p^{-1} and λ_{De}, respectively.

In order to investigate the temporal modulational instability of a large amplitude electromagnetic wave, we suppose

$$\psi = (\psi_0 + \psi_1)\exp(-i\delta t), \qquad (60)$$

where ψ_0 is real and denotes the constant amplitude of the pump, ψ_1 ($\ll \psi_0$) is the amplitude of the perturbation, and δ is a nonlinear frequency shift caused by the nonlinear interaction. For the forced Raman interaction, we insert for N_e^R from (54) into (59), use (60) to derive from the pump wave equation the nonlinear frequency shift

$$\delta = \delta_R = [(1+\psi_0^2)^{-1/2} - \Delta]/\varepsilon, \qquad (61)$$

and an equation for the evolution of the perturbation

$$i(\varepsilon\partial_t + \vec{v}_g\cdot\nabla)\psi_1 + \beta\nabla^2\psi_1 + \frac{\psi_0^2\psi_2}{2(1+\psi_0^2)^{3/2}} = \frac{\beta\psi_0^2}{2(1+\psi_0^2)}\nabla^2\psi_2, \qquad (62)$$

where $\psi_2 = \psi_1 + \psi_1^*$ and the asterisk denotes the complex conjugate. Substituting $\psi_1 = X + iY$ into (62), separating real and imaginary parts, we obtain

$$-(\varepsilon\partial_t + \vec{v}_g\cdot\nabla)Y + \beta\nabla^2 X + \frac{\psi_0^2 X}{(1+\psi_0^2)^{3/2}} = \frac{\beta\psi_0^2}{(1+\psi_0^2)}\nabla^2 X, \qquad (63)$$

and

$$(\varepsilon \partial_t + \vec{v}_G \cdot \nabla) X + \beta \nabla^2 Y = 0. \quad (64)$$

Assuming that

$$(X,Y) = (\tilde{X},\tilde{Y}) \exp(i\vec{K}\cdot\vec{r} - i\Omega t), \quad (65)$$

we can Fourier analyze (63) and (64), and combine them to obtain the nonlinear dispersion relation valid for an arbitrary-large-amplitude-pump. The result is

$$(\Omega - \vec{K}\cdot\vec{v}_G/\varepsilon)^2 = \frac{\beta K^2}{\varepsilon^2(1+\psi_0^2)}\left[\beta K^2 - \frac{\psi_0^2}{(1+\psi_0^2)^{1/2}}\right], \quad (66)$$

where Ω and $\vec{K}$ are the frequency and wave vector associated with the slow plasma motion. For the modulational instability, we set $\Omega = \vec{K}\cdot\vec{v}_G/\varepsilon + i\gamma_R$ in (66) and obtain the growth rate

$$\gamma_R = \frac{\sqrt{\beta}\, K}{\varepsilon(1+\psi_0)^{1/2}}\left[\frac{\psi_0^2}{(1+\psi_0^2)^{1/2}} - \beta K^2\right]. \quad (67)$$

Threshold is given by

$$\psi_0^2/(1+\psi_0^2)^{1/2} \geq \beta K^2. \quad (68)$$

The above analysis can be repeated to investigate the modulational instability involving the QS interaction. Here we have

$$\delta_Q = \left[(1+\psi_0^2)^{-1/2} \exp(\phi_p) - \Delta\right]/\varepsilon, \quad (69)$$

$$\gamma_Q = \frac{\sqrt{\beta}\, K}{\varepsilon}\left[\frac{\psi_0^2}{1+\psi_0^2}\left[\beta_s + (1+\psi_0^2)^{-1/2}\right]\exp(\phi_p) - \beta K^2\right], \quad (70)$$

and, at threshold,

$$\psi_0^2(1+\psi_0^2)^{-1}\left[\beta_s + (1+\psi_0^2)^{-1/2}\right]\exp(\phi_p) = \beta K^2, \quad (71)$$

where $\phi_p = \beta_s - \beta_s(1+\psi_0^2)^{1/2}$.

B. Filamentation Instability

To study the convective amplification arising from the nonlinear interaction, we consider a spatially slowly varying envelope along the z axis and the filamentation of the radiation in a direction perpendicular to the envelope wave propagation. This, $\vec{k}=\hat{z}k$ and $\nabla_\perp \gg \partial_z$. Subsequently, (59) takes the form

$$i v_G \partial_z \psi + \beta \nabla_\perp^2 \psi + \Delta\psi = N\psi(1+|\psi|^2)^{1/2}. \quad (72)$$

For the filamentation problem, we introduce the ansatz

$$\psi = (\psi_0 + \psi_1)\exp(i\varkappa z), \quad (73)$$

where $\varkappa$ is a nonlinear shift in the wavenumber. The procedure for obtaining the dispersion relation for the filamentation instability is similar to that of the modulational instability. Here, instead of (61), (63) and (64), we have

$$\varkappa = \left[\Delta - (1+\psi_0^2)^{-1/2}\right]/v_G, \quad (74)$$

$$-v_G \partial_z Y + \beta \nabla_\perp^2 X + \frac{\psi_0^2 X}{(1+\psi_0^2)^{3/2}} = \frac{\beta\psi_0^2}{(1+\psi_0^2)}\nabla^2 X, \quad (75)$$

and

$$v_G \partial_z X + \beta \nabla_\perp^2 Y = 0. \quad (76)$$

For X and Y spatially varying as

$$(X,Y) = (\tilde{X},\tilde{Y}) \exp(i\vec{K}_\perp\cdot\vec{r}_\perp + iK_z z), \tag{77}$$

equations (75) and (76) can be Fourier transformed and combined to yield the dispersion relation for the filamentation instability

$$K_z^2 = \frac{\beta K_\perp^2 [\psi_0^2(1+\psi_0^2)^{-1/2} - \beta K_\perp^2]}{v_g^2(1+\psi_0^2) + \beta K_\perp^2 \psi_0^2} . \tag{78}$$

For spatial growth along the direction of wave propagation, we set $K_z=-iK_m(K_m > 0)$. Then from (78) we obtain the amplification rate

$$K_m^R = \frac{\sqrt{\beta}\, K_\perp [\psi_0^2(1+\psi_0^2)^{-1/2} - \beta K_\perp^2]^{1/2}}{[v_g^2(1+\psi_0^2) + \beta K_\perp^2 \psi_0^2]^{1/2}}, \tag{79}$$

with the threshold condition

$$\beta K_\perp^2 \leq \psi_0^2/(1+\psi_0^2)^{1/2}. \tag{80}$$

In order to investigate the filamentation instability caused by the QS perturbations, we insert (57) into (72) and use (73). Here, the nonlinear wave number shift is

$$\varkappa = [\Delta - (1+\psi_0^2)^{-1/2}\exp(\phi_p)]/v_g , \tag{81}$$

whereas the spatial growth rate is found to be

$$K_m^Q = (\sqrt{\beta}\, K_\perp/v_g)\{\psi_0^2(1+\psi_0^2)^{-1/2} \times [\beta_s + (1+\psi_0^2)^{-1/2}]\exp(\phi_p) - \beta K_\perp^2\}. \tag{82}$$

Threshold condition is

$$\beta K_\perp^2 \leq \psi_0^2(1+\psi_0^2)^{-1}[\beta_s + (1+\psi_0^2)^{-1/2}]\exp(\phi_p). \tag{83}$$

C.Numerical Results

For illustrative purposes, we examine the variation of the spatial amplification rate as a function of the incident laser amplitude. An overall dependence of K_m on ψ_0 is shown in Fig. 1 for β =50. k=0.1 and $K_\perp$=0.04. We observe that the QS interaction causes growth for $\psi_0 \leq 0.65$. As the laser amplitude increases to larger values, only the forced Ràman causes wave filamentation. The restricted range of ψ_0 values for wave amplification associated with QS interaction could be attributed to the behaviour of the nonlinear wave number shift $\varkappa$, which in Fig. 2 is plotted against ψ_0 for ω^2/ω_p^2 =1.5 and for all other parameters fixed as in Fig. 1. We find that for QS interaction $\varkappa$ reaches the value of k, wavenumber of the incident laser beam, for ψ_0 in the range of $0.5 \leq \psi_0 \leq 0.7$. It appears that at large pump intensity the nonlinear wave number shift is so large that the wave propagation properties are destroyed and the electromagnetic wave suffers damping. On the other hand, for FR interaction, the nonlinear wavenumber shift is relatively small for $0 \leq \psi_0 \leq 1$, and it does not hinder the amplification of the filamentation instability.

The above graphs are drawn for a CO_2 laser ($k=6\times10^3$ cm^{-1}) and a dense plasma with an unperturbed plasma density $n_o=2\times10^{19}$ cm^{-3}. For plasma-based beat wave accelerators, one considers a tenuous plasma with $n_o=4\times10^{15}$ cm^{-3} and $\omega=50\,\omega_p$. For these values, the magnitude of K_m decreases by a factor a seventy. This is due to the significant increase in the parameter v_g .

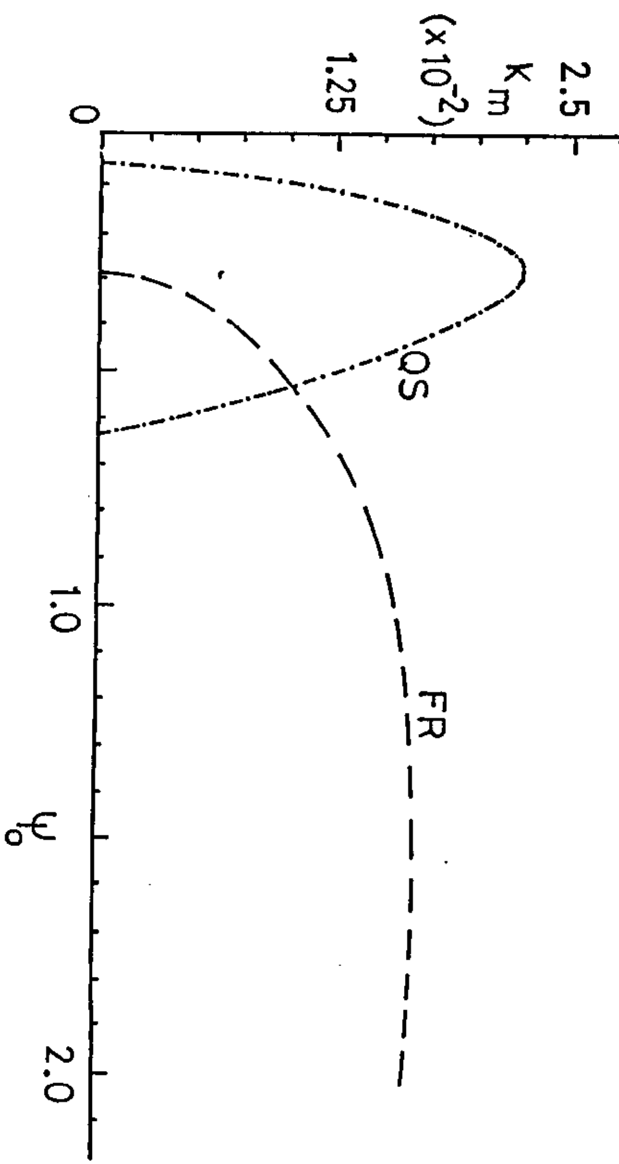
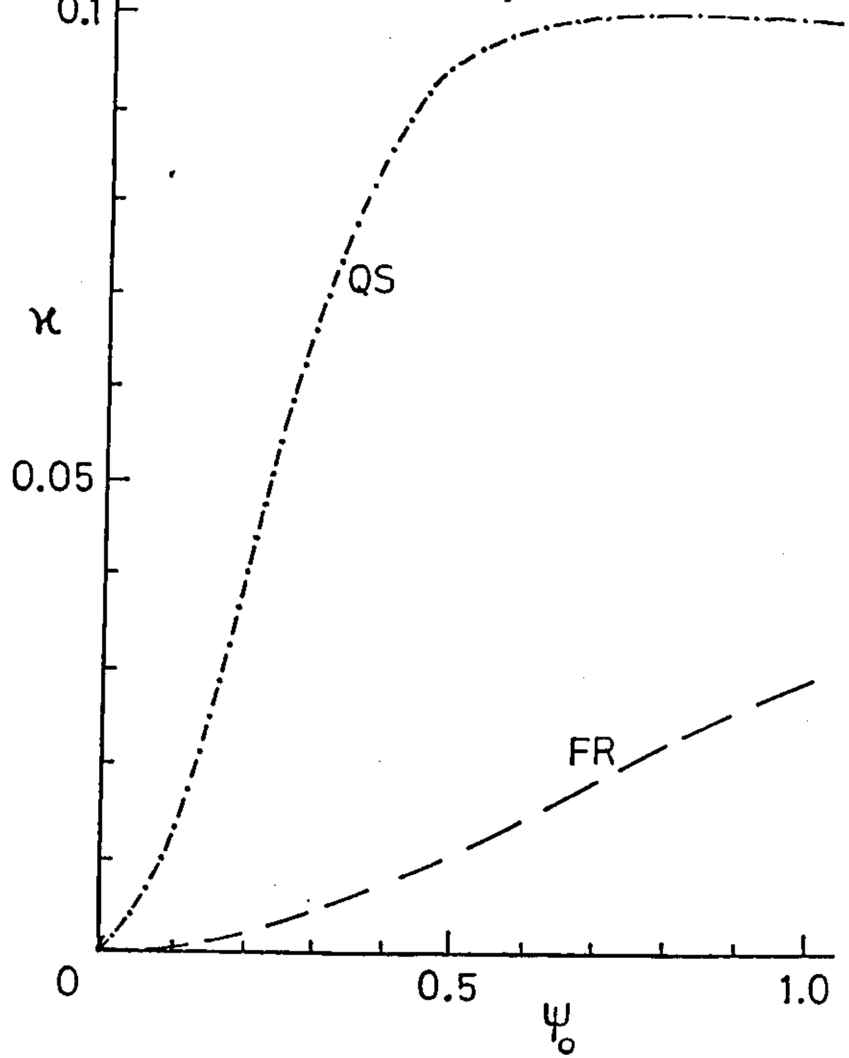


Fig. 1. K_m vs laser amplitude ψ_o for the two plasma responses, with $K_\perp$ = 0.04, β = 50, and σ = 1. FR indicates forced Raman and QS indicates the quasi-static interaction.

Fig. 2. Plot of nonlinear wavenumber shift $\varkappa$ against ψ_o, with all other parameters fixed as in Fig. 1. The curves are labelled as in Fig. 1.



7. SUMMARY

In this talk, we have presented our present understanding of the modulational and filamentational instabilities of a circularly polarized electromagnetic wave in a uniform unmagnetized plasma. We have reviewed the small as well as large amplitude theories paying considerable attention to relativistic electron mass modulation and density fluctuations that are driven by the radiation pressure. Several interesting analytical results for the growth rates and thresholds are presented.

For tutorial purposes, we have concentrated on the instabilities of a single electromagnetic wave. The extension of our theory to multi-electromagnetic waves is straightforward.[22] It is expected that the modulational and filamentation instabilities would grow faster in the presence of multiple electromagnetic waves in plasmas.

The results presented here must provide a better understanding of the nonlinear propagation of electromagnetic waves in laser produced plasmas, ionospheric modification experiments, as well as plasma based beat wave electron accelerators. The modulational and filamentation instabilities can seriously distort the incident laser pulse shapes, with detrimental consequences for the plasma heating and particle acceleration.

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