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MAGNETIC FLUX TUBES

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MAGNETIC FLUX TUBES

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Significance of tubes

a) building blocks for coronal $\underset{\sim}{B}$;

b) presumed to be important for coronal and chromospheric heating, since they provide communication channels between the energetic reservoir of the photosphere and the atmosphere;

c) implications for solar – stellar connections;
" " magnetosphere, etc. ?

d) there are different types of tubes:

isolated tube
(eg. in photosphere)
gives density depletion but enhancement in v_A

Non-isolated tube
(eg. of corona)
a density enhancement leads to a depletion in v_A

Twisted tubes add further complications

e) existence of small scales in plasmas with implications for waves, heating, etc. – – –
establishment of a lengthscale (and timescale)

The ENVIRONMENT of FLUX TUBES

Flux tubes (thin – sunspots) in the Sun's photosphere reside in a <u>dynamic</u> environment, which is superadiabatic. Jostled about by

granules < waves generated / bulk movements of small tubes

supergranules

Interacting with

p-modes

seismological implications

STRUCTURE OF INTENSE TUBES in SOLAR PHOTOSPHERE

size (in photosphere) < best avail. resolutⁿ (≃ 0.3″ ≃ 200 km)

Observations < int. in resolution / low resolution, Fourier transform spectrometer (Stokes parameters)

perties

How strong is <u>B</u>?

$\boxed{1-2\ \text{kG in photosphere}}$ (disputed)

[Stenflo('66, '67) 200–700G, Beckers+Schröter '68 600–1400G,
Howard+Stenflo, Frazier+Stenflo '72 : over 90% in strong form
Stenflo '73 : 1–2 kG in strong field flux
Harvey+Hall '77: 1200–1700 direct determ. iron line in infrared
Tarbell+Title '77 1000–1800
Wiehr '78 : 1500–2200
Solanki+Stenflo '84 : 1400–1700
Stenflo+Harvey '85 : 800–1000 in quiet network
1100–1200 in active region plage

Stenflo et al. '86 : 1400 near $\tau = 1$
1100 near $\tau = 10^{-2}$ i.e. B ↓ with height

Solanki et al. '88 : 2000G at $\tau = 1$, B ↓ with ht
over 90% of net magnetic flux is in kG form in photosphere.
Rayer et al. '89 : 2000G, B ↓ with ht. in manner similar to <u>thin tube approxⁿ</u>;
2a ~ 200 km at $\tau = 1$ (plage)
"turb" v ~ 3 km s⁻¹

$$\boxed{\begin{aligned} \frac{B^2}{2\mu_0} &\gg \frac{1}{2}\rho v^2 \\ \frac{B^2}{2\mu_0} &\sim \frac{1}{2}p_e \end{aligned}}$$

Photospheric flux tubes are <u>elastic</u> $(\beta \sim 1)$, not rigid.

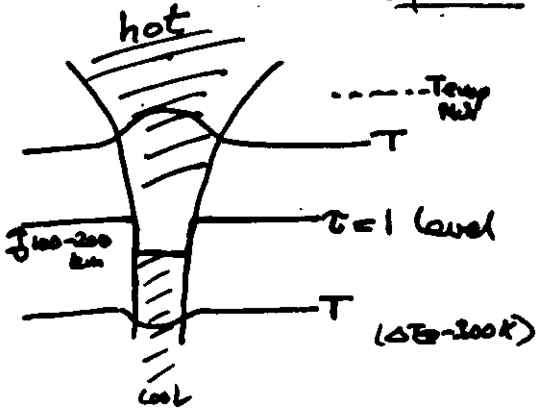
Density/pressure

depletion
tube is partially evacuated

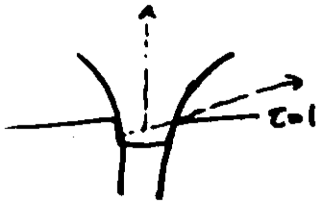
$\rho_0 \sim \frac{1}{2}\rho e$

$p_0 \sim \frac{1}{2} p e$

Temperature - model dependent

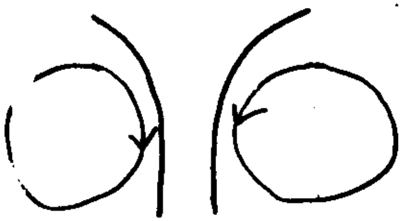


Thermal boundary layers
3-10 km thick



Dynamics

Tube is a dynamical object!
Moved by flows [moved one diameter by granules in 1^m or 2^m]
Surrounded by downdrafts, and possibly swirls (vortices) as seen in granulation
Flow inside tube is small ($< \frac{1}{4}$ km s^{-1} $\ll c_s \sim 10$ km s^{-1}). (Brandt et al '89)



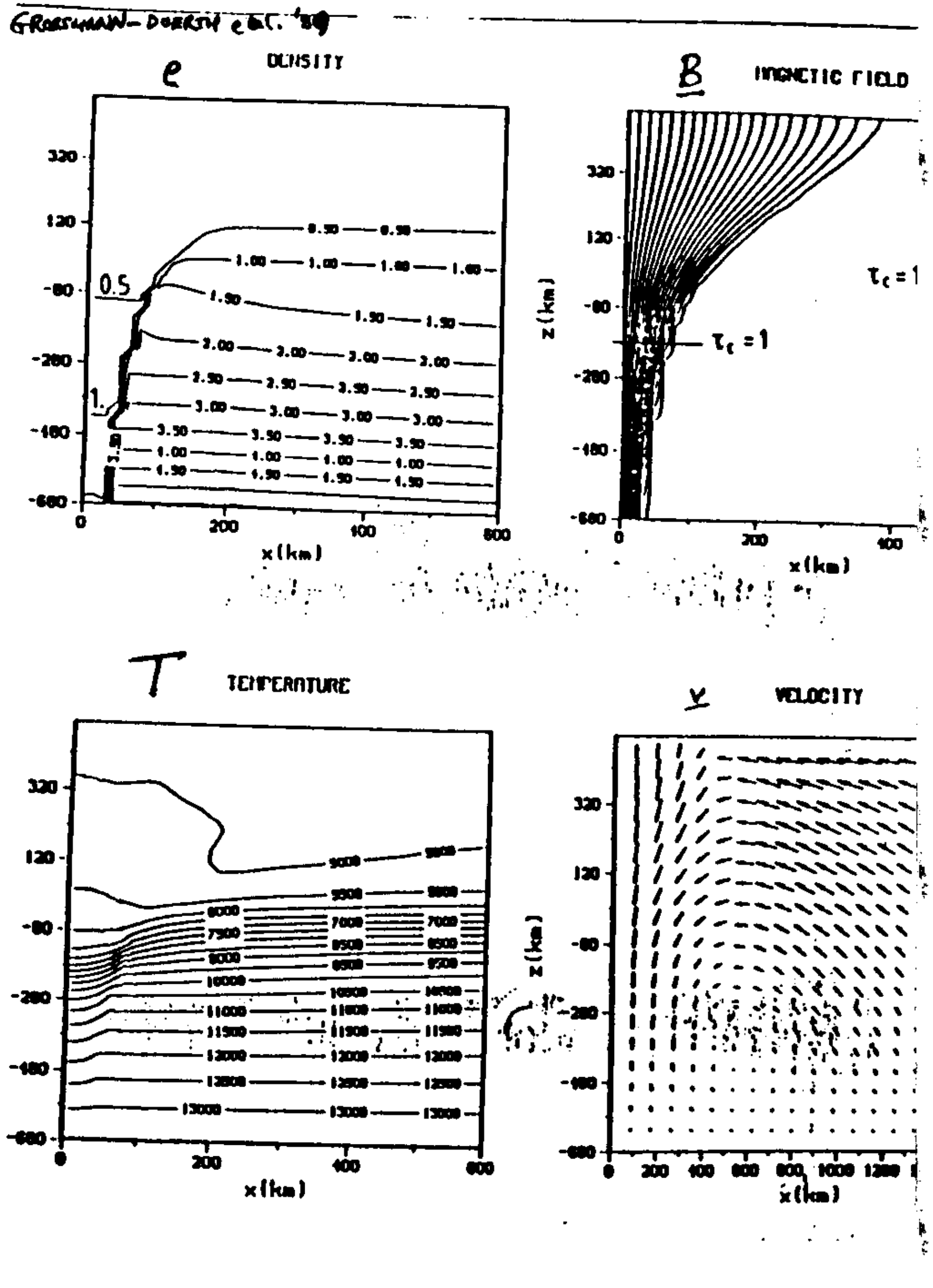


Figure 1: Properties of a model for solar *magnetic elements*. Only half of the flux sheet is shown and the horizontal coordinate has been stretched for bett[er] of the structure (except for the velocity, bottom right). Densities are in units

Oscillations

Some evidence for 5th periods (Giovanelli et al '78)

Stokes polarimetry suggests oscillations to explain profiles (Solanki)

Theoretically, expect

- p-modes to scatter off tubes (Bogdan, Zweibel)
- resonant excitations in the tube by p-modes (Bogdan)
- resonant absorption effects on tube boundaries (Abdo) (perhaps only a boundary layer effect) (cf Hollweg for sunspots)
- overstable oscillations (Hasan, Venkatakrishnan)
- generation of sausage/kink modes which form shocks higher in atmosphere (Danzer et al; Spruit)
- 'exploding' granules sending out shocks which impinge on tubes and on the canopy (Title et al)

etc.

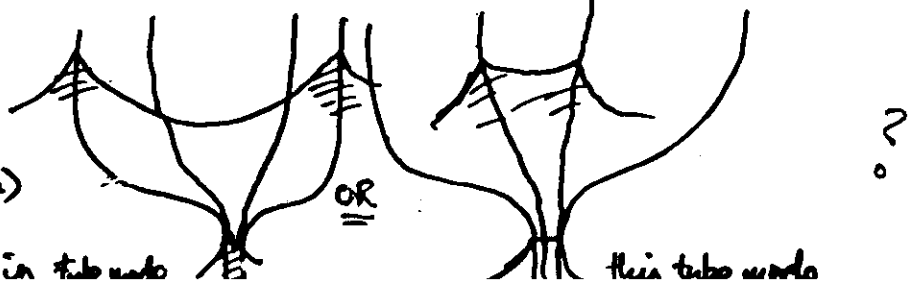
Are tubes the source of spicules?

Mechanism? nonlinear wakes (Hollweg)

solitons (Roberts & Mangeney)

k into
panded
be

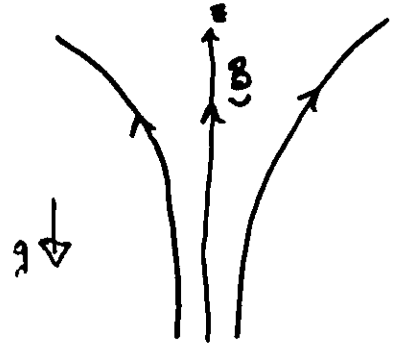
Radiation)



IMPORTANCE : GRAVITY FOR PHOTOSPHERIC TUBES

g is important in photospheric flux tubes.
The pressure scale-height ($p/\rho g$) in the photosphere is about 100 km, so is comparable with the size of thin tubes. This may be contrasted with the corona where a scale-ht. of 10^5 km arises.

The ISOLATED THIN TUBE



E&M. $\Rightarrow$

$$\text{grad}\left(p + \frac{B^2}{2\mu_0}\right) = -\rho g \hat{z} + (\underline{B}.\text{grad})\frac{\underline{B}}{\mu_0}$$

$$p = \frac{k_B}{m}\rho T$$

$$\text{div}\,\underline{B} = 0$$

Taylor series approach (Roberts & Webb '78)

Expand variables about z-axis (assuming $\partial/\partial\theta \equiv 0$)

$$p(r,z) = p^{(0)}(z) + r\,p^{(1)}(z) + r^2 p^{(2)}(z) + \cdots$$

$$B_r(r,z) = \qquad r B_r^{(1)}(z) + \cdots$$

$$B_z(r,z) = B_z^{(0)}(z) + \cdots$$

$$\text{div}\,\underline{B} = 0 \Rightarrow \frac{\partial B_r}{\partial r} + \frac{1}{r}B_r + \frac{\partial B_z}{\partial z} = 0$$

i.e. $\frac{\partial}{\partial z}B_z^{0}(z) + 2B_r^{(1)}(z) + O(r) = 0 \Rightarrow \underline{B} = B_0(z)\,\hat{\underline{z}}$ does not

Thus $\underline{B} = (0,0,B_0(z)) + O(r)$

and so $\frac{1}{2\mu_0}|\underline{B}|^2 \approx \frac{B_0^2}{2\mu_0}$, $(\underline{B}\cdot\text{grad})\frac{\underline{B}}{\mu_0} = \left(B_r\frac{\partial}{\partial r} + B_z\frac{\partial}{\partial z}\right)\frac{\underline{B}}{\mu_0}$

r-compon. of momentum $\Rightarrow$

$$(p^{(1)}(z) + 2rp^{(2)} + \dots) + \frac{1}{2\mu_0}(2rB_r^{(1)} + 2B_z^{(0)}B_z^{(1)} + \dots)$$

$$= O(r)$$

i.e. $p^{(1)}(z) + \frac{1}{\mu_0}B_z^{(0)}B_z^{(1)} = 0$

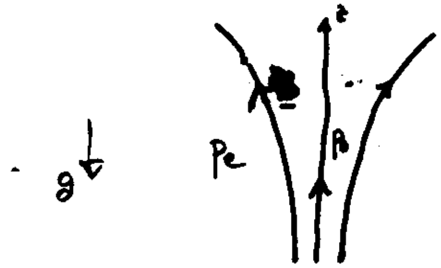
i.e. $p + \frac{B^2}{2\mu_0} = \left(p^0(z) + \frac{B_0^2(z)}{2\mu_0}\right) + O(r^2)$

With total pressure balance on the boundary of the tube we have (to lowest order in r)

$$p^0(z) + \frac{B_0^2(z)}{2\mu_0} = p_e$$ — external pressure field

z-comp. of mom. $\frac{d}{dz}\left(p^0 + \frac{B_0^2}{2\mu_0}\right) = -g\rho^0(z) + \frac{B_0}{\mu_0}\frac{B_0'(z)}{\mu_0}$

i.e. $\frac{d}{dz}p^0 = -g\rho^{(0)}$ — hydrostatic equation



To lowest order in r:

$$\boxed{p_0' = -g\rho_0 \quad p_e' = -g\rho_e \quad p_0 + \frac{B_0^2}{2\mu_0} = p_e}$$

If we now assume that pressure scale-ht Λ_0 inside and outside the tube are the same, we obtain

$$p_0(z) = p_0(0)e^{-n}, \quad \rho_0(z) = \rho_0(0)\frac{\Lambda_0(0)}{\Lambda_0(z)}e^{-n} \qquad \Lambda_0(z) = \frac{p_0}{\rho_0 g}$$

$$B_0(z) = B_0(0)e^{-n/2}, \quad \text{area } A_0(z) = A_0(0)e^{n/2}$$

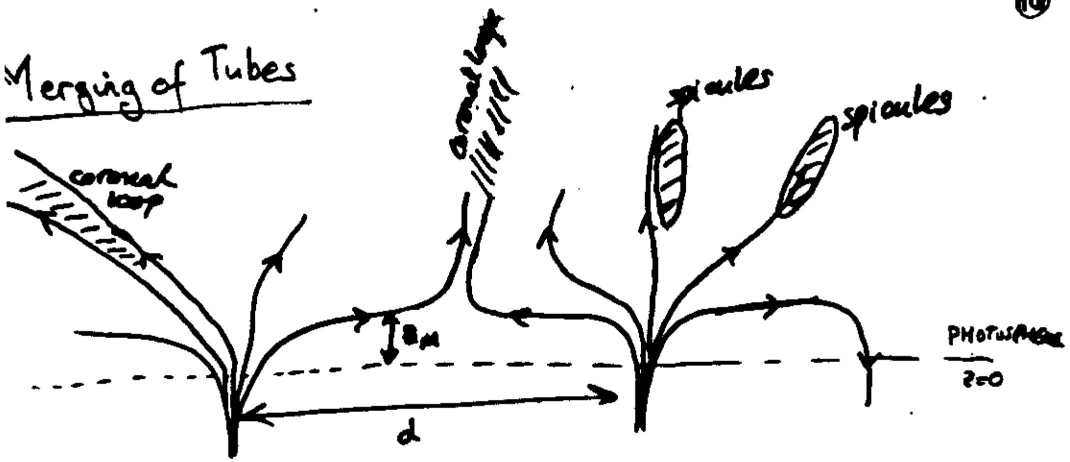
i.e. area $\propto p_0^{-1/2}(z)$, radius $\propto p_0^{-1/4}(z)$

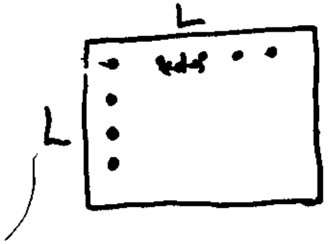
Here $n = \int_0^z \frac{dz}{\Lambda_0(z)}$. (If isothermal, $n = \frac{z}{\Lambda_0}$)

Note that $\beta = \frac{p_0(z)}{(B_0^2/2\mu_0)}$ is independent of depth.

Note: Thin tube approxn. seems to provide a good representation of $\underline{B}$ (Solanki et al. '88) i.e. B ↓ with height at about the right rate, and B is constant across tube with a

Merging of Tubes





'box' of side L, contains n^2 isolated tubes with field strength B_0 and radius a_0 (at $z=0$), separated from one another by distance d.

$$\text{Flux} = \bar{B} L^2 , \quad \bar{B} \equiv \text{mean field}$$
$$= \pi a^2 B_0 n^2$$

With $L = nd$, we get $\quad d = \left(\pi \frac{B_0}{\bar{B}}\right)^{1/2} a.$

But for an isothermal atmosphere

$$B_0(z) = B_0(0) e^{-z/2\Lambda_0}, \quad a(z) = a_0 e^{z/4\Lambda_0}$$

With tube expanding to $\frac{1}{2}d$ when $z = z_m$ (the merging ht)

we obtain

$$\frac{1}{2} d = a_0 e^{z_m/4\Lambda_0}, \text{ i.e. } z_m = 4\Lambda_0 \log_e\left(\frac{d}{2a_0}\right)$$

i.e. $\boxed{z_m \approx 2\Lambda_0 \log_e\left(B_0/\bar{B}\right)}$ (Note: weak dep...)

Examples

Take $a = 200$ km, $\Lambda_0 = 150$ km, $B_0 = 1.5$ kG

Quiet region	Active region
$\bar{B} = 5$ G	$\bar{B} = 100$ G
$\Rightarrow z_m = 1600$ km	$\Rightarrow z_m = 740$ km
$d = 6000$ km	$d = 1400$ km
$n^2 = 25$ in area size of supergranule	$n^2 = 460$ per supergr.

More elaborate calculations : Anzer + Galloway; Pizzo; Solanki, Pneuman

Observations (Giovanelli, Jones)

Suggest lower canopy heights.

[2 component chromospheres : tubes, no tubes ↳ cool CO gas — Ayres, Solanki]

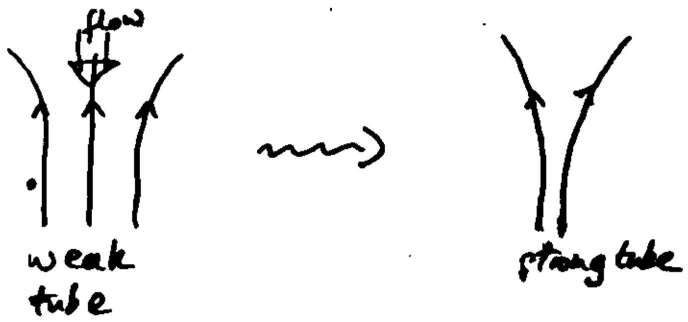
The birth, life and death of a photospheric flux tube

Birth

[Thomas]

Supergranules and granules expel magnetic field from the interiors, where it can concentrate in downdrafts. This explains the early stages of formation but cannot explain the ~~ir~~ high field strengths (1–2 kG). However, the upper reaches of the convection zone is strongly superadiabatic. Consequently, the gas inside the moderate strength tube is subject to convective instability; Motions are along the tube.

Motions down the stratified tube carry lighter gas to the deeper layers of the tube, allowing the external pressure field to compress the tube further. Strong fields are thus formed.



The 'collapse' of a weak tube to form a strong tube as a consequent of the convective instability in the tube (Parker '79; ~~Spruit Zweibel '79~~ Spruit '79)

Middle-aged Life

– in a continual state of excitation

Formed tube subject to the vagaries of its environment.
Tubes are elastic ($\beta \sim 1$).
Shuffled about by granules; buffeted by granules
Interactions with p-modes
A continual source of oscillations?

Death

How tubes "die" is not clear? The vanishing of tube doesn't necessarily ~~mark~~ mark its demise!
Reconnection, submergence, etc. probably involved

DYNAMICS

Basic Speeds

Given the sound and Alfvén speeds, what are the fundamental speeds of a tube?

Sausage mode

(Taylor 60s ?)
(Defouw '76
Roberts+Webb '78)

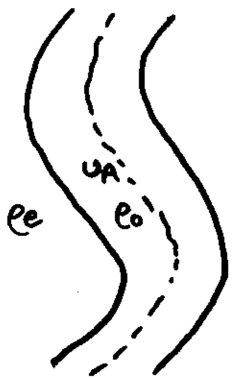
$$\boxed{c_T < c_0, v_A}$$

$$c_T^2 = \frac{c_0^2 v_A^2}{c_0^2 + v_A^2}$$

$$\frac{1}{c_T^2} = \frac{1}{c_0^2} + \frac{1}{v_A^2}$$

Kink mode

Kruskal+Schwarzschild '54
Ryutova '75; Spruit '81
Parker '79

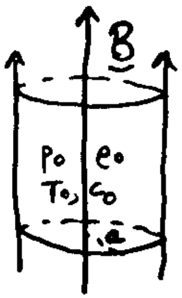


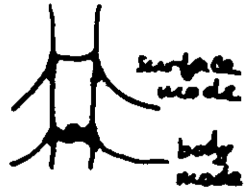
$$\boxed{c_k < v_A}$$

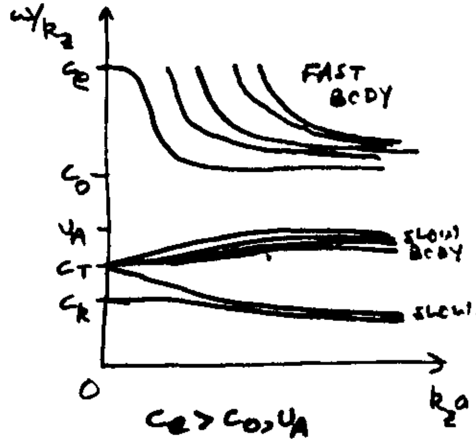
$$c_k^2 = \left(\frac{\rho_0}{\rho_0 + \rho_e}\right) v_A^2$$

Reduced below v_A because the tube displaces about an equal amount of the surrounding gas (density ρ_e).

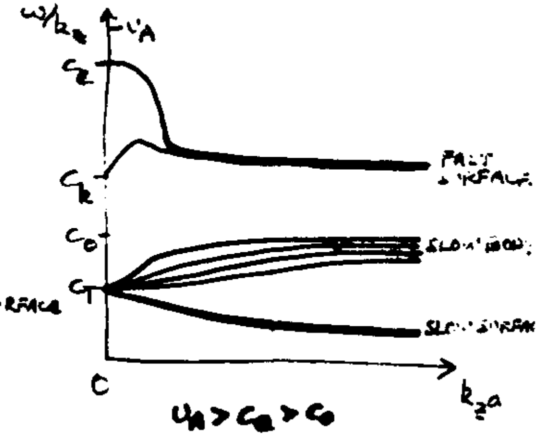
Waves in a cylindrical flux tube ($g = 0$)







$c_e > c_0, v_A$



$v_A > c_e > c_0$

E.g. $c_k = 3.7\ \mathrm{km\,s^{-1}}$, $c_T = 4.5$, $v_A = 5.3$, $c_0 = 8.6$, $c_e = 9.7\ \mathrm{km\,s^{-1}}$ (from Rosie's '86 sunspot model)

$c_T = 5.7$, $c_0 = 6.7$, $c_k = 7.8$, $c_e = 10.5$, $v_A = 12.6\ \mathrm{km\,s^{-1}}$

Dispersion Reln.

$$\boxed{\rho_0 (k_z^2 v_A^2 - \omega^2) m_e \frac{K_n'(m_e a)}{K_n(m_e a)} + \rho_e \omega^2 n_0 \frac{J_n'(n_0 a)}{J_n(n_0 a)} = 0}$$

$$n_0^2 = \frac{(\omega^2 - k_z^2 c_0^2)(\omega^2 - k_z^2 v_A^2)}{(c_0^2 + v_A^2)(\omega^2 - k_z^2 c_T^2)}, \quad m_e^2 = \frac{k_z^2 c_e^2 - \omega^2}{c_e^2} \underline{>0 \text{ assumed}}$$

cf. usual magnetoacoustic dispersion reln.

$$\boxed{\omega^4 - k^2(c_0^2 + v_A^2)\omega^2 + k^2 k_z^2 c_0^2 v_A^2 = 0} \quad k^2 = k_z^2 + k_\perp^2$$

Thin Tube Equations

(sausage mode)

$v_z = v(z,t)$, $p = p(z,t)$, etc.

$$\boxed{\begin{aligned} &\frac{\partial}{\partial t}\rho A + \frac{\partial}{\partial z}\rho v A = 0 \\ &\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \\ &\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)p = \frac{\gamma p}{\rho}\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)\rho \\ &p + \frac{B^2}{2\mu} = p_e \\ &BA = \text{constant} \end{aligned}}$$

elastic tube

area-pressure relns for a magnetic tube

Roberts-Webb 89

Linear disturbances

$\left(\frac{\partial p_e}{\partial t} = 0\right)$

$$\boxed{\frac{\partial^2 Q}{\partial t^2} - c_T^2(z)\frac{\partial^2 Q}{\partial z^2} + \omega_v^2(z)\,Q = 0}$$

Klein–Gordon Eqn (B. & R. '82)

Q is related to v.

$c_T = \frac{c_s v_A}{(c_s^2 + v_A^2)^{1/2}}$; frequency ω_v is complicated

If atmosphere ISOTHERMAL :

$\beta = 2\mu_0 p/B^2$

$$\boxed{\omega_v^2 = \left[\left(\frac{9}{4} - \frac{2}{\gamma}\right) - \left(\frac{3}{2} - \frac{2}{\gamma}\right)\frac{\beta}{\beta + \frac{2}{\gamma}}\right]\omega_a^2}$$

GEOMETRY ELASTICITY

$\omega_a \equiv \frac{c_s}{2\Lambda_0}$ acoustic cutoff

ω_v is reduced by the elasticity of the tube; a rigid tube is $\beta = \infty$. buoyancy

Instability

Klein-Gordon eqn can be rewritten in the form

$$\boxed{\frac{B_0}{\rho}\frac{d}{dz}\left(\frac{\rho_0 c_T^2}{B_0}\frac{dv}{dz}\right) + \left\{\omega^2 - \omega_g^2\left(\frac{c_T^2}{v_A^2} + \frac{\gamma c_T^2}{2c_0^2}\right)\right\}v = 0}$$

$\omega_g^2 = \frac{g}{\Lambda_0}\left(\Lambda_0' + \frac{\gamma - 1}{\gamma}\right)$ buoyancy frequency

With suitable boundary condns. on v, e.g. $v=0$ at two levels we have a Sturm–Liouville problem.

Hence a sufficient condn. for stability ($\omega^2 > 0$) is

$\omega_g^2 > 0$ (Schwarzschild criterion)

A local solution of (*) subject to $v=0$ at $z=0, -d$ gives (approx soln.)

$$v = e^{z/4\Lambda_0}\sin\left(\frac{n\pi z}{d}\right)$$

$$\frac{\omega^2}{c_T^2} = \frac{n^2\pi^2}{d^2} + \frac{1}{16\Lambda_0^2} + \left(\frac{1}{v_A^2} + \frac{\gamma}{2c_0^2}\right)\omega_g^2$$

with $\omega^2 = 0$ at (for d large)

$$\beta = \beta_c, \quad \boxed{1 + \beta_c \equiv \frac{1}{\gamma\left|\Lambda_0' + \frac{\gamma-1}{\gamma}\right|}} = \frac{1}{\gamma|\omega_g^2|}\left(\frac{g}{\Lambda_0}\right)$$

strength of superadiabaticity determines critical field strength

Kink Mode — also satisfies a Klein-Gordon eqn, viz.

$$\boxed{\frac{\partial^2 Q}{\partial t^2} - c_k^2 \frac{\partial^2 Q}{\partial z^2} + \Omega_k^2 Q = 0}$$

(Spruit '81)

with $\Omega_k^2 = \frac{c_k^2}{4\Lambda_0^2}\left(\frac{1}{4} + \Lambda_0'\right)$

Comparison Isothermal atmosphere, $c_0 = v_A = 7.5\,\mathrm{km\,s^{-1}}$, $\gamma = 5/3$, $\Lambda_0 = 125\,\mathrm{km}$

WAVE	SPEED	cutoff $\Omega/2\pi$ (and period)	e-folds in distance
sausage	$5.3\,\mathrm{km\,s^{-1}}$	4.8 mHz, 208 s	500 km
kink	$4.5\,\mathrm{km\,s^{-1}}$	1.4 mHz, 700 s	500 km
sound wave in rigid tube (exponential)	$7.5\,\mathrm{km\,s^{-1}}$	4.9 mHz, 203	500 km
sound in straight tube	$7.5\,\mathrm{km\,s^{-1}}$	4.8 mHz, 208 s	250 km

N.B. $\Omega_{\text{sausage}} \gg \Omega_{\text{kink}}$

so <u>kink modes are likely to be readily generated and propagated</u> in atmosphere (Spruit)

Pulse propagation (Hollweg; Rae + Roberts)

SUNSPOTS

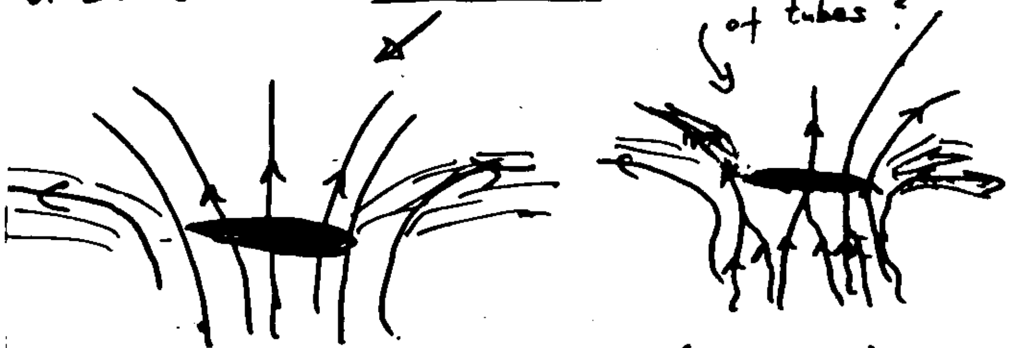
Perfect place to study mhd but there are difficulties.

1. Amplitudes are low. "Active Sun so quiet; the quiet Sun so active!" (Beckers, Zirin)

2. What do sunspots look like? We don't know! 3D geometry complicated, uncertain.

   Not uniform: eg. umbral dots (small regions of brightness spread throughout umbra), penumbral filaments

3. Do we have a monolithic tube or a spaghetti collection of tubes?



(Parker '79)

4. Sunspot seismology may provide an answer.

Waves in sunspots

(a) 3^m modes — seen across umbra, at all heights
(τ : 100 – 200 s)
- strong form "umbral flashes" (Beckers + Tallant '69)
- not found in developing or decaying spots
- no correlation between $2a$ and τ apparent.
- no correlation ... much stronger in chromosphere
- $v \sim 0.1 \text{ km s}^{-1}$ in photosphere, much stronger in chromosphere

(b) 5^m modes — across umbra but restricted to photosphere and below; strongly related to p-modes of quiet Sun ($\lesssim 50\%$)
- spots act as a sink for p-modes (Braun et al. '87, '88); seismological aspects important (Thomas et al. '82)

(c) penumbral waves
- periods 4 – 5 min. (sometimes larger)
- sometimes exhibiting coherent wavefronts (seen with the Krovdo) emanating from edge of umbra (running penumbral waves)
- $v \sim 1 \text{ km s}^{-1}$; speed 20–35 km s^{-1}
- τ : 200 – 300 s (1400 s reported in one case by Lites '88)

(Lites '88)
- $\tau \sim 286\text{ s}$ ($\nu \sim 3.5 \text{ mHz}$)
- $\tau \sim 500\text{ s}$ ($\nu \sim 2.0 \text{ mHz}$) } in upper photosphere (modes predom. $\Pi^2 B$)

$\nu \sim 3.$ mHz only in penumbral chromosphere; do not show char. patterns of p-modes

Single tube model (no g) suggests (Evans + R '89):
[g≠0 needs to be done. But see Scheuer + Thomas 81, 82; Zhugzhda 72]

(a) 3^m oscillns. are slow body modes.

Can develop a theory for the effect of $g \neq 0$ on these modes
(Roberts + Small '84? unpubl.; Moreno-Insertis & Spruit '88, preprint; unstable aspects
cf. Syrovatskii + Zhugzhda '67
→ Klein – Gordon eqn. again, with speed c_T
but ω_v^2 (the cutoff freq.) is changed appropriate to the different geometry.
→ form shocks [umbral flashes: Beckers + Tarrant] (Lites)

Driven by overstable oscillns. (supergranules give wrong; granules give right τ; would favour edges of p-modes wrong τ)

(b) 5^m oscillations

Driven by p-modes (Thomas '81)
Associated with the fast body mode of tube in lower layers (where $v_A < c$)
fast body modes not permitted above $v_A = c_{se}$

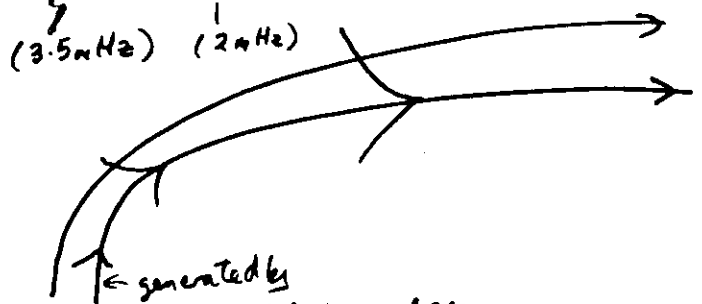
(c) Spots as a sink for p-modes

Why?
Abdelatif and Thomas '87 – resonant transmission of sound waves
Hollweg '88 – resonant absorption (a la coronal heating) on spot boundary
Evans + R. '89 – excitation of trapped fast waves (confined to propagate up/down tube), occurring
... $\sim 10 \Rightarrow 2 \sim 7000$ km

(c) running penumbral waves

— p-modes modified by magnetic field (Nye, Thomas 74, 76)

— fast and slow magnetoacoustic waves (Small+R '84)
(3.5 mHz) (2 mHz)



← generated by granules (giving surface waves a scale of ~10³ km) or p-modes (τ ~ 300 s)

penetration depth of fast surf. wave greater than slow wave (Miles + R, '89)

Theoretically speculation (Evans + Roberts '89):