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I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

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SPRING COLLEGE ON PLASMA PHYSICS

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PARTICLE FOCUSING IN PLASMA WAVES AND PLASMA LENSES

T. Katsouleas

Department of Physics
University of California, Los Angeles
405 Hilgard Avenue
Los Angeles 90024
U.S.A.

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"LARGE AMP. WAVES IN PLASMAS"

LECTURE NOTES:

PARTICLE FOCUSING IN PLASMA WAVES AND PLASMA LENSES

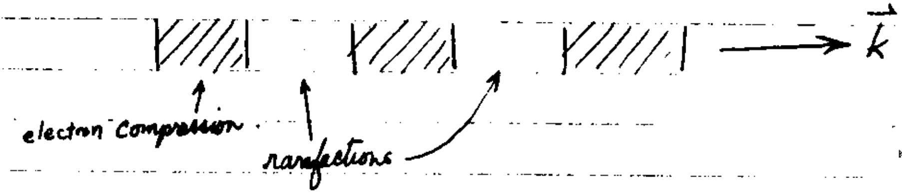
by
T. KATSOULEAS
UCLA, LOS ANGELES, CA – USA

I. FOCUSING FIELDS OF A PLASMA WAVE

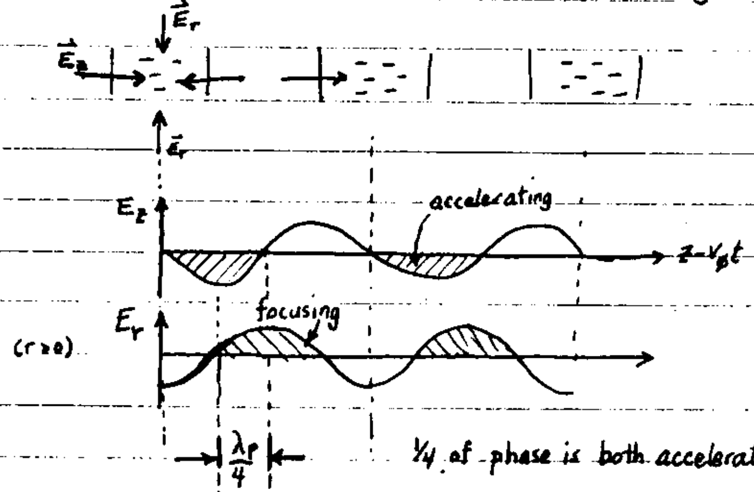
Thus far, you have heard several lectures on large amplitude plasma waves and their associated accelerating (longitudinal) fields. Now let us consider their (transverse) focusing/defocusing fields. These fields are important for the beam quality of plasma wave accelerators, for wigglers in advanced FEL schemes, & for use at the final focus of a linear collider (plasma lens).

physical picture:

a plasma wave is a density wave



the electric field points everywhere toward the excess negative charge



¼ of phase is both accelerating & focu

more quantitatively:

considers $\phi = \phi_0 (1 - r^2/a^2)\cos(k_p z - \omega_p t)$ (r

$$\Rightarrow \begin{cases} E_z = -\partial\phi/\partial z = k_p \phi_0 (1 - r^2/a^2)\sin(k_p z - \omega_p t) \\ E_r = -\partial\phi/\partial r = \phi_0 (2r/a^2)\cos(k_p z - \omega_p t) \end{cases}$$

AN ASIDE ON THE RELATION BETWEEN E_r and E_z

for the electrostatic wave

$$\frac{\partial E_r}{\partial z} = -\frac{\partial^2 \phi}{\partial r \partial z} = \frac{\partial E_z}{\partial r}$$

this is a special case of a general theorem —

PANOFSKY-WENZEL THEOREM

for any fields that are functions of $z-ct$, r, θ (eg, wakefields)

then $\boxed{\dfrac{\partial W_{\parallel}}{\partial r} = \dfrac{\partial W_{\perp}}{\partial z}}$

where $\vec{W}$ is the Lorentz force $\vec{E} + \dfrac{\vec{V} \times \vec{B}}{c}$ on a particle moving at $\vec{V} \simeq c\hat{z}$.

[brief proof: Faraday's Law $\hat{\theta} \cdot \left[\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \right]$

$\Rightarrow \dfrac{\partial E_r}{\partial z} - \dfrac{\partial E_z}{\partial r} = -\dfrac{1}{c}\dfrac{\partial B_\theta}{\partial t}$; $-\dfrac{1}{c}\dfrac{\partial}{\partial t} B_\theta(z-ct) = \dfrac{\partial}{\partial z} B_\theta(z-ct)$

$\Rightarrow \partial E_z / \partial r = \dfrac{\partial}{\partial z}(E_r - B_\theta)$]

HOMEWORK QUESTION:

IN THE Beat Wave Accelerator DRIVEN BY GAUSSIAN PUMPS ($E_0^2 \sim A e^{-r^2/2\sigma^2}$), the center ($r=0$) detunes first while the outer parts of the plasma wave are still growing. This produces an E_z-field of the form

. APPLY THE P-W THEOREM TO FIND WHICH REGIONS (in r) ARE FOCUSING. THAT IS, WHERE WILL ACCELERATED PARTICLES BE?

Notes: 1) E_r & E_z are $\pi/2$ out of phase

2) $\left| E_r / E_z \right| \sim \dfrac{2}{k_p a}$ $\Rightarrow$ $k_p a \gg 1$ $E_r/E_z \sim 0$ 1-D Waves

$\boxed{k_p a \sim 1 \quad E_r \sim \Theta(E_z)}$ STRONG FOCUSING!

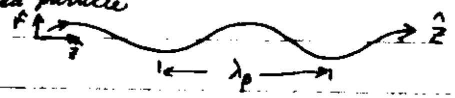
$k_p a \ll 1$ $E_r > E_z$

(recall $k_p = \omega_p/c$ for beat wave accelerator, etc.)

IMPORTANCE

A. BETATRON OSCILLATIONS

an accelerated particle



Oscillates as

$\gamma m \ddot{r} \sim (-eE_r/r) r$

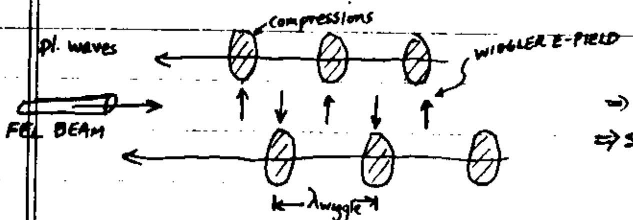
$$d^2 r / dz^2 + \underbrace{\left(\frac{eE_r/r}{\gamma m c^2}\right)}_{\lambda_\beta^{-2}} r = 0$$

emittance of accel. beam:

$\varepsilon \sim r\theta \sim r \; r/\lambda_\beta \propto E_r^{1/2}$

E_r too large $\Rightarrow$ emittance growth (poor beam quality)

B. FEL WIGGLER SCHEME

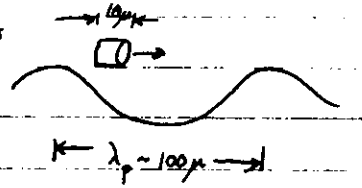


$\Rightarrow$ STRONG WIGGLER (Θ Megagauss)

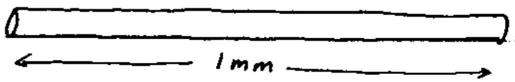
$\Rightarrow$ SHORT WAVELENGTH (Θ 100μ)

C. FINAL FOCUS OF A PARTICLE BEAM

For short beams

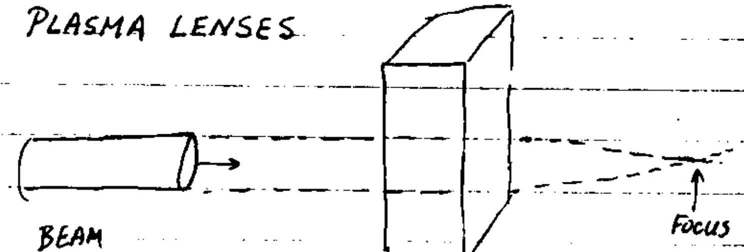


But typically (HEP) beams from high-energy accelerators are long



So plasma wave won't work....

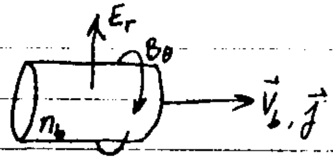
II. PLASMA LENSES



PASSIVE PLASMA — NO WAVES

physical picture

in vacuum

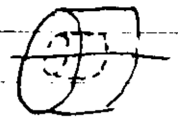


$$E_r \sim 2\pi n_b e r$$

from Gauss' Law

$\oint E \cdot dA = 4\pi q_{enclosed}$

$E \cdot 2\pi r l = 4\pi n_b e \pi r^2 l$



$$B_\theta \sim 2\pi n_b e r \frac{V_b}{c}$$

from Ampere's Law

$\oint \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} \int \vec{j} \cdot d\vec{A}$

$B \cdot 2\pi r = \frac{4\pi}{c} (n_b e V_b) \cdot \pi r^2$

$$F_r = e\left(\vec{E} + \frac{\vec{V}_b \times \vec{B}}{c}\right)_r = 2\pi n_b e^2 r \left(1 - V_b^2/c^2\right) \approx 0$$

NO NET FOCUSING OR DEFOCUSING

in plasma

A. DENSE PLASMA ($n_0 >> n_b$)

i) PLASMA ELECTRONS SPACE-CHARGE SHIELD THE $\vec{E}$-FIELD OF BEA[M]

(PROVIDED THAT $\boxed{L_{BEAM} >> c/\omega_p}$ so they have time to move)

ii) PLASMA ELECTRONS DON'T SHIELD $\vec{B}$

(PROVIDED THAT $\boxed{r_{beam} \le c/\omega_p}$ so return current flows outside

$$\Rightarrow \quad F_r = 0 + e\frac{\vec{V}_b \times \vec{B}_\theta}{c} \approx \boxed{-2\pi n_b e^2 r = F_r}$$

B. UNDERDENSE PLASMA ($n_0 < n_b$)

ALL PLASMA e^- BLOWN OUT BY e^- BEAM CHARGE

NET FOCUSING DUE TO REMAINING ION CHARGE

$$F_r = -2\pi n_i e^2 r \approx \boxed{-2\pi n_0 e^2 r = F_r}$$

Quantitative comparisons – PLASMA vs. MAGNET LENS

$$F_r/r \simeq -2\pi n_b e^2 \simeq -3\times10^{-9}\ \text{Stat-Volts}/\text{cm}^2 \times n_b\ [\text{cm}^{-3}]$$

$$\equiv -3\ \text{MegaGauss/cm} \times \frac{n_b}{10^{15}\,\text{cm}^{-3}}$$

For SLAC beam $n_b \sim \mathcal{O}(10^{15}\,\text{cm}^{-3}) \Rightarrow F_r/r \sim \underline{\mathcal{O}(3\,\text{MG/cm})}$!

For Conventional Focusing

- Quadrupole magnets $\quad F_r/r \sim \underline{\mathcal{O}(5\,\text{kG/cm})}$!

So PLASMA LENSES are very strong.

To see what this does for us and why it is important we must understand some basic principles of beam optics/lenses.

Neglecting aberrations, the spot size produced by a lens (a^*) scales as $\quad a^* \propto (F_r/r)^{-1}$,

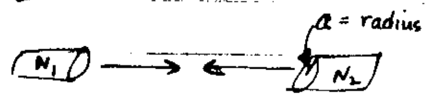
AND THE EVENT RATE OF A COLLIDER SCALES AS

$$\mathcal{L} \propto a^{*-2} \propto (F_r/r)^2$$

THIS SUGGESTS AN IMPROVEMENT OF MANY ORDERS FROM PLASMA LENSES. UNFORTUNATELY WE CANNOT NEGLECT ABERRATIONS (NOR SOME OTHER IMPORTANT PARAMETER DEPENDENCES), SO READ ON...

SOME BASIC LENS & COLLIDER CONSIDERATIONS

Purpose of final focus is to increase the event rate in a collider



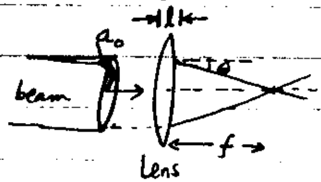
$$\text{Event rate} = \sigma_{\text{cross section}} \times \mathcal{L}_{\text{luminosity}} \propto \frac{N_1 N_2}{a^2} \propto \frac{1}{a^2}\ !$$

Spot size (a) after a lens is limited by

i) Lens strength / beam emittance
ii) Lens aberrations

i) Lens strength Limit

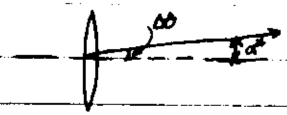
Stronger Lens $\Rightarrow$ Shorter focal length $\Rightarrow$ smaller spot



$$f = \frac{a_0}{\theta} \approx \frac{a_0}{P_\perp/P_\parallel} \approx \frac{a_0 \gamma m c}{F_r \Delta t}$$

($\Delta t = \ell/c$)

Final Spot size (a^*) determined by natural angular spread (Emittance) of beam ($\Delta\theta$)



$$a^* = f\,\Delta\theta = f\,(\varepsilon/a_0)$$

$$\Rightarrow a^* \approx \frac{\gamma m c^2 \cdot \varepsilon}{(F_r/r)\,\ell\, a_0} \propto \frac{1}{(F_r/r)}$$

However, this is somewhat misleading since $F_r/r \propto n_b \propto 1/a_0^2$.
So plasma lenses, unlike Quad. lenses, must start w/ small a_0 to give strong F_r.
Thus, we should compare $a_0 F_r/r$ not F_r/r when comparing lenses. In this case, plasma lenses typically offer modest, but not insignificant, improvements (see example later)

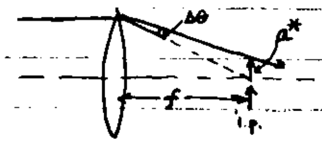
ii) *Aberration Limit*

from previous page $f = \dfrac{\gamma m c^2}{\ell (F_r/r)}$

$\Rightarrow$ if F_r/r = constant, all radii particles focus to same point

If $F_r = -kr + c_1 r^2 + c_2 r^3$,

then $\longrightarrow$ spherical aberrations



$$a^* = f\,\Delta\theta = f\theta\,\frac{\Delta\theta}{\theta} \cong a_0\,\frac{\Delta(F_r/r)}{(F_r/r)} = a_0 \times \text{fractional aberrations}$$

CASE A) OVERDENSE PLASMA – $F_r/r \propto n_b(r)$

$$\frac{\Delta(F_r/r)}{(F_r/r)} \sim 20\text{–}30\% \quad \text{for } e^- \text{ n } e^+$$

CASE B) UNDERDENSE LENS

$F_r/r \propto n_0 =$ constant (for e^-)

$$\Rightarrow \frac{\Delta F_r/r}{F_r/r} < 3\% \quad \text{for } e^-$$

$$\sim 20\% \quad \text{for } e^+$$

- Event Rate (Luminosity) Enhancement From Plasma Lenses Scales as

$$\frac{\mathcal{L}}{\mathcal{L}_0} \cong \frac{a_0^2}{a^{*2}} \sim \left[\frac{\Delta(F/r)}{F/r}\right]^{-2} \lesssim 10\text{–}25 \quad \text{For Case A}$$

$$\lesssim 25+ \quad \text{For Case B}$$

$\rightarrow$ SIGNIFICANT IMPROVEMENTS POSSIBLE MERELY BY INSERTING A THIN O(mm) PLASMA BEFORE THE COLLISION