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SPRING COLLEGE ON PLASMA PHYSICS

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ABSTRACT

THEORY OF PLASMA VORTICES IS BRIEFLY REVIEWED, AND THEIR INTERACTION WITH

- HIGH FREQUENCY WAVES
- ELECTRON BEAMS

IS DISCUSSED. POSSIBILITY OF TWO DIMENSIONAL SELF ORGANIZATION IN FULLY DEVELOPED TURBULENCE IS DEMONSTRATED

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D. JOVANOVIĆ, H.L. PECSELI, J.J. RASMUSSEN
PHYS. FLUIDS (to be published)

MOTIVATION

- BETTER UNDERSTANDING OF TWO- AND THREE-DIMENSIONAL TURBULENCE IN MAGNETIZED PLASMA
- IS THERE A COLLAPSE IN MHD ?
- PARTICLE TRANSPORT IN MAGNETICALLY CONFINED PLASMAS
- MHD & ASTROPHYSICAL PLASMAS – THEORY OF MAGNETIC FLUX TUBES

3)

FLUID VORTICES

THE MOST GENERAL DEFINITION :

VORTEX IS A DOMAIN IN FLUID IN WHICH :

$$\boxed{\nabla \times \vec{v} \neq 0}$$

i.e. DOMAIN IN WHICH THERE EXIST <u>CLOSED STREAMLINES</u> .

EXISTENCE OF VORTICES REQUIRES FORCE ON A FLUID ELEMENT PERPENDICULAR TO ITS VELOCITY

- VISCOSITY (FORMATION OF EDDIES ; DISSIPATION STRUCTURES)
- CORIOLIS FORCE , LORENTZ FORCE etc.

WELL KNOWN NATURAL PHENOMENA :
GREAT RED SPOT ON JUPITER ,
SEA CURRENTS & TRADE WINDS
COFFEE ROTATION AFTER WE STIR THE CONTENT OF OUR CUP
etc, etc.

4)

SIMPLE EXAMPLE :

- $\vec{v} = \vec{e}_z \times \nabla \varphi(x,y)$, φ - STREAM FUNCTION
- FORCE -FREE FLUID
  (e.g. superfluid helium in which by the action of a miracle velocity field $\vec{v} = \vec{e}_z \times \nabla\varphi$ was created at $t = -\infty$)

MOMENTUM EQUATION :

$$\left(\frac{\partial}{\partial t} + \vec{v}\nabla\right)\vec{v} = 0 \quad / \mathrm{div}$$

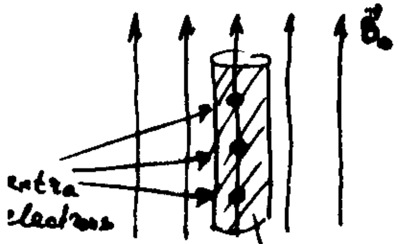
$$\left[\frac{\partial}{\partial t} + (\vec{e}_z \times \nabla\varphi)\cdot\nabla\right]\nabla_\perp^2\varphi = 0 \quad \text{Euler equation}$$

IN PLASMA PHYSICS : CONVECTIVE CELL

$$\vec{E} = -\nabla\phi \;, \quad \frac{1}{\Omega_i}\left(\frac{\partial}{\partial t} + v\nabla\right) << 1$$

$$\left(\frac{\partial}{\partial t} + v_e\nabla\right)v_e = \frac{e}{m_e}\left(-\nabla\phi + \vec{v}_e \times \vec{e}_z B_0\right)$$

$$\Rightarrow \vec{v} = \frac{1}{B_0}\vec{e}_z \times \nabla\phi \;; \left[\frac{\partial}{\partial t} + \frac{1}{B_0}(\vec{e}_z \times \nabla\phi)\cdot\nabla\right]\nabla_\perp^2\phi = 0$$



$$\left[\frac{\partial}{\partial t} + (\vec{e}_z \times \nabla\varphi)\cdot\nabla\right] \nabla^2\varphi = 0$$

STATIONARY SOLUTION :

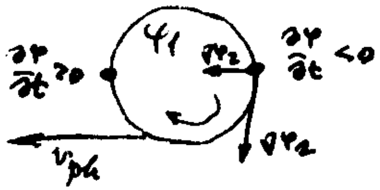
ANY ONE DIMENSIONAL STREAMFUNCTION

e.g. $\varphi = \varphi(r)$ (cylindrical symmetry)

INTERACTION OF TWO MONOPOLAR VORTICES

$\varphi = \varphi_1 + \varphi_2 + \delta\varphi$

$\varphi_1 > 0$, $\varphi_2 < 0$



$$\frac{\partial\delta\varphi}{\partial t} + \vec{e}_z\cdot\left(\nabla\varphi_1 \times \nabla\nabla^2\varphi_2 + \nabla\varphi_2 \times \nabla\nabla^2\varphi_1\right) = 0$$

- INTERACTION OF TWO OPPOSITELY ROTATING VORTICES LEADS TO THEIR PROPAGATION IN THE SAME DIRECTION
- PHASE VELOCITY IS AMPLITUDE DEPENDENT

$\Rightarrow$ MOVING DOUBLE VORTEX 
(stable ??)

THIS IS A GENERAL FEATURE OF ALL NONLINEAR EQUATION WITH VECTOR NONLINEARITIES AND

PARTICLE TRANSPORT, WEAK TURBULENCE
SELF ORGANIZATION, VORTICES

COLLISIONAL TRANSPORT
DIFFUSION COEFFICIENT

$D \sim \frac{\lambda^2}{\tau}$ = colission frequency $\cdot$ (Debye length)^2

IN MAGNETIZED PLASMA, WEAK B FIELD:

$D \sim \frac{\rho_L^2}{\tau}$ = colission frequency $\cdot$ (Larmor radius)^2

STRONG B FIELD :

$D \simeq \frac{cT}{16 e B_0}$ (Bohm diffusion) ? ? !

TURBULENT TRANSPORT
QUASILINEAR THEORY (SINGLE PARTICLE MOVING ALONG TRAJECTORY WITH SCALE LENGTH MUCH LONGER THAN LARMOR RADIUS),
ELECTROSTATIC CASE :

$$D = \frac{1}{B_0^2}\int dv_\parallel f_M(v_\parallel) \int \frac{d\omega\, dk_\parallel}{-i(\omega - k_\parallel v_\parallel)} \left|\nabla\phi(\omega, k_\parallel)\right|^2$$

plug in the weak turbulence spectrum !

7) DRIFT WAVES

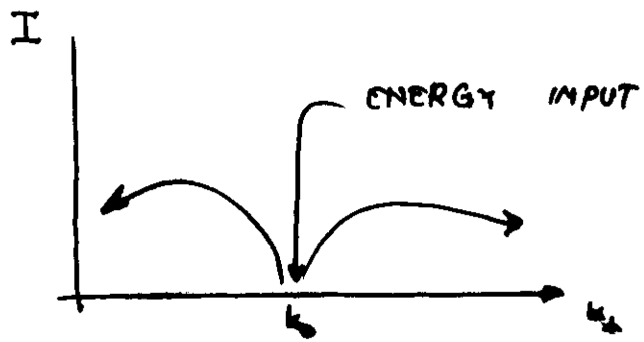
- ELECTROSTATIC
- COLD IONS, BOLTZMANN ELECTRONS
- UNSTABLE ON TEMPERATURE GRADIENTS

$$\omega = k_y \cdot \frac{d}{dx} \ln N_0 \cdot \frac{v_{Te}^2}{\omega_{ce}} \left[1 + \frac{k_\perp^2 v_{Te}^2}{\omega_{pe}^2} \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2} \right) \right]^{-1}$$

- KNOWN WEAK TURBULENCE SPECTRUM

D. EFF BY ORDERS OF MAGNITUDE ?

WHAT WENT WRONG ?
PHASES ARE NOT RANDOM !



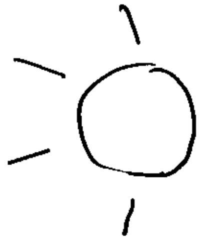
CASCADING TO BOTH LARGER AND SMALLER $k_\perp$

- SELF ORGANIZATION ??
- COMPUTER SIMULATION (OKUDA & DAWSON, HORTON)

⑧ HOW CAN VORTICES CONTRIBUTE TO TRANSPORT ENHANCEMENT ?

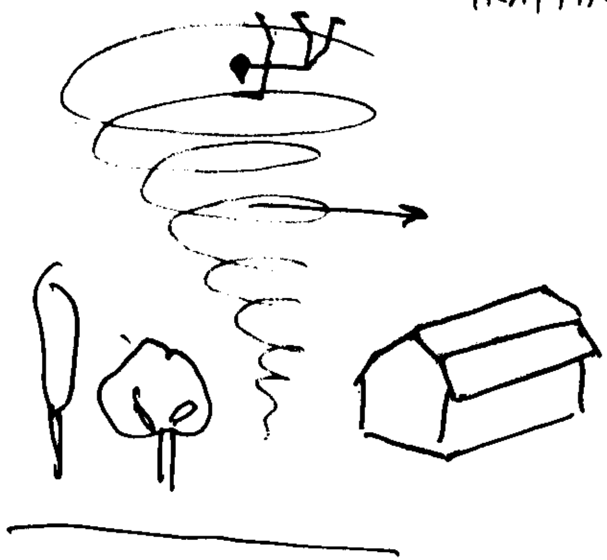
STATIONARY VORTICES

"COLLISION DOMINATED"



MOVING VORTICES

"TRAPPING EFFECTS"



NONLINEAR DRIFT WAVES & VORTICES

BOLTZMANN ELECTRONS: $n_e = n_0 \exp\left(-\frac{e_e \phi}{T_e}\right)$

QUASINEUTRALITY: $n_i \simeq n_e \quad (\omega_{pi}^2 \gg \Omega_i^2)$

TWO-DIMENSIONAL IONS: $v_{zi} = 0$

DRIFT APPROXIMATION: $\frac{d}{dt} \ll \Omega_i$

$$\vec{v}_{\perp i} = \frac{1}{B_0}\left\{ \vec{e}_z \times \nabla\phi - \frac{1}{\Omega_i}\left[\frac{\partial}{\partial t} + \frac{1}{B_0}(\vec{e}_z \times \nabla\phi)\cdot\nabla\right]\nabla_\perp^2 \phi \right\}$$

ION CONTINUITY:

$$\left[\frac{\partial}{\partial t} + \frac{1}{B_0}(\vec{e}_z \times \nabla\phi)\cdot\nabla\right]\left(\log n_i - \frac{\nabla^2\phi}{\Omega_i B_0}\right) = 0$$

HASEGAWA-MIMA equation

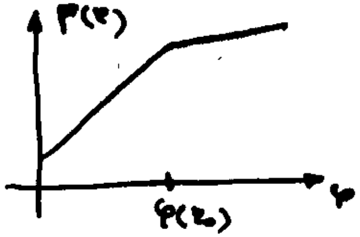
TRAVELLING SOLUTION: $\frac{\partial}{\partial t} = -v_x \frac{\partial}{\partial x} = \frac{1}{B_0}\left[\vec{e}_z \times \nabla(B_0 v_x y)\right]\cdot\nabla$

$$\frac{1}{B_0}\left[\vec{e}_z \times \nabla(\phi + B_0 v_x y)\right]\cdot\nabla\left(\phi + B_0 u y - \lambda_T^2 \nabla^2 \phi\right) = 0$$

$$u = \frac{v_{Te}^2}{\Omega_e L_n}, \quad \lambda_T^2 = \frac{m_e}{m_i}\cdot\frac{v_{Te}^2}{\Omega_i^2}$$

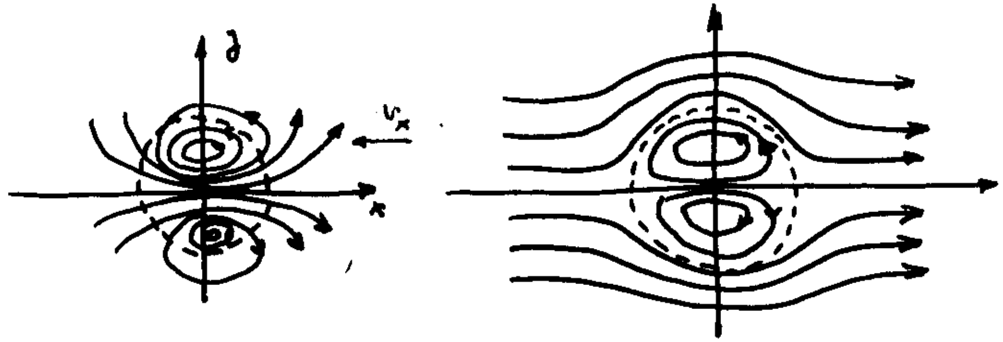
$$\phi + B_0 u y - \lambda_T^2 \nabla^2 \phi = F(\phi + B_0 v_x y) \text{ — arbitrary } F!$$

- F part-by-part linear
- $(\phi + B_0 v_x y)_{r=r_0}$ = const



$$(\lambda_T^2 \nabla^2 + F - 1)\left(\phi + B_0 \frac{F v_x - u}{F-1} y\right) = 0$$

$$\phi = B_0 \sin\theta \cdot \begin{cases} \frac{v_x + u}{A^{(-)2}} \cdot \left[a \frac{J_1(A^{(-)} r)}{J_1(A^{(-)} r_0)} - r\left(1 + \frac{A^{(+)2}}{A^{(-)2}}\right)\right] & r < r_0 \\ -\frac{v_x + u}{A^{(+)2}} \cdot a \frac{K_1(A^{(+)} r)}{K_1(A^{(+)} r_0)}, & r > r_0 \end{cases}$$



STATIONARY FRAME — MOVING FRAME

CONSERVED QUANTITIES:

$$S = \frac{1}{2}\int dx\,dy \left[(1 - \lambda_T^2 \nabla^2)\phi\right]^2 \qquad \text{ENSTROPHY}$$

$$W = \frac{1}{2}\int dx\,dy \left[\phi^2 + \lambda_T^2 (\nabla\phi)^2\right] \qquad \text{ENERGY}$$

$$\vec{P} = \int dx\,dy\, \vec{r}\,(1 - \lambda_T^2 \nabla_\perp^2)\phi \qquad \text{MOMENTUM}$$

$$L = \int dx\,dy\, \phi \qquad \text{ANGULAR MOMENTUM}$$

- FREE VORTEX PARAMETERS: v_x, r_0
- AMPLITUDE PROPORTIONAL TO v_x

HASEGAWA-MIMA VORTICES ARE VERY ROBUST

- LINEARLY STABLE (Laedke & Spatchek)
- NONLINEAR STABILITY SHOWN FOR SOME CLASSES OF PERTURBATIONS (Laedke & Spatchek)
- OFTEN SURVIVE COLLISIONS WITH OTHER VORTICES

OVERTAKING COLLISION

HEAD-ON COLLISION

(TANIUTI & HASEGAWA)

DRIFT-WAVE VORTICES AND STRONG UPPER-HYBRID TURBULENCE

- INTERACTION OF A STRONG UPPER-HYBRID PUMP WITH LOW FREQUENCY DENSITY PERTURBATIONS ASSOCIATED WITH ION CYCLOTRON WAVES IN LOW β PLASMAS YIELDS NONLINEAR SCHRÖDINGER EQ. IN ONE DIMENSIONAL CASE
- NLS SOLITONS UNSTABLE TO PERPENDICULAR PERTURBATIONS
- LOW FREQUENCY CONVECTIVE DERIVATIVE $\frac{1}{B_0}(\vec{e}_z \times \nabla\phi)\cdot\nabla$ PROVIDES SUCH PERTURBATION
- PHYSICAL PICTURE IN TWO DIMENSIONS IS NOT CLEAR

- UPPER HYBRID PUMP
- WEAKLY MAGNETIZED PLASMA $\omega_{pe}^2 \gg \Omega_i^2$
- COUPLING TO LOW FREQUENCY DENSITY PERTURB.
- KEEP LOW FREQUENCY CONVECTIVE DERIVATIVES (ASSOCIATED WITH DRIFT WAVES)

$$\nabla\cdot\left\{\left[2i\omega_{pe}\left(\frac{\partial}{\partial t}+\cancel{\vec{v}_{LF}\nabla}\right)-\Omega_e^2-\omega_{pe}^2\left(\frac{y}{L}+\frac{n_{e\,LF}}{N_0}\right)+3v_{Te}^2\nabla^2\right]\cdot\nabla\phi_{HF}\right\}=0$$ H.F. Poisson

$$\left[\frac{\partial}{\partial t}+\frac{1}{B_0}(\vec{e}_z\times\nabla\phi_{LF})\right]\left(\frac{y}{L}+\frac{n_{e\,LF}}{N_0}-\frac{\nabla^2\phi_{LF}}{\Omega_i B_0}\right)=0$$ L.F. ion continuity

$$\phi_{LF}+\phi_p+\frac{T_e}{n_0 e_e}n_{e\,LF}=0$$ Boltzmann electrons

$$\phi_p=\frac{e_e}{2m_e\omega_{pe}^2}\left|\nabla\phi_{HF}\right|^2$$ ponderomotive force

in one-dimensional case vector nonlinearity drops out

$$n_{e\,LF}(\omega,\vec{k}_\perp)=-\frac{e_e}{2T_e}\,\frac{\lambda_T^2k_\perp^2\omega-u k_x}{(\lambda_T^2k_\perp^2+1)\omega-u k_x}\left|\nabla\phi_{HF}\right|^2$$

THEN :

$$\left[2i\omega_{pe}\frac{\partial}{\partial t}-\Omega_e^2-\omega_{pe}^2\left(\frac{y}{L}+\frac{\alpha}{N_0}\left|E_{HF}\right|^2\right)+3v_{Te}^2\frac{\partial^2}{\partial x^2}\right]E_{HF}=($$

– THIS IS A NONLINEAR SCHREDINGER EQUATION

TWO DIMENSIONAL CASE

- TRAVELLING SOLUTION $\frac{\partial}{\partial t}=-v_x\frac{\partial}{\partial x}$
- LONG WAVELENGTH PUMP

$$\nabla\phi_{HF}=-\vec{e}_xE_0+\nabla\delta\phi\ ,\quad E_0=\text{const}$$

$$\Phi_p=\frac{eE_0^2}{2m_e\omega_{pe}^2}\left[1-\frac{1}{E_0}\frac{\partial}{\partial x}(\delta\phi+\delta\phi^*)\right]$$

- EXPAND $n_{e\,LF}$ INTO CYLINDRICAL HARMONICS

$$(\nabla_\perp^2-A_1^2)(\nabla_\perp^2-A_2^2)(n_{e\,LF}^{(1)}+ax)=0$$ equation for the first harmonic

$$A_1^2+A_2^2=\frac{1}{\lambda_T^2}\left[1+F-\frac{\omega_{pi}^2}{9\Omega_i^2}\left(\frac{v_x^2}{v_{Te}^2}+\frac{3}{2}\frac{e\phi_{p0}}{T_e}\right)\right]$$

$$A_1^2A_2^2=\frac{-1}{\lambda_T^4}\cdot\frac{\omega_{pi}^2}{9\Omega_i^2}\left[\frac{v_x^2}{v_{Te}^2}(F+1)+\frac{3}{2}\frac{e\phi_{p0}}{T_e}\cdot F\right]$$

$$a=\frac{eN_0B_0}{2T_e}\cdot\frac{u+Fv_x}{F+1+\frac{3}{2}\frac{e\phi_{p0}}{mv_x^2}}$$

$$n_{e\,LF}=\sin\theta\cdot\begin{cases}-ar+d_1I_1(A_1^{(1)}r)+d_2I_1(A_2^{(1)}r)\ , & r<r_0\\ \beta_1K_1(A_1^{(1)}r)+\beta_2K_1(A_2^{(1)}r)\ , & r>r_0\end{cases}+\Delta n$$

- DOUBLE VORTEX SOLUTION
- HASEGAWA – MIMA MODE + NEW NONLINEAR MODE (NOTE TWO BESSEL FUNCTIONS IN THE SOLUTION)
- TWO DIMENSIONAL, SPATIALLY WELL LOCALIZED
- DIMENSION SAME ORDER AS WIDTH OF THE NLS

MHD VORTICES (LOW β CASE)

- $\beta = \dfrac{2 n_0 T_e}{c^2 \varepsilon_0 B_0^2} << \dfrac{m_e}{m_i}$ — ONLY TORSIONAL ALFVEN WAVES
- COLD IONS ; TWO DIMENSIONAL IONS
- WARM ELECTRONS (BUT NO ACOUSTIC EFFECTS)
- SMALL PARAMETERS :

$$\frac{1}{\Omega_i}\frac{d}{dt} \; ; \; \frac{1}{\nabla_\perp}\frac{\partial}{\partial z} \; ; \; \frac{\delta B_\perp}{B_0} \; ; \; \frac{\delta n}{n_0} \; ; \; \frac{m_e}{m_i} \; ; \; \frac{c^2}{\omega_{pe}^2}\nabla_\perp^2$$

- REDUCED MHD EQUATIONS (STRAUSS 1076) FOR LARGE ASPECT RATIO TOKAMAKS
- SYSTEMATIC EXPANSION IN THE POWERS OF THE INVERSE ASPECT RATIO TO FIRST ORDER.

KINK MODES, DESTRUCTION OF FLUX SURFACES, DISRUPTIONS ETC.

CONVINIENT FOR NUMERICAL TREATMENT ; HAMILTONIAN STRUCTURE

NONIDEAL EFFECTS ??

GYROKINETIC DESCRIPTION OF ELECTRONS

KINETIC EFFECTS ALONG $\vec{B}$ FIELD LINES

DRIFT – KINETIC EQUATION (LOW β):

$$\frac{\partial f(v_\parallel)}{\partial t} + \nabla\left[\left(v_\parallel \frac{\vec{B}}{|\vec{B}|} + \frac{\vec{E}\times\vec{B}}{B^2}\right) f(v_\parallel)\right] + \frac{\vec{E}\cdot\vec{B}}{|\vec{B}|}\frac{\partial f(v_\parallel)}{\partial v_\parallel} = 0$$

$$\vec{E} = -\nabla\phi + \frac{\partial \psi_z}{\partial t}$$

$$\vec{B} = B_0 \vec{e}_z + \vec{e}_z \times \nabla \psi_z$$

DRIFT – KINETIC EQUATION :

$$\left\{\frac{\partial}{\partial t} + v_\parallel \frac{\partial}{\partial z} + \frac{1}{B_0}\left[\vec{e}_z\times\nabla(\phi + v_\parallel \psi_z)\right]\cdot\nabla\right\} f(v_\parallel) +$$

$$+ \frac{n_0 e}{m_e}\frac{\partial f_M(v_\parallel)}{\partial v_\parallel}\left\{\left[\frac{\partial}{\partial t} + \frac{1}{B_0}(\vec{e}_z\times\nabla\phi)\cdot\nabla\right]\psi_z - \frac{\partial\phi}{\partial z}\right\} = 0$$

ZEROTH MOMENT + ION CONTINUITY + POISSON'S :

$$\left[\frac{\partial}{\partial t} + \frac{1}{B_0}(\vec{e}_z\times\nabla\phi)\cdot\nabla\right]\nabla^2\phi - \left[\frac{\partial}{\partial z} + \frac{1}{B_0}(\vec{e}_z\times\nabla\psi_z)\cdot\nabla\right] v_A^2 \nabla_\perp^2 \psi_z = 0$$

FIRST MOMENT + AMPERE'S LAW :

$$\left[\frac{\partial}{\partial t} + \frac{1}{B_0}(\vec{e}_z\times\nabla\phi)\cdot\nabla\right]\psi_z - \frac{\partial\phi}{\partial z} = \left[\frac{\partial}{\partial z} + \frac{1}{B_0}(\vec{e}_z\times\nabla\psi_z)\cdot\nabla\right]\frac{p_\parallel}{n_0 e}$$

$$p_\parallel = \int dv_\parallel \cdot m_e v_\parallel^2 \cdot f(v_\parallel) \qquad << n_0 e \phi$$

- $p_\parallel$ IS A SMALL CORRECTION

IN LOW β PLASMA $p_\parallel$ ARISES DUE TO THE INTERACTION OF $\vec{E}_z$ WITH RESONANT PARTICLES

(4) TRAVELLING SOLUTION :

$$X' = X - \int^{t} dt\, v_x(\epsilon t, \epsilon z) + \int^{z} dz\, \frac{v_x(\epsilon t, \epsilon z)}{v_z(\epsilon t, \epsilon z)}$$

DOUBLE VORTEX :

$$\phi = B_0 v_x z_0 \sin\vartheta \cdot \begin{cases} -\frac{z}{z_0} + \frac{J_1(\gamma_1 \frac{z}{z_0})}{\gamma_1 J_0(\gamma_1)} + \frac{J_1(\gamma_2 \frac{z}{z_0})}{\gamma_2 J_0(\gamma_2)} , & z < z_0 \\ -\frac{z_0}{z} , & z > z_0 \end{cases}$$

$$J_1(\gamma_1) = J_1(\gamma_2) = 0$$

$$f(v_\parallel) = \frac{n_0 e}{m_e} \frac{\partial f_M}{\partial v_\parallel} \left[\frac{H(z)}{v_\parallel - v_z} (\phi + v_z \psi_z) + (H(z) - 1)\left(\psi_z - \frac{v_x}{v_z} B_0 y\right) \right]$$

$$H(z) = \begin{cases} v_z^2 / c_A^2 & z < z_0 \\ 1 & z > z_0 \end{cases}$$

THEN $p_\parallel$ IS DUE TO RESONANT PARTICLES $v_\parallel = v_z$:

$$p_\parallel = n_0 e v_z^2 \frac{\partial f_M(v_z)}{\partial v_z} \cdot \int_{-\infty}^{\infty} \frac{dx''}{x'' - x'} \cdot H(\sqrt{x''^2 + y^2}) \cdot (\phi + v_z \psi_z)_{x'', y}$$

- VORTEX PARAMETERS v_x, v_z, z_0 ARE WEAKLY t, z DEPENDENT (ADIABATIC APPROXIMATION)
- EVOLUTION EQUATIONS FOR VORTEX PARAMETERS FROM CONSERVATION LAWS

5)

$$\frac{\partial S}{\partial t} = 0$$

$$\frac{\partial W}{\partial t} + \frac{\partial P}{\partial z} = R$$

$$\frac{1}{c_A^2} \frac{\partial P}{\partial t} + \frac{\partial W}{\partial z} = \frac{R}{v_z}$$

WHERE :

$$S = \frac{1}{2} \int dx dy \left(\frac{e}{\epsilon^2 \epsilon_0} n_e + \frac{1}{c_A^2} \nabla_\perp^2 \phi \right)^2 \quad \text{enstrophy}$$

$$W = \frac{1}{2} \int dx dy \left[\frac{1}{c_A^2} (\nabla_\perp \phi)^2 + (\nabla_\perp \psi_z)^2 + \frac{c^2}{\omega_{pe}^2} (\nabla_\perp^2 \psi_z)^2 \right] \quad \text{energy}$$

$$P = - \int dx dy\, \nabla\phi \cdot \nabla\psi_z \quad \text{Poynting vector}$$

$$R = \int dx dy\, \nabla^2 \psi_z \left[\frac{\partial}{\partial z} + \frac{1}{B_0} (e_z \times \nabla\psi_z) \cdot \nabla \right] \frac{p_\parallel}{n_0 e} \quad \text{power exchanged with particles}$$

- VORTEX IS DAMPED WHEN $\frac{\partial f_M(v_z)}{\partial v_z} < 0$
- NO INTERACTION FOR $v_z = c_A$
- v_x AND VORTEX AMPLITUDE REMAIN CONSTANT
- v_z APPROACHES c_A
- VORTEX SPREADS, $z_0 \to \infty$, AS $t \to \infty$

- FOR $\frac{\partial f_M(v_z)}{\partial v_z} > 0$ VORTEX IS EXCITED BY RESONANT PARTICLES !!

(EXCITATION OF VORTICES BY PARTICLE BEAMS WAS SEEN IN COMPUTER SIMULATIONS, BUT NO THEORY WAS AVAILABLE).

3)

$$\frac{\partial S}{\partial t} = 0$$

$$\frac{\partial W}{\partial t} + \frac{\partial P}{\partial z} = R$$

$$\frac{1}{c_A^2}\frac{\partial P}{\partial t} + \frac{\partial W}{\partial z} = \frac{R}{v_z}$$

WHERE :

$$S = \frac{1}{2}\int dx\,dy \left(\frac{e}{e^2\varepsilon_0} n_e + \frac{1}{c_A^2}\nabla_\perp^2 \phi\right)^2$$ enstrophy

$$W = \frac{1}{2}\int dx\,dy \left[\frac{1}{c_A^2}(\nabla_\perp\phi)^2 + (\nabla_\perp \psi_z)^2 + \frac{c^2}{\omega_{pe}^2}(\nabla_\perp^2 \psi_z)^2\right]$$ energy

$$P = -\int dx\,dy\ \nabla\phi\cdot\nabla\psi_z$$ Poynting vector

$$R = \int dx\,dy\ \nabla^2\psi_z \left[\frac{\partial}{\partial z} + \frac{1}{B_0}(e_z\times\nabla\psi_z)\cdot\nabla\right]\frac{p_0}{ne}$$ power exchanged with particles

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