INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4-SMR 393/31

SPRING COLLEGE ON PLASMA PHYSICS

15 May - 9 June 1989

WAVE ENVELOPE SIMULATIONS OF THE BEAT-WAVE ACCELERATOR

P. Gibbon

Imperial College
London
England

WAVE-ENVELOPE SIMULATIONS
OF THE
BEAT-WAVE ACCELERATOR

Paul Gibbon

Imperial College, London

A. R. Bell (I.C.)

Major Unresolved Issues

1) Focussing of laser energy
   - overcome rayleigh length
     $L_R \ll L_{dephasing}$

2) Particle acceleration
   - trajectories in 3D
   - effect of wave dynamics

3) Coupling efficiency of laser → plasma wave
   - 'pump depletion'
   - plasma wake $\neq 0$

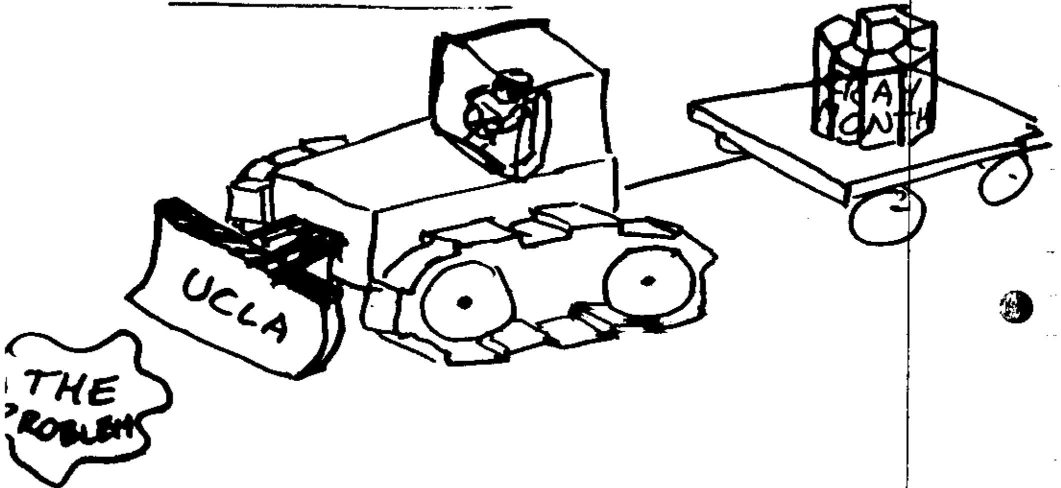
4) Beam loading

What is the most appropriate model for the BWA?

1) Analytical
   - Saturation
   - 1D cascading
   - 2D Focussing thresholds + Ansätze

$\Rightarrow$ Isolates physics, but 'whole accelerator' intractable

2) Particle-in-cell (2D PIC)



3) Wave-envelope (NLSE, + P.W.)

NEMESIS: BWA 'SIMULATOR'

- Numerical solution of 2D envelope equations for e.m. cascade & plasma wave

Can address:
- Laser focussing
- Particle acc^n
- Efficiency laser $\rightarrow$ p.w.

- Tabletop GeV accelerator

$$I\lambda^2 = 10^{16} - 10^{17} \ \mathrm{W cm^{-2} \mu m^2}$$

$$\omega_0/\omega_p = 100 - 300$$

20 – 50 cascade modes

Pulse width $\sigma \simeq 5 - 20 \ c/\omega_p$

Simulation time: $cT \sim (1-5) L_{dp}$

$\sim 10^5 \ c/\omega_p$

ASSUMPTIONS

- No ion motion $n_i = n_0$; $V_i = 0$ } modulational instabili / channel formation (⇒ density detuning)

- Pumps nearly parallel.
  Can't model true optical arrangement

- Neglect wave vector & phase mismatches due to different dispersion relations in em cascade : $\omega_m^2 = c^2 k_m^2 + \omega_p^2$

- Large spot size $\sigma \gg \lambda_p$, so that 2D effects on plasma wave can be neglected. $E_r \ll E_z$

2D Envelope Equations

In frame following pulse $(z-ct, x) \to (\xi, x)$

1) Cascade $m = (-20 \dots 0, 1 \dots 21)$

$$\nu_m \frac{\partial a_m}{\partial t} = \underbrace{\frac{i}{2}\frac{\partial^2 a_m}{\partial x^2}}_{\text{Diffraction}} - \frac{i}{8}\underbrace{|\varepsilon|^2 a_m}_{\text{'Resonant' focussing}} - \underbrace{\frac{i}{8}\sum_{s,q} a_{m+s} a_q a^*_{q+s}}_{\text{self-focussing}}$$

$$+ \frac{1}{4}\underbrace{(\varepsilon^* a_{m+1} - \varepsilon a_{m-1})}_{\text{cascading}}$$

2) Plasma wave longitudinal E-field

$$\frac{\partial \varepsilon}{\partial \xi} = \underbrace{-i\delta\varepsilon}_{\text{Mismatch}} + \underbrace{\frac{3i}{16}|\varepsilon|^2 \varepsilon}_{\substack{\text{Rel. mass} \\ \gamma_{\parallel}}} + \underbrace{\frac{i}{8}\sum_m \left(|a_m|^2 \varepsilon - a_m a^*_{m-2}\varepsilon^*\right)}_{\substack{\text{Rel. mass} \\ \gamma_{\perp}}}$$

$$\delta \equiv \frac{\omega_p^2 - \omega^2}{2\omega^2} \qquad = \frac{1}{4}\sum_m a_m a^*_{m-1}$$

FOCUSSING OF LASER LIGHT IN BEAT-WAVE ACCELERATORS

COMPETING EFFECTS:

① Relativistic 'quiver' of electrons in laser field

② Relativistic motion of electrons in plasma wave field

③ Raman cascading: transfer of energy from pumps to em sidebands

Mickey-Mouse Theory of Cascade-Focussing

– P. Gibbon, PhD Thesis 1988

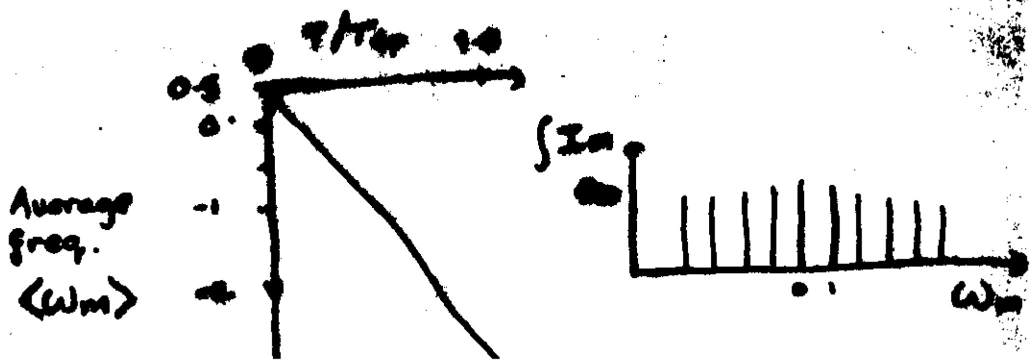
- cascading is <u>unavoidable</u>

$$\omega_m \rightarrow \omega_{m-1} + \omega_p$$

$$\omega_m + \omega_p \rightarrow \omega_{m+1}$$

Phase mismatches ~ $O\left(\frac{\omega_p}{\omega_0}\right)^3$, so large number of modes may be excited.

<u>Example</u>



How does cascading cause refraction?

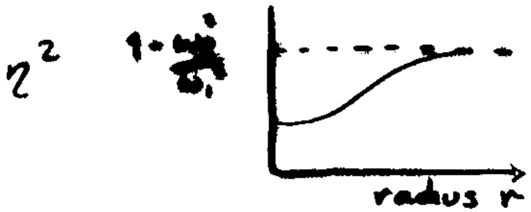
Consider: $\omega_0 + \omega_p \rightarrow \omega_1$

Find refractive index of daughter wave:

$$\eta^2 = 1 - \frac{\omega_p^2}{\omega_1^2} - \beta \frac{\omega_p^2}{\omega_1^2} a_0 \frac{\delta n}{n_0}(\cos\theta - i\sin\theta)$$

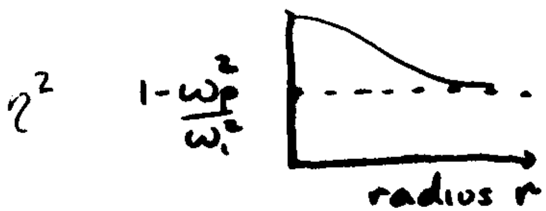
θ is the phase lag of the p.w. relative to the beat envelope of the pumps.

1) Plasma wave relativistically saturated & no frequency mismatch: $(\theta = \pi/2)$

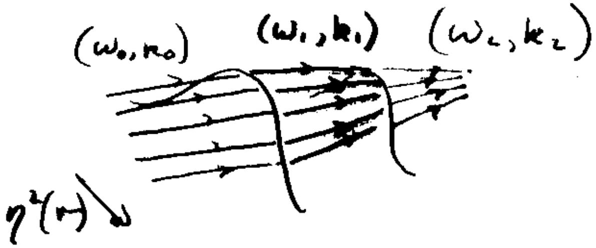


DEFOCUSSING

2) Plasma wave driven with $\Delta\omega < \omega_p$: $\left(\theta = -\frac{\pi}{2}\right)$



FOCUSSING



Not self-focussing:

It is the daughter waves that suffer refraction.

All interactions behave similarly:

$$\omega_1 + \omega_p \rightarrow \omega_2$$
$$\omega_0 + \omega_p \rightarrow \omega_1$$
$$\omega_1 \rightarrow \omega_0 + \omega_p$$
$$\omega_2 \rightarrow \omega_1 + \omega_p$$

CALCULATION OF THRESHOLDS

ENVELOPE EQUATIONS
FOR LIGHT WAVES AND PLASMA WAVE

↓

'MOMENT' EQUATION FOR
AVERAGE WIDTH OF BEAM σ

↓

VIRIAL THEOREM

$$\frac{\partial^2 \langle \sigma^2 \rangle}{\partial t^2} = \underbrace{D}_{\text{Diffraction}} + \underbrace{C}_{\text{Cascading}} + \underbrace{R^e}_{\text{Plasma wave foc.}} + \underbrace{R^a}_{\text{Rel. foc}}$$

↓

EVALUATE FOR RANGE
OF $I\lambda^2$, σ_0

↓

FOCUSSING THRESHOLDS

Find :

$$N \frac{\partial^2}{\partial t^2} \langle x^2 \rangle = D + C + R^e + R^a$$

$$D = \frac{2}{\omega_0} \int \left| \frac{\partial a_m}{\partial x} \right|^2 d^2x \qquad \text{Diffraction}$$

$$C = -\frac{1}{\omega_0} \int x \, \mathrm{Im} \left[\frac{\partial e}{\partial x} a_m a^*_{m+1} \right] d^2x \qquad \text{Cascading}$$

$$R^e = \frac{1}{4\omega_0} \int x \, \mathrm{Re} \left[\frac{\partial}{\partial x} |e|^2 |a_m|^2 - \frac{\partial e^2}{\partial x^2} a_m a^*_{m+2} \right] \qquad \text{Resonant foc.}$$

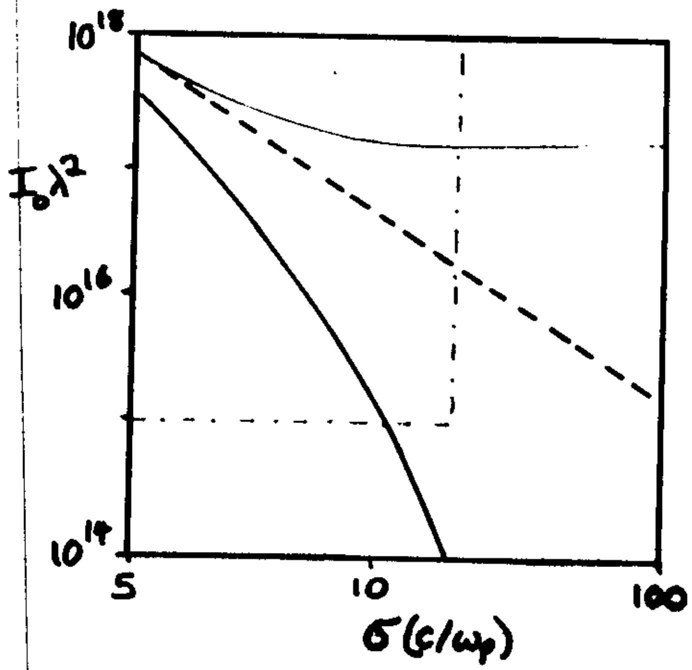
$$R^a = -\frac{1}{8\omega_0} \int Q^a d^2x \qquad \text{Self-foc.}$$

$$Q^a = \sum_{s,q} \left| a_q a^*_{q+s} \right|^2 \qquad \left[\text{Sum conv. for } m \right]$$

<u>But</u> Can't integrate because D, C etc are all time-dep. Evaluating RHS at <u>$t=0$</u> determines whether beam <u>INITIALLY</u>

FOCUSSING THRESHOLDS

Calculated from 'virial theorem' $\frac{\partial^2}{\partial t^2}\langle\sigma^2\rangle = c$



— RESONANTLY DRIVEN PLASMA WAVE

— PLASMA WAVE WITH FREQUENCY MISMATCH

$$\Delta n/n \simeq 0.06\,(I_{16}\lambda_N)^{2/3}$$

- - - NO PLASMA WAVE ([illegible])

What are the options for self-trapped propagation?

1) Small-amplitude plasma wave. $(\tau \ll \tau_{rel})$

— Relativistic focussing applies

$P \gtrsim 5\left(\frac{\omega_0}{\omega_p}\right)^2 GW$; $I_{16}\lambda_\mu^2\left(\frac{\sigma}{10}\right)^2 \gtrsim 18$

2) Large-amplitude, resonant plasma wave $(\tau \approx \tau_{rel})$

— Cascading causes defocusing, so must be above irradiance $I_{16}\lambda_\mu^2 \gtrsim$ [illegible]

3) Large-amplitude p.w. driven below ω_p.

— Cascade-foc. dominates

$$\boxed{I_{16}\lambda_\mu^2\left(\frac{\sigma}{10}\right)^6 \gtrsim 0.27}$$

$\delta = 2\delta_{opt}$

(Gibbon & Bell. Phys Rev Lett 1988)

NUMERICAL EXAMPLES

Note: Relativistic focusing included

(1) RESONANT PLASMON

$$I_0 = 10^{16}\ \mathrm{W\,cm^{-2}}$$

$$\lambda = 1\ \mu\mathrm{m}$$

$$\frac{\omega_0}{\omega_p} = 100$$

Pulse width $\sigma = 10\ c/\omega_p$

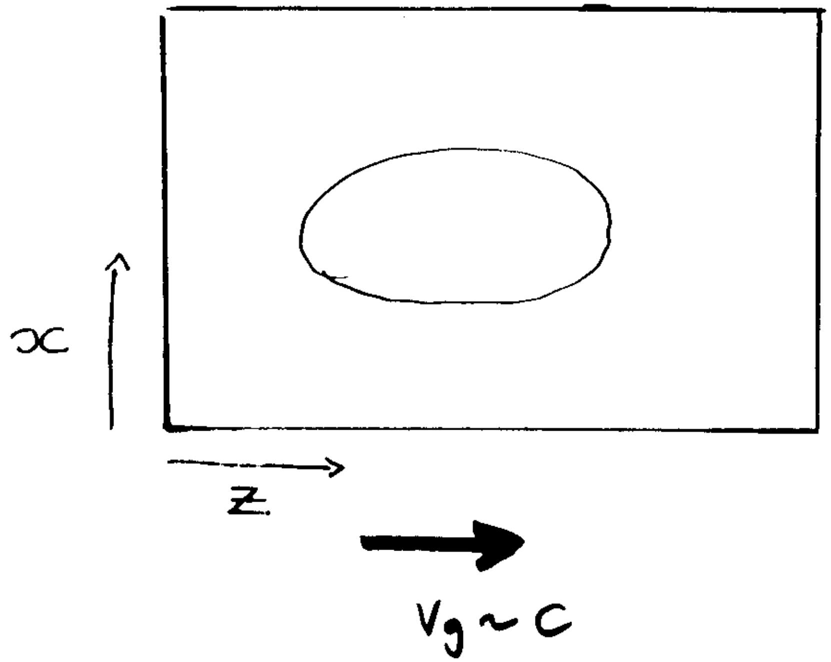
Pulse length $\tau \simeq 400\ c/\omega_p$

(= Rosenbluth-Liu period)

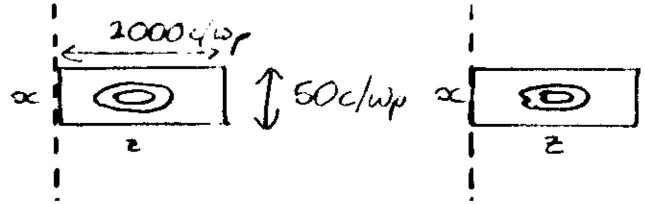
Grid ratio $(z:x) = 10:1$

Run time $T = 2\times10^4\ \omega_p^{-1}$

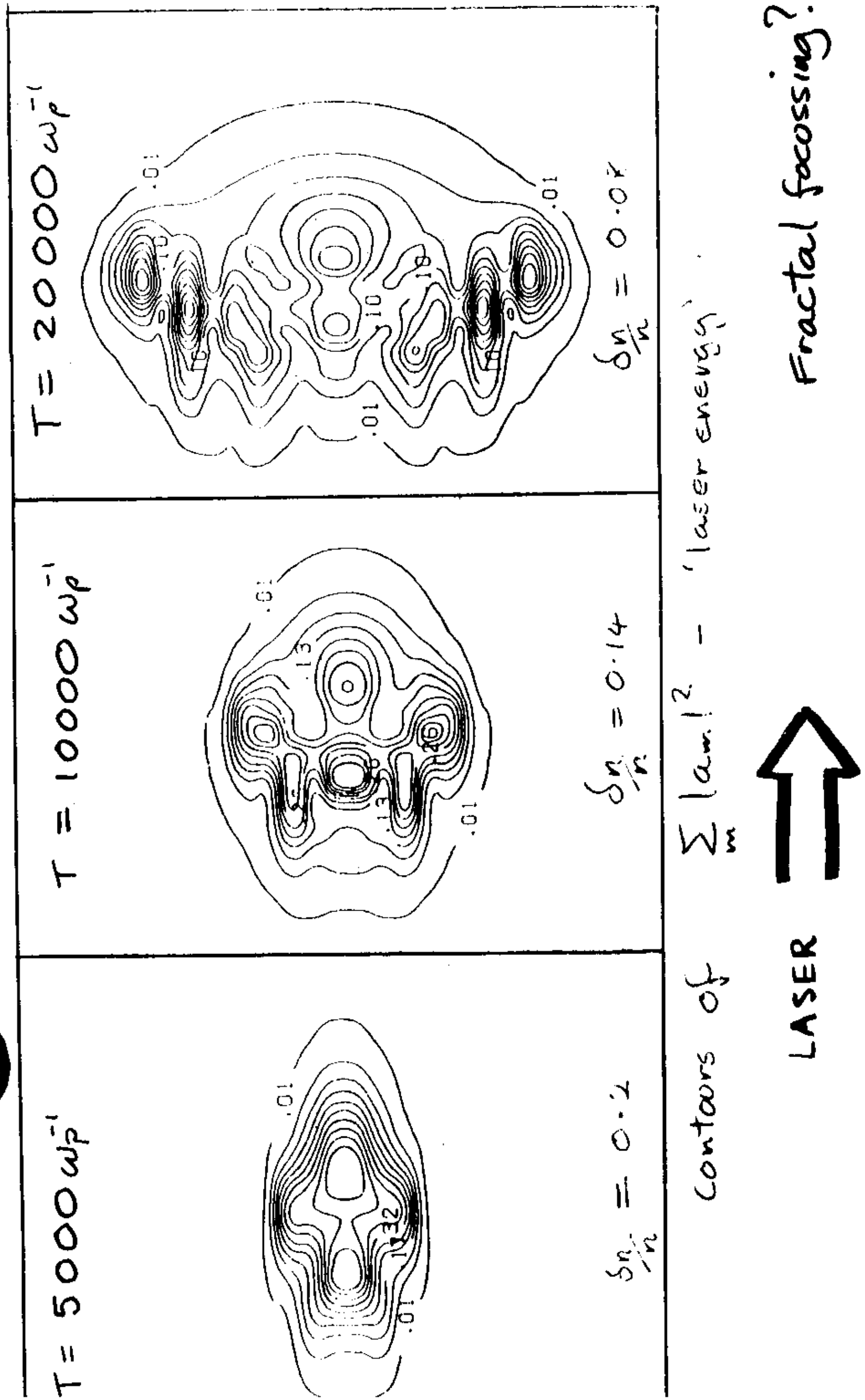
GEOMETRY



Computational box moves with pulse



1. Resonant Plasmon ($\delta = 0$)



2 DETUNED PLASMON

$$\delta = 1.66\,\delta_{opt}$$
$$= 0.03$$

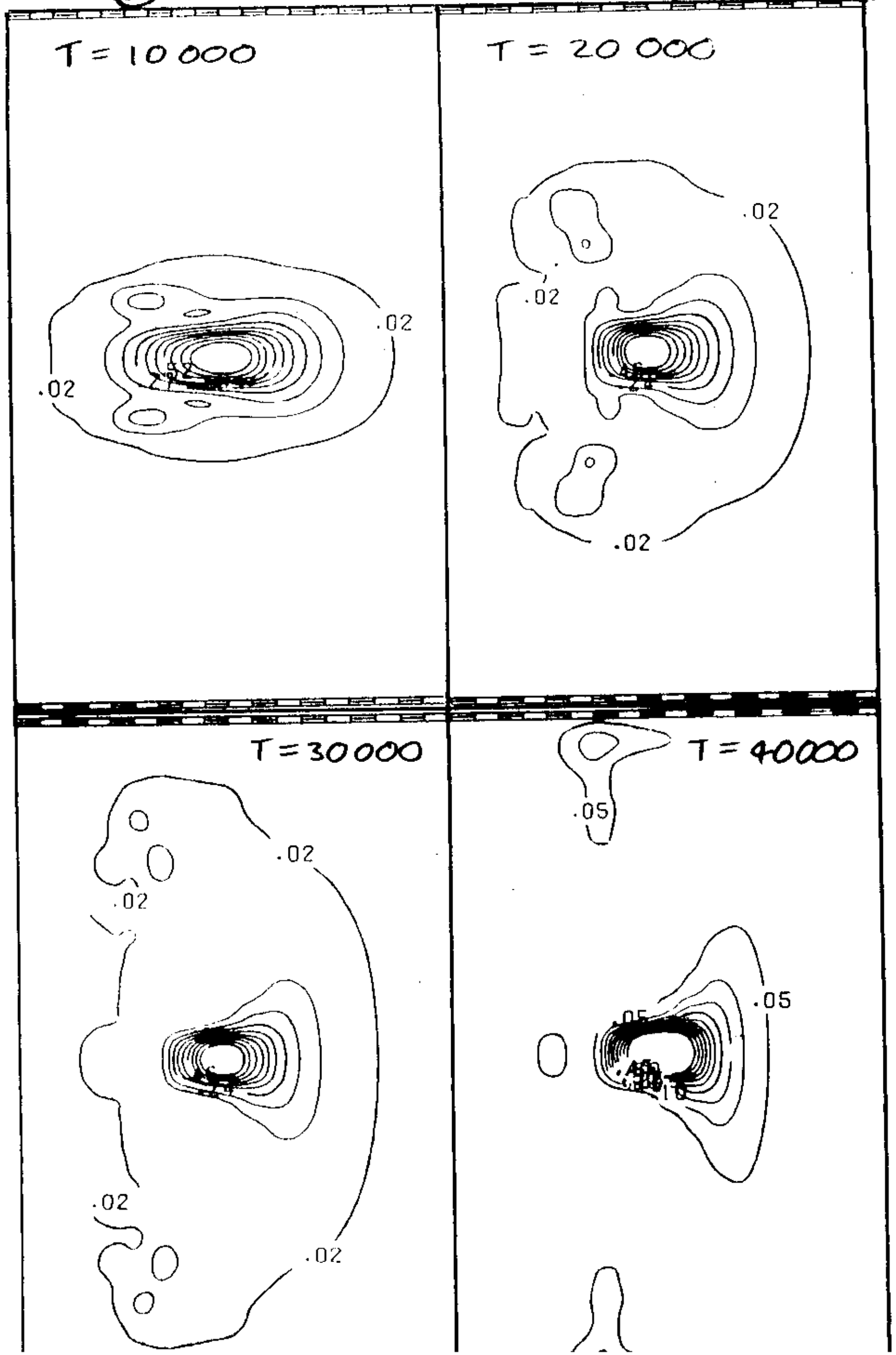
Density mismatch $\left(\frac{\Delta n}{n} = 6\% \right)$

$$\left.\frac{\delta n}{n}\right)_{max} \simeq 0.13 \quad \sim \text{CONSTANT}$$

Run time $T = 4 \times 10^4\,\omega_p^{-1}$

$$= \frac{4}{3}\,L_{stage}\ !$$

② DETUNED PLASMON ($\delta = 1.7 \times \delta_{opt}$)



CONCLUSIONS

- New (?) non-relativistic focussing effect identified.
- Threshold for cascade-focussing much lower than for relativistic.
- Self-trapped propagation over accelerator stage length (but only by driving plasma wave below ω_p).