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SPRING COLLEGE ON PLASMA PHYSICS

15 May - 9 June 1989

PLASMA UNDULATORS

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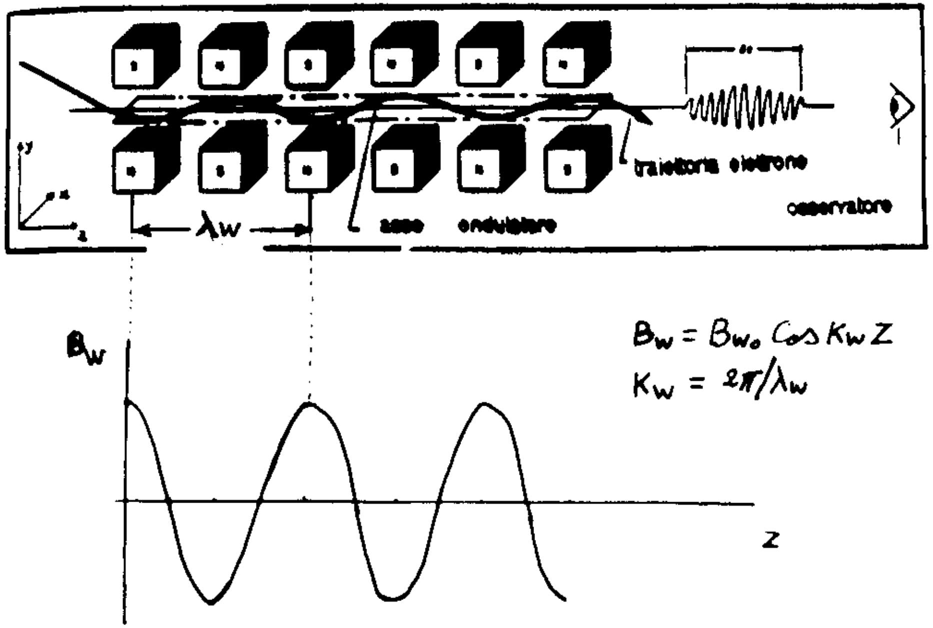
* Presented by R. Fedele

1. INTRODUCTION

WHAT IS A F.E.L. ?

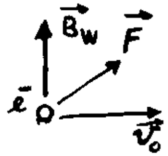


$B_w = B_{w_0} \cos K_w Z$

$K_w = 2\pi/\lambda_w$

CHARACTERISTICS :

- RELATIVISTIC ELECTRON BEAM
- UNDULATOR → space-varying magnetic field
- INITIAL INCOHERENT EMITTED RADIATION



$\vec{F} = -\frac{e}{c}\,\vec{v}_0 \times \vec{B}_w \propto \cos K_w z$

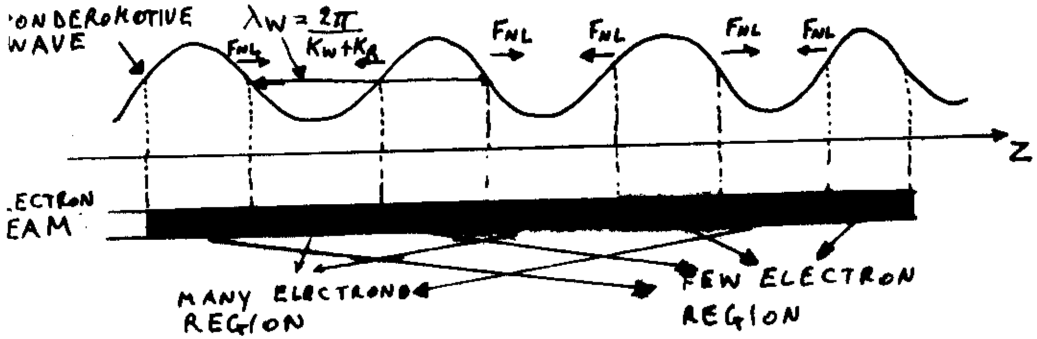
$z = z_0 + v_0 t \qquad v_0 \sim c$

$$B_W = B_{W0} \cos(K_W Z_0 + \omega_W t)$$

$$\omega_W \equiv K_W v_0 \sim K_W c$$

THE ELECTRONS SEE A BACKWARD E.M. WAVE WITH WAVENUMBER K_W AND FREQUENCY ω_W. IN THE TRANSVERSE PLANE ($\perp \hat{z}$) THEY OSCILLATE AND PRODUCE, AT ONCE, INCOHERENT E.M. RADIATION.

• STIMULATED COHERENT EMISSION



BEATING BETWEEN THE UNDULATOR MAGNETIC FIELD A_W AND THE E.M. RADIATION FIELD A_R : $\vec{A}_W + \vec{A}_R \Rightarrow$ A PONDEROMOTIVE FORCE APPEARS

$$\vec{F}_{NL} \propto -\vec{\nabla}\,|\vec{A}_W + \vec{A}_R|^2$$

THIS FORCE PRODUCES A SELF-BUNCHING OF THE ELECTRON BEAM
THE PARTICLES ARE FORCED TO OSCILLATE IN PHASE

COMPTON REGIME: THE PONDEROMOTIVE EFFECT IS LARGER THAN THE EFFECT DUE TO THE SPACE CHARGE OF THE BEAM.

THE COHERENT RADIATION WAVELENGTH

$$\lambda_R = \frac{\lambda_W}{2\gamma_0^2}$$

(Simplified expression)

IN PRINCIPLE WE CAN PRODUCE ANY E.M. FREQUENCY BY VARYING BOTH λ_W AND γ_0

BUT SOME PROBLEMS ARISE

IF WE WANT TO REACH VERY SMALL WAVELENGTHS, SAY $\lambda_R \sim 10^2 \div 10^3$ Å

$$\lambda_W < 1\,mm$$

or

$$mc^2\gamma_0 \sim 1 \div 10\ GeV$$

LIMITATIONS IN TERMS OF COST, TECHNOLOGY, AND TIME :

a) $\lambda_W \sim 1\,cm$: IT IS DIFFICULT TO REDUCE λ_W UNDER 1 cm

- A VERY SMALL DISTANCE BETWEEN THE MAGNETS BOTH REDUCES AND DEFORMS THE USABLE PART OF THE MAGNETIC FIELD IN THE UNDULATOR

b) γ_0 VERY LARGE : BIG PARTICLE ACCELERATORS

HOW DO WE OVERCOME THESE LIMITATIONS, IN ORDER TO REACH SMALL COHERENT RADIATION WAVELENGTHS WITH LOW ENERGY ELECTRON BEAMS ?

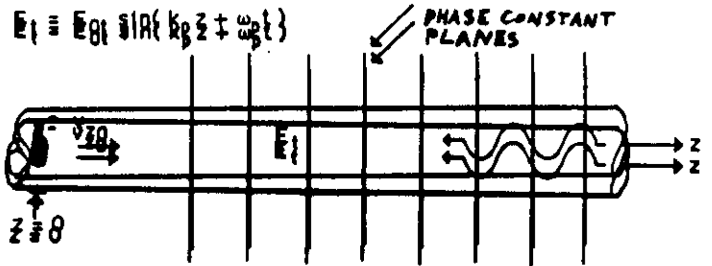
A SOLUTION OF GREAT INTEREST AT THIS MOMENT MAY BE REPRESENTED BY PLASMA UNDULATORS

2. WHAT IS A PLASMA UNDULATOR ?

THE FIRST IDEA CAME FROM U.C.L.A. GROUPE
(C. Joshi, T. Katsouleas, J.M. Dawson, Y.T. Yan, and J.M. Slater, IEEE Trans. on Q. E., QE-23, 1571 (1987); C. Joshi, T. Katsouleas, J.M. Dawson, Y.T. Yan, and F.F. Chen, Proc. 1987 AIP Conf., Washington)

BASIC IDEA (a possible sheme)

A RELATIVIST e^- BEAM IS LAUNCHED AGAINST A LARGE AMPLITUDE PLASMA WAVE.



THE ELECTRONS OF THE BEAM ARE "WIGGLED" BY THE ELECTRIC FIELD OF THE PLASMA WAVE IN A WAY THAT'S PERFECTLY ANALOGOUS TO THE WAY A MAGNETIC UNDULATOR WIGGLES SUCH BEAM.

LORENTZ TRANSFORMATION :

FOR AN ELECTRON OF THE BEAM A TIME-VARYING ELECTRIC FIELD IS EQUIVALENT TO A SPACE-VARYING MAGNETIC FIELD.

WHAT IS ANALOGOUS TO λ_W IN A PLASMA UNDULATOR ?

MAGNETIC UNDULATOR $\quad K_W Z = K_W Z_0 + K_W v_0 t \equiv K_W Z_0 + \omega_w t$

PLASMA UNDULATOR $\quad K_p Z + \omega_p t = K_p Z_0 + (\omega_p + K_p v_0) t$

Plasma wave phase velocity

$$v_{ph} \equiv \frac{\omega_p}{K_p}$$

in general $\quad v_{ph} \neq c$

- COMPARING M.U. AND P.U. :

$$\lambda_W \longrightarrow \frac{\lambda_p}{1+\beta_{ph}} \qquad \beta_{ph} \equiv v_{ph}/c$$

MAGNETIC UNDULATOR $\quad \lambda_R = \dfrac{\lambda_W}{2\gamma_0^2}$

PLASMA UNDULATOR $\quad \lambda_R = \dfrac{\lambda_p}{2(1+\beta_{ph})\gamma_0^2}$

In particular, for $\beta_{ph} = 1$

$$\lambda_R = \frac{\lambda_p}{4\gamma_0^2}$$

- SOME ESTIMATES

Plasma density : $n_0 = 10^{17} \div 10^{13}\ cm^{-3}$
$(\lambda_p = 100\,\mu m \div 1\ cm)$

In particular for $\lambda_p = 100\,\mu m$ and $\beta_{ph} = 1$ and $\gamma_0 = 50\ (25\ MeV) \div 16\ (8\ MeV)$:

$$\lambda_R = 10^2\ \text{Å} \div 10^3\ \text{Å}$$

3. HOW TO PRODUCE A LARGE AMPLITUDE PLASMA WAVE?

WE KNOW THREE MECHANISMS:

- PLASMA BEAT WAVE EXCITATION
- PLASMA WAKE FIELD EXCITATION
- LASER WAKE FIELD EXCITATION.

● WE CONSIDER HERE THE FIRST ONE ONLY

PBW
- LASER REGIME ⟶ VERY HIGH PLASMA FIELDS (1 - 100 GV/m)
- MICROWAVE REGIME ⟶ MODEST PLASMA FIELDS ($10^4 - 10^5$ V/m)

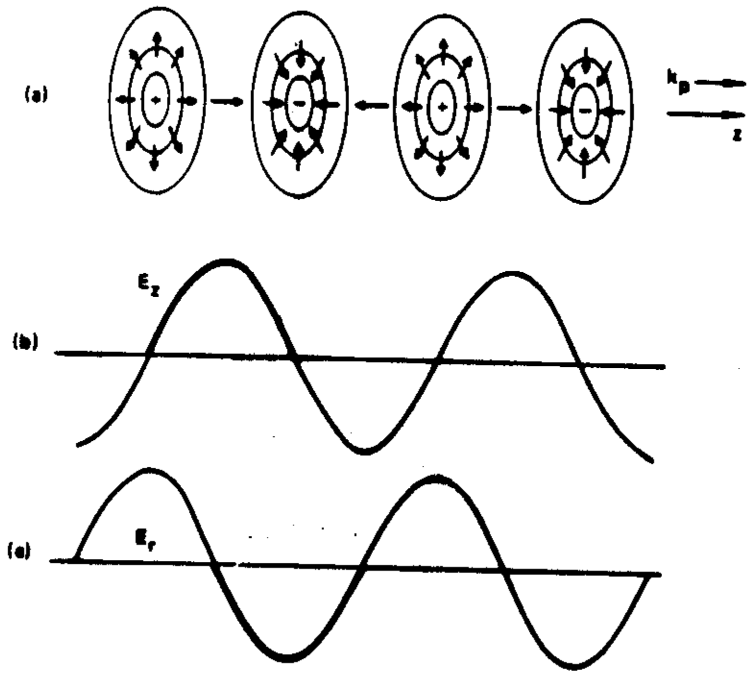
(U. DE ANGELIS, R. FEDELE, G. MIANO, AND C. NAPPI, Plasma Phys. Contr. Fus. **29**, 789, 1987)

↓

IMPORTANCE FOR ACMON Project at INFN and UNIVERSITY OF NAPLES, R.A.L., and POLYTECHNIQUES OF LOSANNA:

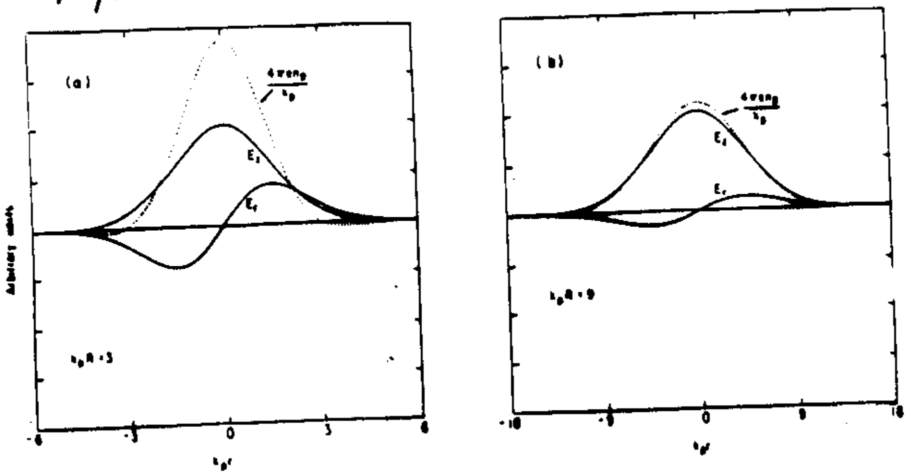
Microwave open resonator for PBW excitation scaled experiment.

SINCE THE E.M. PUMPS FOR P.B.W. EXCITATION HAVE AN AMPLITUDE RADIAL PROFILE, UNDER THE USUAL RESONANCE CONDITION $\omega_1 - \omega_2 = \omega_p$ WE HAVE A RADIAL ELECTRIC FIELD IS GENERATED IN THE PLASMA.



SO BOTH THE LONGITUDINAL AND THE RADIAL COMPONENTS OF THE FIELD INCREASE IN AMPLITUDE AT THE SAME TIME. THIS PROCESS CONTINUES UNTIL RELATIVISTIC SATURATION IS REACHED

- GAUSSIAN PUMP PROFILE (R. FEDELE, U. DE ANGELIS, AND T. KATSOULEAS, Phys. Rev. A 33, 4412 (1986))



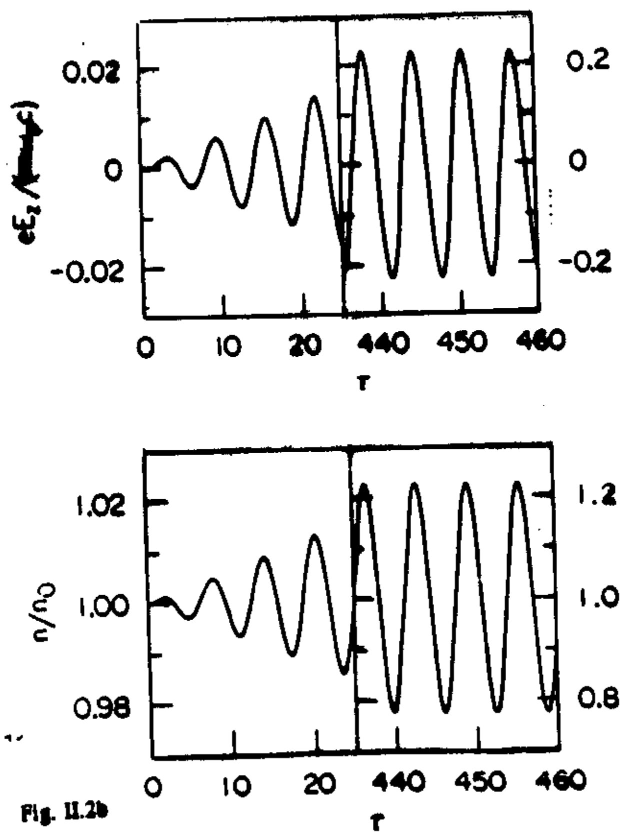


Fig. II.2b

(A.J. NOBLE, Phys. Rev A32, 460 (1985))

- E.M. PUMPS: TEM00 MODES

$$\nabla \times \bar{E} = 0 \Rightarrow \left|\frac{E_{0r}}{E_{0z}}\right| \sim \frac{\lambda_p r}{\pi R^2} \qquad R = \text{radius of the e.m. beams}$$

$$\text{if } \lambda_p \sim R, \quad E_{0r} \sim E_{0z}$$

$$\mathcal{E}_s^2 \cdot \tau_s \sim 8\pi \qquad \mathcal{E}_s \equiv \frac{E_{sat}}{E_D} = \frac{E_{sat}}{\left(\frac{m\omega_p c}{e}\right)}$$

$$\nu_0 = \frac{e E_{pump}}{m\omega c} \qquad \omega = \frac{\omega_1 + \omega_2}{2}$$

$$\tau_s \sim 7.4\, \nu_0^{-4/3}$$

IN THIS SCHEME WE CONSIDER A TRAVELLING e^- BEAM ALONG AN AXIS PARALLEL TO Z AT $r = R/\sqrt{2}$ WHERE E_r IS MAXIMUM:

$$\vec{E}_p = \left(\hat{r}\, E_{0r} + \hat{z}\, E_{0z}\right) \exp\left[i(k_p z + \omega_p t\right] + c.c. \equiv \vec{E}_p(z,t)$$

$$\frac{E_{0r}}{E_D} \simeq 0.21 \frac{\tau_s}{k_p R} \nu_0^2$$

$$\frac{E_{0z}}{E_D} \simeq 0.15\, \tau_s\, \nu_0^2$$

FOR THE F.E.L. ACTION THE INTERACTION BETWEEN THE BEAM AND E_{0r} IS <u>RESONANT</u>, BUT THE INTERACTION BETWEEN THE BEAM AND E_{0z} IS <u>NOT RESONANT</u>

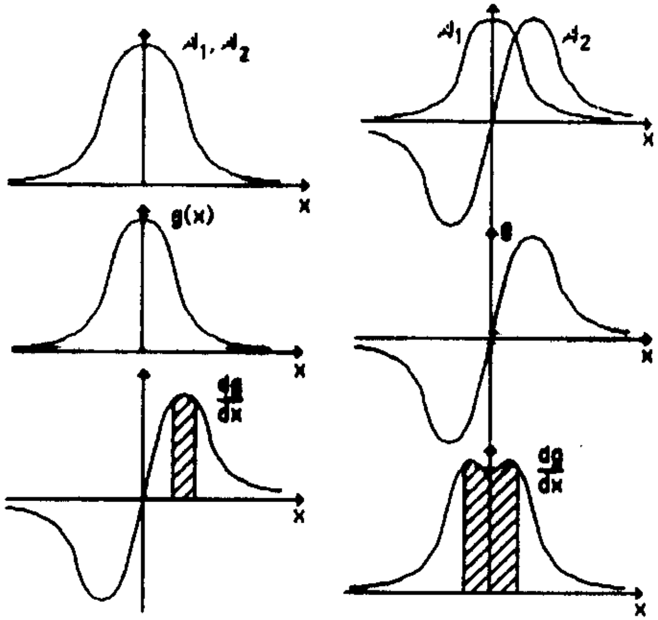
$$\mathbf{E} = - E_D \cdot (\omega_p t) \frac{1}{\beta_f^2 k_p} \mathrm{grad}\left[\frac{\nu^2(r,z)}{4} \sin(k_p z - \omega_p t + \Delta\Phi_0) \right].$$

- E_D Dawson limit electric field: $E_D \equiv 4\pi e n_0/k_p$;
- $\beta_f \equiv \omega_p/(k_p c)$.

If $\nu^2(r,z)$ does not depend on z coordinate (in the waist of the external electromagnetic waves) then we obtain for axjsymmetric pumps:

$$\mathbf{E} = \begin{cases} -E_D \dfrac{1}{\beta_f^2 k_p} \dfrac{d}{dr}(\nu^2(r)) \cdot (\omega_p t) \sin(k_p z - \omega_p t + \Delta\Phi_0)\, \mathbf{e}_r , \\[2ex] -E_D \dfrac{1}{\beta_f^2} \dfrac{\nu^2}{4} \cdot (\omega_p t)\, \cos(k_p z - \omega_p t + \Delta\Phi_0)\, \mathbf{e}_z . \end{cases}$$

$$\nu^2 = g\,\nu_c^2$$



• PUMPS: GAUSSIAN + "ANTIGAUSSIAN" PROFILES

- MORE STABLE ELECTRON BEAM MOTION THRU THE PLASMA UNDER THE ACTION OF THE ELECTRIC FIELD OF THE PLASMA WAVE

- [illegible]

(10). LINEAR FLUID THEORY OF A F.E.L. WITH A PLASMA UNDULATOR

$$m \frac{d}{dt}(\gamma \vec{v}) = - e\vec{E}_p - e\vec{E} - \frac{e}{c}\vec{v} \times \vec{B}$$

$$\frac{d\gamma}{dt} = - \frac{e}{mc^2} \vec{v} \cdot (\vec{E}_p + \vec{E})$$

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\vec{\nabla} \cdot \vec{E} = -4\pi e n \qquad \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} - \frac{4\pi}{c} e n \vec{v}$$

THE ANALYSIS IS VERY SIMILAR TO THE ANALYSIS GIVEN FOR MAGNETIC UNDULATORS (for example see P. Sprangle, C.M. Tang and C.W. Roberson, Nucl. Inst. Meth. Phys. Research A 239, 1, 1985)

FRAMEWORK OF THE PLASMA UNDULATOR TREATMENT (for more details see R. Fedele, M.R. Masullo, G. Miano, and V.G. Vaccaro, Proc. 5th CNEQP, Florence, 1988):

1) EACH QUANTITY DEPENDS ON Z AND t ONLY
2) INTRODUCING THE FOUR-POTENTIAL $(\phi, \vec{A}_R)$

$$\vec{A}_R(z,t) = (A_{Rx}(z,t), A_{Ry}(z,t), 0)$$

$$\vec{E}(z,t) = -\frac{1}{c}\frac{\partial \vec{A}_R(z,t)}{\partial t} - \vec{\nabla}\phi(z,t) \equiv \underbrace{\vec{E}_R}_{\text{Radiation}} - \underbrace{\hat{z}\frac{\partial \phi}{\partial z}}_{\text{space charge}}$$

$$\vec{B}(z,t) = \vec{\nabla} \times \vec{A}_R(z,t)$$

3) IN ORDER TO LINEARIZE, WE SUPPOSE

$$\vec{v}(z,t) = \hat{z}\, v_0 + \delta \vec{v}(z,t) \qquad (v_0 \sim c)$$

$$n(z,t) = n_b + \delta n(z,t)$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}$$

AND ASSUME

$$A_R = A_{R0} \exp[i(kz - \omega t)] + c.c.$$

$$\delta n = (\delta n)_0 \exp[i(k'z - \omega' t)] + c.c.$$

IN SUCH A WAY THAT:

$$\omega' = \omega_R - \omega_p \qquad \text{(resonance conditions)}$$
$$k' = k_R + k_p$$

NEGLECTING THE NONRESONANT COUPLING IN THE LORENTZ-MAXWELL SISTEM WE FINALLY GET:

$$\begin{cases} \left(\dfrac{d^2}{dt^2} + \dfrac{\omega_b^2}{\gamma_z^2 \gamma_0}\right)\delta n = \dfrac{e n_b}{m \gamma_0} \dfrac{\partial}{\partial z}\left(\dfrac{\partial}{\partial z} + \dfrac{v_0}{c^2}\dfrac{\partial}{\partial t}\right)\left[\left(\dfrac{v_{pt}}{c}\right) A_R\right] \\ \left(\dfrac{\partial^2}{\partial z^2} - \dfrac{1}{c^2}\dfrac{\partial^2}{\partial z^2} - \dfrac{\omega_b^2}{\gamma_0 c^2}\right) A_R = 4\pi e \left(\dfrac{v_{pt}^*}{c}\right)\delta n \\ \vec{v}_{pt} = \dfrac{i e E_{0r}\, \hat{r}}{m \gamma_0 (\omega_p + v_0 k_p)} \exp[i(k_p z + \omega_p t)] + c.c. \equiv \vec{v}_W \end{cases}$$

$$\equiv \vec{v}_{pt0} \exp[i(k_p z + \omega_p t)] + c.c.$$

THE COUPLING BETWEEN THE E.M. RADIATION FIELD AND PLASMA WAVE EXCITES THE BEAM DENSITY PERTURBATION AND THIS, IN TURN, EXCITES THE COHERENT E.M. RADIATION FIELD, BY COUPLING WITH THE PLASMA WAVE

- THE PONDEROMOTIVE POTENTIAL

$$\phi_{NL} = \frac{e n_b}{m \gamma_0} \frac{v_{pt}}{c} A_R \propto \exp[i(k_R + k_p) z - i(\omega_R - \omega_p) t]$$

- $\frac{v_{pt}^*}{c}\delta n \propto \exp[i(k' - k_p) - i(\omega' + \omega_p) t]$

- $\gamma_0 = \gamma_z \left[1 + \left(\frac{e E_{0r}}{m(\omega_p + v_0 k_p)}\right)^2\right]^{1/2} \equiv \gamma_z (1 + K^2)^{1/2}$

COMPTON REGIME:

- MAGNETIC UNDULATOR

- $\beta_W \gg \beta_{CRIT}$

$\beta_W \equiv \frac{e A_W}{\gamma_0 m_0 c^2}$ (magnitude of the electron wiggle velocity)

$\beta_{CRIT} \equiv \left[\frac{2\omega_b}{c k_W \gamma_0^{7/2}}\right]^{1/2} (1 + K^2)^{3/4}$

$K = \frac{e A_W}{m_0 c^2}$ (magnetic strength)

$k_W = 2\pi / \lambda_W$

$\omega_b = \left(\frac{4\pi e^2 n_b}{m}\right)^{1/2}$ (beam plasma frequency)

- $\lambda_R = \frac{\lambda_W}{2\gamma_0^2}(1 + K^2)$

- INTRINSIC EFFICIENCY $\eta = \frac{P_{out}}{P_{in}} = \frac{\Delta E}{(\gamma_0 - 1) m c^2}$

$$\eta = 0.07 \left(\frac{I_{[kA]}\, \lambda_W^2\, \beta_W^2}{\gamma_0\, \sigma^2}\right)^{1/3} \propto \frac{1}{\sigma^{2/3}}$$

PLASMA UNDULATOR

$$\beta_W = \frac{\varepsilon_s}{(1+\beta_{ph})\gamma_0} \qquad \varepsilon_s = \varepsilon_s(\beta_{ph})$$

$$\beta_{CRIT} = \left[\frac{2\omega_b}{(1+\beta_{ph})\, c K_p\, \gamma_0^{7/2}}\right]^{1/2} \left[1+\left(\frac{\varepsilon_s}{1+\beta_{ph}}\right)^2\right]^{3/4}$$

$$\lambda_R = \frac{1}{1+\beta_{ph}}\,\frac{\lambda_p}{2\gamma_0^2}\left[1+\left(\frac{\varepsilon_s}{1+\beta_{ph}}\right)^2\right]$$

Typically for the P.B.W. excitation $\left(\frac{\varepsilon_s}{1+\beta_{ph}}\right)^2 << 1$

$$\lambda_R \simeq \frac{1}{1+\beta_{ph}}\,\frac{\lambda_p}{2\gamma_0^2}$$

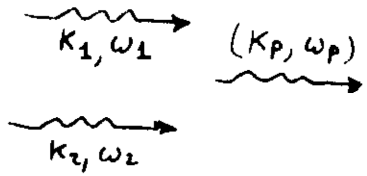
$$\beta_{CRIT} \simeq \left[\frac{2\omega_b}{(1+\beta_{ph})\, c K_p\, \gamma_0^{7/2}}\right]^{1/2}$$

$$\eta = 0.07\left[\frac{I_{[KA]}\,\lambda_p^2\,\varepsilon_s^2}{(1+\beta_{ph})^4\,\gamma_0^3\,\sigma^2}\right]^{1/3}$$

5. TWO POSSIBLE SCHEMES

I) $K_p = K_1 - K_2$ (FORWARD)

$$\lambda_p^{I} = \frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2} \sim \frac{\lambda_0^2}{\Delta\lambda}$$



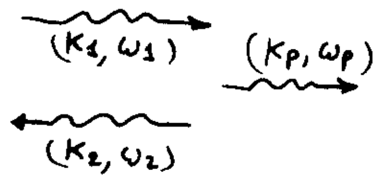
$$\beta_{ph}^{I} \sim 1$$

$$\lambda_R^{I} = \frac{\lambda_p^{I}}{4\gamma_0^2}$$

$$\eta^{I} = 0.028\left[\frac{I_{[KA]}\,{\lambda_p^{I}}^2\,\varepsilon_s^2}{\gamma_0^3\,\sigma^2}\right]^{1/3}$$

II) $K_p = K_1 + K_2$ (FORWARD - BACKWARD)

$$\lambda_p^{II} = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2} \sim \frac{\lambda_0}{2}$$



$$\beta_{ph}^{II} << 1$$

$$\lambda_R^{II} = \frac{\lambda_p^{II}}{2\gamma_0^2}$$

$$\eta^{II} = 0.07\left[\frac{I_{[KA]}\,{\lambda_p^{II}}^2\,\varepsilon_s^2}{\gamma_0^3\,\sigma^2}\right]^{1/3}$$

NOTE THAT $\lambda_p^{I} > \lambda_p^{II}$

PUMPS	BEAMS	SCHEME	λ_w (mm)	λ_R (mm)	P_{out} (MW)	B_W ($\times 10^{-3}$)	B_{CAV} ($\times 10^{-3}$)
$\lambda_1 = 9.6\,\mu m$, $\lambda_2 = 10.6\,\mu m$	$\gamma_0 = 20$, $I = 0.1\,kA$, $\sigma = 13.5\,\mu m$, $n_b = 3.6 \times 10^{13}\,cm^{-3}$	$K_1 - K_2$	100	62.5	1.5	12.5	2.24
		$K_1 + K_2$	5	6.25	0.5	25	0.70
	$\gamma_0 = 50$, $I = 0.25\,kA$, $\sigma = 5.5\,\mu m$, $n_b = 5.4 \times 10^{16}\,cm^{-3}$	$K_1 - K_2$	100	10	9.4	5	0.9
		$K_1 + K_2$	5	1	3.2	10	0.27
$\lambda_1 = 10.3\,\mu m$, $\lambda_2 = 10.6\,\mu m$	$\gamma_0 = 20$	$K_1 - K_2$	364	227.5	3.5	12.5	4.3
		$K_1 + K_2$	5	6.25	0.5	25	0.70
	$\gamma_0 = 50$	$K_1 - K_2$	364	36.4	22.5	5	1.68
		$K_1 + K_2$	5	1	3.2	10	0.27

$E_s = 0.5 \qquad n_0 = 10^{17}\,cm^{-3}$

6. THE ROLE OF THE EMITTANCE

(a). A CONDITION CAN BE USED TO ESTABLISH A CONNECTION BETWEEN THE WAVELENGTH λ_R AND THE ELECTRON BEAM TRANSVERSE EMITTANCE ε (C. Pellegrini, Nucl. Instr. Method, 177, 227, 1980):

$$\varepsilon \lesssim 0.6\,\lambda_R$$

WITH THE PRESENT TECHNOLOGY THERE IS A LIMITATION IN GENERATING VERY SMALL WAVELENGTH E.M. RADIATION

FOR $\gamma_0 = 20$ $\qquad \varepsilon = 4.24 \times 10^{-7}\,m \cdot rad$

FOR $\gamma_0 = 50$ $\qquad \varepsilon = 6.90 \times 10^{-8}\,m \cdot rad$

STRONG LIMITATION!

(b). $$\eta \propto \sigma^{-2/3}$$

THE EMITTANCE SPREAD CAUSES A RIDUCTION OF THE EFFICIENCY

WE CANNOT REDUCE THE EMITTANCE BUT WE CAN PARTIALLY CONTROL σ.

HOW ?

BY MEANS OF THE SELF-PINCHING OF THE BEAM:
THE PLASMA SCREENS THE BEAM SPACE CHARGE AND THE MAGNETIC FORCE ON THE BEAM DUE TO THE BEAM CURRENT MAKES THE SELF-PINCH

- THE KAPCHINSKIJ-VLADIMIRSKIJ EQUATION:

$$\frac{d^2\sigma}{dz^2} + K_F\sigma - \varepsilon^2\sigma^{-3} = 0$$

$$K_F = \frac{2\pi e^2 n_b}{m\gamma_0 c^2} = \left(\frac{1}{\gamma_0}\frac{N r_e}{\sigma_z}\right)\frac{1}{\sigma^2}$$ *focusing strength*

EQUILIBRIUM CONDITION

$$K_F\sigma_{eq} = \varepsilon^2\sigma_{eq}^{-3}$$

$$\sigma_{eq}^2 = \frac{\varepsilon^2\gamma_0}{r_e(N/\sigma_z)}$$

r_e = *classical electron radius*

$$\frac{N}{\sigma_z} = (2\pi)^{3/2} n_b \sigma_0^2$$

$$\sigma_0 = \sigma(z=0)$$

FOR $\gamma_0 = 20$, $n_b = 3.6\times10^{15}\,cm^{-3}$, $\sigma_0 = 13.5\,\mu m$

$$\sigma_{eq} \simeq 13\,\mu m \;!$$

7. CONCLUSIONS

- ON THE BASIS OF THE THEORY OF MAGNETIC UNDULATORS FOR F.E.L. WE HAVE ANALYZED THE USE OF A LARGE AMPLITUDE WAVE AS AN ELECTROSTATIC UNDULATOR
- WE HAVE REVIEWED THE ORIGINAL IDEA OF THE PLASMA UNDULATOR AND CONSIDERED SOME PARTICULAR PLASMA UNDULATOR CONFIGURATIONS.
- BY USING LOW ENERGY ELECTRON BEAMS WE HAVE SHOWED THE POSSIBILITY TO GENERATE SMALL WAVELENGTH E.M. RADIATION, BUT THE LIMITATION DUE TO THE EMITTANCE HAS BEEN DISCUSSED
- THE FLUID THEORY FOR THE P.U., IN COMPLETE ANALOGY WITH THE ORDINARY UNDULATORS HAS BEEN DISCUSSED
- AN ANALYSIS FOR THE INTRINSIC EFFICIENCY, WITH RESPECT THE EMITTANCE AND THE SELF-PINCHING HAS SHOWN THAT THE SELF-PINCHING CAN BALANCE THE EMITTANCE SPREAD EFFECT