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STIMULATED SCATTERING OF LARGE AMPLITUDE WAVES IN THE IONSPHERE A THEORETICAL REVIEW

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LECTURE NOTES

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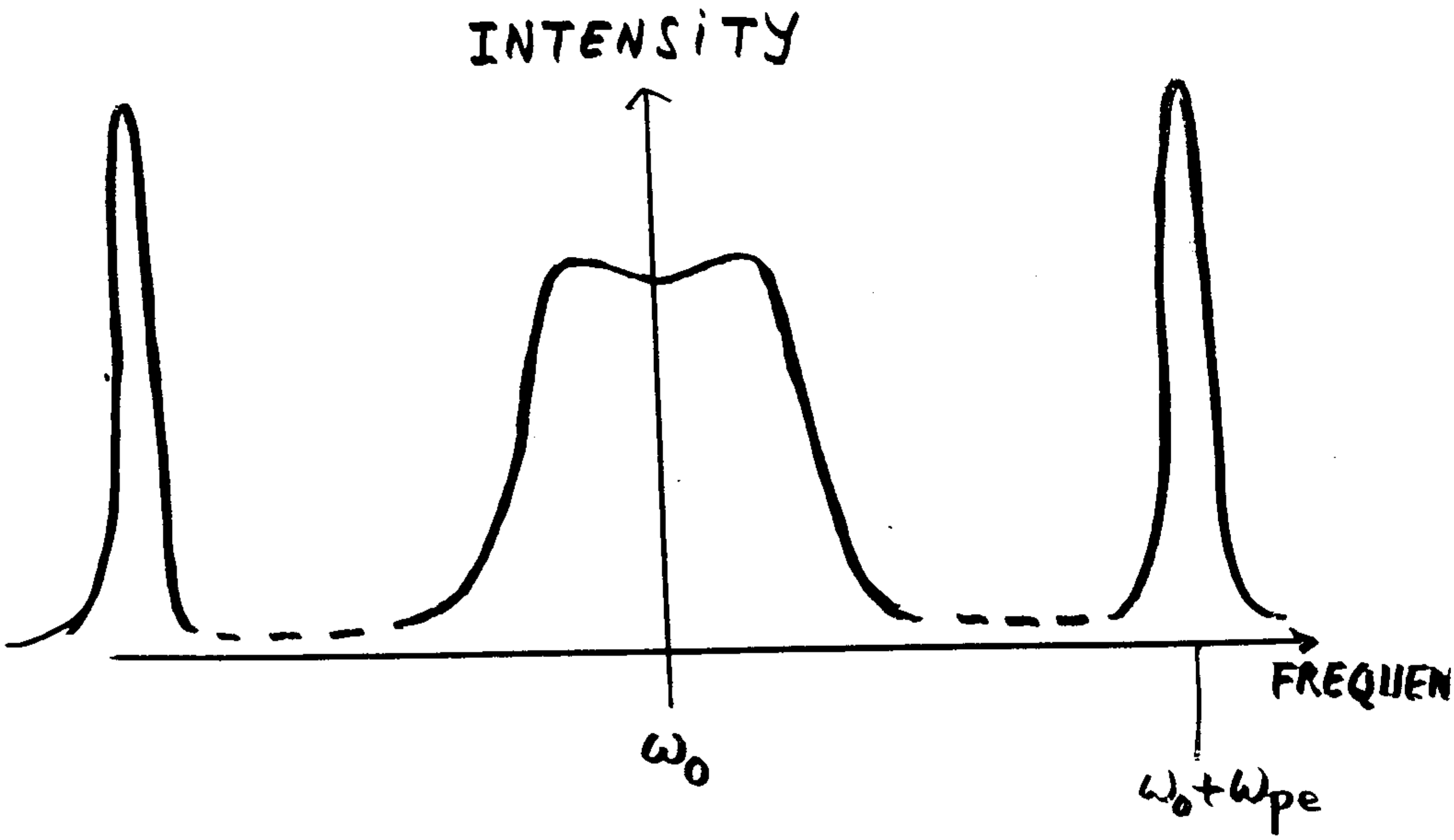

$$\omega_0^2 = \omega_{pe}^2 + k_0^2 c^2$$

$$\omega_\pm = \omega \pm \omega_0$$

$$\underset{\sim}{k}_\pm = \underset{\sim}{k} \pm \underset{\sim}{k}_0$$

$$c^2\, \underset{\sim}{k}_\pm \times (\underset{\sim}{k}_\pm \times \underset{\sim}{E}_\pm) + (\omega_\pm^2 - \omega_{pe}^2)\, \underset{\sim}{E}_\pm \approx \omega_{pe}^2 \frac{\delta n}{n_0} \underset{\sim}{E}_0$$

$$\varepsilon(\omega, \underset{\sim}{k})\, \delta n = -\frac{k^2 \varepsilon_0}{m_e \omega_0^2} \chi_e (1 + \chi_i) \left(\underset{\sim}{E}_{0+} \cdot \underset{\sim}{E}_- + \underset{\sim}{E}_{0-} \cdot \underset{\sim}{E}_+ \right)$$

$$\varepsilon(\omega, \underset{\sim}{k}) \equiv 1 + \chi_e(\omega, \underset{\sim}{k}) + \chi_i(\omega, \underset{\sim}{k})$$

if $\omega_0 >> \omega_{pe}$

$$\frac{1}{\chi_e} + \frac{1}{1+\chi_i} = \frac{k^2 |\underset{\sim}{k}_+ \times \underset{\sim}{v}_0|^2}{k_+^2 (k_+^2 c^2 - \omega_+^2 + \omega_{pe}^2)} + \frac{k^2 |\underset{\sim}{k}_- \times \underset{\sim}{v}_0|^2}{k_-^2 (k_-^2 c^2 - \omega_-^2 + \omega_{pe}^2)}$$

$$\underset{\sim}{v}_0 = \frac{q_e \underset{\approx}{E}_0}{m_e \omega_0}$$

if $k v_{ti} < \omega \ll k v_{te}$

$$\chi_e \approx \frac{1}{k^2 \lambda_D^2} \left(1 + i \sqrt{\frac{\pi}{2}} \frac{\omega}{k v_{te}} \right)$$

$$\chi_i \approx - \frac{\omega_{pi}^2}{\omega^2} + \frac{i\sqrt{\pi/2}}{k^2 \lambda_D^2} \frac{T_e}{T_i} \frac{\omega}{k v_{ti}} e^{-\frac{\omega^2}{2k^2 v_{ti}^2}}$$

$$\omega_0 >> \omega_{ce}$$

$$\frac{1}{\chi_e} + \frac{1}{1+\chi_i} = k^2 \sum_{+,-} \left[\frac{|\underset{\sim}{k}_\pm \times \underset{\sim}{v}_0|^2}{k_\pm^2 \left(k_\pm^2 c^2 - \omega_\pm^2 + \omega_{pe}^2 - \frac{i\nu_e \omega_{pe}^2}{\omega_\pm} \right)} \right.$$

$$\left. - \frac{|\underset{\sim}{k}_\pm \cdot \underset{\sim}{v}_0|^2}{k_\pm^2 \omega_\pm^2 \varepsilon(\omega_\pm, \underset{\sim}{k}_\pm)} \right]$$

$$\chi = \frac{\omega_p^2}{k^2 v_t^2} \left\{ 1 - \sum_{n=-\infty}^{\infty} I_n(b) e^{-b} \left[\omega - \right. \right.$$

$$\left. \left. - \omega^* \left(1 - \frac{n\omega}{b\omega_c} \right) \right] \int_{-\infty}^{\infty} \frac{F_z \, dv_z}{\omega - k_z v_z - n\omega_c} \right\}$$

$$b = \frac{k_\perp^2 v_t^2}{\omega_c^2}$$

$$\omega^* = - \frac{k_y \mathcal{H} v_t^2}{\omega_c}$$

$$\mathcal{H} = - \frac{1}{n_0} \frac{\partial n_0}{\partial x}$$

$$\chi_i = 0$$

$$\chi_e \approx -\frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2} - \frac{k_z^2}{k^2}\frac{\omega_{pe}^2}{\omega^2}$$

$\omega_{pe} \gg \omega_{ce} \Rightarrow$

$$\omega_1 \approx \pm\omega_{pe}\left(1 + \frac{\omega_{ce}^2}{2\omega_{pe}^2}\sin^2\alpha\right)$$

$$\omega_2 \approx \pm\omega_{ce}\left(1 - \frac{\omega_{ce}^2}{2\omega_{pe}^2}\sin^2\alpha\right)\cos\alpha$$

Raman scattering

$$(\mathrm{Im}\,\omega)_{max} \approx \frac{v_0}{c}\sqrt{\omega_0\omega_{pe}}$$

Resonance line scattering

$$\left(\frac{v_0}{c}\right)^2 = \frac{\Gamma_2\Gamma_-}{\omega_2\omega_0}\frac{(\omega_2^2-\omega_1^2)}{(\omega_2^2-\omega_{ce}^2)\sin^2\varphi\cos^2\theta}$$

$$\begin{cases} \dfrac{\partial n}{\partial t} + \nabla \cdot n\mathbf{v} = 0 \\[2ex] \dfrac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = \dfrac{q}{m}\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}_0\right) - \dfrac{v_t^2}{n}\nabla n - \nu\mathbf{v} \end{cases}$$

$$\begin{cases} \dfrac{\partial n_1}{\partial t} + \nabla\cdot\left(n_0\mathbf{v}_1 + n_1\mathbf{v}_d\right) = 0 \\[2ex] \dfrac{\partial \mathbf{v}_1}{\partial t} + \mathbf{v}_d\cdot\nabla\mathbf{v}_1 - \dfrac{q}{m}\left(\mathbf{E}_1 + \mathbf{v}_1\times\mathbf{B}_0\right) + \dfrac{v_t^2}{n_0}\nabla n_1 - \\[2ex] \quad - \dfrac{v_t^2 n_1}{n_0^2}\nabla n_0 + \nu\mathbf{v}_1 = -\nabla\dfrac{v_d^2}{2} \end{cases}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \mu_0\sum n q\mathbf{v} + \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$$

$$\begin{cases} \dfrac{1}{1+\chi_i} \approx -\dfrac{i\omega\nu_i}{\omega_{pi}^2} \\[2ex] \dfrac{1}{\chi_e} \approx -\dfrac{i(\omega - \underset{\sim}{k}\cdot\underset{\sim}{v}_{de})\,\omega_{ce}^2}{\omega_{pe}^2\,\nu_e} \end{cases}$$

$$\omega \simeq \frac{\underset{\sim}{k}\cdot\underset{\sim}{v}_{de}}{1+\psi}$$

$$\psi = \left|\frac{\nu_i\,\nu_e}{\omega_{ci}\,\omega_{ce}}\right|$$

$$\frac{\partial n}{\partial t} + \nabla \cdot n \mathbf{v} = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{\nabla n T}{m n} - \nu \mathbf{v}$$

$$\frac{3}{2}\left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T\right) + T \nabla \cdot \mathbf{v} = \frac{R_z}{n_0} \nabla_z^2 T + \frac{R_\perp}{n_0} \nabla_\perp^2 T$$

$$- \frac{3}{2\tau}(T - T_g) + m \nu \mathbf{v}^2$$

$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \sum n q \mathbf{v} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

$$\frac{1}{1+\chi_i} + \frac{1}{\chi_e} = -\left(1 + \frac{4i}{3\tilde{\omega}}\nu_e\right)\frac{k^2 v_{ts}^2}{\omega_{pe}^2}$$

$$v_{ts}^2 = -\omega_{pe}^2 \sum_{+,-} \left\{ \frac{|\underset{\sim}{k}_\pm \times \underset{\sim}{v}_0|^2}{k_\pm^2\left(k_\pm^2 c^2 - \omega_\pm^2 + \omega_{pe}^2 + i\frac{\nu_e \omega_{pe}^2}{\omega_0}\right)} - \frac{|\underset{\sim}{k}_\pm \cdot \underset{\sim}{v}_0|^2}{k_\pm^2 \omega_\pm^2 \varepsilon(\omega_\pm, \underset{\sim}{k}_\pm)} \right\}$$

$$\tilde{\omega} = \omega - \underset{\sim}{k}\cdot\underset{\sim}{v}_{de} + \frac{i}{\tau} + \frac{2i}{3n_0}\left(k_z^2 R_{ze} + k_\perp^2 R_{\perp e}\right)$$

EXCITATION OF LOW-FREQUENCY ELECTROSTATIC FLUCTUATIONS BY MODULATION OF ELECTROMAGNETIC WAVES IN THE IONOSPHERE

$$\frac{k_0^2 c^2}{\omega_0^2} = 1 - \sum \frac{\omega_p^2}{\omega_0(\omega_0 + \omega_c)}$$

$$i\left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z}\right)E + \frac{v_{gz}'}{2} \frac{\partial^2}{\partial z^2} E + \frac{v_{g\perp}'}{2} \nabla_\perp^2 E = \Delta E$$

$$v_g = \frac{\partial \omega_0}{\partial k_0}$$

$$v_{gz}' = \frac{\partial^2 \omega_0}{\partial k_0^2}$$

$$v_{g\perp}' = \left.\frac{\partial^2 \omega_0}{\partial k_{0\perp}^2}\right|_{k_{0\perp}=0}$$

$$\Delta = \frac{v_g}{2 k_0 c^2} \sum \frac{\omega_0 \omega_p^2}{\omega_0 + \omega_c}\left(\frac{\delta n}{n_0} - \frac{k_0 \omega_c \, \delta v_z}{\omega_0(\omega_0 + \omega_c)}\right)$$

$$\underset{\sim}{E} = E_0\left[\hat{\underset{\sim}{x}}\cos(\omega_0 t - k_0 z) + \hat{y}\sin(\omega_0 t - k_0 z)\right]$$

$$\underset{\sim}{B} = B_{z0}\hat{\underset{\sim}{z}} + B_{\perp 0}\left[\hat{y}\cos(\omega_0 t - k_0 z) - \hat{\underset{\sim}{x}}\sin(\omega_0 t - k_0 z)\right]$$

$$\omega_0^2 \frac{\omega_{ci}}{\omega_0 + \omega_{ci}} = k_0^2 c_A^2$$

$$\omega^2 - k^2 c_s^2 - \frac{B_{\perp 0}^2}{B_{z0}^2} k^2 c_A^2 = \frac{B_{\perp 0}^2}{2 B_{z0}^2} k^2 c_A^4 \left[\frac{(k - k_0)^2}{\mathcal{E}_+} + \frac{(k + k_0)^2}{\mathcal{E}_-}\right]$$

$$\mathcal{E}_\pm = \frac{(\omega_0 \mp \omega)^2 \omega_{ci}}{\omega_0 \mp \omega + \omega_{ci}} - (k_0 \mp k)^2 c_A^2$$

$\omega << \omega_0$

$$\omega^2 = k^2 c_s^2 - \frac{k_0^2 k^2 c_A^2}{k^2 - 4 k_0^2} \frac{B_{\perp 0}^2}{B_{z0}^2}$$

DRIFT MODES $(\omega, \underset{\sim}{k})$

$$\chi_i + \chi_e = 0$$

$$\chi_i = -\frac{\omega_{pi}^2}{\omega(\omega + i\nu_i) - k^2 v_{ti}^2}$$

$$\chi_e = \left[\frac{k^2 v_{te}^2}{\omega_{pe}^2} - \frac{i k^2 \omega_{ce}(\omega - \underset{\sim}{k}\cdot v_{de})}{\omega_{pe}^2 \nu_e \left(k^2 - \frac{i q_e (\underset{\sim}{k} \times \underset{\sim}{B}_0)\cdot \nabla n_0}{n_0 m_e \nu_e}\right)}\right]^{-1}$$

$$\left(\frac{1}{\chi_i} + \frac{1}{\chi_e}\right)\frac{\delta n}{n_0} = -\frac{4i}{3}\frac{\nu_e}{\tilde{\omega}_e}\frac{k^2 |E|^2}{\omega_{pe}^2 B_{z0}^2}$$

$\tilde{\omega}_e = \omega - \underset{\sim}{k}\cdot v_{de}$ + damping terms

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$$\Delta \approx -\frac{k_0}{2} v_g \frac{\delta n}{n_0}$$

(SHUKLA; STENFLO, KARPMAN)

$$i\left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z}\right)E + \frac{v_{gz}'}{2}\frac{\partial^2}{\partial z^2}E + \frac{v_g}{4k_0}\nabla_\perp^2 E = \Delta E$$

$$v_g = \frac{2k_0 c_A^2}{\omega_0}\frac{(\omega_0+\omega_{ci})^2}{\omega_{ci}(\omega_0+2\omega_{ci})}$$

$$v_{gz}' = \left[1 - \frac{v_g^2}{c_A^2}\frac{\omega_{ci}^3}{(\omega_0+\omega_{ci})^3}\right]\frac{v_g}{k_0}$$

$$\Delta \simeq -\frac{k_0 v_g}{2}N + \omega_0 b - k_0\frac{\partial}{\partial t}\left(\frac{\partial}{\partial z}\right)^{-1}(N-b)$$

$$N = \frac{\delta n}{n_0}$$

$$b = \frac{\delta B_z}{B_{z0}}$$

$$\frac{\partial}{\partial t} + \cdots \underset{\sim}{v} = 0$$

$$\frac{\partial \underset{\sim}{v}}{\partial t} + \frac{c_A^2}{B_0^2} \underset{\sim}{B_0} \times (\nabla \times \underset{\sim}{B}) + c_s^2 \underset{\sim}{\nabla} N = \underset{\sim}{f} =$$

$$= \frac{q_i^2}{m_i^2 \omega_{ci} (\omega_0 + \omega_{ci})} \left(\frac{\partial}{\partial z} + \frac{2}{v_g} \frac{\partial}{\partial t} \right) |E|^2 \underset{\sim}{\hat{z}} - \frac{q_i^2}{m_i^2 (\omega_0 + \omega_{ci})^2} \nabla_\perp |E|^2$$

$$\frac{\partial \underset{\sim}{B}}{\partial t} - \nabla \times (\underset{\sim}{v} \times \underset{\sim}{B_0}) = 0$$

$$\frac{\partial^2}{\partial t^2} (N - b) - c_s^2 \frac{\partial^2 N}{\partial z^2} = - \frac{q_i^2}{m_i^2 \omega_{ci} (\omega_0 + \omega_{ci})} \left(\frac{\partial^2}{\partial z^2} + \frac{2}{v_g} \frac{\partial}{\partial t} \frac{\partial}{\partial z} \right) |E|^2$$

$$\left(\frac{\partial^2}{\partial t^2} - c_A^2 \nabla^2 \right) b - c_s^2 \nabla_\perp^2 N = \frac{q_i^2}{m_i^2 (\omega_0 + \omega_{ci})^2} \nabla_\perp^2 |E|^2$$